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Thermodynamics of Fluid Flow

Problem 1

In the exercise Problem 5, you have already stated your comments on the effects of (a) variation of number of stages, and (b) variation of stage efficiency on the performance of a multistage compression process, depending on the results of Problems 1–4. Now, include the solved examples of this chapter, with repeated variation of such parameters with more number of stages and more number of values of the parameters to reinforce and fine-tune your comments.

Solution: The solved Example 2.6 is chosen for the analysis. The example is of a four stage air compressor with a stage pressure ratio of 1.3 and a stage efficiency of 0.9, drawing air at 100 kPa and 300 K. The example is now solved with stage efficiencies of 0.8, 0.83, and 0.86. Further, with a stage pressure ratio of 1.3 and four stages, the final pressure is 285.61 kPa. Between these pressure limits (namely, 100 kPa and 285.61 kPa), the example is repeated for three stages with stage efficiencies of 0.8, 0.83, 0.86, and 0.9. All the results are tabulated as shown.

Results of four stage compressor:

p_1 = 100 kPa, p_2 = 130 kPa, p_3 = 169 kPa, p_4 = 219.7 kPa, p_5 = 285.61 kPa

1. Stage efficiency = 0.8. The different temperatures are

T_1 = 300 K

$T_{2'}$ = 323.352 K T_2 = 329.19 K

$T_{3'}$ = 354.8136 K T_3 = 361.2195 K

$T_{4'}$ = 389.3362 K T_4 = 396.365 K

$T_{5'}$ = 427.2178 K T_5 = 434.931 K $T_{5''}$ = 404.889 K

Overall efficiency = $\dfrac{404.889-300}{437.931-300} = 0.777353$

Sum of isentropic stage works = 107.9451 kJ/kg

Single stage work = 105.31 kJ/kg

Extra work required, due to pre-heat effect = 2.6351 kJ/kg

2. Stage efficiency = 0.83. The different temperatures are

T_1 = 300 K

$T_{2'}$ = 323.352 K T_2 = 328.13433 K

$T_{3'}$ = 353.675 K T_3 = 358.907 K

$T_{4'}$ = 386.844 K T_4 = 392.566 K

$T_{5'}$ = 423.1225 K T_5 = 429.381 K $T_{5''}$ = 404.889 K

Overall efficiency = $\dfrac{404.889-300}{429.381-300}=0.8107$

Sum of isentropic stage works = 107.38617 kJ/kg

Single stage work = 105.31 kJ/kg

Extra work required, due to pre-heat effect = 2.07617 kJ/kg

3. Stage efficiency = 0.86. The different temperatures are

T_1 = 300 K

$T_{2'}$ = 323.352 K T_2 = 327.153 K

$T_{3'}$ = 352.619 K T_3 = 356.764 K

$T_{4'}$ = 384.534 K T_4 = 389.055 K

$T_{5'}$ = 419.338 K T_5 = 424.268 K $T_{5''}$ = 404.889 K

Overall efficiency = $\dfrac{404.889-300}{424.268-300}=0.84406$

Sum of isentropic stage works = 106.871 kJ/kg

Single stage work = 105.31 kJ/kg

Extra work required, due to pre-heat effect = 1.561 kJ/kg

4. Stage efficiency = 0.9.

This case is the solved one, with the following results.

Overall efficiency = 0.8886

Extra work required, due to pre-heat effect = 1.345 kJ/kg

Results of the three-stage compressor:

Initial pressure = 100 kPa; final pressure = 285.61 kPa.

Overall pressure ratio = 2.8561

When there are three stages, each stage pressure ratio = $(2.8561)^{1/3}$ = 1.4188

Hence, p_1 = 100 kPa, p_2 = 142 kPa, p_3 = 201.3 kPa, p_4 = 285.61 kPa

Isentropic temperature ratio per stage = $(1.4188)^{((\gamma-1)/\gamma)} = 1.105106$

