4

General Analysis of Turbomachines

Problem 1

The 8 solved examples and the 10 exercise problems of this chapter make up a variety of turbomachines. With a broad bifurcation under radial flow and axial flow machines, prepare a table of all the parameters such as speed, specific speed, specific work, head or pressure, degree of reaction, angles, and efficiencies. Find the gaps in the information and obtain those values, if it is possible. For example, if specific speed is neither a part of data nor has been asked as part of answers, then, it may be calculated and provided in the table. Are there any particular observations to be made from the table?

Solution: First the values are tabulated for radial flow machines:

Example/Problem	D_1 **(m)**	D_2 **(m)**	β_1	β_2	N **(rpm)**	α_1	W **(J/kg)**	R	U_1 **(m/s)**	U_2 **(m/s)**	V_1 **(m/s)**	**Pump/Turbine**
4.1	0.05	0.125	50°	70°	1500	90°	79.74		3.93	9.82		Pump
4.2	0.5	0.15	61.94°	45.46°	1500	20°	1291.6	0.58	39.27	11.78	35	Turbine
4.3	0.36	0.54			3000	90°	2400	0.833	56.57			Pump
4.4	0.10	0.20	40°	70°	3280	90°	1000	0.8	17.18			Pump
4.5	0.812	0.406	65°	50°	1000	25°	2314	0.361	42.554		60	Turbine
1	0.045	0.1		65°	2500	90°	130.7	0.84	5.81	13.09	10	Pump
2	0.065	0.15	70°	85°	1440		115.93	0.695	4.9	11.31	13.46	Pump
3	0.75	0.3	60.4°	55°	1500	28°	469.43	0.053	19.635	7.854	31.856	Turbine
10	0.8	0.4	105.3°	50°	375	27.16°	286.46	0.8	15.7	7.85	–	Turbine

Some observations from the table:

1. The speeds in the problems/examples are randomly chosen. Pumps have to run at speeds which are generally of induction motors, that is, synchronous speed less slip. It is seen here that except in Problem 2, other pumps do not have such speeds. Further, turbines have to run at synchronous speeds; and it is seen that all turbines have the synchronous speeds.

2. Entry of fluid is at smaller diameters in pumps and at larger diameters in turbines. This is in agreement with usual practice.

3. The entry of fluid in a radially outward flow machine is invariably without whirl component (α_1 = 90°). This is reinforced in the problems.

4. The problems cover a wide range of specific works. However, all of them imply that the fluid is liquid (water). If it were gaseous (air, steam), the speeds would have been much higher.

5. The sizes of machines are rather small. For a hydro-electric project, depending on the field data, the sizes of rotors can be huge (3–4 m). The pump sizes cover a wide range; the pump of solved Example 4.3, with a speed of 3000 rpm and 54 cm diameter, seems to be a high-duty pump suitable for town-water supply.

6. The outlet blade angles, β_2, in pumps are around 65° and 85°. This may be particularly noted.

Now, the values are tabulated for all the axial flow machines:

Example/Problem	D_{tip}	D_{hub} **(m)**	β_{1t}	β_{2t}	N **(rpm)**	α_1	W **(J/kg)**	R	U_1 **(m/s)**	U_2 **(m/s)**	V_1 **(m/s)**	Pump/Turbine
	Or D_{mean}								U_{mean}			
4.8			76.9°	45.5°		30°	2914	0.25	60		80	Turbine
4.9	0.5		37.38°	65°	750	90°	248.1	0.68	19.635		15	Pump
4.10	1.28		40°	25°	1500	25°	35480	0.5	100.66		250	Turbine
4	1.323		60°		1500	30°	21600	0.5	103.92		180	Turbine
5	0.85		50°	25°	2200	20°	13130	0.214	97.95		150	Turbine
6	0.85		49.3°	49.3°	3000	32.72°	44032				320	Turbine
7	0.3	0.1	28.44°	48.44°	2880		472.8	0.74	45.25	15.08	16.334	Blower
8	0.3	0.04	30°	60°	600		44.416				5.44	Fan
9	0.45		29.5°	70.6°	6000		126800	0.6			80	Compressor

Observations:

1. In the axial flow machines, referred to in the examples and problems, pump, fan, blower and compressor are only one each. The remaining five problems/examples are on turbines.

