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INTRODUCTION TO CELLULAR MOBILE RADIO COMMUNICATION

The subject of this book is code division multiple access (CDMA) communications. A major application of CDMA is wireless communication including mobile radio. In this chapter we introduce the basic concepts of mobile radio systems, including cellular concepts, consider the general structure of a cellular system, and study different principles of multiple-access (time, frequency, and code division) and spread spectrum concepts.

This chapter begins with an overview of the principles of cellular radio systems. Next, given the focus on simultaneous wideband transmission of all users over a common frequency spectrum, we consider direct-sequence CDMA systems, frequency-hopped CDMA systems, and pulse position-hopped CDMA systems. The chapter concludes with a description of this book. The book is devoted to the analysis of different aspects of CDMA communication. Given the rapid and continuing growth of cellular radio systems throughout the world, CDMA digital cellular radio systems will be the widest-deployed form of spread spectrum systems for voice and data communication. It is a major technology of the twenty-first century.

1.1 CELLULAR MOBILE RADIO SYSTEMS

A cellular radio system provides a wireless connection to the public telephone network for any user location within the radio range of the system. The term *mobile* has traditionally been used to classify a radio terminal that can be moved during

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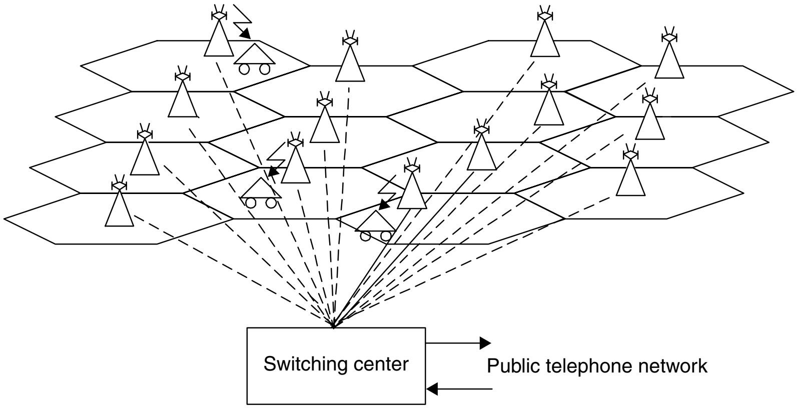

Figure 1.1. An illustration of a cellular system.

communication. Cellular systems accommodate a large number of mobile units over a large area within a limited frequency spectrum. There are several types of radio transmission systems. We consider only *full duplex systems*. These are communication systems that allow simultaneous two-way communication. Transmission and reception for a full duplex system are typically on two different channels, so the user may constantly transmit while receiving signals from another user.

Figure 1.1 shows a basic cellular system that consists of *mobiles*, *base stations*, and a *switching center*. Each mobile communicates via radio with one or more base stations. A call from a user can be transferred from one base station to another during the call. The process of transferring is called *handoff*.

Each mobile contains a *transceiver* (transmitter and receiver), an antenna, and control circuitry. The base stations consist of several transmitters and receivers, which simultaneously handle full duplex communications and generally have towers that support several transmitting and receiving antennas. The base station connects the simultaneous mobile calls via telephone lines, microwave links, or fiber-optic cables to the switching center. The switching center coordinates the activity of all of the base stations and connects the entire cellular system to the public telephone network.

The channels used for transmission from the base station to the mobiles are called *forward* or *downlink channels*, and the channels used for transmission from the mobiles to the base station are called *reverse* or *uplink channels*. The two channels responsible for call initiation and service request are the *forward control channel* and *reverse control channel*.

Once a call is in progress, the switching center adjusts the transmitted power of the mobile (this process is called *power control*[1]) and changes the channel of the mobile and base station (handoff) to maintain call quality as the mobile moves in and out of range of a given base station.

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[1] Sometimes the mobile adjusts the transmitted power by measuring the power of the received signal (so-called *open-loop power control*).

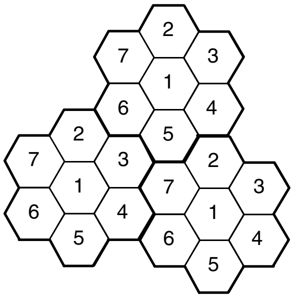

Figure 1.2. An illustration of the cellular frequency reuse concept.

The cellular concept was a major breakthrough in solving the problem of spectral congestion. It offered high system capacity with a limited spectrum allocation. In a modern conventional mobile radio communication system, each base station is allocated a portion of the total number of channels available to the entire system and nearby base stations are assigned different groups of channels so that all the available channels are assigned to a relatively small number of neighboring base stations. Neighboring base stations are assigned different groups of channels so that interference between the users in different cells is small.

The idealized allocation of cellular channels is illustrated in Figure 1.2, in which the cells are shown as contiguous hexagons. Cells labeled with the same number use the same group of channels. The same channels are never reused in contiguous cells but may be reused by noncontiguous cells. The κ cells that collectively use the complete set of available frequencies is called a *cluster*. In Figure 1.2, a cell cluster is outlined in bold and replicated over the coverage area. Two cells that employ the same allocation, and hence can interfere with each other, are separated by more than one cell diameter.

The factor κ is called the *cluster size* and is typically equal to 3, 4, 7, or 12. To maximize the capacity over a given coverage area we have to choose the smallest possible value of κ. The factor $1/\kappa$ is called the *frequency reuse factor* of a cellular system. In Figure 1.2 the cluster size is equal to 7, and the frequency reuse factor is equal to 1/7.

EXAMPLE 1.1

The American analog technology standard, known as Advanced Mobile Phone Service (AMPS), employs frequency modulation and occupies a 30-*kHz frequency slot for each voice channel [47]. Suppose that a total of* 25-*MHz bandwidth is allocated to a particular cellular radio communication system with cluster size* 7. *How many channels per cell does the system provide?*

Solution

Allocation of 12.5 *MHz each for forward and reverse links provides a little more than* 400 *channels in each direction for the total system, and correspondingly a little less than* 60 *per cell.*

The other-cell interference can be reduced by employing sectored antennas at the base station, with each sector using different frequency bands. However, using sectored antennas does not increase the number of slots and consequently the frequency reuse factor is not increased.

A multiple access system that is more tolerant to interference can be designed by using digital modulation techniques at the transmitter (including both source coding and channel error-correcting coding) and the corresponding signal processing techniques at the receiver.

1.2 FREQUENCY DIVISION AND TIME DIVISION MULTIPLE ACCESS

Multiple access schemes are used to allow many mobile users to share simultaneously a common bandwidth. As mentioned above, a full duplex communication system typically provides two distinct bands of frequencies (channels) for every user. The forward band provides traffic from the base station to the mobile, and the reverse band provides traffic from the mobile to the base station. Therefore, any duplex channel actually consists of two simplex channels.

Frequency division multiple access (FDMA) and *time division multiple access (TDMA)* are the two major access techniques used to share the available bandwidth in a conventional mobile radio communication systems.

Frequency division multiple access assigns individual channels (frequency bands) to individual users. It can be seen from Figure 1.3 that each user is allocated a unique frequency band. These bands are assigned on demand to users who request service. During the period of the call, no other user can share the same frequency band. The bandwidths of FDMA channels are relatively narrow (25–30 kHz) as each channel supports only one call per carrier. That is, FDMA is usually implemented in narrowband systems. If an FDMA channel is not in use (for example, during pauses in telephone conversation) it sits idle and cannot be used by other users to increase the system capacity.

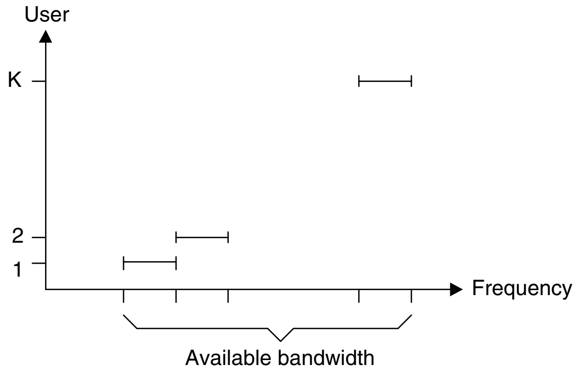

Figure 1.3. FDMA scheme in which different users are assigned different frequency bands.

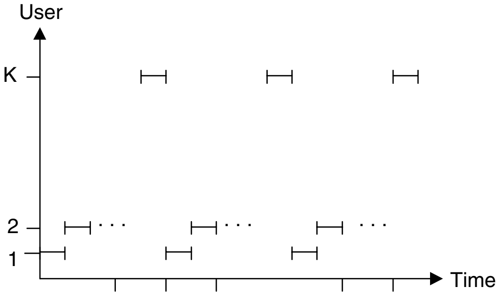

Figure 1.4. TDMA scheme in which each user occupies a cyclically repeating time slot.

Time division multiple access systems divide the transmission time into time slots, and in each slot only one user is allowed to either transmit or receive. It can be seen from Figure 1.4 that each user occupies cyclically repeating wording, so a channel may be thought of as a particular time slot that reoccurs at slot locations in every frame. Unlike in FDMA systems, which can accommodate analog frequency modulation (FM), digital data and digital modulation must be used with TDMA.

TDMA shares a single carrier frequency with several users, where each user makes use of nonoverlapping time slots. Analogously to FDMA, if a channel is not in use, then the corresponding time slots sit idle and cannot be used by other users. Data transmission for users of a TDMA system is not continuous but occurs in bursts. Because of burst transmission, synchronization overhead is required in TDMA systems. In addition, guard slots are necessary to separate users. Generally, the complexity of TDMA mobile systems is higher compared with FDMA systems.

EXAMPLE 1.2

The global system for mobile communications (GSM) utilizes the frequency band 935–960 *MHz for the forward link and frequency range* 890–915 *MHz for the reverse link. Each* 25-*MHz band is broken into radio channels of* 200 *kHz. Each radio channel consists of eight time slots. If no guard band is assumed, find the number of simultaneous users that can be accommodated in GSM. How many users can be accommodated if a guard band of* 100 *kHz is provided at the upper and the lower end of the GSM spectrum?*

Solution

The number of simultaneous users that can be accommodated in GSM in the first case is equal to

$$\frac{25 \cdot 10^6}{(200 \cdot 10^3)/8} = 1000$$

In the second case the number of simultaneous users is equal to 992.

Each user of a conventional multiple access system, based on the FDMA or the TDMA principle, is supplied with certain resources, such as frequency or time slots, or both, which are disjoint from those of any other user. In this system, the multiple access channel reduces to a multiplicity of single point-to-point channels. The transmission rate in each channel is limited only by the bandwidth and time allocated to it, the channel degradation caused by background noise, multipath fading, and shadowing effects.

Viterbi [47] pointed out that this solution suffers from three weaknesses. The first weakness is that it assumes that all users transmit continuously. However, in a two-person conversation, the percentage of time that a speaker is active, that is, talking, ranges from 35% to 50%. In TDMA or FDMA systems, reallocation of the channel for such brief periods requires rapid circuit switching between the two users, which is practically impossible.

