Chapter 3: Probability and Distributions

3.4 Binomial Distribution

(a) Probability that exactly 5 customers will return the items.

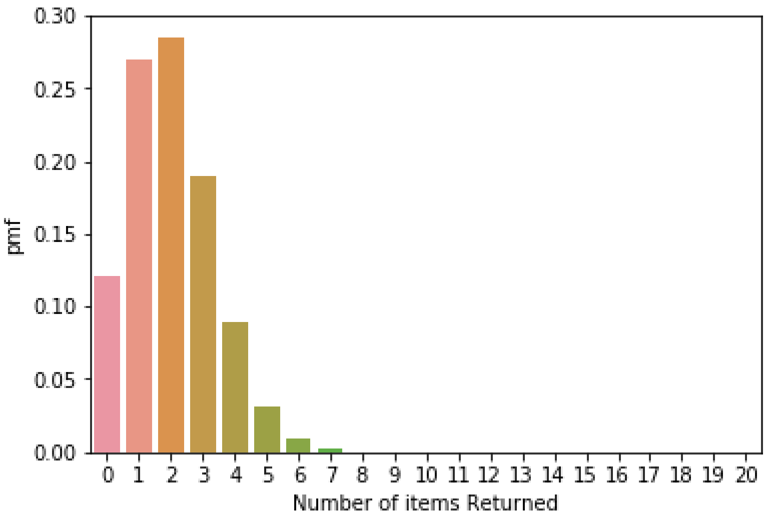
```
import warnings
warnings.filterwarnings('ignore')
```

```
from scipy import stats

stats.binom.pmf( 5, 20, 0.1 )
```

```
0.03192136111995428
```

```
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sn
```

```
# range(0,21) returns all values from 0 to 20 (excluding 21)
pmf_df = pd.DataFrame( { 'success': range(0,21),
                         'pmf': list(stats.binom.pmf( range(0,21),
                                                      20,
                                                      0.1 ) ) } )

sn.barplot( x = pmf_df.success, y = pmf_df.pmf )
plt.ylabel('pmf')
plt.xlabel('Number of items Returned');
```



(b) Probability that a maximum of 5 customers will return the items.

```
stats.binom.cdf( 5, 20, 0.1 )
```

```
0.988746865835491
```

(c) Probability that more than 5 customers will return the items purchased by them.

```
1 - stats.binom.cdf( 5, 20, 0.1 )
```

```
0.011253134164509015
```

(d) Average number of customers who are likely to return the items and the variance and the standard deviation of the number of returns.

```
mean, var = stats.binom.stats(20, 0.1)
print( "Average: ", mean , " Variance:", var)
```

```
Average:  2.0  Variance: 1.8
```

3.5 Poisson Distribution

Example of Poisson distribution

(a) Calculate the probability that the number of calls will be maximum 5.

```
stats.poisson.cdf(5, 10)
```

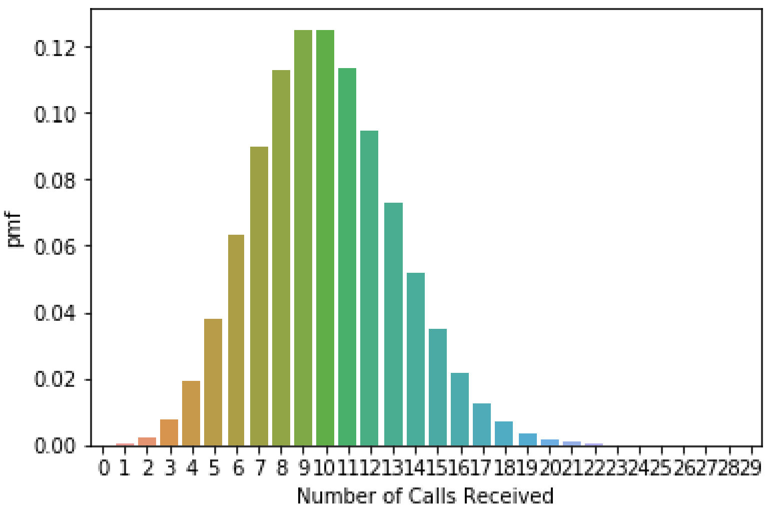
```
0.06708596287903189
```

(b) Calculate the probability that the number of calls over a 3-hour period will exceed 30.

```
1 - stats.poisson.cdf(30, 30)
```

```
0.45164848742208863
```

```
pmf_df = pd.DataFrame( { 'success': range(0,30),
                         'pmf': list(stats.poisson.pmf( range(0,30),
                                                        10) ) } )

sn.barplot( x = pmf_df.success, y = pmf_df.pmf );
plt.xlabel('Number of Calls Received');
```