2. The least specific work is that of the fan (44.416 J/kg of Problem 8). This is the maximum duty that is expected of a fan/blower.

3. The specific work of the pump (Example 4.9) is rather low at 248.1 J/kg. But one has to remember that the pump handles water/liquid, with a density almost a thousand times that of air. For the equal volume flow rates (m^3/s), a pump handles almost 1000 times the mass flow rates as that of air. Hence, the power of the pump can be very high.

4. Except the fan, pump, and a turbine (which have speeds of 600 rpm, 750 rpm, and 1500 rpm, respectively), all other machines have higher speeds. Almost all of them have gaseous working fluids, although one of them (Problem 4 with 1500 rpm) is a water turbine. This is acceptable.

5. The specific works are spread over a really large range of values. Just to get a feel of it, the values are quoted again, in descending order:

$$126.8\,\text{kJ/kg}$$
$$44.032\,\text{kJ/kg},\ 35.48\,\text{kJ/kg},\ 21.6\,\text{kJ/kg},\ 13.13\,\text{kJ/kg}$$
$$2.914\,\text{kJ/kg},\ 2.914\,\text{kJ/kg},\ 0.4728\,\text{kJ/kg},\ 0.248\,\text{kJ/kg}$$
$$0.044416\,\text{kJ/kg}$$

These above values may be correlated with the speed and the size of the machines.

Problem 2

In order to study the effect of the outlet blade angle β_2 on the performance of a radial flow machine, calculate the specific work W and degree of reaction R for an impeller with the inlet diameter D_1= 3 cm, outlet diameter D_2= 6 cm, speed N = 1000 rpm, inlet blade angle β_1 = 50°, and a series of values of β_2 (15°, 20°, 22°, 24°, 25°, etc.). Neglect the effects of slip and losses. Now plot the values of W and R, so obtained, on a base of β_2. If required, extend the values of β_2 for this "experiment". Write your observations from the plot. Can you correlate your answer with any equation in the text?

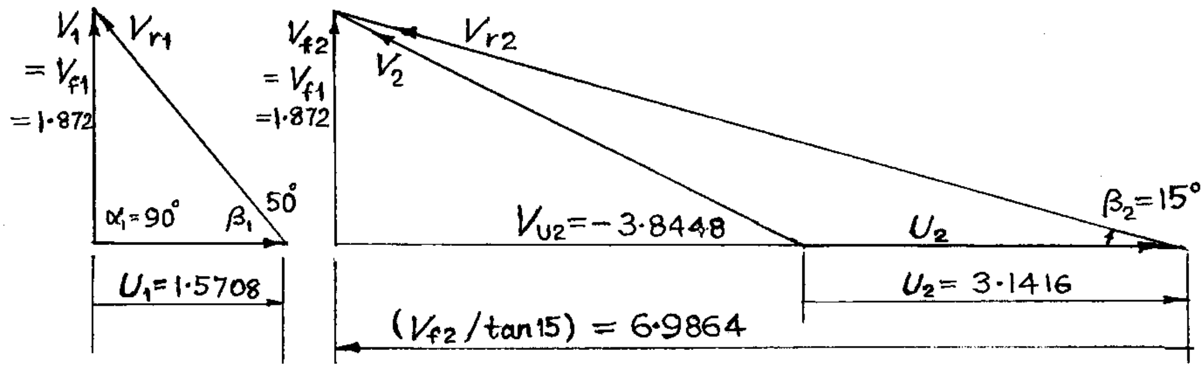
Solution: It is suggested that the effect of outlet blade angle β_2 on the performance (W, R) of a radial flow machine is required to be studied, with the following parameters fixed as: D_1 = 0.03 m, D_2 = 0.06 m, β_1 = 50°, and N = 1000 rpm. It is assumed that $V_{f1} = V_{f2}$, and α_1 = 90°.

To start with, we find the blade velocities at inlet and outlet, then draw the inlet velocity triangle and the outlet velocity triangle with one value of β_2, that is, β_2 = 15°.

We have,

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.03 \times 1000}{60} = 1.5708\,\text{m/s}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.06 \times 1000}{60} = 3.1416\,\text{m/s}$$



Velocity triangles, Problem 2 of project oriented questions, Chapter 4 (Effect of variation of β_2).

