1.1 MOTIVATION OF SPEECH COMPRESSION

According to information theory, the minimum bitrate at which the condition of distortionless transmission of any source signal is possible is determined by the entropy of the speech source message. Note, however, that in practical terms the source rate corresponding to the entropy is only asymptotically achievable, for the encoding memory length or delay tends to infinity. Any further compression is associated with information loss or coding distortion. Many practical source compression techniques employ lossy coding, which typically guarantees further bitrate economy at the cost of nearly imperceptible speech, audio, video, and other source representation degradation.

Note that the optimum Shannonian source encoder generates a perfectly uncorrelated source-coded stream, in which all the source redundancy has been removed. Therefore, the encoded source symbols—which in most practical cases are constituted by binary bits—are independent, and each one has the same significance. Having the same significance implies that the corruption of any of the source-encoded symbols results in identical source signal distortion over imperfect channels.

Under these conditions, according to Shannon’s fundamental work [33–35], the best protection against transmission errors is achieved if source and channel coding are treated as separate entities. When using a block code of length \( N \) channel-coded symbols in order to encode \( K \) source symbols with a coding rate of \( R = K/N \), the symbol error rate can be rendered arbitrarily low if \( N \) tends to infinity and the coding rate to \( K/N \). This condition also implies an infinite coding delay. Based on the above considerations and on the assumption of Additive White Gaussian Noise (AWGN) channels, source and channel coding have historically been separately optimized.

In designing a telecommunications system, one of the most salient parameters is the number of subscribers that can be accommodated by the transmission media utilized. Whether it is a time division multiplex (TDM) or a frequency division multiplex (FDM) system, whether it is analog or digital, the number of subscribers is limited by the channel capacity needed for one speech channel. If the channel capacity demand of the speech channels is
halved, the total number of subscribers can be doubled. This gain becomes particularly important in applications like power- and band-limited satellite or mobile radio channels, where the demand for free channels overshadows the inevitable cost constraints imposed by a more complex low-bitrate speech codec. In the framework of the basic limitations of state-of-art very large scale integrated (VLSI) circuit technology, the design of a speech codec is based on an optimum tradeoff between lowest bitrate and highest quality, at the price of lowest complexity, cost, and system delay. Analysis of these contradictory factors pervades all our forthcoming discussions.

1.2 BASIC CHARACTERIZATION OF SPEECH SIGNALS

In contrast to deterministic signals, random signals, such as speech, music, video, and other information signals, cannot be described by the help of analytical formulas. They are typically characterized by the help of statistical functions. The power spectral density (PSD), autocorrelation function (ACF), cumulative distribution function (CDF), and probability density function (PDF) are some of the most frequent ones invoked.

Transmitting speech information is one of the fundamental aims of telecommunications, and in this book we concentrate mainly on the efficient encoding of speech signals. The human vocal apparatus has been portrayed in many books dealing with the human anatomy and has also been treated in references dealing with speech processing [5, 17, 22]. Hence, here we dispense with its portrayal and simply note that human speech is generated by emitting sound pressure waves, radiated primarily from the lips, although significant energy emanates through sounds from the nostrils, throat, and the like.

The air compressed by the lungs excites the vocal cords in two typical modes. When generating voiced sounds, the vocal cords vibrate and generate a high-energy quasi-periodic speech waveform, while in the case of lower energy unvoiced sounds, the vocal cords do not participate in the voice production and the source behaves similarly to a noise generator. In a somewhat simplistic approach, the excitation signal denoted by $E(z)$ is then filtered through the vocal apparatus, which behaves like a spectral shaping filter with a transfer function of $H(z) = 1/A(z)$ that is constituted by the spectral shaping action of the glottis, which is defined as the opening between the vocal folds. Further spectral shaping is carried out by the vocal tract, lip radiation characteristics, and so on. This simplified speech production model is shown in Figure 1.1.

Typical voiced and unvoiced speech waveform segments are shown in Figures 1.2 and 1.3, respectively, along with their corresponding power densities. Clearly, the unvoiced segment appears to have a significantly lower magnitude, which is also reflected by its PSD. Observe in Figure 1.3 that the low-energy, noise-like unvoiced signal has a rather flat PSD, which is similar to that of white noise. In general, the flatter the signal’s spectrum, the more unpredictable it becomes, and so it is not amenable to signal compression or redundancy removal.

In contrast, the voiced segment shown in Figure 1.2 is quasi-periodic in the time-domain, and it has an approximately 80-sample periodicity, identified by the positions of the largest time-domain signal peaks, which corresponds to $80 \times 125 \mu s = 10$ ms.

This interval is referred to as the pitch period and it is also often expressed in terms of the pitch frequency $\rho$, which in this example is $\rho = 1/(10$ ms$) = 100$ Hz. In the case of male speakers, the typical pitch frequency range is between 40 and 120 Hz, whereas for females it can be as high as 300–400 Hz. Observe furthermore that within each pitch period there is a gradually decaying oscillation, which is associated with the excitation and gradually decaying vibration of the vocal cords.
A perfectly periodic time-domain signal would have a line spectrum, but since the voiced speech signal is quasi-periodic with a frequency of \( p \) (rather than being perfectly periodic) its spectrum in Figure 1.2 exhibits somewhat widened but distinctive spectral needles at frequencies of \( n \times p \), rather than being perfectly periodic. As a second phenomenon, we can also observe three, sometimes four, spectral envelope peaks. In our voiced spectrum of Figure 1.2, these **formant frequencies** are observable around 500 Hz, 1500 Hz, and 2700 Hz, and they are the manifestation of the resonances of the vocal tract at these frequencies. In contrast, the unvoiced segment of Figure 1.3 does not have a formant structure; rather it has a more dominant high-pass nature, exhibiting a peak around 2500 Hz. Observe, furthermore, that its energy is much lower than that of the voiced segment of Figure 1.2.

It is equally instructive to study the ACF of voiced and unvoiced segments, which are portrayed on an expanded scale in Figures 1.4 and 1.5, respectively. The voiced ACF shows a set of periodic peaks at displacements of about 20 samples, corresponding to \( 20 \times 125 \mu s = 2.5 \) ms, which coincides with the positive quasi-periodic time-domain segments. Following four monotonously decaying peaks, there is a more dominant one around a displacement of 80 samples, which indicates the pitch periodicity. The periodic nature of the ACF can therefore, for example, be exploited to detect and measure the pitch
Chapter 1 • Speech Signals and Introduction to Speech Coding

Unvoiced Speech "xwdl" (male)

Power Density Spectrum

Voiced Speech "xwdl" (male)

Autocorrelation Function

**Figure 1.3** Typical unvoiced speech segment and its PSD for a male speaker.

**Figure 1.4** Typical voiced speech segment and its ACF for a male speaker.
periodicity in a range of applications, such as speech codecs and voice activity detectors. Observe, however, that the first peak at a displacement of 20 samples is about as high as the one near 80. Hence, a reliable pitch detector has to attempt to identify and rank all these peaks in order of prominence, exploiting also the a priori knowledge as to the expected range of pitch frequencies. Recall, too, that, according to the Wiener-Khintshin Theorem, the ACF is the Fourier transform pair of the PSD of Figure 1.2.