The second weakness is the relatively low frequency reuse factor of FDMA and TDMA. As we can see from Example 1.1 the frequency reuse factor 1/7 reduces the number of channels per cell in AMPS from 400 to less than 60.

Using antenna sectorization (Fig. 1.5) for reducing interference does not increase system capacity. As an example, a cell site with a three-sectored antenna has an interference that is approximately one-third of the interference received by an omnidirectional antenna. Even with this technique, the interference power received at a given base station from reused channels in other cells is only about 18 dB below the signal power received from the desired user of the same channel in the given cell. Reuse factors as large as 1/4 and even 1/3 have been considered and even used, but decreasing the distance between interfering cells increases the other-cell interference to the point of unacceptable signal quality.

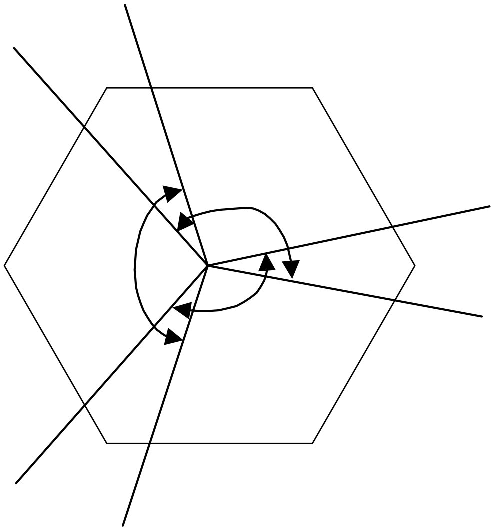

Figure 1.5. A three-sectored antenna in a single isolated cell.

A third source of performance degradation, which is common to all multiple access systems, particularly in terrestrial environments, is fading. Fading is caused by interference between two or more versions of the transmitted signal that arrive at the receiver at slightly different time. This phenomenon is particularly severe when each channel is allocated a narrow bandwidth, as for FDMA systems.

1.3 DIRECT SEQUENCE CDMA

A completely different approach, realized in CDMA systems, does not attempt to allocate disjoint frequency or time resources to each user. Instead the system allocates all resources to all active users.

In *direct sequence* (DS) CDMA systems, the narrowband message signal is multiplied by a very large-bandwidth signal called the *spreading signal*. All users in a DS CDMA system use the same carrier frequency and may transmit simultaneously. Each user has its own spreading signal, which is approximately orthogonal to the spreading signals of all other users. The receiver performs a correlation operation to detect the message addressed to a given user. The signals from other users appear as noise due to decorrelation. For detecting the message signal, the receiver requires the spreading signal used by the transmitter. Each user operates independently with no knowledge of the other users *(uncoordinated transmission)*.

Potentially, CDMA systems provide a larger *radio channel capacity* than FDMA and TDMA systems. The radio channel capacity (not to be confused with *Shannon's channel capacity*, see Chapter 8) can be defined as the maximum number K_0 of simultaneous users that can be provided in a fixed frequency band. Radio channel capacity is a measure of the *spectrum efficiency* of a wireless system. This parameter is determined by the *required signal-to-noise ratio* at the input of the receiver and by the channel bandwidth W.

To explain the principle of DS CDMA let us consider a simple example. Suppose that two users, user 1 and user 2, located the same distance from the base station, wish to send the *information* (or *data*) *sequences*[2] $\boldsymbol{u}^{(1)} = u_0^{(1)}, u_1^{(1)}, u_2^{(1)}, u_3^{(1)} = 1, -1, -1, 1$ and $\boldsymbol{u}^{(2)} = u_0^{(2)}, u_1^{(2)}, u_2^{(2)}, u_3^{(2)} = -1, 1, -1, -1$, respectively, to the base station. First, user 1 maps the data sequence $\boldsymbol{u}^{(1)}$ into the data signal $u^{(1)}(t)$, and user 2 maps $\boldsymbol{u}^{(2)}$ into the data signal $u^{(2)}(t)$, such that the real number 1 corresponds to a positive rectangular pulse of unit amplitude and duration T, and the real number -1 corresponds to a negative rectangular pulse of the same amplitude and same duration (Fig. 1.6a). Then both users synchronously transmit the data signals over the *multiple access adding channel*. Because each pulse corresponds to the transmission of one bit, the transmission rate $R = 1/T$ (bit/s) for each user and the overall rate is $2/T$ (bit/s).

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[2] In information-theoretic literature, binary sequences consist of symbols from the binary logical alphabet {0, 1}. In CDMA applications it is more convenient to use the binary real number alphabet $\{1, -1\}$. The mapping $0 \rightleftarrows 1$, $1 \rightleftarrows -1$ establishes a one-to-one correspondence between sequences of binary logical symbols and sequences of binary real numbers (see also Chapter 4).

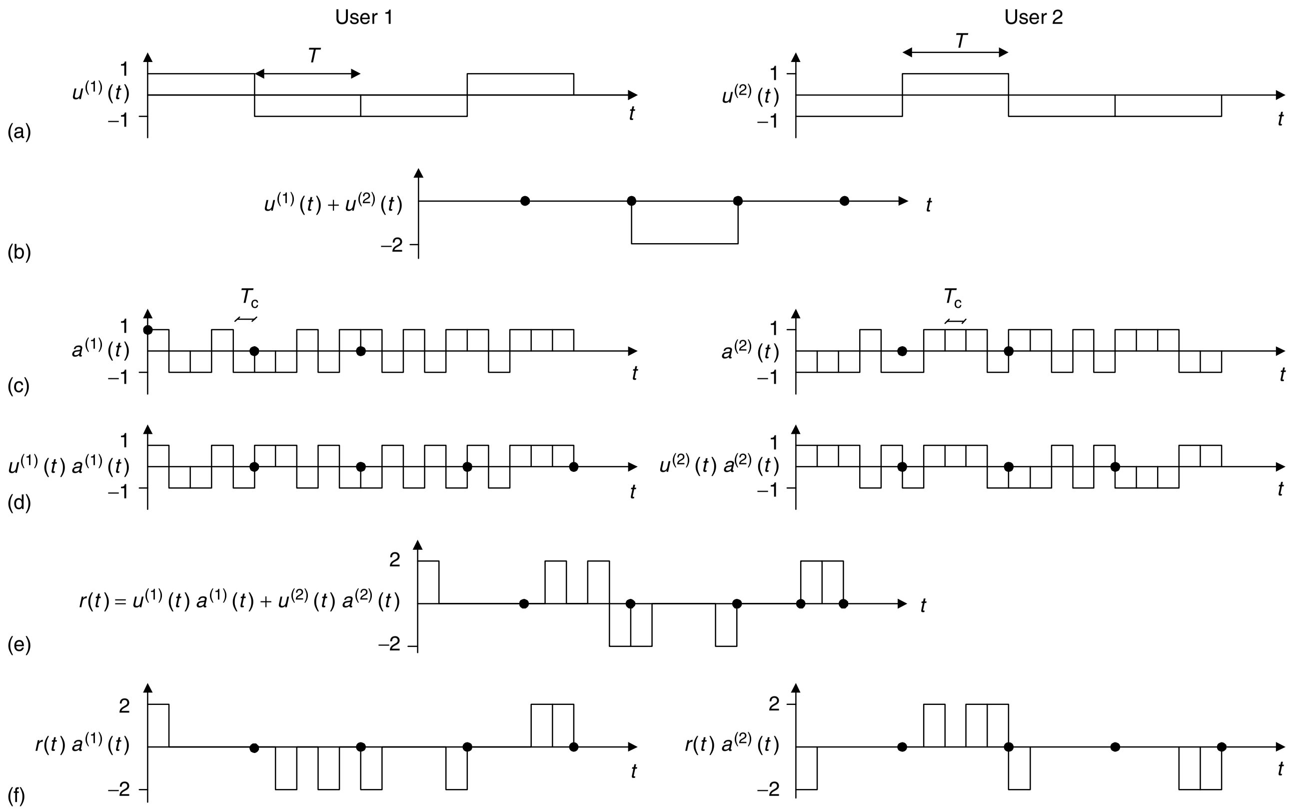

Figure 1.6. Example of the transmission over an adding channel, synchronous case.

If the *propagation delay* and the *attenuation* in the channel for both signals are the same, the output of the adding channel, that is, the input of the base station receiver, is the sum of identically attenuated transmitted signals. In our example the received signal is nonzero only in the third interval (Fig. 1.6b). Then the receiver cannot decide which pulses were sent by the users in the first, second, and fourth intervals, but it knows that in the third interval both of the users have sent negative pulses, and correspondingly $u_2^{(1)} = -1$, $u_2^{(2)} = -1$.

Suppose now that instead of sending the data signals $u^{(1)}(t)$ and $u^{(2)}(t)$ directly over the multiple access adding channel, the users first *spread* them, that is, multiply them by the *spreading signals* $a^{(1)}(t)$ and $a^{(2)}(t)$, respectively. The signals $a^{(1)}(t)$ and $a^{(2)}(t)$, presented in Figure 1.6c, are sequences of positive and negative unit amplitude rectangular pulses of duration T_c, $T_\mathrm{c} < T$ (in our example $T_\mathrm{c} = T/4$). These pulses are called *chips*, and T_c is called the *chip duration*. We will always consider the case when the ratio $T/T_\mathrm{c} = N$ is an integer. The spread signals $u^{(1)}(t) \cdot a^{(1)}(t)$ and $u^{(2)}(t) \cdot a^{(2)}(t)$ (Fig. 1.6d) are sent over the adding channel. The received signal $r(t) = u^{(1)}(t) \cdot a^{(1)}(t) + u^{(2)}(t) \cdot a^{(2)}(t)$ is presented in Figure 1.6e.

As we will see in Chapter 2, the bandwidth of the signal formed by the sequence of positive and negative pulses of duration T is proportional to $1/T$. Therefore, the bandwidth of the signals $u^{(k)}(t)$, $k = 1, 2$, is proportional to the transmission rate R and the bandwidth W of the spread signals is proportional to $1/T_\mathrm{c}$. The ratio $T/T_\mathrm{c} \approx W/R$ that characterizes the increase of the bandwidth by spreading is called the *spreading factor* or *processing gain*.

The base station receiver *despreads* the received signal $r(t)$, that is, multiplies $r(t)$ by the spreading signals $a^{(1)}(t)$ and $a^{(2)}(t)$. The results of despreading are given in Figure 1.6f. It is obvious that the receiver can correctly decide which data sequences were transmitted by the users in each of the four intervals.