3.6 Exponential Distribution

Example of Exponential Distribution

(a) Calculate the probability that the system will fail before 1000 hours.

```
stats.expon.cdf(1000,
                loc = 1/1000,
                scale = 1000)
```

```
0.6321201909489326
```

(b) Calculate the probability that it will not fail up to 2000 hours.

```
1 - stats.expon.cdf(2000,
                loc = 1/1000,
                scale = 1000)
```

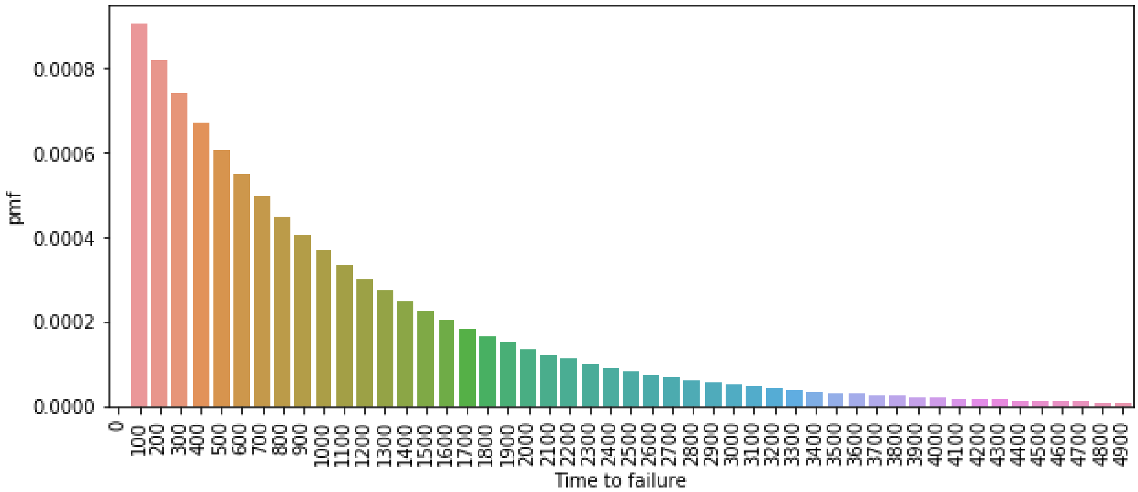
```
0.13533541857196352
```

(c) Calculate the time by which 10% of the system will fail (that is, calculate P10 life).

```
stats.expon.ppf(.1,
                loc = 1/1000,
                scale = 1000)
```

```
105.36151565782632
```

```
pmf_df = pd.DataFrame( { 'success': range(0,5000, 100),
                         'pmf':
                        list(stats.expon.pdf( range(0,5000, 100),
                                              loc = 1/1000,
                                              scale = 1000 ) ) } )

plt.figure( figsize=(10,4))
sn.barplot( x = pmf_df.success, y = pmf_df.pmf )
plt.xticks(rotation=90);
plt.xlabel('Time to failure');
```



3.7 Normal Distribution

```
import pandas as pd
import numpy as np
import warnings

# Setting precision level to 4 to show only upto 4 decimal points
pd.option_context('display.precision', 2)

beml_df = pd.read_csv( 'BEML.csv' )
beml_df[0:5]
```

	Date	Open	High	Low	Last	Close	Total Trade Quantity	Turnover (Lacs)
0	2010-01-04	1121.0	1151.00	1121.00	1134.0	1135.60	101651.0	1157.18
1	2010-01-05	1146.8	1149.00	1128.75	1135.0	1134.60	59504.0	676.47
2	2010-01-06	1140.0	1164.25	1130.05	1137.0	1139.60	128908.0	1482.84
3	2010-01-07	1142.0	1159.40	1119.20	1141.0	1144.15	117871.0	1352.98
4	2010-01-08	1156.0	1172.00	1140.00	1141.2	1144.05	170063.0	1971.42

```
glaxo_df = pd.read_csv( 'GLAXO.csv' )
glaxo_df[0:5]
```