By contrast, the unvoiced segment of Figure 1.5 has a much more rapidly decaying ACF, indicating no inherent correlation between adjacent samples and no long-term periodicity. Its sinc-function-like ACF is akin to that of band-limited white noise. The wider ACF of the voiced segment suggests predictability over a time interval of some 3–400 μs. Since the human speech is voiced for about two-thirds of the time, redundancy can be removed from it, using predictive techniques in order to reduce the bitrate required for its transmission.

Having characterized the basic features of speech signals, let us now focus our attention on their digital encoding. Intuitively, it can be expected that the higher the encoder/decoder (codec) complexity, the lower the achievable bitrate and the higher the encoding delay. This is because more redundancy can be removed by considering longer speech segments and employing more sophisticated signal processing techniques.

### 1.3 CLASSIFICATION OF SPEECH CODECS

Speech coding methods can be broadly categorized as **waveform coding**, **vocoding**, and **hybrid coding**. The principle of these codecs is considered later in this chapter, while the most prominent subclass of hybrid codecs known as analysis-by-synthesis schemes are revisited in detail in Chapter 3 and features throughout the book. Their basic differences become explicit in Figure 1.6, where the speech quality versus bitrate performance of these
codec families is portrayed in qualitative terms. The bitrate is plotted on a logarithmic axis, and the speech quality classes “poor to excellent” broadly correspond to the five-point mean opinion score (MOS) scale values of 2–5 defined by the CCITT, which was recently renamed the International Telecommunications Union (ITU). We will more frequently refer to this diagram and to these codec families during our further discourse in order to allocate various codecs on this plane. Hence, only a rudimentary interpretation is offered here.

1.3.1 Waveform Coding [10]

Waveform codecs have been comprehensively characterized by Jayant and Noll [10]; therefore the spirit of virtually all treatises on the subject follows their approach. Our discussion is no exception.

In general, waveform codecs are designed to be signal independent. They are designed to map the input waveform of the encoder into a facsimile-like replica of it at the output of the decoder. Because of this advantage, they can also encode a secondary type of information such as signaling tones, voice band data, or even music. Because of this signal transparency, their coding efficiency is usually quite modest. The coding efficiency can be improved by exploiting some statistical signal properties, if the codec parameters are optimized for the most likely categories of input signals, while still maintaining good quality for other types of signals as well. The waveform codecs can be further subdivided into time-domain waveform codecs and frequency-domain waveform codecs.

1.3.1.1 Time-Domain Waveform Coding. The most well-known representative of signal-independent time-domain waveform coding is the A-law companded pulse code modulation (PCM) scheme. This coding has been standardized by the CCITT at 64 kbit/s, using non-linear companding characteristics to result in near-constant signal-to-noise ratio...
(SNR) over the total input dynamic range. More explicitly, the nonlinear companding compresses large-input samples and expands small ones. Upon quantizing this companded signal, large-input samples will tolerate higher quantization noise than small samples.

Also well-known is the 32 kbit/s adaptive differential PCM (ADPCM) scheme standardized in the ITU Recommendation G.721 (which is the topic of Section 2.7) and the adaptive delta modulation (ADM) arrangement, where usually the most recent signal sample or a linear combination of the last few samples is used to form an estimate of the current one. Then their difference signal, the prediction residual, is computed and encoded with a reduced number of bits, since it has a lower variance than the incoming signal. This estimation process is actually linear prediction with fixed coefficients. However, owing to the nonstationary statistics of speech, a fixed predictor cannot consistently characterize the changing spectral envelope of speech signals. Adaptive predictive coding (APC) schemes utilize two different time-varying predictors to describe speech signals more accurately: a short-term predictor (STP) and a long-term predictor (LTP). We will show that the STP is utilized to model the speech spectral envelope, while the LTP is employed to model the line-spectrum-like fine structure representing the voicing information due to quasi-periodic voiced speech.

All in all, time-domain waveform codecs treat the speech signal to be encoded as a full-band signal and attempt to map it into as close a replica of the input as possible. The difference among various coding schemes is in their degree and way of using prediction to reduce the variance of the signal to be encoded, so as to reduce the number of bits necessary to represent it.

1.3.1.2 Frequency Domain Waveform Coding. In frequency-domain waveform codecs, the input signal undergoes a more or less accurate short-time spectral analysis. The signal is split into a number of sub-bands, and the individual sub-band signals are then encoded by using different numbers of bits in order to obey rate-distortion theory on the basis of their prominence. The various methods differ in their accuracies of spectral resolution and in the bit-allocation principle (fixed, adaptive, semi-adaptive). Two well-known representatives of this class are sub-band coding (SBC) and adaptive transform coding (ATC).

1.3.2 Vocoder

The philosophy of vocoders is based on a priori knowledge of the way the speech signal to be encoded was generated at the signal source by a speaker, which was portrayed in Figure 1.1. The air compressed by the lungs excites the vocal cords in two typical modes. When generating voiced sounds, they vibrate and generate a quasi-periodic speech waveform, while in the case of lower energy unvoiced sounds they do not participate in the voice production and the source behaves similarly to a noise generator. The excitation signal denoted by $E(z)$ in $z$-domain is then filtered through the vocal apparatus, which behaves like a spectral shaping filter with a transfer function of $H(z) = 1/A(z)$ that is constituted by the spectral shaping action of the glotti, vocal tract, lip radiation characteristics, and so on.

Accordingly, instead of attempting to produce a close replica of the input signal at the output of the decoder, the appropriate set of source parameters is found in order to characterize the input signal sufficiently closely for a given duration of time. First, a decision must be made as to whether the current speech segment to be encoded is voiced or unvoiced. Then the corresponding source parameters must be specified. In the case of voiced sounds, the source parameter is the time between periodic vocal tract excitation pulses, which is often referred to as the pitch $p$. In the case of unvoiced sounds, the variance or power of the noise-
like excitation must be determined. These parameters are quantized and transmitted to the decoder in order to synthesize a replica of the original signal.

The simplest source codec arising from the speech production model is depicted in Figure 1.7. The encoder is a simple speech analyzer, determining the current source parameters. After initial speech segmentation, it computes the linear predictive filter coefficients \( a_i, i = 1 \ldots p \), which characterize the spectral shaping transfer function \( H(z) \). A voiced/unvoiced decision is carried out, and the corresponding pitch frequency and noise energy parameters are determined. These are then quantized, multiplexed, and transmitted to the speech decoder, which is a speech synthesizer.