The spreading signal $a^{(k)}(t)$, $k = 1, 2$, can be generated by mapping the *spreading sequences* $\boldsymbol{a}^{(k)} = a_0^{(k)}, a_1^{(k)}, \ldots a_n^{(k)}, \ldots, a_n^{(k)} \in \{1, -1\}$ into sequences of positive and negative pulses, analogous to mapping the data sequence $\boldsymbol{u}^{(k)}$, $k = 1, 2$, into the data signal $u^{(k)}(t)$. Suppose now that we repeat each symbol $u_n^{(k)}$ of the data sequence $\boldsymbol{u}^{(k)}$ N times, $N = T/T_\mathrm{c} = W/R$, to get a sequence $\boldsymbol{v}^{(k)} = v_0^{(k)}, v_1^{(k)}, \ldots v_n^{(k)} \ldots$ where $v_n^{(k)} = u_{\lfloor n/N \rfloor}^{(k)}$. Here $\lfloor x \rfloor$ means the largest integer that is less or equal to x. (For example, in Fig. 1.6 we have $N = 4$.) Then we multiply symbols of the sequence $\boldsymbol{v}^{(k)}$ by symbols of the sequence $\boldsymbol{a}^{(k)}$. We get the sequence

$$\boldsymbol{v}^{(k)} * \boldsymbol{a}^{(k)} \overset{\mathrm{def}}{=} v_0^{(k)} a_0^{(k)}, v_1^{(k)} a_1^{(k)}, \ldots, v_n^{(k)} a_n^{(k)}, \ldots \tag{1.1}$$

If we map the symbols of the sequence $\boldsymbol{v}^{(k)} * \boldsymbol{a}^{(k)}$ into a sequence of positive and negative pulses, as we did before, we get the spread signals $u^{(k)}(t) \cdot a^{(k)}(t)$, $k = 1, 2$. This is an alternative way of spreading.

The operation of repeating the symbol $u_n^{(k)} N$ times can be considered as *encoding*. The code is called the *repetition code*[3]; it consists of two *codewords*: N is the *block length* and $r = 1/N$ (bit/symbol) is the *code rate*. In the general, we will consider more complicated code constructions. Obviously, for rectangular pulses the operations of mapping sequences into signals and multiplication of signals/sequences are permutable, but for nonrectangular pulses these operations are, generally speaking, not permutable. Below we will consider both ways of generating spread signals.

Figure 1.6 corresponds to the *synchronous model* of the transmission, when the received signals from both transmitters are in the same phase. But the situation would not differ significantly in the *asynchronous case* (Fig. 1.7), when the received signals are in different phases. Using the same procedure of despreading as in the synchronous case, the receiver can even more easily recover both transmitted sequences $\boldsymbol{u}^{(1)}$ and $\boldsymbol{u}^{(2)}$. The necessary condition of the correct despreading is the knowledge of the phases of both transmitted signals $u^{(1)}(t) \cdot a^{(1)}(t)$ and $u^{(2)}(t) \cdot a^{(2)}(t)$. In other words, although the transmitters of the different users can be unsynchronized, the transmitter and the receiver corresponding to a particular user should be *synchronized*.

In general, we do not have two but K simultaneous active users and they operate asynchronously. A realistic model of the received signal should also include additive white Gaussian noise (AWGN) $\xi(t)$. The received (baseband) signal is

$$r(t) = \sum_{k=1}^{K} \sqrt{P^{(k)}} u^{(k)}(t - \delta^{(k)}) a^{(k)}(t - \delta^{(k)}) + \xi(t) \tag{1.2}$$

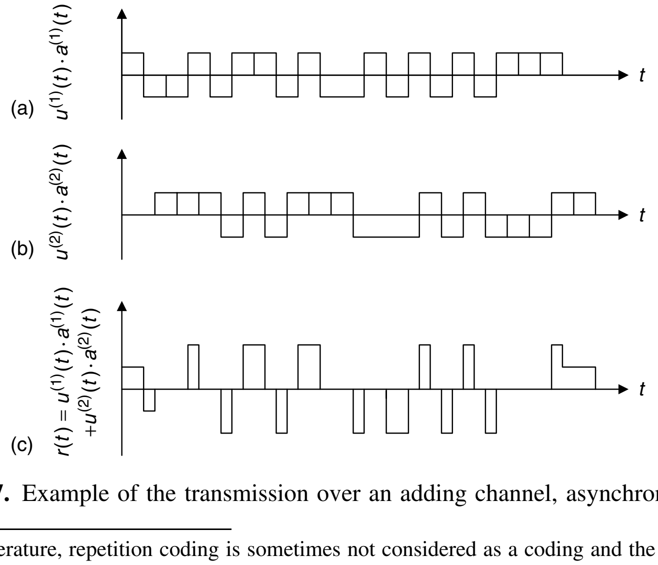

Figure 1.7. Example of the transmission over an adding channel, asynchronous case.

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[3] In the literature, repetition coding is sometimes not considered as a coding and the transmission is called *uncoded transmission*.

where $P^{(k)}$ is the power of the signal from the kth user at the base station and $\delta^{(k)}$ is the kth user's *time offset*. The time offset values $\delta^{(k)}$ characterize asynchronism between different users, propagation delay, etc. If we are interested in the reception of the information from the kth user, we will present the received signal (1.2) as

$$r(t) = \sqrt{P^{(k)}}u^{(k)}(t - \delta^{(k)})a^{(k)}(t - \delta^{(k)}) + \xi^{(k)}(t) \tag{1.3}$$

where the total noise

$$\xi^{(k)}(t) = \sum_{k' \neq k} \sqrt{P^{(k')}}u^{(k')}(t - \delta^{(k')})a^{(k')}(t - \delta^{(k')}) + \xi(t) \tag{1.4}$$

includes the interference from the $(K - 1)$ other active users and additive noise. If the receiver is synchronized with the kth user, that is, $\delta^{(k)}$ is known, the despreading of the signal, that is, multiplication by $a^{(k)}(t - \delta^{(k)})$, reduces the problem in the case of repetition coding to detection of the known signal in noise (see Chapter 3) or, in the case of more complicated codes, to the decoding problem (see Chapter 4).

We emphasize that the model of uplink communication in the DS CDMA system considered here is the *information-theoretic* model. The model that is studied in *communication theory* describes processes in the transmitter-receiver, particularly the processes of *modulation-demodulation*, in more detail.

The receiver for binary DS CDMA signaling schemes can have one of two equivalently performing structures, a correlator implementation and a matched-filter implementation (see Chapters 2 and 3). The correlator receiver performs a correlation operation with all possible signals sampling at the end of each T-second signaling interval and comparing the outputs of the correlators. In the matched-filter receiver, correlators are replaced by matched filters.

The model of uplink DS CDMA communication with K users is presented in Figures 1.8 and 1.9. The base station receiver includes K demodulators synchronized with the modulators of the K transmitters. Assuming perfect synchronization, the output of the kth demodulator, $k = 1, 2, \dots K$, is the sequence $\{v_n^{(k)} a_n^{(k)} + \xi_n^{(k)}\}$, where the noise components $\xi_n^{(k)}$ are contributions of all other active users and AWGN. Despreading consists of multiplication by the spreading sequence $\{a_n^{(k)}\}$. The input of the kth decoder is the sequence

$$\{v_n^{(k)} + \xi_n^{(k)} a_n^{(k)}\} \tag{1.5}$$

The output of the kth decoder is the decoded information sequence $\{\hat{u}_n^{(k)}\}$.

Using power control, the switching center can adjust the powers of transmitted signals such that the powers of the received signals would be approximately the same. If the power control is perfect the power of the received signal equals P independently from the user, that is, $P^{(k)} = P$, $k = 1, 2, \dots K$. Each receiver at the base station of a single-cell communication system receives a

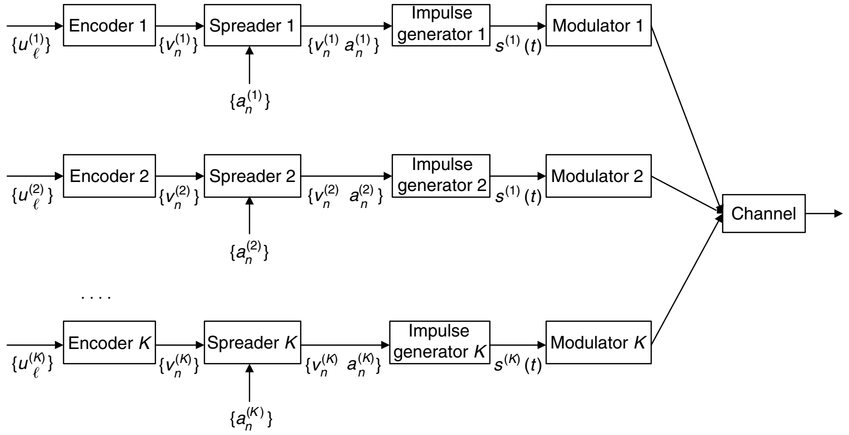

Figure 1.8. The model of uplink transmission in the DS CDMA system.

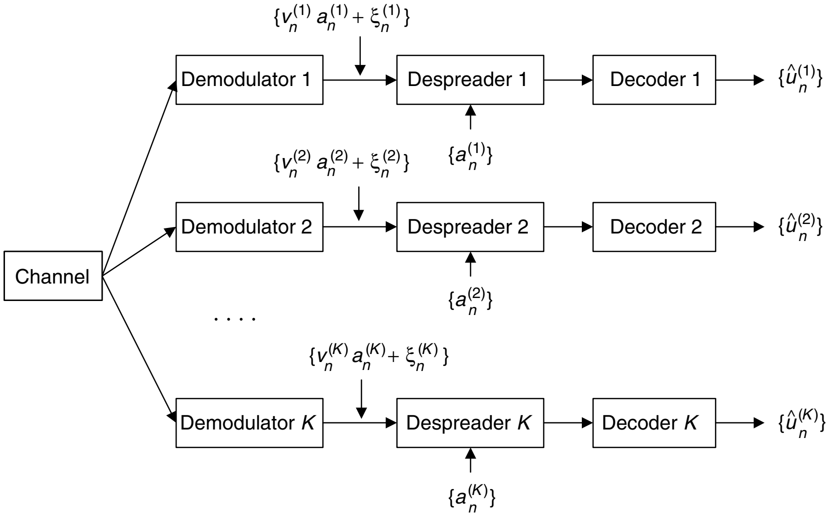

Figure 1.9. The model of the base station receiver of the DS CDMA system.

composite waveform containing the desired signal of power P, the component due to background AWGN $\xi(t)$, and the other-user interference component of power $P(K-1)$. Then the average one-sided total noise power spectral density[4] becomes

$$I_0 = (K-1)\,\frac{P}{W} + N_0 \tag{1.6}$$

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[4] In this book we will later use only two-sided power spectral density, which for modulated signals is equal to half of the one-sided power spectral density.

where N_0 is the one-sided power spectral density of the AWGN and W is the signal bandwidth.