	Date	Open	High	Low	Last	Close	Total Trade Quantity	Turnover (Lacs)
0	2010-01-04	1613.00	1629.10	1602.00	1629.0	1625.65	9365.0	151.74
1	2010-01-05	1639.95	1639.95	1611.05	1620.0	1616.80	38148.0	622.58
2	2010-01-06	1618.00	1644.00	1617.00	1639.0	1638.50	36519.0	595.09
3	2010-01-07	1645.00	1654.00	1636.00	1648.0	1648.70	12809.0	211.00
4	2010-01-08	1650.00	1650.00	1626.55	1640.0	1639.80	28035.0	459.11

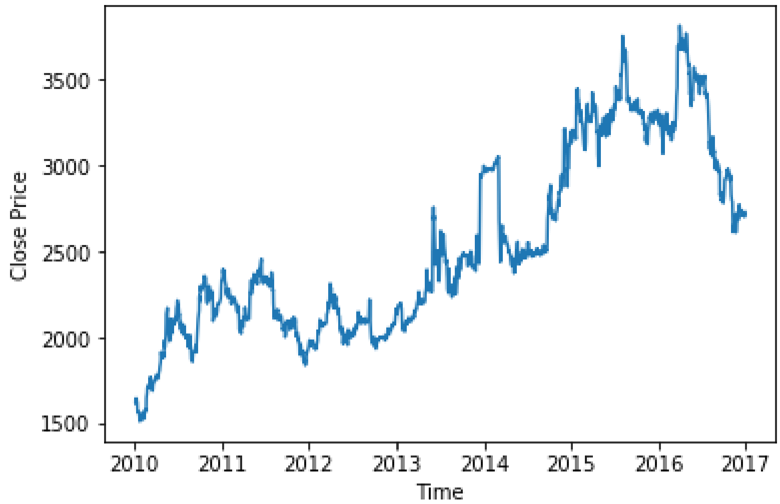
```
beml_df = beml_df[['Date', 'Close']]
glaxo_df = glaxo_df[['Date', 'Close']]
```

```
glaxo_df = glaxo_df.set_index(pd.DatetimeIndex(glaxo_df['Date']) )
beml_df = beml_df.set_index(pd.DatetimeIndex(beml_df['Date']) )
```

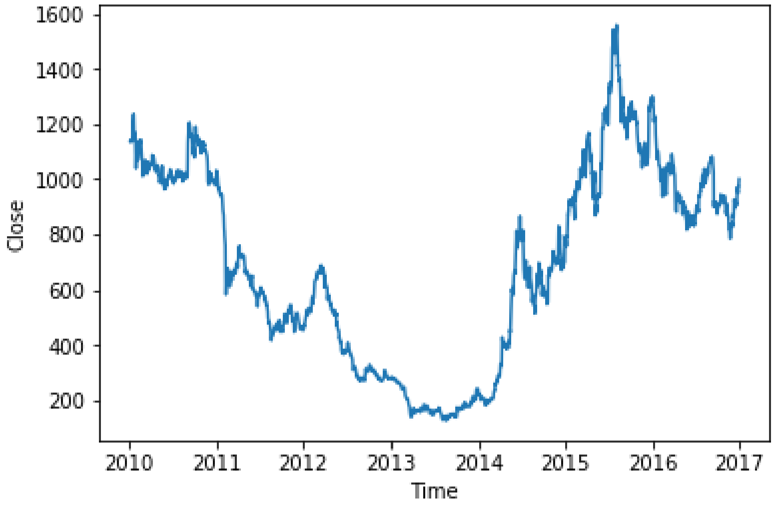
```
glaxo_df.head(5)
```

	Date	Close
Date		
2010-01-04	2010-01-04	1625.65
2010-01-05	2010-01-05	1616.80
2010-01-06	2010-01-06	1638.50
2010-01-07	2010-01-07	1648.70
2010-01-08	2010-01-08	1639.80

```
import matplotlib.pyplot as plt
import seaborn as sn
%matplotlib inline

plt.plot( glaxo_df.Close );
plt.xlabel( 'Time' );
plt.ylabel( 'Close Price' );
```



```
plt.plot( beml_df.Close );
plt.xlabel( 'Time' );
plt.ylabel( 'Close' );
```