The associated speech quality of this type of systems may be predetermined by the adequacy of the source model, rather than by the accuracy of the quantization of these parameters. This means that the speech quality of source codecs cannot simply be enhanced by increasing the accuracy of the quantization, that is, the bitrate, which is evidenced by the saturating MOS curve of Figure 1.6. Their speech quality is fundamentally limited by the fidelity of the model used. The main advantage of the above vocoding techniques is their low bitrate, with the penalty of relatively low, synthetic speech quality. A well-known representative of this class of vocoders is the 2400 bps American Military Standard LPC-10 codec.

In linear predictive coding (LPC), often more complex excitation models are used to describe the voice-generating source. Once the vocal apparatus has been described by the help of its spectral domain transfer function \( H(z) \), the central problem of coding is to decide how to find the simplest adequate excitation for high-quality parametric speech representation. Strictly speaking, this separable model represents a gross simplification of the vocal apparatus, but it provides the only practical approach to the problem. Vocoding techniques can also be categorized into frequency-domain and time-domain subclasses. However, frequency-domain vocoders are usually more effective than their time-domain counterparts.

1.3.3 Hybrid Coding

Hybrid coding methods are an attractive tradeoff between waveform coding and source coding, both in terms of speech quality and transmission bitrate, although usually at the price of higher complexity. Every speech coding method, combining waveform and source coding methods in order to improve the speech quality and reduce the bitrate, falls into this broad category. However, adaptive predictive time domain techniques used to describe the human spectral shaping tract, combined with an accurate model of the excitation signal, play the most prominent role in this category. The most important family of hybrid codecs, often referred to as analysis-by-synthesis (AbS) codecs, are ubiquitous at the time of writing. Hence, they are
1.4 WAVEFORM OF CODING [10]

1.4.1 Digitization of Speech

The waveform coding of speech and video signals was comprehensively—in fact exhaustively—documented by Jayant and Noll in their classic monograph [10], and hence any treatise on the topic invariably follows a similar approach. This section endeavors to provide a rudimentary overview of waveform coding following the spirit of Jayant and Noll [10]. In general, waveform codecs are designed to be signal independent. They are designed to map the input waveform of the encoder into a facsimile-like replica of it at the output of the decoder. Because of this advantageous property, they can also encode secondary types of information such as signaling tones, voice band data, or even music. Naturally, because of this transparency, their coding efficiency is usually quite modest. The coding efficiency can be improved by exploiting some statistical signal properties, if the codec parameters are optimized for the most likely categories of input signals, while still maintaining good quality for other types of signals.

As noted earlier the waveform codecs can be further subdivided into time-domain waveform codecs and frequency-domain waveform codecs. Let us initially consider the first category. The digitization of analog source signals, such as speech, for example, requires the following steps (see Figure 1.8); the corresponding waveforms are shown in Figure 1.9.

- **Anti-aliasing low-pass filtering** (LPF) is necessary in order to bandlimit the signal to a bandwidth of $B$ before sampling. In case of speech signals, about 1% of the energy resides above 4 kHz and only a negligible proportion above 7 kHz. Hence, commentary quality speech links, which are also often referred to as wideband speech systems, typically bandlimit the speech signal to 7–8 kHz. Conventional telephone systems usually employ a bandwidth limitation of 0.3–3.4 kHz, which results only in a minor speech degradation, hardly perceptible by the untrained listener.

- The band-limited speech is sampled according to the **Nyquist Theorem**, as seen in Figure 1.8, which requires a minimum sampling frequency of $f_{\text{Nyquist}} = 2 \cdot B$. This process introduces time-discrete samples. Due to sampling, the original speech spectrum is replicated at multiples of the sampling frequency. This is why the previous bandlimitation was necessary—in order to prevent aliasing or frequency-domain overlapping of the spectral lobes. If this condition is met, the original analog speech signal can be restored from its samples by passing the samples through a low-pass filter (LPF) with a bandwidth of $B$. In conventional speech systems, typically a sampling frequency of 8 kHz corresponding to a sampling interval of 125 μs is used.

![Figure 1.8 Digitization of analogue speech signals.](image-url)
Lastly, **amplitude discretization or quantization** must be invoked, according to Figure 1.8, which requires an analog to digital (A/D) converter. The output bits of the quantizer can be converted to a serial bit stream for transmission over digital links.

### 1.4.2 Quantization Characteristics

Figure 1.9 shows that the original speech signal is contaminated during the quantization process by quantization noise. The severity of contamination is a function of the signal’s distribution, the quantizer’s resolution, and its transfer characteristic.

The family of **linear quantizers** exhibits a linear transfer function within its dynamic range and saturation above that. They divide the input signal’s dynamic range into a number of uniformly or nonuniformly spaced quantization intervals, as seen in Figure 1.10, and assign an $R$-bit word to each **reconstruction level**, which represents the legitimate output values. In Figure 1.10, according to $R = 3$ there are $2^3 = 8$ reconstruction levels, and a **mid-tread quantizer** is featured, where the quantizer’s output is zero, if the input signal is zero. In the case of the **mid-riser quantizer**, the transfer function exhibits a level change at the abscissa value of zero. Note that the quantization error characteristic of the quantizers is also shown in Figure 1.10. As expected when the quantizer characteristic saturates at its maximum output level, the quantization error increases without limit.

The difference between the uniform and nonuniform quantizer characteristics in Figure 1.10 is that the uniform quantizer maintains a constant maximum error across its total dynamic range, whereas the nonuniform quantizer employs unequal quantization intervals (quantiles) in order to allow larger granular error, where the input signal is larger. Hence, the nonuniform quantizer exhibits a near-constant signal-to-noise ratio (SNR) across its dynamic range. This may allow us to reduce the number of quantization bits and the required transmission rate, while maintaining perceptually unimpaired speech quality.

In summary, linear quantizers are conceptually and implementationally simple and impose no restrictions on the analog input signal’s statistical characteristics such as the probability density function (PDF). Clearly, they do not require a priori knowledge of the input signal. Note, however, that other PDF-dependent quantizers perform better in terms of...
1.4.3 Quantization Noise and Rate-Distortion Theory

Observe in Figure 1.10 that the instantaneous quantization error \( e(x) \) is dependent on the instantaneous input signal level. In other words, \( e(x) \) is nonuniform across the quantizer’s dynamic range and some amplitudes are represented without quantization error, if they happen to be on a reconstruction level, while others are associated with larger errors. If the input signal’s dynamic range exceeds the quantizer’s linear range, the quantizer’s output voltage saturates at its maximum level and the quantization error may become arbitrarily high. Hence, knowledge of the input signal’s statistical distribution is important for minimizing the overall \textbf{granular and overload distortion}. The quantized version \( \hat{x}(t) \) of the input signal \( x(t) \) can be computed as:

\[
\hat{x}(t) = x(t) + e(t),
\]

where \( e(t) \) is the quantization error.