As we will see later, the important parameter that is the figure merit of the digital modem is *bit energy-to-noise density ratio* (for brevity we will call this parameter *signal-to-noise ratio, SNR*)

$$\frac{E_b}{I_0} = \frac{P}{I_0 R}, \tag{1.7}$$

where $E_b = P/R$ is the received energy per bit. Combining (1.6) and (1.7) we get

$$\frac{E_b}{I_0} = \frac{P/R}{(K-1)\dfrac{P}{W} + N_0} = \frac{W/R}{(K-1) + \dfrac{N_0 W}{P}} \tag{1.8}$$

From (1.8) follows the next formula for the radio channel capacity K_0 of a single-cell CDMA system:

$$K_0 = 1 + \frac{W/R}{E_b/I_0} - \frac{N_0 W}{P} = 1 + \frac{W/R}{E_b/I_0} - \frac{W/R}{E_b/N_0} \tag{1.9}$$

or, because we usually can neglect the influence of the background AWGN,

$$K_0 \approx 1 + \frac{W/R}{E_b/I_0} \tag{1.10}$$

The ratio W/R (in Hz/bit/s) was defined above as the *spreading factor* or the *processing gain*. Typical values of W/R range from one hundred (20 dB) to one million (60 dB). The required signal-to-noise ratio depends on the type of error-correcting coding used, the type of noise, and the limitations on the output probability of error. Under the condition that the number of active users K is large, we may consider the total noise as Gaussian noise of one-sided power spectral density I_0. Then, if the trivial repetition code is used, the bit error probability is the same as for uncoded transmission, that is,

$$P_b = Q\left(\sqrt{\frac{2E_b}{I_0}}\right) \tag{1.11}$$

where the Q function defined by the integral

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-y^2/2)\,\mathrm{d}y \tag{1.12}$$

can be upperbounded by the inequality (Problem 1.5)

$$Q(x) \leq \frac{1}{2}\exp(-x^2/2), \ \ x \geq 0 \tag{1.13}$$

If the repetition code is used, we get from Formulas (1.10)–(1.13)

$$K_0 \approx 1 - \frac{W/R}{\ln 2P_\mathrm{b}} \tag{1.14}$$

For the voice channel the required bit error rate is in the range 10^{-3}–10^{-4}, and the required signal-to-noise ratio E_b/I_0 is in the interval 4–8 dB, depending on the error correction code.

EXAMPLE 1.3

If the repetition code is used in the communication system and the required bit error rate is 10^{-4}*, what is the required signal-to-noise ratio? What is the required* E_b/I_0 *if* $P_\mathrm{b} = 10^{-5}$*?*

Solution

Using Formula (1.11) we get that to $P_\mathrm{b} = 10^{-4}$ *corresponds* $E_\mathrm{b}/I_0 = 6.92$ *(*8.40 dB*) and to* $P_\mathrm{b} = 10^{-5}$ *corresponds* $E_\mathrm{b}/I_0 = 9.09$ (9.59 dB). *If we use the upper bound (1.13) we get that if* $E_\mathrm{b}/I_0 = 6.12$*, then* $P_\mathrm{b} < 4.96 \cdot 10^{-4}$*, and if* $E_\mathrm{b}/I_0 = 9.09$*, then* $P_\mathrm{b} < 5.6 \cdot 10^{-5}$.

Suppose further that two more processing features are added to the CDMA system to diminish interference. The first involves the monitoring of users' activity such that each transmitter is switched off or reduces its power during the periods of no user activity. In a two-way telephone conversation, the activity of each of the speakers is $1/\gamma_\mathrm{v}$, where $\gamma_\mathrm{v} \approx 8/3 \approx 2.67$; therefore, we can, in principle, reduce the interference noise in Formulas (1.6)–(1.10) by the factor γ_v. The parameter γ_v is called the *voice activity gain*.

Similarly, if we assume that the population of mobiles is uniformly distributed in the area of the single isolated cell, employing a sectored antenna reduces the interference noise by the *antenna gain* γ_a. For a three-sectored antenna, this gain is less than 3 and can be estimated as $\gamma_\mathrm{a} \approx 2.4$.

To calculate the capacity of the entire CDMA system, not only of a single isolated cell, we have to include in I_0 the one-sided power spectral density N_oc of the other-cell interference noise. Let us suppose that the frequency reuse factor of the CDMA system is equal to 1, that is, all users in all cells employ the common spectral allocation of W Hz. It was shown previously [47] that the total interference from the users in all the other cells equals approximately 0.6 of that caused by all the users in the given cell *(other-cell relative interference factor* f *is* equal to 0.6*)*, that is, $N_\mathrm{oc} = 0.6(K-1)P/W$. Thus, in consideration of the total system capacity, the interference term of I_0 should be increased by the factor 1.6. Finally, introducing the voice activity and antenna gain factors,

γ_{v} and γ_{a}, and the other-cell relative interference factor, f, into the total noise power spectral density expression yields

$$I_0 = (K-1)\frac{P}{W}\,\frac{1+f}{\gamma_{\mathrm{v}}\gamma_{\mathrm{a}}} + N_0 \tag{1.15}$$

Thus, analogously to Formula (1.9), we get the following expression for the total radio channel capacity of the CDMA system [47]:

$$K_0 = 1 + \frac{W/R}{E_{\mathrm{b}}/I_0}\,\frac{\gamma_{\mathrm{v}}\gamma_{\mathrm{a}}}{1+f} - \frac{N_0 W}{P}\,\frac{\gamma_{\mathrm{v}}\gamma_{\mathrm{a}}}{1+f} = 1 + \frac{W/R}{E_{\mathrm{b}}/I_0}\,\frac{\gamma_{\mathrm{v}}\gamma_{\mathrm{a}}}{1+f} - \frac{W/R}{E_{\mathrm{b}}/N_0}\,\frac{\gamma_{\mathrm{v}}\gamma_{\mathrm{a}}}{1+f} \tag{1.16}$$

or, if the AWGN is negligible,

$$K_0 \approx 1 + \frac{W/R}{E_{\mathrm{b}}/I_0}\,\frac{\gamma_{\mathrm{v}}\gamma_{\mathrm{a}}}{1+f} \tag{1.17}$$

EXAMPLE 1.4 [47]

Consider a cellular system with voice activity gain $\gamma_{\mathrm{v}} = 2.67$*, antenna gain* $\gamma_{\mathrm{a}} = 2.4$*, required signal-to-noise ratio* $E_{\mathrm{b}}/I_0 = 4$ *(6 dB), and other-cell relative interference factor* $f = 0.6$*. What is the radio channel capacity of this system?*

Solution

Using Formula (1.17), we get

$$K_0 \approx W/R$$

The radio channel capacity is approximately equal to the spreading factor.

In Example 1.1 and Example 1.2 we mentioned two standards, AMPS and GSM. They standardize non-CDMA systems. The first DS CDMA system standardized as Interim Standard 95 (IS-95) [44] was adopted in 1993. IS-95 is specified for uplink operation in 824–849 MHz and for downlink in 869–894 MHz.

EXAMPLE 1.5

Each channel of the CDMA system IS-95 occupies 1.25 MHz of the spectrum on each one-way link. Bands of 25 MHz are available in each direction. The maximum user rate is $R = 9.6$ *kb/s. If a minimum acceptable* E_{b}/I_0 *is 6 dB, determine the capacity of a CDMA system using*

a) *Omnidirectional base station antennas and no voice activity detection and*
b) *Three-sectored antennas at the base station with* $\gamma_{\mathrm{a}} = 2.4$ *and voice activity detection with* $\gamma_{\mathrm{v}} = 2.67$

The received signal power P *is* 10^{-11} *W, the one-sided AWGN power spectral density* $N_0 = 10^{-17}$ *W/Hz, and the other-cell relative interference factor* $f = 0.6$*.*

Solution

From Formula (1.17) we have for each channel

a) $K_0 = 1 + \dfrac{1.25 \cdot 10^6/9.6 \cdot 10^3}{4 \cdot 1.6} - \dfrac{10^{-17} \cdot 1.25 \cdot 10^6}{1.6 \cdot 10^{-11}} \approx$
$1 + 18.8 - 0.8 = 19$

b) $K_0 \approx 1 + 18.8 \cdot 2.4 \cdot 2.67 - 0.8 \cdot 2.4 \cdot 2.67 =$
$1 + 120.3 - 5.1 \approx 115$

Because the system has $25/1.25 = 20$ *channels in each link, the total capacity is equal to* 380 *in the first case and* 2300 *in the second case.*

Our last example of this section concerns the third-generation (3G) mobile communication systems, based on wideband CDMA (WCDMA) [55]. For WCDMA there are available bands 1920–1980 MHz in reverse direction and 2110–2170 MHz in forward direction, that is, 60 MHz in each direction. The speech codec in WCDMA employs the Adaptive Multi-Rate (AMR) technique standardized in 1999. It has eight source rates, from 4.75 kb/s up to 12.2 kb/s.

EXAMPLE 1.6

Each channel of the WCDMA system occupies 5 *MHz of the spectrum on each link. Assume that the user rate* 12.2 *kb/s. The other parameters are the same as in Example 1.5. Find the capacity of the WCDMA system under the given conditions.*

Solution

From Formula (1.17) we get for each channel

$$\begin{aligned} K_0 &= 1 + \frac{5 \cdot 10^6/12.2 \cdot 10^3}{4 \cdot 1.6} \cdot 2.4 \cdot 2.6 - \frac{10^{-17} \cdot 5 \cdot 10^6}{1.6 \cdot 10^{-11}} \cdot 2.4 \cdot 2.6 \\ &= 1 + 409 - 20 = 390 \end{aligned}$$

Under the given conditions, the total capacity of the WCDMA system equals $60 \cdot 390/5 = 4680$.

The mathematical model of the CDMA system considered above is a model of many-to-one transmission. Strictly speaking, it describes only reverse link transmission. In Chapter 3 we show that Formula (1.17) for the radio channel capacity is valid also for forward link. The forward link transmission that is one-to-many transmission has some advantages in comparison to many-to-one transmission. First, the signals transmitted to different users can be synchronized and accommodated by a pilot signal, such that the users can use coherent receivers. For the reverse link, a pilot signal is not always used because of power limitations. Second, because the transmitter knows the transmitted information sequences of all the users, it can in principle use this information in the encoding process, and improve the performance of the overall system. In this case we can talk about *coordinated transmission* or *partially coordinated transmission*. We consider this problem in Chapter 9.

In the DS CDMA system each of the active users occupies in each time instance all wideband channels. In the next section we consider a system in which the wideband channel is divided into narrow frequency bands. Each of the active users occupies in each time instance only one band and periodically changes this band.

1.4 FREQUENCY-HOPPED CDMA

Conventional frequency-hopped (FH) CDMA is a digital multiple access system in which individual users select one of Q frequencies within a wideband channel as carrier frequency. The pseudorandom changes of the carrier frequencies randomize the occupancy of a specific band at any given time, thereby allowing for multiple access over a wide range of frequencies. In a conventional FH CDMA system, the total hopping bandwidth W is divided into Q narrow bands each of bandwidth B, where $B = W/Q$. Each of the Q bands is defined as a spectral region with a central frequency called the *carrier frequency*. The set of possible carrier frequencies is called the *hopset*. The bandwidth B of a band used in a hopset is called the *instantaneous bandwidth*. The bandwidth of the spectrum W over which the hopping occurs is called the *total hopping bandwidth*. Information is sent by hopping the carrier frequency according to the pseudorandom law, which is known to the desired receiver. In each hop, a small set of code symbols is sent with conventional narrowband modulation before the carrier frequency hops again. The time duration between hops is called *hop duration* or *hopping period* and is denoted by T_{c}. The time duration between transmission of two consecutive symbols is T.