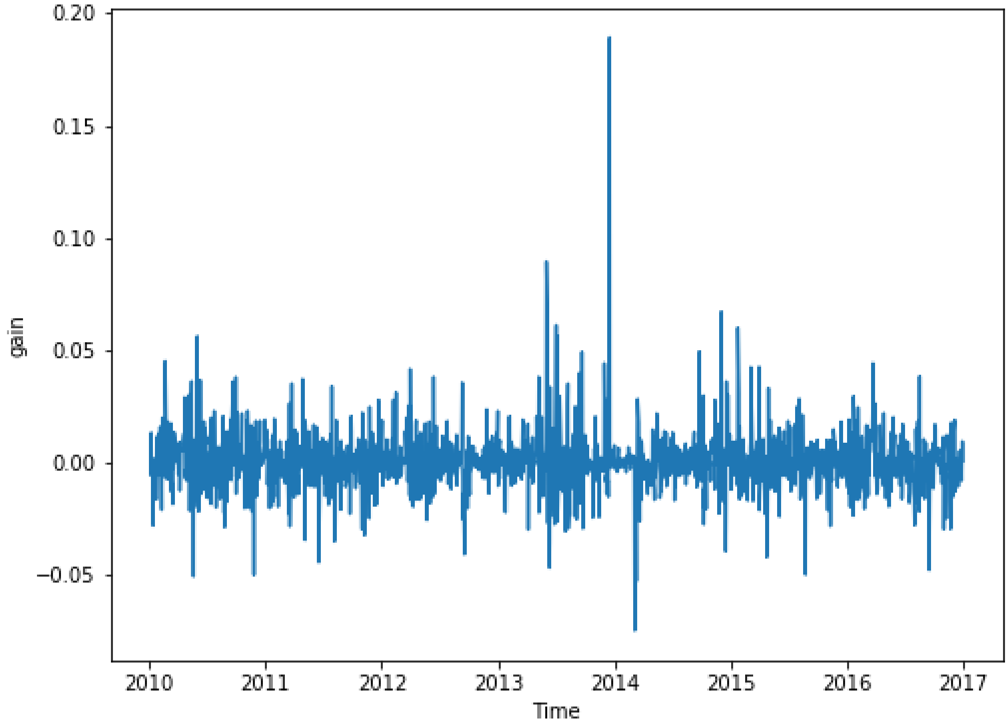



```
glaxo_df['gain'] = glaxo_df.Close.pct_change( periods = 1 )
beml_df['gain'] = beml_df.Close.pct_change( periods = 1 )
glaxo_df.head( 5 )
```

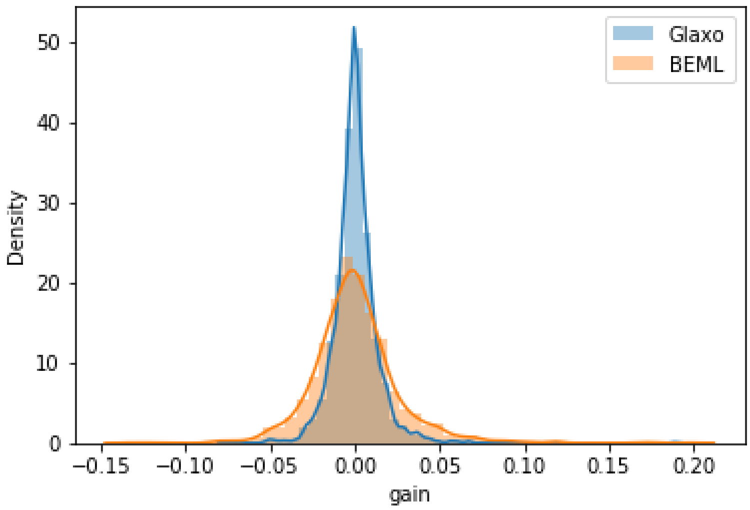
	Date	Close	gain
Date			
2010-01-04	2010-01-04	1625.65	NaN
2010-01-05	2010-01-05	1616.80	-0.005444
2010-01-06	2010-01-06	1638.50	0.013422
2010-01-07	2010-01-07	1648.70	0.006225
2010-01-08	2010-01-08	1639.80	-0.005398

```
glaxo_df = glaxo_df.dropna()
beml_df = beml_df.dropna()
```

```
plt.figure( figsize = ( 8, 6 ));
plt.plot( glaxo_df.index, glaxo_df.gain  );
plt.xlabel( 'Time' );
plt.ylabel( 'gain' );
```




```
sn.distplot( glaxo_df.gain, label = 'Glaxo'  );
sn.distplot( beml_df.gain, label = 'BEML'  );
plt.xlabel( 'gain' );
plt.ylabel( 'Density' );
plt.legend();
```