If no amplitude discretization is used for a source signal, a sampled analog source has formally an infinite entropy, requiring an infinite transmission rate, which is underpinned by the formal application of Equation 1.2. If the analog speech samples are quantized to \( R \)-bit accuracy, there are \( q = 2^R \) different legitimate samples, each of which has a probability of occurrence \( p_i, i = 1, 2 \ldots q \). It is known from information theory that the \( R \) bit/symbol
channel capacity requirement can be further reduced using entropy coding to the value of the source's entropy given by:

\[ H(x) = - \sum_{i=1}^{q} p_i \cdot \log_2 p_i, \]  

without inflicting any further coding impairment, if an infinite delay entropy-coding scheme is acceptable. Since this is not the case in interactive speech conversations, we are more interested in quantifying the coding distortion, when using \( R \) bits per speech sample.

An important general result of information theory is the rate-distortion theorem, which quantifies the minimum required average bitrate \( R_D \) in terms of bit/sample in order to represent a random variable (rv) with less than \( D \) distortion. Explicitly, for an rv \( x \) with variance of \( \sigma_x^2 \) and quantized value \( \hat{x} \), the distortion is defined as the mean squared error (mse) expression given by:

\[ D = E[(x - \hat{x})^2] = E[e^2(t)] \]  

where \( E \) represents the expected value.

Observe that if \( R_D = 0 \) bits are used to quantize the quantity \( x \), then the distortion is given by the signal's variance \( D = \sigma_x^2 \). If, however, more than zero bits are used, that is, \( R_D > 0 \), then intuitively one additional bit is needed every time we want to halve the root mean squared (rms) value of \( D \), or quadruple the signal-to-noise ratio of \( \text{SNR} = \frac{\sigma_x^2}{D} \), which suggests a logarithmic relation between \( R_D \) and \( D \). After Shannon and Gallager, we can write for a Gaussian distributed source signal that:

\[ R_D = \frac{1}{2} \log_2 \frac{\sigma_x^2}{D} \quad \text{if} \quad D \leq \sigma_x^2. \]  

Upon combining \( R_D = 0 \) and \( R_D > 0 \) into one equation, we arrive at:

\[ R_D = \begin{cases} \frac{1}{2} \log_2 \frac{\sigma_x^2}{D} & D < \sigma_x^2, \\ 0 & D \geq \sigma_x^2. \end{cases} \]  

The qualitative or stylized relationship of \( D \) versus \( R_D \) inferred from Equation 1.5 is shown in Figure 1.11.

In order to quantify the variance of the quantization error, it is reasonable to assume that if the quantization interval \( q \) is small and no quantizer overload is incurred, then \( e(t) \) is uniformly distributed in the interval \([-q/2, q/2]\). If the quantizer's linear dynamic range is limited to \([\pm V]\), then for a uniform quantizer the quantization interval can be expressed as \( q = 2V/2^R_D \), where \( R_D \) is the number of quantization bits. The quantization error variance can then be computed by squaring the instantaneous error magnitude \( e \) and weighting its

![Figure 1.11 Stylized distortion (D) versus coding rate (R_D) curve.](image)
contribution with its probability of occurrence expressed by the help of its PDF \( p(e) = \frac{1}{q} \) and finally integrating or averaging it over the range of \([-q/2, q/2]\) as follows:

\[
\sigma_e^2 = \int_{-q/2}^{q/2} e^2 p(e)de = \int_{-q/2}^{q/2} e^2 \frac{1}{q} de
\]

\[
= \frac{1}{q} \left[ \frac{e^3}{3} \right]_{-q/2}^{q/2} = \left( \frac{q^3}{8} + \frac{q^3}{8} \right) \cdot \frac{1}{3q} = \frac{q^2}{12}, (1.6)
\]

which corresponds to an RMS quantizer noise of \( \frac{q}{\sqrt{12}} \approx 0.3q \). In the case of uniform quantizers, we can substitute \( q = 2V/2^{R_D} \) into Equation 1.6—where \( R_D \) is the number of bits used for encoding—giving the noise variance in the following form:

\[
\sigma_q^2 = \frac{q^2}{12} = \frac{1}{12} \left( \frac{2V}{2^{R_D}} \right)^2 = \frac{1}{2} \frac{V^2}{2^{2R_D}}. (1.7)
\]

Similarly, assuming a uniform signal PDF, the signal’s variance becomes:

\[
\sigma_x^2 = \int_{-\infty}^{\infty} x^2 p(x)dx = \int_{-\infty}^{\infty} x^2 \frac{1}{2V} dx = \frac{1}{2V} \left[ \frac{x^3}{3} \right]_{-V}^{V} = \frac{1}{6V} \cdot 2V^3 = \frac{E^2}{3}. (1.8)
\]

Then the SNR can be computed as:

\[
SNR = \frac{\sigma_x^2}{\sigma_q^2} = \frac{V^2}{3} \cdot \frac{2^{2R_D}}{V^2} \cdot 3 = 2^{2R_D}, (1.9)
\]

which can be expressed in terms of \( dB \) as follows:

\[
SNR_{dB} = 10 \cdot \log_{10} 2^{2R_D} = 20R_D \cdot \log_{10} 2
\]

\[
SNR_{dB} \approx 6.02 \cdot R_D [dB]. (1.10)
\]

This simple result is useful for quick SNR estimates, and it is also intuitively plausible, since every new bit used halves the quantization error and hence doubles the SNR. In practice, the speech PDF is highly nonuniform, and the quantizer’s dynamic range cannot be fully exploited, in order to minimize the quantizer characteristic or, synonymously, dynamic range overload error. Hence, Equation 1.10 overestimates the expected SNR.

### 1.4.4 Nonuniform Quantization for a Known PDF: Companding

If the input signal’s PDF is known and can be considered stationary, higher SNR can be achieved by appropriately matched nonuniform quantization (NUQ) than in case of uniform quantizers. The input signal’s dynamic range is partitioned into nonuniformly spaced segments as we have seen in Figure 1.10, where the quantization intervals are more dense near the origin, in order to quantize the typically high-probability low-magnitude samples more accurately. In contrast, the lower probability signal PDF tails are less accurately quantized. In contrast to uniform quantization, where the maximum error was constant across the quantizer’s dynamic range, for nonuniform quantizers the SNR becomes more or less constant across the signal’s dynamic range.

It is intuitively advantageous to render the width of the quantization intervals or quantiles inversely proportional to the signal PDF, since a larger quantization error is affordable in the case of infrequent signal samples and vice versa. Two different approaches
have been proposed, for example, by Jayant and Noll [10] in order to minimize the total quantization distortion in the case of nonuniform signal PDFs.

One system model is shown in Figure 1.12, where the input signal is first compressed using a **nonlinear compander** characteristic and then uniformly quantized. The original signal can be recovered using an expander at the decoder, which exhibits an inverse characteristic with respect to that of the compander. This approach will be considered first, while the design of the minimum mean squared error (mmse) nonuniform quantizer using the Lloyd-Max [36–38] algorithm will be portrayed during our later discussions.