Usually in FH CDMA frequency shift-keying (FSK) is used. If in FH CDMA system q-FSK is used, then each of the Q bands is divided into q subbands and during each hop one or several of the central frequencies of the subbands within the band can be sent. We will also call each frequency subband the *transmission channel*. We denote the total number Qq of transmission channels by M. If binary FSK (BFSK) is used, $M = 2Q$ and the pair of possible instantaneous frequencies changes with each hop.

At the receiver side, after the frequency hopping has been removed from the received signal, the resulting signal is said to be *dehopped*. Before demodulation, the dehopped signal is applied to a conventional receiver. If another user transmits in the same band at the same time in a FH CDMA system, a *collision* can occur.

Frequency hopping can be classified as slow or fast. *Slow frequency hopping* occurs if one or more q-ary symbols are transmitted in the interval between frequency hops. Thus slow frequency hopping implies that the symbol rate $1/T$ exceeds the hopping rate $1/T_{\mathrm{c}}$. *Fast frequency hopping* occurs if there is more than one frequency hop during one symbol transmission time. If other users occupied the same frequency band in the same time, the probability of incorrect transmission of the corresponding information symbols would become high. Therefore, it is advisable to combine frequency hopping with *interleaving* and *coding*.

Figure 1.10a illustrates slow frequency hopping if FSK is used in the system and $Q = 4$, $q = 4$, and $M = 16$. In this figure the instantaneous frequency subbands (transmission channels) are shown as a function of the time. The 4-ary symbol transmission time T is equal to $T_c/3$, where T_c is the hop duration. Two bits are collected each T second, and one of four frequencies is generated by the modulator. This frequency is translated to one of $Q = 4$ frequency hop bands by the FH modulator. In this example, a frequency hop occurs after each group of 3 symbols or when 6 bits have been transmitted. The dehopped signal is shown in Figure 1.10b.

A representation of a transmitted signal for a fast frequency-hopped system is illustrated in Figure 1.11. The output of the data modulator is one of the tones as before, but now time T of the transmission of one group of 2 bits is subdivided into $T/T_c = 4$ chips (hops). In this example, each pair of bits is transmitted during 4 carrier frequency hops.

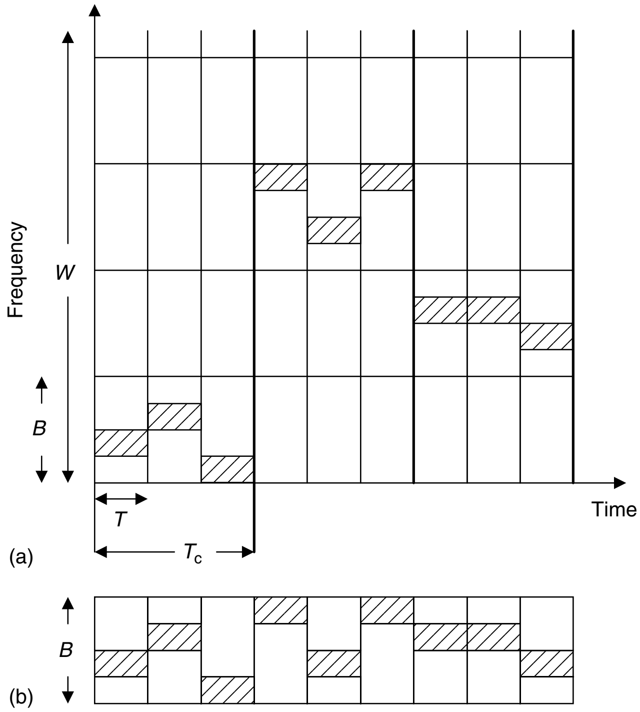


Figure 1.10. Illustration of FSK slow-frequency-hopped spread spectrum system. (a) transmitted signal; (b) dehopped signal. (4-FSK modulation)

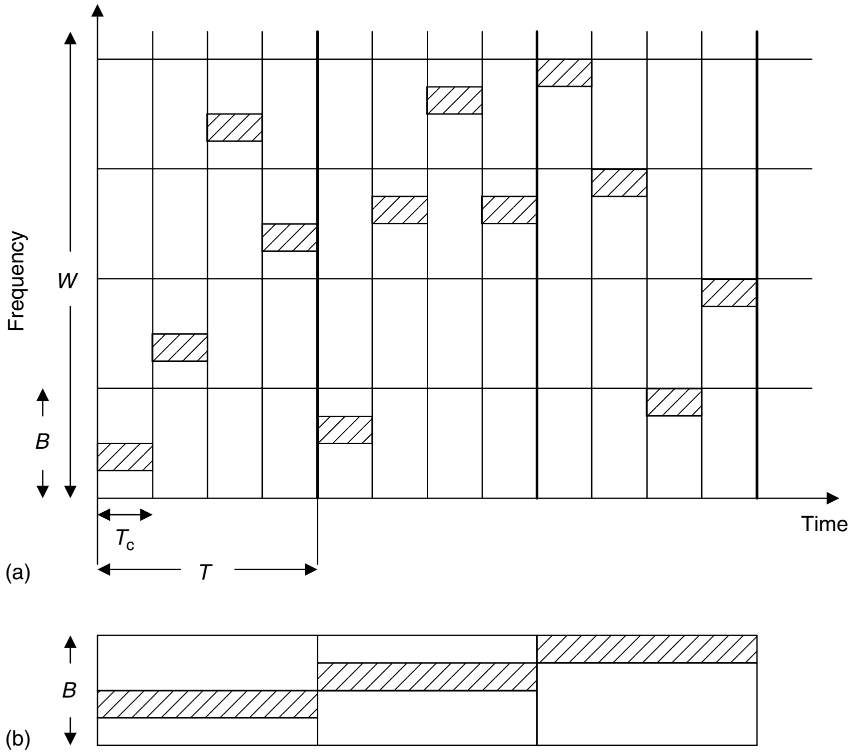


Figure 1.11. Illustration of FSK fast-frequency-hopped spread spectrum system (a) transmitted signal; (b) receiver down-converter output.

In this book, we will consider a scheme slightly different from the conventional scheme of FH CDMA. We will not distinguish between frequency hopping caused by changing of the carrier frequency and frequency hopping caused by transmission of a new symbol. In other words, we will consider any instantaneous frequency (transmission channel) change to be a *hop*. Correspondently, we modify the terminology and call the set of all M possible instantaneous frequencies as a *hopset*. The value M, the ratio of the total hopping bandwidth to the instantaneous bandwidth, is called the *hopset size*. The *hop duration* T_c is defined as the time interval between two consecutive instantaneous frequency changes. Then for the slow frequency hopping scheme in Figure 1.10, the hop duration is equal to one symbol transmission time T and will be labeled as T_c. The time interval between two consecutive carrier frequency changes should be omitted. For both frequency hopping schemes in Figures 1.10 and 1.11, the instantaneous bandwidth should be decreased four times. This modification of the FH CDMA scheme is quite natural, because a modern digital FH CDMA system uses coding and the information bit rate is, as a rule, lower than the hopping rate.

In contrast to a DS CDMA signaling scheme, which uses matched-filter or correlator receivers, we assume that a FH CDMA system uses a *radiometer* as the receiver. A radiometer detects energy received in an instantaneous frequency band by filtering to this bandwidth, squaring the output of the filter, integrating the output of the squarer for time T_c, and comparing the output of the integrator with a threshold. If the integrator output is above a present threshold, the signal is declared present in this instantaneous frequency band; otherwise, the signal is declared absent. Let us assume that there is no additive noise in the channel and that the users are chip synchronized, that is, frequency hops of the received signal occur in the moments nT_c, $n = \ldots 0, 1, 2, \ldots$. Then we may choose zero threshold and the radiometer declares the presence of the signal in the instantaneous band if and only if one OR more users occupy this band. Such a receiver is also called an *OR receiver*.

To explain the mechanism of FH CDMA we consider a simple example. Suppose that the number of users is $K = 2$ and BFSK with an OR receiver is used. The total bandwidth W is divided into two subbands (transmission channels), left and right, $M = q = 2$, and the transmission time is divided into time slots of duration T_c. Each of the users occupies the left channel if it would like to transmit 1 in a given time slot and the right channel if it would like to transmit -1 (Fig. 1.12a). The users are chip synchronized (synchronous reception). If we apply the TDMA principle (see Section 1.2), we assign, for example, even time slots to user 1 and odd time slots to user 2. This gives the overall transmission rate 1 (bits/time slot) conditioned that both of the users are active all the time and the information symbols are equiprobable. But if both users are active only 40% of the time, the average overall transmission rate is only 0.4 (bits/time slot).

Now suppose that both of the users may occupy all time slots (Fig. 1.12b). If either the first user or the second one or both of them transmit in the given subband the radiometer detects this event. If both of the users transmit the

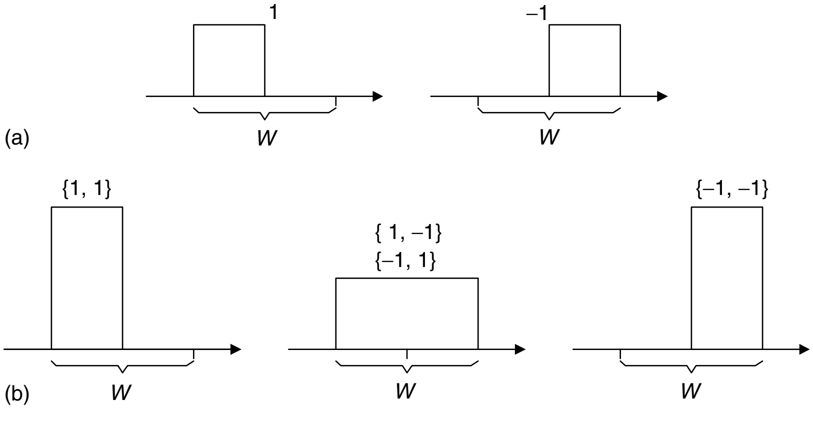


Figure 1.12. Illustration of FH: (a) binary FSK transmission; (b) FH CDMA transmission with two users.

same symbol, energy would be detected only in one of the subbands and the receiver determines which symbol was transmitted by the users. If the symbols are equiprobable we can say that the receiver gets 2 bits of information. If the users transmit different symbols, the radiometer detects energy in both subbands and can not decide which symbol was transmitted. The receiver gets no information. Conditioned that at least one of the users is active, the average transmission rate is still 1 (bits/time slot). If both of the users are active 40% of the time, the average transmission rate is 0.64 (bits/time slot), which is essentially higher than when the TDMA system is used.