3.7.2 Mean and Variance

```
print( "Daily gain of Glaxo")
print("---------------------")
print( "Mean: ", round(glaxo_df.gain.mean(), 4) )
print( "Standard Deviation: ", round(glaxo_df.gain.std(), 4) )
```

```
Daily gain of Glaxo
---------------------
Mean:  0.0004
Standard Deviation:  0.0134
```

```
print( "Daily gain of BEML")
print("---------------------")
print( "Mean: ", round(beml_df.gain.mean(), 4) )
print( "Standard Deviation: ", round(beml_df.gain.std(), 4) )
```

```
Daily gain of BEML
---------------------
Mean:  0.0003
Standard Deviation:  0.0264
```

```
beml_df.gain.describe()
```

```
count    1738.000000
mean        0.000271
std         0.026431
min        -0.133940
25%        -0.013736
50%        -0.001541
75%         0.011985
max         0.198329
Name: gain, dtype: float64
```

3.7.3 Confidence Interval

```
from scipy import stats

glaxo_df_ci = stats.norm.interval(0.95,
                                  loc = glaxo_df.gain.mean(),
                                  scale = glaxo_df.gain.std())

print( "Gain at 95% confidence interval is:", np.round(glaxo_df_ci, 4) )
```

```
Gain at 95% confidence interval is: [-0.0258  0.0266]
```

```
beml_df_ci = stats.norm.interval( 0.95,
                loc=beml_df.gain.mean(),
                scale=beml_df.gain.std())

print( "Gain at 95% confidence interval is:", np.round(beml_df_ci, 4) )
```

```
Gain at 95% confidence interval is: [-0.0515  0.0521]
```

3.7.4 Cumulative Probability Distribution

```
print( "Probability of making 2% gain or higher in Glaxo: ")
stats.norm.cdf( -0.02,
             loc=glaxo_df.gain.mean(),
             scale=glaxo_df.gain.std())
```

```
Probability of making 2% gain or higher in Glaxo:

0.06352488667177397
```

```
print( "Probability of making 2% gain or higher in BEML: ")
stats.norm.cdf( -0.02,
             loc=beml_df.gain.mean(),
             scale=beml_df.gain.std())
```

```
Probability of making 2% gain or higher in BEML:

0.22155987503755292
```

```
print( "Probability of making 2% gain or higher in Glaxo: ",
            1 - stats.norm.cdf( 0.02,
             loc=glaxo_df.gain.mean(),
             scale=glaxo_df.gain.std()) )

print( "Probability of making 2% gain or higher in BEML: ",
            1 - stats.norm.cdf( 0.02,
             loc=beml_df.gain.mean(),
             scale=beml_df.gain.std()) )
```

```
Probability of making 2% gain or higher in Glaxo:  0.071045114576185
68
Probability of making 2% gain or higher in BEML:  0.2276982948407534
3
```

3.9 Hypothesis Test

3.9.1 z-test

```
passport_df = pd.read_csv('passport.csv')

passport_df.head(5)
```

	processing_time
0	16.0
1	16.0
2	30.0
3	37.0
4	25.0

```
print(list(passport_df.processing_time))
```

```
[16.0, 16.0, 30.0, 37.0, 25.0, 22.0, 19.0, 35.0, 27.0, 32.0, 34.0, 2
8.0, 24.0, 35.0, 24.0, 21.0, 32.0, 29.0, 24.0, 35.0, 28.0, 29.0, 18.
0, 31.0, 28.0, 33.0, 32.0, 24.0, 25.0, 22.0, 21.0, 27.0, 41.0, 23.0,
23.0, 16.0, 24.0, 38.0, 26.0, 28.0]
```

```
import math

def z_test( pop_mean, pop_var, sample ):
    z_score = (sample.mean() - pop_mean)/(pop_var/math.sqrt(len(sample)))
    return z_score, stats.norm.cdf(z_score)
```

```
z_test( 30, 12.5, passport_df.processing_time )
```

```
(-1.4925950555994747, 0.06777160919961511)
```