The qualitative effect of nonlinear compression on the signal’s PDF is portrayed in Figure 1.13, where it becomes explicit why the compressed PDF can be quantized by a uniform quantizer. Observe that the compander has a more gentle slope, where larger quantization intervals are expected in the uncompressed signal’s amplitude range and vice versa. This implies that the compander’s slope is proportional to the quantization interval density and inversely proportional to the step-size of any given input signal amplitude.

Following Bennett’s approach [39], Jayant and Noll [10] have shown that if the signal’s PDF $p(x)$ is a smooth, known function and sufficiently fine quantization is used—implying that $R \geq 6$—then the quantization error variance can be expressed as:

$$\sigma_q^2 \approx \frac{q^2}{12} \int_{-x_{\text{min}}}^{x_{\text{max}}} \frac{p(x)}{|C(x)|^2} \, dx,$$

(1.11)

where $\dot{C}(x) = dC(x)/dx$ represents the slope of the compander’s characteristic. Where the input signal’s PDF $p(x)$ is high, the $\sigma_q^2$ contributions are also high due to the high probability
of occurrence of such signal amplitudes. This effect can be mitigated using a compander exhibiting a high gradient in this interval, since the factor $1/|\hat{C}(x)|^2$ de-weights the error contributions due to the highly peaked PDF near the origin. For an optimum compander characteristic $C(x)$, all quantiles give the same distortion contribution.

Jayant and Noll [10] have also shown that the minimum quantization error variance is achieved by the compander characteristic given by:

$$C(x) = x_{\text{max}} \int_{x_{\text{min}}}^{x_{\text{max}}} \frac{\sqrt{p(x)}dx}{\int_{x_{\text{min}}}^{x_{\text{max}}} \sqrt{p(x)}dx}, \quad (1.12)$$

where the denominator constitutes a normalizing factor. Hence, a simple practical compander design algorithm can be devised by evaluating the signal’s histogram in order to estimate the PDF $p(x)$ and by graphically integrating $\sqrt{p(x)}$ according to Equation 1.12 up to the abscissa value $x$, yielding the companding characteristic at the ordinate value $C(x)$.

Although this technique minimizes the quantization error variance or maximizes the SNR when there is a known signal PDF, if the input signal’s PDF or variance is time-variant, the compander’s performance degrades. In many practical scenarios, this is the case; hence it is often advantageous to optimize the compander’s characteristic to maximize the SNR independently of the shape of the PDF. Then no compander mismatch penalty is incurred. In order achieve this, the quantization error variance $\sigma_q^2$ must be rendered proportional to the value of the input signal $x(t)$ across its dynamic range, implying that large signal samples will have larger quantization error than small samples. This issue is the topic of the next section.

1.4.5 PDF-Independent Quantization using Logarithmic Compression

The input signal’s variance is given in the case of an arbitrary PDF $p(x)$ as follows:

$$\sigma_x^2 = \int_{-\infty}^{\infty} x^2 p(x)dx. \quad (1.13)$$

Assuming zero saturation distortion, the SNR can be expressed from Equations 1.11 and 1.13 as follows:

$$\text{SNR} = \frac{\sigma_x^2}{\sigma_q^2} = \frac{\int_{x_{\text{min}}}^{x_{\text{max}}} \sigma_x^2 p(x)dx}{\frac{q^2}{12} \int_{x_{\text{min}}}^{x_{\text{max}}} (p(x)/|\hat{C}(x)|^2)dx}. \quad (1.14)$$

In order to maintain an SNR value that is independent of the signal’s PDF $p(x)$, the numerator of Equation 1.14 must be a constant times the denominator, which is equivalent to requiring that:

$$|\hat{C}(x)|^2 = \left| \frac{K}{x} \right|^2, \quad (1.15)$$

or alternatively that:

$$\hat{C}(x) = K/x \quad (1.16)$$
and hence:

\[ C(x) = \int_0^x \frac{K}{z} \, dz = K \cdot \ln x + A. \]  
\[ (1.17) \]

This compander characteristic is shown at the left-hand side of Figure 1.14, and it ensures a constant SNR across the signal’s dynamic range, regardless of the shape of the signal’s PDF. Intuitively, large signals can have large errors, while small signal must maintain a low distortion, which gives a constant SNR for different input signal levels.

Jayant and Noll also note that the constant \( A \) in Equation 1.17 allows for a vertical compander characteristic shift in order to satisfy the boundary condition of matching \( x_{\text{max}} \) and \( y_{\text{max}} \), yielding \( y = y_{\text{max}} \), when \( x = x_{\text{max}} \). Explicitly:

\[ y_{\text{max}} = C(x_{\text{max}}) = K \cdot \ln x_{\text{max}} + A. \]  
\[ (1.18) \]

Upon normalizing Equation 1.17 to \( y_{\text{max}} \), we arrive at:

\[ \frac{y}{y_{\text{max}}} = \frac{C(x)}{y_{\text{max}}} = \frac{K \cdot \ln x + A}{K \cdot \ln x_{\text{max}} + A}. \]  
\[ (1.19) \]

It is convenient to introduce an arbitrary constant \( B \), in order to be able to express \( A \) as \( A = K \cdot \ln B \), since then Equation 1.19 can be written as:

\[ \frac{y}{y_{\text{max}}} = \frac{K \cdot \ln x + K \cdot \ln B}{K \cdot \ln x_{\text{max}} + K \cdot \ln B} = \frac{\ln x B}{\ln x_{\text{max}} B}. \]  
\[ (1.20) \]

Equation 1.20 can be further simplified upon rendering its denominator unity by stipulating \( x_{\text{max}} \cdot B = e^1 \), which yields \( B = e/x_{\text{max}} \). Then Equation 1.20 simplifies to

\[ \frac{y}{y_{\text{max}}} = \frac{\ln x e/x_{\text{max}}}{\ln e} = \ln \left( \frac{e \cdot x}{x_{\text{max}}} \right). \]  
\[ (1.21) \]

which now gives \( y = y_{\text{max}} \), when \( x = x_{\text{max}} \). This logarithmic characteristic, which is shown at the left-hand side of Figure 1.14, must be rendered symmetric with respect to the y-axis, which we achieve upon introducing the signum(x) = sgn(x) function:

\[ \frac{y}{y_{\text{max}}} = \frac{C(x)}{y_{\text{max}}} = \ln \left( \frac{e \cdot |x|}{x_{\text{max}}} \right) \text{sgn}(x). \]  
\[ (1.22) \]

This symmetric function is shown in the center of Figure 1.14. However, a further problem is that the logarithmic function is noncontinuous at zero. Thus, around zero amplitude a linear

---

**Figure 1.14** Stylized companding characteristic for a near-optimal quantizer.
section is introduced in order to ensure a seamless positive-negative transition in the compression characteristic.