1. Stage efficiency = 0.8. The different temperatures are

T_1 = 300 K

$T_{2'}$ = 331.53 K T_2 = 339.415 K

$T_{3'}$ = 375.089 K T_3 = 384.088 K

$T_{4'}$ = 424.369 K T_4 = 434.44 K $T_{4''}$ = 404.889 K

Overall efficiency = $\dfrac{404.889-300}{434.44-300} = 0.7802$

Sum of isentropic stage works = 107.915 kJ/kg

Single stage work = 105.31 kJ/kg

Extra work required, due to pre-heat effect = 2.605 kJ/kg

2. Stage efficiency = 0.83. The different temperatures are

T_1 = 300 K

$T_{2'}$ = 331.53 K T_2 = 337.99 K

$T_{3'}$ = 373.515 K T_3 = 380.79 K

$T_{4'}$ = 420.814 K T_4 = 429.012 K $T_{4''}$ = 404.889 K

Overall efficiency = $\dfrac{404.889-300}{429.012-300} = 0.813$

Sum of isentropic stage works = 107.079 kJ/kg

Single stage work = 105.31 kJ/kg

Extra work required, due to pre-heat effect = 1.769 kJ/kg

3. Stage efficiency = 0.86. The different temperatures are

T_1 = 300 K

$T_{2'}$ = 331.53 K T_2 = 336.663 K

$T_{3'}$ = 372.05 K T_3 = 377.808 K

$T_{4'}$ = 417.52 K T_4 = 423.983 K $T_{4''}$ = 404.889 K

Overall efficiency = $\dfrac{404.889-300}{423.983-300} = 0.846$

Sum of isentropic stage works = 106.629 kJ/kg

Single stage work = 105.31 kJ/kg

Extra work required, due to pre-heat effect = 1.319 kJ/kg

4. Stage efficiency = 0.9. The different temperatures are

T_1 = 300 K

$T_{2'}$ = 331.53 K T_2 = 335.033 K

$T_{3'}$ = 370.247 K T_3 = 374.16 K

$T_{4'}$ = 413.486 K T_4 = 417.856 K $T_{4''}$ = 404.889 K

Overall efficiency = $\dfrac{404.889-300}{417.856-300} = 0.8898$

Sum of isentropic stage works = 106.07 kJ/kg

Single stage work = 105.31 kJ/kg

Extra work required, due to pre-heat effect = 0.79 kJ/kg

The results obtained as above are now tabulated in order to clearly visualize the effect of the number of stages and the stage efficiencies:

Final Results of the Air Compressor

No. of Stages	Stage Efficiency	Overall Efficiency	Extra Work due to Pre-Heat Effect
3	0.8	0.7802	2.605
3	0.83	0.813	1.769
3	0.86	0.846	1.319
3	0.9	0.88998	0.76
4	0.8	0.777353	2.6351
4	0.83	0.8107	2.07617
4	0.86	0.84406	1.561
4	0.9	0.8886	1.345

Comments: The results amply demonstrate and reinforce the observations made, in the exercise Problem 5, namely,

(1) Increased stage efficiency increases overall efficiency also; extra work required is reduced.

(2) Increase in number of stages reduces the overall efficiency, requiring more extra work due to pre-heat.

(3) There is likely to be an optimum number of stages which gives better results.

Problem 2

In the exercise Problem 10, you have already stated your comments on the effects of (a) variation of number of stages, and (b) variation of stage efficiency on the performance of a multistage expansion process, depending on the results of Problems 6–9. Now, include the solved examples of this chapter, with repeated variation of such parameters with more number of stages and more number of values of the parameters, to reinforce and fine-tune your comments.

Solution: The solved Example 2.8 is chosen for the analysis. The example is of a four-stage gas turbine with inlet conditions of 600 kPa, 1000 K and outlet pressure of 100 kPa, and a stage efficiency of 0.86. This example is now solved with stage efficiencies 0.8, 0.83, and 0.9. This is again repeated with four-stages with the stage efficiencies of 0.8, 0.83, 0.86, and 0.9.

Results of the four-stage turbine:

Overall pressure ratio is (1/6)

Stage pressure ratio is $(1/6)^{1/4} = 0.63894$

The pressure levels are

$p_1 = 600$ kPa, $p_2 = 383.366$ kPa, $p_3 = 244.95$ kPa, $p_4 = 156.5$ kPa, and $p_5 = 100$ kPa

The stage temperature ratio is $\left(\frac{1}{6}\right)^{((\gamma-1)/\gamma)} = 0.879873$

1. Stage efficiency = 0.8. The different temperatures are

$T_1 = 1000$ K

$T_{2'} = 879.873$ K $T_2 = 903.8984$ K

$T_{3'} = 795.316$ K $T_3 = 817.012$ K

$T_{4'} = 718.885$ K $T_4 = 738.51$ K

$T_{5'} = 649.8$ K $T_5 = 667.54$ K $T_{5''} = 599.35$ K

Overall efficiency = $\dfrac{1000-667.54}{1000-599.35} = 0.8298$

Sum of isentropic stage works = 417.21 kJ/kg

Single stage work = 402.25 kJ/kg

Reheat factor = 1.0372

2. Stage efficiency = 0.83. The different temperatures are

$T_1 = 1000$ K

$T_{2'} = 879.873$ K $T_2 = 900.29$ K

$T_{3'} = 792.145$ K $T_3 = 810.53$ K

$T_{4'} = 713.163$ K $T_4 = 729.72$ K

$T_{5'} = 642.057$ K $T_5 = 656.96$ K $T_{5''} = 599.35$ K

Overall efficiency = $\dfrac{1000 - 656.96}{1000 - 599.35} = 0.8562$

Sum of isentropic stage works = 414.96 kJ/kg

Single stage work = 402.25 kJ/kg

Reheat factor = 1.0316

3. Stage efficiency = 0.86

This case is the solved one, with the following results:

Overall efficiency = 0.8823

Reheat factor = 1.026

4. Stage efficiency = 0.9. The different temperatures are

$T_1 = 1000$ K

$T_{2'} = 879.873$ K $T_2 = 891.88$ K

$T_{3'} = 784.746$ K $T_3 = 795.46$ K

$T_{4'} = 699.903$ K $T_4 = 709.46$ K

$T_{5'} = 624.234$ K $T_5 = 632.76$ K $T_{5''} = 599.35$ K

Overall efficiency = $\dfrac{1000 - 632.76}{1000 - 599.35} = 0.9166$

Sum of isentropic stage works = 409.68 kJ/kg

Single stage work = 402.25 kJ/kg

Reheat factor = 1.0185

Results of three-stage turbine:

Overall pressure ratio = 1/6

Stage pressure ratio = $(1/6)^{1/3} = 0.55032$

The pressure levels are

p_1 = 600 kPa, p_2 = 330.193 kPa, p_3 = 181.712 kPa, p_4 = 100 kPa

Isentropic temperature ratio for a stage = $(0.55032)^{((\gamma-1)/\gamma)} = 0.843129$

1. Stage efficiency = 0.8. The different temperatures are

T_1 = 1000 K

$T_{2'}$ = 843.129 K T_2 = 874.5 K

$T_{3'}$ = 737.319 K T_3 = 764.755 K

$T_{4'}$ = 644.787 K T_4 = 668.781 K $T_{4''}$ = 599.35 K

Overall efficiency = $\dfrac{1000-668.781}{1000-599.35} = 0.8267$

Sum of isentropic stage works = 415.676 kJ/kg

Single stage work = 402.25 kJ/kg

Reheat factor = 1.0332

2. Stage efficiency = 0.83. The different temperatures are

T_1 = 1000 K

$T_{2'}$ = 843.129 K T_2 = 869.797 K

$T_{3'}$ = 733.351 K T_3 = 756.547 K

$T_{4'}$ = 637.867 K T_4 = 658.042 K $T_{4''}$ = 599.35 K

Overall efficiency = $\dfrac{1000-658.042}{1000-599.35} = 0.8535$

Sum of isentropic stage works = 413.645 kJ/kg

Single stage work = 402.25 kJ/kg

Reheat factor = 1.02813

3. Stage efficiency = 0.86. The different temperatures are

T_1 = 1000 K

$T_{2'}$ = 843.129 K T_2 = 865.09 K

$T_{3'}$ = 729.383 K T_3 = 748.38 K

$T_{4'} = 630.983$ K $T_4 = 647.42$ K $T_{4''} = 599.35$ K

Overall efficiency = $\dfrac{1000 - 647.42}{1000 - 599.35} = 0.88$

Sum of isentropic stage works = 411.615 kJ/kg

Single stage work = 402.25 kJ/kg

Reheat factor = 1.0233

4. Stage efficiency = 0.9. The different temperatures are

$T_1 = 1000$ K

$T_{2'} = 843.129$ K $T_2 = 858.82$ K

$T_{3'} = 724.093$ K $T_3 = 737.57$ K

$T_{4'} = 621.863$ K $T_4 = 633.41$ K $T_{4''} = 599.35$ K

Overall efficiency = $\dfrac{1000 - 633.41}{1000 - 599.35} = 0.915$

Sum of isentropic stage works = 408.934 kJ/kg

Single stage work = 402.25 kJ/kg

Reheat factor = 1.0166

Final Results of the Gas Turbine

No. of Stages	Stage Efficiency	Overall Efficiency	Reheat Factor
3	0.8	0.8267	1.0332
3	0.83	0.8535	1.02813
3	0.86	0.88	1.0233
3	0.9	0.915	1.0166
4	0.8	0.8289	1.0372
4	0.83	0.8562	1.0316
4	0.86	0.8823	1.026
4	0.9	0.9166	1.0185

Comments: The results once again demonstrate and reinforce the observations made in the exercise Problem 10, namely,

(1) Increased stage efficiencies increase the overall efficiencies also; but the reheat factors decrease.

(2) Increasing of number of stages also increases the overall efficiencies and the reheat factors.

(3) Since increasing of stages involves higher costs, a marginal increase of overall efficiency may not be enough to justify the investment.