3.9.2 One sample t-test

```
bollywood_movies_df = pd.read_csv( 'bollywoodmovies.csv' )
```

```
bollywood_movies_df.head(5)
```

	production_cost
0	601
1	627
2	330
3	364
4	562

```
print(list(bollywood_movies_df.production_cost))
```

```
[601, 627, 330, 364, 562, 353, 583, 254, 528, 470, 125, 60, 101, 11
0, 60, 252, 281, 227, 484, 402, 408, 601, 593, 729, 402, 530, 708, 5
99, 439, 762, 292, 636, 444, 286, 636, 667, 252, 335, 457, 632]
```

```
stats.ttest_1samp( bollywood_movies_df.production_cost, 500 )
```

```
Ttest_1sampResult(statistic=-2.284553287266754, pvalue=0.02786255640
67618)
```

3.9.3 Two sample t-test

```
healthdrink_yes_df = pd.read_excel( 'healthdrink.xlsx', 'healthdrink_yes')
```

```
healthdrink_yes_df.head(5)
```

	height_increase
0	8.6
1	5.8
2	10.2
3	8.5
4	6.8

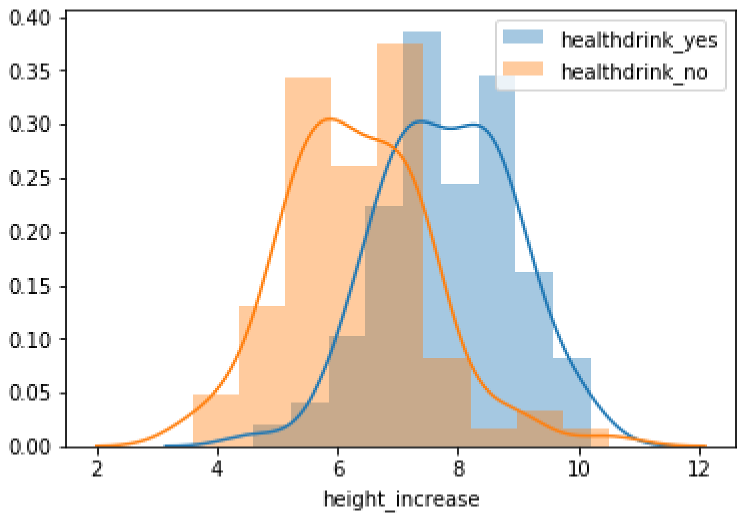
```
healthdrink_yes_df.columns
```

```
Index(['height_increase'], dtype='object')
```

```
healthdrink_no_df = pd.read_excel( 'healthdrink.xlsx', 'healthdrink_no')
healthdrink_no_df.head(5)
```

	height_increase
0	5.3
1	9.0
2	5.7
3	5.5
4	5.4

```
sn.distplot( healthdrink_yes_df['height_increase'], label ='healthdrink_yes' )
sn.distplot( healthdrink_no_df['height_increase'], label ='healthdrink_no' )
plt.legend();
```



```
stats.ttest_ind(healthdrink_yes_df['height_increase'],
                healthdrink_no_df['height_increase'])
```