Although the average transmission rate in this case is higher than in the time division case, parts of the transmitted symbols vanish. These symbols can be reconstructed if the system uses coding. The following, more complicated example shows how we can do this.

EXAMPLE 1.7

Consider the synchronous FH CDMA system using BFSK with K users, which are active all the time. The hopset size is M, $q = 2$, $Q = M/2$, *and the probability that a user chooses a particular band is equal to* $1/Q$. *Each user transmits one bit of information in N time slots using rate* $r = 1/N$ *block length N repetition code and an OR receiver. If the user transmits a* 1 *it occupies the left subband of the band, and if it transmits a* -1 *it occupies the right subband. Assume that the symbols* 1 *and* -1 *are equiprobable. Consider the case* $K = 100$, $N = 98$, $M = 36$. *What is the overall transmission rate* r_{overall} *(in bits per time slot)? What is the bit error probability* P_{b}*?*

Solution

Because each active user transmits one bit in N time slots, the individual user transmission rate is $1/N$ *(bits/time slot), and the overall transmission rate is* $r_{\text{overall}} = K/N$ *(bits/time slot). In our case* $r_{\text{overall}} = 1.02$.

Now we estimate the error probability. Let the first user be the reference user and the other users be jammers. Suppose that the first user transmits the symbol 1. *Then the radiometer always detects energy in the left subband of the band in which the first user transmits in the nth time slot. The receiver makes a correct decision, if at least for one n,* $n = 0, 1, \ldots, N-1$, *the right subband of this band would not be occupied by one of the* $(K-1)$ *jammers. The probability of this event for the nth subband,* $n = 0, 1, \ldots, N-1$, *is*

$$\pi = \left(1 - \frac{1}{M}\right)^{K-1}$$

and the probability of the complementary event is

$$1 - \pi = 1 - \left(1 - \frac{1}{M}\right)^{K-1}$$

Then the probability that the receiver cannot make a single decision on the information bit sent by the first user is $(1-\pi)^N$. *Assuming that in the case of a tie the receiver makes a random decision, we have*

$$P_b = \frac{1}{2}\left[1-\left(1-\frac{1}{M}\right)^{K-1}\right]^N$$

This probability does not depend on which user is the reference user and on which information symbol was sent. For $N = 98$, $K = 100$, *and* $M = 36$ *we have* $P_b = 10^{-3}$.

For $M \gg 1$,

$$P_b \approx \frac{1}{2}\left[1-\left(1-\frac{1}{M}\right)^{M\cdot\frac{K-1}{M}}\right]^N \approx \frac{1}{2}\left(1-e^{-\frac{K-1}{M}}\right)^N \tag{1.18}$$

In Section 1.3 we defined the processing gain of a DS CDMA system as the bandwidth expansion factor, or equivalently, the number of chips per information bit. In the general case the processing gain represents the advantage gained over the jammer that is obtained by expanding the bandwidth of the transmitted signal. The users' transmission rate R of the FH CDMA system of Example 1.7 is equal to $1/NT_c$. If the system did not use the spread spectrum technique, it would occupy a bandwidth of order $1/NT_c$. Because the FH CDMA system occupies a bandwidth W of order M/T_c, we can estimate the FH CDMA system processing gain as $W/R = MN$.

EXAMPLE 1.8

Under the same condition as in Example 1.7, find the maximal number of active users in the FH CDMA system if the number of active users $K = \lambda M + 1$, *where* λ *is a positive factor,* $M \gg 1$, *and* P_b *is given. Assume that* $T_c = 2/B = M/W$. *Find* λ *that maximizes* K.

Solution

From Formula (1.18) it follows that for $M \gg 1$

$$(1-e^{-\lambda})^N = 2P_b$$

or

$$N\ln(1-e^{-\lambda}) = \ln(2P_b) \tag{1.19}$$

But

$$N = \frac{T}{T_c} = \frac{W}{MR} = \frac{W\lambda}{(K-1)R} \tag{1.20}$$

From Formulas (1.19) and (1.20) we have

$$K = 1 + \frac{W/R}{\ln(2P_\mathrm{b})}\,\lambda \ln(1 - e^{-\lambda}) \tag{1.21}$$

The maximum of the right side is at $\lambda = \ln 2$, *and for this* λ *we get the number of active users (radio channel capacity)*

$$K_0 = 1 - \frac{W/R}{\ln(2P_\mathrm{b})}\,(\ln 2)^2 \approx 1 - \frac{W/R}{\ln(2P_\mathrm{b})}\,0.48 \tag{1.22}$$

Analogously to the DS CDMA case the radio channel capacity of the FH CDMA system is proportional to the spreading factor W/R.

The number of active users in the FH CDMA system given by (1.22) is about two times less than the radio channel capacity in the DS CDMA system given by (1.14), that is, the efficiency of the FH CDMA is about half of that of DS CDMA. We note that Formula (1.22) is derived under very idealized assumptions (absence of additive noise), and the real capacity of FH CDMA system with the OR receiver is even less. Even if we used in FH a noncoherent receiver (see Chapter 3) instead of the OR receiver, the capacity of the FH CDMA system would be still less than the capacity of the analogous DS CDMA system.

However, FH CDMA systems have an important advantage in comparison with DS CDMA systems. In the DS CDMA system, the signals have a large instantaneous bandwidth. The complexity and cost of the transmitter increases as the instantaneous bandwidth of the signal grows. In an FH CDMA system this bandwidth is much smaller and the complexity of the equipment can be smaller. Thus, although the efficiency of DS CDMA is higher than that of FH CDMA, this advantage is overshadowed by the greater band spreading achievable with FH technology.

FH CDMA can be considered as a counterpart to FDMA. In the next section we consider a counterpart to TDMA, "time-hopped" or *pulse position-hopped CDMA*.

1.5 PULSE POSITION-HOPPED CDMA

Digital radio transmission has traditionally been based on the concept that the carrier frequency is much larger than the bandwidth of the transmitted signal. When the required bandwidth is of the order of 100 MHz this approach encounters many obstacles. Typically, the transmitter would operate at a carrier frequency above 10 GHz, and thus would suffer from absorption by rain and fog. A different technique is the *impulse radio*, multiple access modulated by a pulse position hopping (PPH). The impulse radio technique is also denoted *ultrawideband* transmission. Impulse radio communicates with pulses of very short duration, typically on the order of a nanosecond, thereby spreading the energy of the radio signal very thinly up to a few gigahertz. It is a promising technique for short-range and indoor communication.

The main advantages of impulse radio are as follows. In an impulse radio system, the transmitted signal is a dithered pulse train without a sinusoidal carrier and, hence, carrier recovery at the receiver is not required. As we mentioned above, in an ultrawideband system, such as impulse radio, fading is not nearly as serious a problem as it is for narrowband systems. Impulse radio systems can operate at variable bit rates by changing the number of pulses used to transmit one bit of information. We note that DS CDMA and FH CDMA use more complex bit rate variation techniques.

In a typical PPH format used by impulse radio the output signal of the kth, $k = 1, 2, \ldots, K$, transmitter is a sequence of monocycle waveforms $h_{T_c}(t - \tau_n^{(k)})$, $n = 0, 1, \ldots$, where t is the clock time of the transmitter, T_c is the pulse duration, and $\tau_n^{(k)}$ is the time delay. Because the pulses have electromagnetic origin, the average pulse amplitude is equal to zero.

In Figure 1.13, we present two examples of $h_{T_c}(t)$, which will be considered in a theoretical treatment. These are:

a) Manchester pulse:

$$h_{T_c}(t) = \begin{cases} -1, & -T_c/2 < t < 0 \\ 1, & 0 < t < T_c/2 \\ 0, & \text{otherwise} \end{cases} \tag{1.23}$$

b) differentiated Gaussian pulse:

$$h_{T_c}(t) = A_{dg} t \exp\left(-\frac{t^2}{4\gamma T_c^2}\right) \tag{1.24}$$

where A_{dg} and γ are normalizing constants (see Chapter 2), and T_c is pulse duration. In PPH CDMA applications we will only study Manchester pulses.

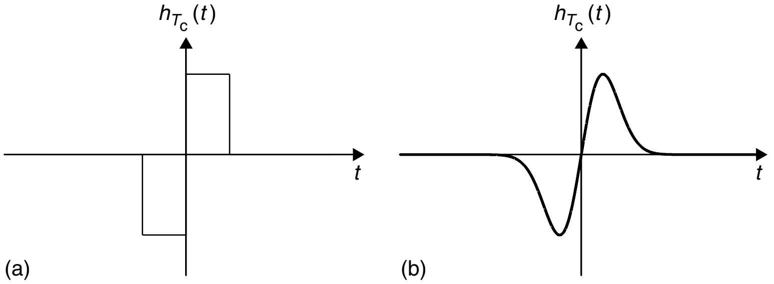


Figure 1.13. Examples of monocycle waveforms that model pulses in impulse radio: (a) Manchester pulse; (b) differentiated Gaussian pulse.

Consider again the uplink transmission. Suppose that the transmission time is divided into frames of duration T_f, $T_\text{f} = MT_\text{c}$, $M \gg 1$, M is even. The kth user, $k = 1, 2, \ldots, K$, transmits the nth bit $v_n^{(k)}$ of the code sequence $\boldsymbol{v}^{(k)} = v_0^{(k)}, v_1^{(k)}, \ldots, v_{N-1}^{(k)}$, where $v_n^{(k)} \in \{+1, -1\}$, in the nth frame. The users are synchronized such that signals that were sent by the users in the nth frame will be received by the receiver in the same time interval $(t, t + T_\text{f})$.

Each frame is divided into Q time slots, $Q = M/2$, of length $2T_\text{c}$, and each slot is divided into two subslots of length T_c (Fig. 1.14).

An individual user selects one of Q time slots within a frame to transmit the code symbol. Each slot can be choosen with probability $1/Q$. If the code symbol is 1, then the user transmits a pulse in the left subslot, if the code symbol is -1, then the user transmits a pulse in the right subslot.

The signal waveform transmitted by the kth user in the nth frame is

$$\tilde{s}_n^{(k)}(t) = h_{T_\text{c}}\left(t - nT_\text{f} - \tau_n^{(k)} + v_n^{(k)}\frac{T_\text{c}}{2}\right) \tag{1.25}$$

where $v_n^{(k)} \in \{+1, -1\}$ is the nth code symbol of the kth user, $\tau_n^{(k)} = 2a_n^{(k)}T_\text{c}$ is the *addressable pulse position shift*, and $a_n^{(k)}$ is an integer-valued random variable uniformly distributed on the set $\{0, 1, \ldots, Q - 1\}$. The ratio $T_\text{f}/T_\text{c} = M$ is the *hopset size*.