Two practical logarithmic compander characteristics that satisfy the above requirements have emerged. In the United States the $\mu$-law compander was standardized [40–42], while in Europe the A-law compander was proposed [4]. The corresponding stylized logarithmic compander characteristic is depicted at the right-hand side of Figure 1.14.

### 1.4.5.1 The $\mu$-Law Compander

This companding characteristic is given by:

$$y = C(x) = y_{\text{max}} \cdot \frac{\ln[1 + \mu \cdot (|x|/x_{\text{max}})]}{\ln(1 + \mu)} \cdot \text{sgn}(x).$$  \hspace{1cm} (1.23)

Upon inferring from the $\log(1 + z)$ function that

$$\log(1 + z) \approx z \text{ if } z \ll 1,$$  \hspace{1cm} (1.24)

in the case of small and large signals, respectively, we have from Equation 1.23 that:

$$y = C(x) = \begin{cases} 
  y_{\text{max}} \cdot \frac{\mu \cdot (|x|/x_{\text{max}})}{\ln \mu} & \text{if } \mu \cdot \frac{|x|}{x_{\text{max}}} \ll 1 \\
  y_{\text{max}} \cdot \frac{\ln[\mu \cdot (|x|/x_{\text{max}})]}{\ln \mu} & \text{if } \mu \cdot \frac{|x|}{x_{\text{max}}} \gg 1
\end{cases}$$  \hspace{1cm} (1.25)

which is a linear function of the normalized input signal $x/x_{\text{max}}$ for small signals and a logarithmic function for large signals. The $\mu \cdot |x|/x_{\text{max}} = 1$ value can be considered to be the breakpoint between the small- and large-signal operation, and the $|x| = x_{\text{max}}/\mu$ is the corresponding abscissa value. In order to emphasize the logarithmic nature of the characteristic, $\mu$ must be large, which reduces the abscissa value of the beginning of the logarithmic section. The optimum value of $\mu$ may be dependent on the quantizer resolution $R$, and for $R = 8$ the American standard pulse code modulation (PCM) speech transmission system recommends $\mu = 255$.

Following the approach proposed by Jayant and Noll [10], the SNR of the $\mu$-law compander can be derived upon substituting $y = C_{\mu}(x)$ from Equation 1.23 into the general SNR formula of Equation 1.14:

$$y = C_{\mu}(x) = y_{\text{max}} \cdot \frac{\ln[1 + \mu(|x|/x_{\text{max}})]}{\ln(1 + \mu)} \cdot \text{sgn}(x)$$  \hspace{1cm} (1.26)

$$\hat{C}_{\mu}(x) = \frac{y_{\text{max}}}{\ln(1 + \mu)} \cdot \frac{1}{1 + \mu(|x|/x_{\text{max}})} \cdot \mu \left(\frac{1}{x_{\text{max}}}\right).$$  \hspace{1cm} (1.27)

For large-input signals we have $\mu(|x|/x_{\text{max}}) \gg 1$, and hence:

$$\hat{C}_{\mu}(x) \approx \frac{y_{\text{max}}}{\ln \mu} \cdot \frac{1}{x}.$$  \hspace{1cm} (1.28)

Upon substituting

$$\frac{1}{C_{\mu}(x)} = \frac{\ln \mu}{y_{\text{max}}} \cdot x$$  \hspace{1cm} (1.29)
Chapter 1 • Speech Signals and Introduction to Speech Coding

in Equation 1.14 we arrive at:

\[
SNR = \frac{\int_{-x_{\text{max}}}^{x_{\text{max}}} x^2 p(x)dx}{q^2 \int_{-x_{\text{max}}}^{x_{\text{max}}} (\ln \mu)^2 x^2 p(x)dx}
\]

\[
= \frac{1}{q^2 (\ln \mu)^2} = 3 \left( \frac{2y_{\text{max}}}{q} \right)^2 \cdot \left( \frac{1}{\ln \mu} \right)^2
\]

\[
= 3 \cdot 2^{2R} \cdot \left( \frac{1}{\ln \mu} \right)^2. \tag{1.30}
\]

Upon exploiting that \(2y_{\text{max}}/q = 2^R\) represents the number of quantization levels and expressing the above equation in terms of dB, we get:

\[
SNR_{\text{dB}}^R = 6.02 \cdot R + 4.77 - 20 \log_{10}(\ln(1 + \mu)), \tag{1.31}
\]

which gives an SNR of about 38 dB in the case of the American standard system using \(R = 8\) and \(\mu = 255\). Recall that under the assumption of no quantizer characteristic overload and a uniformly distributed input signal, the corresponding SNR estimate would yield \(6.02 \cdot 8 \approx 48\) dB. Note, however, that in practical terms this SNR is never achieved, since the input signal does not have a uniform distribution and saturation distortion is also often incurred.

1.4.5.2 The A-law Compander. Another practical logarithmic compander characteristic is the \textbf{A-Law Compander} [4], which was standardized by the ITU and is used throughout Europe:

\[
y = C(x) = \begin{cases} 
 y_{\text{max}} \cdot \frac{A(|x|/x_{\text{max}})}{1 + \ln A} \cdot \text{sgn}(x); & 0 < |x| < \frac{1}{x_{\text{max}} < A} \\
 y_{\text{max}} \cdot \frac{1 + \ln[A(|x|/x_{\text{max}})]}{1 + \ln A} \cdot \text{sgn}(x); & \frac{1}{A} < |x| < 1
\end{cases} \tag{1.32}
\]

where \(A = 87.56\). Similarly to the \(\mu\)-law characteristic, it has a linear region near the origin and a logarithmic section above the breakpoint \(|x| = x_{\text{max}}/A\). Note, however, that in case of \(R = 8\) bits \(A < \mu\), hence, the A-law characteristic’s linear-logarithmic breakpoint is at a higher input value than that of the \(\mu\)-law characteristic.

Again, substituting:

\[
\frac{1}{C_A(x)} = \frac{(1 + \ln A)}{y_{\text{max}}} \cdot x \tag{1.33}
\]
into Equation 1.14 and exploiting that $2y_{\text{max}}/q = 2^R$ represents the number of quantization levels, we have:

$$SNR = \frac{\int_{-y_{\text{max}}}^{y_{\text{max}}} x^2 p(x) dx}{\frac{q^2}{12} \int_{-y_{\text{max}}}^{y_{\text{max}}} \left( \frac{1 + \ln A}{y_{\text{max}}} \right)^2 x^2 p(x) dx}$$

$$= \frac{1}{\frac{q^2}{12} \left( \frac{1 + \ln A}{y_{\text{max}}} \right)^2} = 3 \left( \frac{2y_{\text{max}}}{q} \right)^2 \left( \frac{1}{(1 + \ln A)} \right)^2$$

$$= 3 \cdot 2^{2R} \cdot \left( \frac{1}{(1 + \ln A)} \right)^2.$$  \hspace{1cm} (1.34)

Upon expressing Equation 1.34 in terms of dB, we arrive at:

$$SNR_{\text{dB}}^4 = 6.02 \cdot R + 4.77 - 20 \log_{10}(1 + \ln A),$$  \hspace{1cm} (1.35)

which, similarly to the $\mu$-law compander, gives an SNR of about 38 dB in case of the European standard PCM speech transmission system using $R = 8$ and $A = 87.56$.