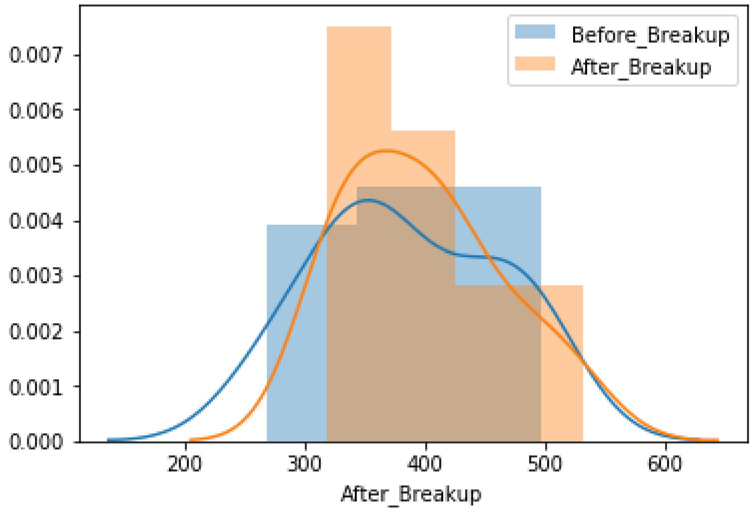
```
Ttest_indResult(statistic=8.131675069083359, pvalue=1.19769859226394
6e-13)
```

3.9.4 Paired sample t-test

```
breakups_df = pd.read_csv( 'breakups.csv' )
breakups_df.head(5)
```

	Before_Breakup	After_Breakup
0	470	408
1	354	439
2	496	321
3	351	437
4	349	335

```
sn.distplot( breakups_df['Before_Breakup'], label ='Before_Breakup' )
sn.distplot( breakups_df['After_Breakup'], label ='After_Breakup' )
plt.legend();
```




```
stats.ttest_rel( breakups_df['Before_Breakup'], breakups_df['After_Breakup'] )
```

```
Ttest_relResult(statistic=-0.5375404241815105, pvalue=0.597134673829
2477)
```

3.9.5 Chi-squre Test of Independence

```
## Observed frequencies
f_obs = [190, 185, 90, 35]
## Expected frquencies from the percentages expected
f_exp = [500*0.35, 500*0.4, 500*.2, 500*0.05]
print( f_exp )
```

```
[175.0, 200.0, 100.0, 25.0]
```

```
stats.chisquare( f_obs, f_exp )
```

```
Power_divergenceResult(statistic=7.410714285714286, pvalue=0.0598975
1420084909)
```

3.10 Analysis of Variance (ANOVA)

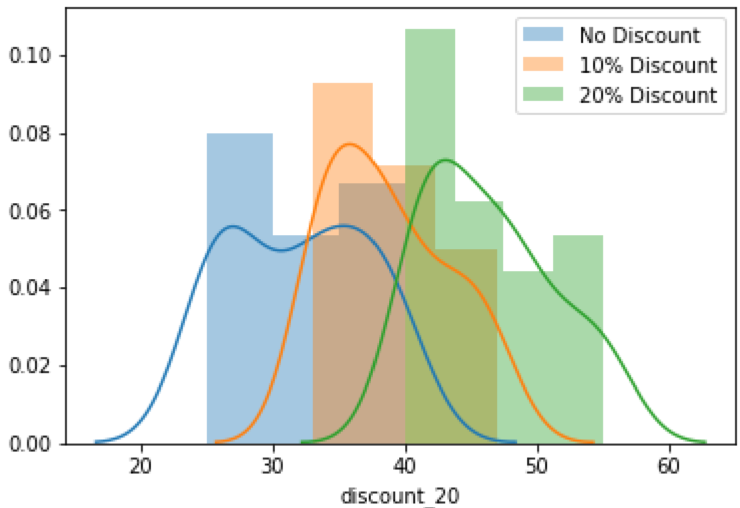
3.10.1 Example of One-Way ANOVA

```
onestop_df = pd.read_csv('onestop.csv')
onestop_df.head(5)
```

	discount_0	discount_10	discount_20
0	39	34	42
1	32	41	43
2	25	45	44
3	25	39	46
4	37	38	41

Let's visualize the distribution of group using distribution plot.

```
sn.distplot(onestop_df['discount_0'], label = 'No Discount')
sn.distplot(onestop_df['discount_10'], label = '10% Discount')
sn.distplot(onestop_df['discount_20'], label = '20% Discount')
plt.legend();
```




```
from scipy.stats import f_oneway

f_oneway( onestop_df['discount_0'],
          onestop_df['discount_10'],
          onestop_df['discount_20'])
```

```
F_onewayResult(statistic=65.86986401283694, pvalue=3.821500669725641
e-18)
```