This is the time-domain analog of the FH CDMA transmission model considered in the previous section. As receiver we can use both the correlator receiver and the OR receiver. If the users are synchronized and use rate $R = 1/N$ block length N repetition codes with OR receivers then the bit error probability is [compare with Formula (1.18)]

$$P_\text{b} = \frac{1}{2}\left[1 - \left(1 - \frac{1}{M}\right)^{K-1}\right]^N \approx \frac{1}{2}\,(1 - e^{-\lambda})^N \tag{1.26}$$

where $\lambda = (K - 1)/M$. The radio channel capacity reaches its maximum for $\lambda = \ln 2$. The transmission rate R of this PPH CDMA system is equal to $1/NT_\text{f} = 1/NMT_\text{c}$. Because the transmission rate of the corresponding system not using

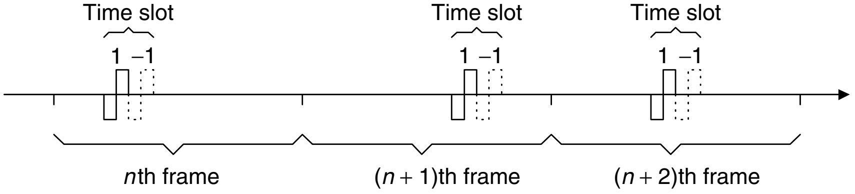


Figure 1.14. Illustration of PPH CDMA transmission.

the spread spectrum technique is $1/T_c$, we define the processing gain as NM, which equals $NT_f/T_c = W/2R$, where we defined the bandwidth[5] $W = 2/T_c$. Analogously to (1.22) we get the radio channel capacity of the PPH CDMA system with OR receiver

$$K_0 \approx 1 - 0.48\frac{W/2R}{\ln(2P_b)} \tag{1.27}$$

Note that if it were possible to use unimodal pulses of duration $T_c = 1/W$ in the PPH transmission, the radio channel capacity would be approximately two times larger.

As we mentioned above, the PPH CDMA signals can be processed by the correlator receiver. Consider again the same model of the PPH CDMA system, but now assuming that the users are not required to be synchronized and may use an arbitrary code rate r length N block code. For simplicity we suppose that the system has perfect power control, that is, the power $P^{(k)}$ of the received signal from the kth user equals P independently of the user. Then the received signal from the kth user is

$$\sum_{n=0}^{N-1} \sqrt{P}\, h_{T_c}\left[t - nT_f - \tau_n^{(k)} + v_n^{(k)}\frac{T_c}{2} - \delta^{(k)}\right] \tag{1.28}$$

where $\delta^{(k)}$ is the kth user's time offset, including propagation delay, asynchronism between the users, etc. The total received signal is

$$r(t) = \sum_{k=1}^{K}\sum_{n=0}^{N-1} \sqrt{P}\, h_{T_c}\left(t - nT_f - \tau_n^{(k)} + v_n\,\frac{T_c}{2} - \delta^{(k)}\right) \tag{1.29}$$

Each receiver at the base station receives a composite waveform containing the desired signal of power P, the component due to background AWGN of one-sided spectral density N_0, and the other user interference component. All the energy PT_c of the desired signal in the nth frame is concentrated in the time window of size $2T_c$,

$$nT_f + \tau_n^{(k)} - T_c + \delta^{(k)} \le t < nT_f + \tau_n^{(k)} + T_c + \delta^{(k)} \tag{1.30}$$

which is only $2T_c/T_f$ part of the frame. Therefore, the average contribution of each of the $(K-1)$ interfering users to the total noise energy is equal to $2PT_c^2/T_f$. The average signal-to-interference ratio per chip is then equal to

$$\frac{PT_c}{2(K-1)PT_c^2/T_f} \tag{1.31}$$

---

[5] In general, we use the definition $W = 1/T_c$, but for bimodal Manchester and differentiated Gaussian pulses we make an exception.

and the average signal-to-interference ratio per bit is

$$\frac{E_\mathrm{b}}{I_0} = \frac{PT_\mathrm{c}Nr}{2(K-1)PT_\mathrm{c}^2/T_\mathrm{f}} \tag{1.32}$$

Because the receiver operates in the window of size $2T_\mathrm{c}$, the AWGN contribution to the total noise power spectral density is $2N_0$. Then Equality (1.32) can be modified and we obtain

$$\frac{E_\mathrm{b}}{I_0} = \frac{PT_\mathrm{c}Nr}{2(K-1)PT_\mathrm{c}^2/T_\mathrm{f} + 2N_0} \tag{1.33}$$

Because $W = 2/T_\mathrm{c}$, $R = r/NT_\mathrm{f}$, we get from Formula (1.33)

$$\frac{E_\mathrm{b}}{I_0} = \frac{W/4R}{(K-1) + N_0T_\mathrm{f}/PT_\mathrm{c}^2}. \tag{1.34}$$

From (1.34) follows the next formula for the radio channel capacity of the PPH CDMA system:

$$K_0 = 1 + \frac{W/4R}{E_\mathrm{b}/I_0} - \frac{N_0T_\mathrm{f}}{PT_\mathrm{c}^2} \tag{1.35}$$

For the repetition code $P_\mathrm{b} \approx \frac{1}{2}\exp(-E_\mathrm{b}/I_0)$ and if we neglect AWGN we finally have

$$K_0 \approx 1 - \frac{W/4R}{\ln(2P_\mathrm{b})} \tag{1.36}$$

A comparison of (1.27) and (1.36) shows that PPH CDMA systems with OR and correlator receivers have approximately equal capacities. However, we must note that Formulas (1.22) and (1.27) for the radio channel capacity of the FH CDMA system and the PPH CDMA system with OR receivers are derived under very idealized assumptions. First, we assumed that the users of the systems are synchronized. Asynchronism of the users decreases the capacity about two times. At the same time, asynchronism of the users does not affect the capacity of the PPH CDMA system with correlator receivers. Second, and more important, even small additive noise essentially decreases the capacities of FH CDMA and PPH CDMA systems with OR receivers but practically does not affect the capacity of the PPH CDMA system with correlator receivers. On the other hand, FH CDMA and PPH CDMA systems with OR receivers are robust to imperfection of the power control.

The PPH CDMA system described above uses *pulse position modulation* (PPM) format. It is also in principle possible to use the *pulse amplitude on-off modulation* (PAM) format. Then the signal waveform transmitted by the kth user in the nth frame is

$$\tilde{s}_n^{(k)}(t) = \frac{[v_n^{(k)} + 1]}{\sqrt{2}}\, h_{T_c}[t - nT_f - \tau_n^{(k)}] \tag{1.37}$$

where $\tau_n^{(k)}$ is the addressable pulse position shift and $1/\sqrt{2}$ is a normalizing factor.

The PPH CDMA system with on-off PAM is analyzed analogously to the system with PPM.

EXAMPLE 1.9

Consider a single cell PPH CDMA system using impulse radio with pulses of duration $T_c = 10^{-9}$*s. The user transmission rate is* $R = 10$ *kb/s. If a minimum acceptable* E_b/I_0 *is* 7 dB, *determine the capacity of the system. Estimate the influence of the system on the existing radio transmission systems.*

Solution

From Formula (1.36) we have

$$K_0 \approx 2\, \frac{10^9/4 \cdot 10^4}{5} = 10^4 \tag{1.38}$$

This capacity is much higher than the capacity of existing mobile radio systems. This is good news. The bad news is that the system occupies the full frequency range up to gigahertz, where all existing radio systems operate.

The PPH CDMA system can be used not only in cellular and conventional packet radio architectures, but also in *peer-to-peer* architectures. In peer-to-peer networks, connections between two users are not routed through a base station but are established directly. In peer-to-peer systems the synchronization overhead is minimal, especially for long message holding times.

Impulse radio has some other advantages in comparison with traditional carrier frequency-based radio, but, because it operates over the highly populated frequency range below a few gigahertz, it interferes with narrowband radio systems operating in dedicated bands. Potentially there must be a real payoff in the use of PPH CDMA protocol to undertake the difficult problem of coexistence with existing radio systems.

1.6 ORGANIZATION OF THE TEXT

In the previous three sections we described three different CDMA systems, direct-sequence, frequency-hopped, and pulse position-hopped CDMA systems. All three systems use the spread spectrum communication principle. Chapter 2 is an introduction to spread spectrum communication. It is devoted to the description and the generation of spreading signals in different CDMA systems.

Although spread spectrum systems can use both analog and digital modulation, modern spread spectrum communication systems mostly use digital modulation techniques. Digital modulation offers many advantages over analog transmission systems, including greater noise immunity and robustness to channel impairments. DS CDMA most often employs binary-phase shift keying (BPSK) or quadrature-phase shift keying (QPSK). FH CDMA most often employs, as we mentioned above, frequency shift keying (FSK). Corresponding modulation schemes are considered in Chapter 2. The PPH CDMA system does not use a sinusoidal carrier for the transmission of information but uses pulse position or pulse amplitude modulation. These modulation formats are also considered in Chapter 2.

A DS CDMA system spreads a PSK-modulated carrier by a wideband spreading signal. *Coherent* or *noncoherent demodulation* may be employed in the receiver. The spreading sequence in the case of FH CDMA does not directly modulate the FSK signal but is instead used to control the sequence of hopping frequencies. The demodulation of the received signal is in most cases *noncoherent*. The PPH CDMA system uses both a correlator receiver and an OR receiver. Reception of spread spectrum signals is studied in Chapter 3.

Interference is the major limiting factor in the performance of a cellular CDMA system. Sources of interference include other mobiles in the same or neighboring cells, other base stations operating in the same frequency band, or any radio system that inadvertently leaks energy into the cellular frequency band. Because the number of interference sources is usually large, the resulting interference noise on the receiver input can be considered as additive Gaussian noise and conventional methods of signal detection in additive Gaussian noise can be applied. An overview of these methods is given in Chapter 3.

Spread spectrum CDMA provides a considerably higher dimensionality of signals than needed to transmit information by any single user. This is reflected by the high processing gain. A universally effective method of exploiting redundancy to improve performance is forward error-correcting (or error control) coding. Although most known error-correcting methods can be used in CDMA communication, two of them have an advantage in this application. The first, first-order Reed–Muller code (Hadamard sequences), provides high reliability in low rate transmission, which is the case here. The second, convolutional coding with Viterbi decoding, permits exploitation of soft inputs to decrease the required signal-to-noise ratio at the receiver input. Application of coding to CDMA communication is studied in Chapter 4.

The mobile radio channel places fundamental limitations on the performance of cellular communication systems. The transmission path between the transmitter and the receiver can be obstructed by buildings and mountains. Unlike the one-path AWGN channel that is stationary and predictable, the mobile radio channel is a nonstationary multipath channel. Therefore, the analysis of spread spectrum communication in multipath *fading* channels is an essential part of the theory of CDMA communications. Analysis of spread spectrum CDMA communication in fading channels is carried out in Chapter 5.

In time-varying multipath fading channels, diversity can improve performance markedly. Such diversity can be attained by spatial separation such as multipath antennas or by natural multiple-path propagation with a *Rake receiver* or by temporal separation through an *interleaving* process. Interleaving has become an extremely useful technique in modern digital cellular systems and is also studied in Chapter 5.