Further features of the European $A$-law standard system are that the characteristic given by Equation 1.32 is implemented in the form of a 16-segment piecewise linear approximation, as seen in Figure 1.15. The segment retaining the lowest gradient of $\frac{1}{4}$ is at the top end of the input signal’s dynamic range, which covers half of the positive dynamic range, and it is divided into 16 uniformly spaced quantization intervals. The second segment from the top covers a quarter of the positive dynamic range and doubles the top segment’s steepness or gradient to $\frac{1}{2}$, and so on. The bottom segment covers a 64th of the positive dynamic range and has the highest slope of 16 and the finest resolution. The first bit of each $R = 8$-bit PCM codeword represents the sign of the input signal. The next 3 bits specify which segment the input signal belongs to, while the last 4 bits divide a specific segment with 16 uniform-width quantization intervals, as shown in:

<table>
<thead>
<tr>
<th>$b_7$</th>
<th>$b_6$</th>
<th>$b_5$</th>
<th>$b_4$</th>
<th>$b_3$</th>
<th>$b_2$</th>
<th>$b_1$</th>
<th>$b_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sign (segment)</td>
<td>segments</td>
<td>uniform quant. in each segment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This scheme was standardized by the International Telegraph and Telephone Consultative Committee (CCITT) as the G711 Recommendation for the transmission of speech sampled at 8 kHz. Hence the transmission rate becomes $8 \times 8 = 64$ kbit/s (kbps). This results in perceptually unimpaired speech quality, which would require about 12 bits in case of linear quantization.

**1.4.6 Optimum Nonuniform Quantization**

For nonuniform quantizers the quantization error variance is given by:

$$\sigma_q^2 = E[|x - x_q|^2] = \int_{-\infty}^{\infty} e^2(x)p(x)dx,$$  \hspace{1cm} (1.36)
which, again, corresponds to weighting and averaging the quantization error energy over its magnitude range. Assuming an odd-symmetric quantizer transfer function and symmetric PDF $p(x)$, we find that the total quantization distortion power $\sigma_D^2$ is as follows:

$$\sigma_D^2 = 2 \int_0^\infty e^2(x)p(x)dx.$$  \hfill (1.37)

The total distortion can be expressed as the sum of the quantization distortion in the quantizer’s linear range, plus the saturation distortion in its nonlinear range, as follows:

$$\sigma_D^2 = 2 \int_0^{\nu} e^2(x)p(x)dx + 2 \int_{\nu}^{\infty} e^2(x)p(x)dx$$  \hfill (1.38)

or more simply as:

$$\sigma_D^2 = \sigma_q^2 + \sigma_I^2.$$  \hfill (1.39)
In order to emphasize the fact that in case of nonuniform quantization each of the $N$ quantization intervals or quantiles adds a different PDF-weighted contribution to the total quantization distortion, we rewrite the first term of Equation 1.38 as follows:

\[
\sigma_q^2 = \sum_{n=1}^{N} \int_{x_n}^{x_{n+1}} e^2(x)p(x)dx
\]

\[
= \sum_{n=1}^{N} \int_{x_n}^{x_{n+1}} (x - x_n)^2 p(x)dx
\]

\[
= \sum_{n=1}^{N} \int_{x_n}^{x_{n+1}} (x - r_n)^2 p(x)dx,
\]

(1.40)

where $x_n = r_n$ represents the reconstruction levels.

Given a certain number of quantization bits $R$ and the PDF of the input signal, the optimum Lloyd-Max quantizer, which was independently invented by Lloyd [36, 37] and Max [38], determines the set of optimum quantizer decision levels and the corresponding set of quantization levels.

Jayant and Noll [10] have provided a detailed discussion of two different methods of determining the minimum mean squared error (mmse) solution to the problem. One of the solutions is based on an iterative technique of rearranging the decision thresholds and reconstruction levels, while the other one is an approximate solution valid for fine quantizers using a high number of bits per sample. We first present the general approach to minimizing the mse by determining the set of optimum reconstruction levels $r_n$, $n = 1 \ldots N$ and the corresponding decision threshold values $t_n$, $n = 1 \ldots N$.

In general, it is a necessary but not sufficient condition for finding the global minimum of Equation 1.41 for its partial derivatives to become zero. However, if the PDF $p(s)$ is log-concave, that is, the second derivative of its logarithm is negative, then the minimum found is a global one. For the frequently encountered uniform (U), Gaussian (G), and Laplacian (L) PDFs, the log-concave condition is satisfied, but, for example, for Gamma (Γ) PDFs it is not.

Setting the partial derivatives of Equation 1.41 with respect to a specific $r_n$ to zero, only one term in the sum depends on the $r_n$ value considered. Hence we arrive at:

\[
\frac{\partial(\sigma_q^2)}{\partial r_n} = \int_{t_n}^{t_{n+1}} (s - r_n) \cdot p(s)ds = 0, \quad n = 1 \ldots N,
\]

(1.42)

which leads to

\[
\int_{r_n}^{r_{n+1}} s \cdot p(s)ds = r_n \int_{r_n}^{r_{n+1}} p(s)ds,
\]

(1.43)

yielding the optimum reconstruction level $r_n^{opt}$ as follows:

\[
r_n^{opt} = \frac{\int_{r_n}^{r_{n+1}} s \cdot p(s)ds}{\int_{r_n}^{r_{n+1}} p(s)ds}, \quad n = 1 \ldots N.
\]

(1.44)

Note that the above expression depends on the optimum quantization interval thresholds $r_n^{opt}$ and $r_{n+1}^{opt}$. Furthermore, for an arbitrary nonuniform PDF $r_n^{opt}$ is given by the mean value or the center of gravity of $s$ within the quantization interval $n$, rather than by $(r_n^{opt} + r_{n+1}^{opt})/2$. 

Similarly, when computing $\frac{\partial \sigma^2_q}{\partial t_n}$, there are only two terms in Equation 1.41, which contain $t_n$. Therefore, we get:

$$\frac{\partial \sigma^2_q}{\partial t_n} = (t_n - r_{n-1})^2 p(t_n) - (t_n - r_n)^2 p(t_n) = 0 \quad (1.45)$$

leading to:

$$t_n^2 - 2t_n r_{n-1} + r_{n-1}^2 - t_n^2 + 2t_n r_n - r_n^2 = 0. \quad (1.46)$$

Hence, the optimum decision threshold is given by:

$$r_n^{\text{opt}} = (r_n^{\text{opt}} + r_{n-1}^{\text{opt}})/2; \quad n = 2 \ldots N, \quad r_1^{\text{opt}} = -\infty, \quad r_N^{\text{opt}} = \infty \quad (1.47)$$

which is halfway between the optimum reconstruction levels. Since these nonlinear equations are interdependent they can only be solved by recursive iterations, starting from either a uniform quantizer or from a “hand-crafted” initial nonuniform quantizer design.