From inlet velocity triangles,

$$V_{f1} = U_1 \tan 50°$$
$$= 1.872 \text{ m/s}$$

We have,

$$V_{u2} = U_2 \frac{V_{f2}}{\tan \beta_2}$$
$$= 3.1416 - \frac{1.872}{\tan 15}$$
$$= 3.1416 - 6.9864$$
$$= -3.8448$$

Therefore, the specific work is given by ($V_{u1} = 0$)

$$W = -U_2 V_{u2}$$
$$= -(3.1416) \times (-3.8448)$$
$$= +12.08 \text{ J/kg}$$

This means that when β_2 is 15°, the specific work of the machine is actually positive, suggesting that the machine is a turbine. Continuing with various values of β_2, the values of W are tabulated as shown:

β_2	V_{u2} (m/s)	W (J/kg)	Remarks
15	–3.8448	+12.08	Turbine
20	–2.0017	+6.2885	Turbine
22	–1.4918	+4.6866	Turbine
24	–1.063	+3.3395	Turbine

26	–0.6966	+2.1883	Turbine
28	–0.37912	+1.191	Turbine
30	–0.1008	+0.3167	Turbine
32	+0.1458	–0.458	Pump
34	+0.36625	–1.1506	Pump

The specific work becomes zero (zero-work impeller), when V_{u2} reduces to zero, that is, when $U_2 = V_{f2} \tan\beta_2$ or when

$$\beta_2 = \tan^{-1}\left[\frac{U_2}{V_{f2}}\right]$$

$$\beta_2 = \tan^{-1}\left[\frac{3.1416}{1.872}\right] = 30.79°$$

In almost a similar way, we can tabulate the values of the degree of reaction, R, with varying values of β_2 and locate a value of β_2 for which $R = 0$.

Using the expression,

$$R = \left[1 - \frac{V_{u2}}{(2U_2)}\right] \qquad \text{(Eqn. 4)}$$

at $\beta_2 = 20°$, $R = 1.3186$.

The range of values of β_2 is now shifted to about 145°. The following table gives the net results.

β_2	R
145	0.0745
150	–0.16
146	0.0583
147	0.0412
148	0.0232
149	0.004
149.5	–0.0058
149.2	0.0002

The zero value of R is obtained when $\beta_2 = 149.21°$. As we know, zero reaction means that the entire input energy is imparted to the fluid only in the form of kinetic energy, without any generation of pressure. The output, in such a case, is the fluid with very high kinetic energy (depending on diameter, speed and consequently blade velocity at outlet, U_2). The velocity triangles in this case are shown in Figure 4.1.

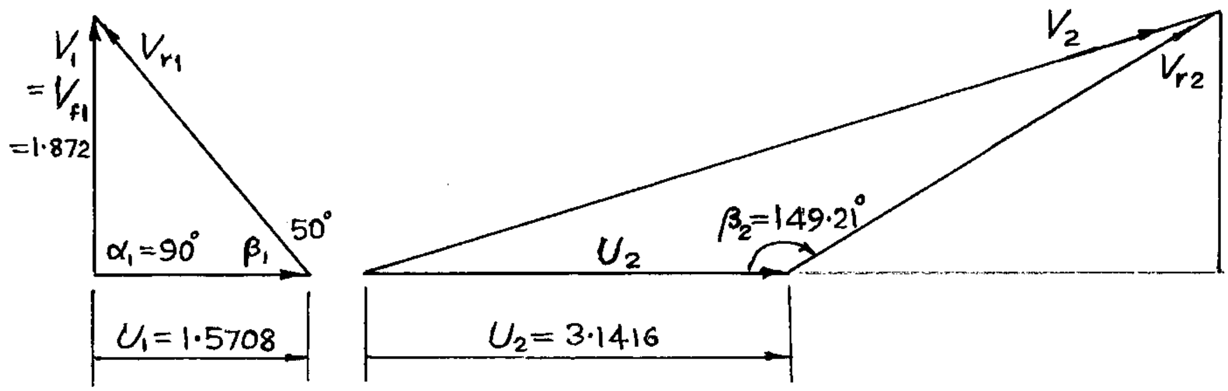


Figure 4.1 Velocity triangles, Problem 2 of project oriented questions, Chapter 4 (Effect on degree of reaction, $R = 0$).