The important issue to be dealt with in developing a spread-spectrum system is the generation of pseudorandom signals. This can be achieved in a variety of ways, but the generating process must be easily implemented and reproducible, because the same process must be generated at the transmitter for spreading (or hopping) and at the receiver for despreading (or dehopping). It is shown in Chapter 6 that generation of pseudorandom spreading or hopping process is most easily implemented by linear binary sequence generators, followed by linear filters. The largest part of the chapter is devoted to comprehensive discussion of maximum-length sequences, which are by far the most widely used spreading codes. The chapter concludes with a short discussion of orthogonal spreading sequences, Gold sequences, and Kasami sequences, which are used in the third-generation wireless networks.

Spread spectrum communication requires that the spreading (hopping) process in the transmitter and the despreading (dehopping) process in the receiver should be *synchronized*. If they are out of synchronization, insufficient signal energy will reach the receiver data demodulator. The task of achieving and maintaining synchronization is always delegated to the receiver. There are two components of the synchronization problem. The first component is the determination of the initial phase of the spreading (hopping) sequence. This part of the problem is called the *acquisition process*. The second component is the problem of maintaining synchronization of the spreading sequence after initial acquisition. This part of the problem is called the *tracking process*. The acquisition and tracking of the signal phase and frequency are performed in the same way as for any digital communication system. We consider the synchronization problems in Chapter 7.

Information-theoretical aspects of multiuser communication are considered in Chapter 8. In particular, we analyze such fundamental performance measures as Shannon capacity of CDMA systems and reliability of the transmission in the system.

In a CDMA system, it is possible for the strongest transmitter to successfully capture the intended receiver, even when many other users are also transmitting. Often, the closest transmitter is able to capture a receiver because of the small propagation path loss. This is called the *near-far effect*. A strong transmitter may make it impossible for the receiver to detect a much weaker transmitter that is attempting to communicate to the same receiver.

In practical cellular communication systems the power levels transmitted by every mobile are under constant control by the serving base stations. This is done to ensure that each mobile transmits the smallest power necessary to maintain a good quality link on the reverse channel. *Power control* not only helps prolong battery life for the user but also reduces reverse channel signal-to-noise ratio in the system.

When a mobile moves into a different cell while a conversation is in progress, the switching center automatically transfers the call to a new channel belonging to the new station. This handoff operation requires that the voice and control signals can be allocated to channels associated with the new base station. The CDMA system provides a handoff capability that cannot be provided by other multiple access systems. Unlike the FDMA and TDMA systems that assign different radio channels during a handoff (called a *hard handoff*), spread spectrum mobiles share the same channel (bandwidth) in every cell. By simultaneously evaluating the received signals from a single user at several neighboring base stations, the switching center may actually decide which version of the user's signal is most probable at a certain time instance. The ability to select between the instantaneous received signals from a variety of base stations is called *soft handoff*. Power control, handoff, and other problems of organizing the spread spectrum multiple access network, such as queueing analysis, are considered in Chapter 9. We also analyze in Chapter 9 two advanced methods of increasing capacity of the CDMA system—successive interference cancellation in the reverse link and user coordination in the forward link. The last section of Chapter 9 gives a short review of the third-generation wireless networks.

1.7 COMMENTS

Spread spectrum communication grew out of research efforts during World War II to provide secure and antijam communication in hostile environment. Starting in 1948 with Shannon's landmark paper "A Mathematical Theory of Communication" [42], information theory became the theoretical base of spread spectrum communication. The idea of the time-hopped CDMA system was expressed in 1949 in a technical memorandum of Bell Telephone Laboratories [35]. The concept of direct sequence spread spectrum was probably first published in 1950 [12]. The Interim Standard 95, adopted in 1993, specifies the spread spectrum communication format and protocols for communication between the base station of a cell and a mobile. It was developed by Qualcomm Inc., a company headed by Jacobs and Viterbi. The theory of CDMA communication was developed by Viterbi, in a number of papers and the classical book [47]. Specification of WCDMA technology has been created in the 3rd Generation Partnership Project (3GPP) started in 1992 [1]–[6]. Application of impulse radio to time-hopped CDMA (PPH CDMA) was described by Scholtz [40]. A general introduction into wireless communication systems is given by Rappaport [39].

PROBLEMS

1.1. If a total of 30 MHz of bandwidth is allocated to a particular cellular radio system that uses 30-kHz simplex channels to provide voice and control channels, compute the number of channels available per cell if a system uses:

a) 4-cell reuse

b) 7-cell reuse

c) 12-cell reuse

If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

1.2. GSM uses a frame structure in which each frame consists of 8 time slots, each time slot contains 156.25 bits, and data are transmitted at 270.833 kb/s over the channel. Find

a) The time duration of a bit

b) The time duration of a slot

c) The time duration of a frame

d) How long must a user occupying a single time slot wait between two consecutive transmissions?

1.3. Let $\xi(t)$ be a zero mean white Gaussian noise process with one-sided power spectral density N_0. Assume that the process $\xi(t)$ is the input signal to a running integrator and let $y(t)$ be the corresponding output, that is,

$$y(t) = \int_{t-T}^{t} \xi(\tau)\, d\tau$$

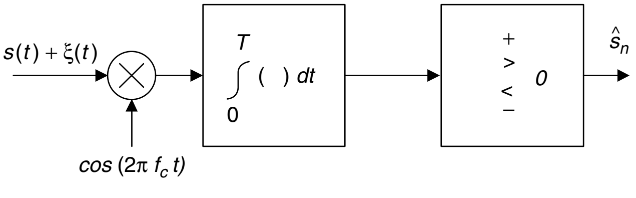
a) Find the mean and variance of $y(t)$. Can you say anything about the probability density function?

b) Find the autocorrelation function and the power spectral density for the process $y(t)$.

1.4. Consider a BPSK system in which the signals are given by

$$s(t) = \pm\sqrt{2}\cos(2\pi f_c t), \ \ 0 \le t \le T$$

Assume that the received signal is corrupted by AWGN with one-sided power spectral density N_0. Show that the probability of error for the



Problem 1.4. Receiver structure for BPSK.

receiver in the figure below is

$$P_\mathrm{b} = Q\left(\sqrt{2\,\frac{E_\mathrm{b}}{N_0}}\right)$$

where $E_\mathrm{b} = T$ is the energy per bit and $Q(x)$ is defined by Formula (1.12).

1.5. Prove the formula

$$Q(x) < \frac{1}{2}e^{-\frac{x^2}{2}},\ \ x \geq 0$$

Hint: The function

$$F(x) \overset{\mathrm{def}}{=} \frac{1}{2}\,e^{-\frac{x^2}{2}} - Q(x)$$

is equal to 0 for $x = 0$, increases in the interval $\left(0, \sqrt{\frac{2}{\pi}}\right)$ strictly decreases in the interval $\left(\sqrt{\frac{2}{\pi}}, \infty\right)$ and goes to 0 if x goes to ∞.

1.6. Find the radio channel capacity of the single-cell DS CDMA system if the total bandwidth of the duplex channel is 25 MHz, the individual transmission rate of the user is $R = 10^4$ bits/s and the required bit error probability is $P_\mathrm{b} = 10^{-5}$. The repetition code is used. Neglect the influence of background noise. Consider the next cases: a) no voice activity detection, omni-directional antennas; b) voice activity detection with $\gamma_\mathrm{v} = 8/3$, 3-sector antennas.

1.7. Find the processing gain W/R if the radio channel capacity is $K_0 = 100$ and the required signal-to-noise ratio at the demodulator input is equal:

a) 3 dB

b) 5 dB

c) 10 dB

d) 20 dB

Neglect the influence of background noise.

1.8. A total of 30 equal-power users are to share a common communication channel by DS CDMA. Each user transmits information at a rate of 10 kbits/s. Determine the minimum chip rate to obtain a bit error probability of 10^{-5} if

a) Additive noise in the receiver can be ignored

b) $E_\mathrm{b}/N_0 = 12$ dB

1.9. If $W = 1.25$ MHz, $R = 9600$ bits/s, and the required E_b/I_0 is 10 dB, determine the maximum number of users that can be supported in a single-cell DS CDMA system using

a) Omnidirectional base station antennas and no voice activity detection

b) Three-sectored antennas at the base station and voice activity detection with $\gamma_v = 8/3$

Assume that the background noise can be neglected.

1.10. Suppose that you are a communication engineer and you have to construct a mobile telephone network with bandwidth $W = 4.2$ MHz in each link. The data rate is $R = 10$ kb/s. You have the following alternatives:

a) A system based on TDMA with parameters
- Frequency reuse factor 1/7
- Each link divided into radio channels of 200 kHz
- Ten users per channel

b) A system based on DS CDMA with parameters
- Frequency reuse factor 1
- Voice activity gain $\gamma_v = 2.5$
- No antenna sectorization
- The required signal-to-noise ratio $E_b/I_0 = 6$ dB
- Other-cell relative interference factor $f = 0.6$

What alternative would you choose?

1.11. In an omnidirectional (single cell, single sector) DS CDMA cellular system, let the required SNR be 10 dB. If 100 users, each with a data rate of 13 kb/s, are to be accommodated, determine the minimum channel bit rate of the spread spectrum chip sequence if

a) Voice activity monitoring is absent, and omni-directional antenna is used,

b) Voice activity is equal to 40%, and omni-directional antenna is used,

c) Voice activity is equal to 40%, and a three-sectored antenna is used.

1.12. Two signals $s_0(t)$ and $s_1(t)$ are said to be *orthogonal* (over the time interval T) if

$$\int_0^T s_0(t)s_1(t)\,\mathrm{d}t = 0$$

Let $s_0(t) = \sqrt{2}\cos(2\pi f_0 t)$ and $s_1(t) = \sqrt{2}\cos(2\pi f_1 t + \varphi_1)$.

a) If $f_0 T = k_0/2$, and $f_1 T = k_1/2$ where k_0, k_1 are integers, show that the smallest value of $|f_1 - f_0|$ that makes the signals orthogonal is $1/T$.

b) If $\varphi_1 = 0$, what is the smallest value of $|f_1 - f_0|$ that makes the signals orthogonal?

c) If $|f_1 - f_0|T \gg 1$, show that the signals can be assumed to be orthogonal.

1.13. Suppose that the FH CDMA system with $K = 30$ synchronized active users uses the same transmission method as in Example 1.7. Let $M = 100$, $q = 2$. What number N of the time slots should be used, if the required bit error probability is $P_b = 10^{-4}$?

1.14. Under the same conditions as in Problem 1.6, find the radio channel capacity, K_0, of the synchronous FH CDMA system. Assume that the system does not use voice activity detection and antenna sectorization.

1.15. Find the radio channel capacity of the PPH CDMA system if the system uses impulse radio with pulses of duration $T_{\mathrm{c}} = 10^{-8}$ s. The user transmission rate is $R = 10$ kb/s, and the required bit error probability is $P_{\mathrm{b}} = 10^{-5}$. The repetition code is used. Neglect the other-cell interference and background noise.