Since most practical signals do not obey any analytically describable distribution, the signal’s PDF typically has to be inferred from a sufficiently large and characteristic training set. Equations 1.44 and 1.47 will also have to be evaluated numerically for the training set. Below we provide a simple practical algorithm that can be easily implemented by the coding practitioner by the help of the flowchart of Figure 1.16.

**Step 1:** Input initial parameters such as the number of quantization bits $R$, maximum number of iteration $I$, dynamic range minimum $t_1$ and maximum $t_N$.

**Step 2:** Generate the initial set of thresholds $r_0^1 \ldots r_0^N$, where the superscript 0 represents the iteration index, either automatically creating a uniform quantizer between $t_1$ and $t_N$ according to the required number of bits $R$, or inputting a “hand-crafted” initial design.

**Step 3:** While $t < T$, where $T$ is the total number of training samples do:

1. Assign the current training sample $s^t$, $t = 1 \ldots T$ to the corresponding quantization interval $[r_n^0 \ldots r_{n+1}^0]$ and increment the sample counter $C[n], n = 1 \ldots N$, holding the number of samples assigned to interval $n$. This corresponds to generating the histogram $p(s)$ of the training set.

2. Evaluate the mse contribution due to assigning $s^t$ to bin$[n]$, that is, $mse^t = (s^t - s_q^t)^2$, and the resultant total accumulated mse, that is, $mse^t = mse^{t-1} + mse^t$.

**Step 4:** Once all training samples have been assigned to their corresponding quantization bins, that is, the experimental PDF $p(s)$ is evaluated, the center of gravity of each bin is computed by summing the training samples in each bin$[n], n = 1 \ldots N$ and then dividing the sum by the number of training samples $C[n]$ in bin$[n]$. This corresponds to the evaluation of Equation 1.44, yielding $r_n$.

**Step 5:** Rearrange the initial quantization thresholds $r_0^1 \ldots r_0^N$ using Equation 1.47 by placing them halfway between the above computed initial reconstruction levels $r_0^0, n = 1 \ldots N$, where again, the superscript 0 represents the iteration index. This step generates the updated set of quantization thresholds $r_1^1 \ldots r_N^1$.

**Step 6:** Evaluate the performance of the current quantizer design in terms of

$$\text{SNR} = 10 \log_{10} \left[ \frac{\sum_{t=1}^T (s^t)^2}{mse^t} \right]$$
Recursion: Repeat **Steps 3–6** by iteratively updating \( r_i^n, t_i^n \) for all bins \( n = 1 \ldots N \), until the iteration index \( i \) reaches its maximum \( I \), while monitoring the quantizer SNR performance improvement given above.

Note that it is important to invoke the algorithm several times, while using a different initial quantizer, in order to ascertain its proper convergence to a global optimum. The inner workings of the algorithm may place the reconstruction levels and thresholds more sparsely, where the PDF \( p(s) \) is low and vice versa. If the input signal’s statistics obey a U, G, L, or \( \Gamma \) distribution, the Lloyd-Max quantizer’s SNR performance can be evaluated using Equations 1.44 and 1.47. Various authors have tabulated the achievable SNR values. Following Max [38], Noll and Zelinski [43] as well as Paez and Glisson [44], both Jayant and Noll [10] as
Table 1.1 Maximum Achievable SNR and mse in Case of Zero-mean, Unit-variance Input \([f(R)]\) for Gaussian (G) and Laplacian (L) PDFs for \(R = 1, 2, \ldots, 7\) © Prentice Hall, Jayant-Noll [10] 1984, p. 135 and Jain [45] 1989, p. 104

<table>
<thead>
<tr>
<th></th>
<th>(R = 1)</th>
<th>(R = 2)</th>
<th>(R = 3)</th>
<th>(R = 4)</th>
<th>(R = 5)</th>
<th>(R = 6)</th>
<th>(R = 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>SNR (dB)</td>
<td>4.40</td>
<td>9.30</td>
<td>14.62</td>
<td>20.22</td>
<td>26.01</td>
<td>31.89</td>
</tr>
<tr>
<td></td>
<td>(f(R))</td>
<td>0.3634</td>
<td>0.1175</td>
<td>0.0345</td>
<td>0.0095</td>
<td>0.0025</td>
<td>0.0006</td>
</tr>
<tr>
<td>L</td>
<td>SNR (dB)</td>
<td>3.01</td>
<td>7.54</td>
<td>12.64</td>
<td>18.13</td>
<td>23.87</td>
<td>29.74</td>
</tr>
<tr>
<td></td>
<td>(f(R))</td>
<td>0.5</td>
<td>0.1762</td>
<td>0.0545</td>
<td>0.0154</td>
<td>0.0041</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

well as Jain [45] collected these SNR values, which we summarised in Table 1.1 for G and L distributions. Jayant and Noll [10] as well as Jain [45] also tabulated the corresponding \(t_n\) and \(r_n\) values for a variety of PDFs and \(R\) values.

Note in Table 1.1 that apart from the achievable maximum SNR values the associated quantizer mse \(f(R)\) is given as a function of the number of quantization bits \(R\). When designing a quantizer for an arbitrary nonunity input variance \(a^2\), the associated quantization thresholds and reconstruction levels must be appropriately scaled by \(\sigma^2\). In case of a large input variance, the reconstruction levels may have to be sparsely spaced in order to cater for the signal’s expanded dynamic range. Therefore the reconstruction mse \(\sigma^2\) must also be scaled by \(\sigma^2\), giving:

\[
\sigma^2 = \sigma^2 \cdot f(R).
\]

Here we curtail our discussion of zero-memory quantization techniques; the interested reader is referred to the excellent in-depth reference [10] by Jayant and Noll for further details. Before we move on to predictive coding techniques, the reader is reminded that in Section 1.2 we observed how both the time- and the frequency-domain features of the speech signal exhibit redundancy. In the next chapter we introduce a simple way of exploiting this redundancy in order to achieve better coding efficiency and reduce the required coding rate from 64 kbps to 32 kbps.

1.5 CHAPTER SUMMARY

In this chapter we provided a rudimentary characterization of voiced and unvoiced speech signals. It was shown that voice speech segments exhibit a quasi-periodic nature and convey significantly more energy than the more noise-like unvoiced segments. Because of their quasi-periodic nature, voiced segments are more predictable; in other words, they are more amenable to compression.

These discussions were followed by a brief introduction to the digitization of speech and to basic waveform coding techniques. The basic principles of logarithmic compression were highlighted, and the optimum nonuniform Max-Lloyd quantization principle was introduced. In the next chapter we introduce the underlying principles of more efficient predictive speech coding techniques.