3

Energy Exchange in Turbomachines

Problem 1

The solved and unsolved examples of this chapter are meant to illustrate the various forms of velocity triangles and the variety of the turbomachines. In addition, their working parameters also present a wide range of values of such parameters. Now, with respect to all the problems, conduct an overall survey of all the parameters such as (a) speed, (b) specific work, (c) specific speed, (d) power, (e) degree of reaction, (f) utilization factor, and (g) speed ratio. Present your findings in the form of a table. What are the different conclusions, you can draw, from this table?

Solution: The solved Examples 3.1–3.10 are only qualitative in nature, just meant to get familiar with velocity triangles. These examples illustrate how the velocity triangles of the radial flow and axial flow machines differ from each other; and how to correlate the blade shapes with the directions of energy flow, fluid flow, and velocity triangles; solved Examples 3.11–3.15 illustrate further steps in the utility of velocity triangles, how to decide the specific work, power, degree of reaction, utilization factor, and axial thrust. The values of parameters of these examples are listed in Table 3.1(a):

Table 3.1(a) Parametric values

Example	W (J/kg)	R	$\in$	U (m/s)
3.11	64	0.5		4, 8
3.12	109.05	0.243	0.62	15
3.13	19470	0.02	0.863	210
3.14	15917	–0.016	0.836	85
3.15	600	0.67	0.5	30

The examples indicate a wide range of values of the parameters. Specific work has a lower value of just 64 J/kg, the upper value is 19470 J/kg. This may be attributed to the blade speeds, 8 m/s–210 m/s. The blade speeds can reach still higher values, when the fluid is gaseous, which is steam or combustion gases in steam or gas turbines, respectively.

The values of the parameters of the machines of exercise problems are also listed in Table 3.1(b):

Table 3.1(b) The values of parameters of the machines of exercise problems

Problem No.	D_1 (m)	D_2 (m)	N (rpm)	U (m/s)	β_1	β_2	α_1	α_1	V_1 (m/s)	V_2 (m/s)	W (kJ/kg)	R	$\in$	ϕ	
1	0.4	0.9	8000	377	41.84	90	90	21.7	150	405.75	142.13	0.5			Radial flow
2	0.6	0.6	3000	94	25	25	13.124	90	193.55	43.95	17.766	0.0	0.948	0.486	Axial flow
3	0.2	0.5	800	21	50	60	90				0.317				Radial flow
4	0.5	0.5	10000	262			25		380	163.84	81.67	0.28	0.86	0.689	Axial flow
5	0.6	0.6	10000	314	29.81	72	90	35	180		80.76	0.589			Axial flow
6	0.9	0.3	1200	57	81.3	57.86	30	90	60		2.938	0.54	0.867		Radial flow
7			1000	37	42.28	28	28		100	69.8	5.128	0.5	0.678	0.367	Axial flow
8				125	40	25	18.6		220	73	28.976	0.2566	0.9159		Axial flow
9	0.1	0.2	2000	21	48.89	80	90		12	22.225	0.395	0.556			Radial flow
10(a)	0.3	0.6	5513	173	30	90	90		50	180.3	30	0.5			Radial flow
(b)			5729	180	29.05	75	90		50	173.94	30	0.5374			Radial flow
11(a)	0.035	0.08	3469	15		65	90		8						Radial flow
(b)			2880	12		65	90		8						
12	Tip 0.45	Hub 0.1	800	19	23, 62.3	40, 90	90								Axial flow pump

Observations

Generally, higher speeds are employed for gaseous fluids.

Problem 1

This is a fairly big size machine, running at 8000 rpm, with a blade tip velocity of 377 m/s. This must be a compressor handling air. If the fluid were liquid, with outlet diameter of 90 cm, the flow rate may be huge and power may be prohibitively large. Alternately, in such cases, the speed may be about 500 or 600 rpm with much lower blade tip velocity; and the machine may be called pump.

Problem 2

As mentioned in the problem statement, this is a turbine; the speed of 3000 rpm suggests that this handles steam or gas. With R = 0, this is an impulse turbine. The utilization factor of 0.948 suggests that this is a well-designed turbine.

Problem 3

This is a typical pump, handling liquid with an input of 0.317 kJ/kg, and running at 800 rpm, this is a small unit.

Problem 4

This is an axial flow turbine with R = 0.28 and $\in$ = 0.86. At 10000 rpm, this is mostly a gas turbine, coupled directly to its own compressor. Due to a lower degree of reaction, the range of expansion of pressure is low; and the outlet pressure of this turbine may be rather high, having further capacity to expand, that is, to generate high-velocity jet, as in jet engines.

Problem 5

This is an axial flow compressor. This has its speed of 10000 rpm, equal to that of the turbine of Problem 4. The specific work of this compressor (80.76 kJ/kg) is slightly less than that of turbine (81.67 kJ/kg), as it should be. These machines, compressor and turbine, together form a gas turbine power plant. Because R = 0.54, the velocity at the outlet of compressor may be useful in the combustion chamber, located between the compressor and turbine.

Problem 6

This is a radially inward flow turbine. With its speed of only 1200 rpm, this may be a water turbine, typically a Francis turbine. The values of R = 0.54 and $\in$ = 0.867 suggest that it is a well-designed turbine.

Problem 7

It is mentioned in the statement of problem that this is a turbine. With a speed of 1000 rpm, inlet velocity of 100 m/s and a speed ratio of 0.367, this is not a water turbine; it is a steam turbine, with $R = 0.5$. The utilization factor of 0.678 can be a little improved upon.

Problem 8

The blade velocity of 125 m/s is specified instead of the speed and diameter of the rotor. The situation is almost same as that of Problem 7; it is clearly specified that relative velocity increases in the rotor, which means that there is a reaction. This is a reaction type steam turbine.

Problem 9

As mentioned in the statement of the problem, this is a radial flow pump. With a speed of 2000 rpm and outlet diameter of 20 cm, this has to be a fairly large pump. To calculate the power, the flow rate is required to be specified. If one-tenth of outlet diameter (20 cm), that is, 2 cm, is taken as the width of blade at outlet, the flow rate works out as 150.8 kg/s and power works out to be approximately 59.56 kW.

Problem 10

This is a radial flow air compressor, as per the statement of the problem. With air velocity at 50 m/s and $W = 30$ kJ/kg, the speed works out to be 5513 rpm; with air as working fluid, this speed is to be expected.

Problem 11

This is a radial flow pump, a small one with the outlet diameter of just 8 cm. To create a head of 16 m of water, this pump has to have a speed of 3469.5 rpm. At an alternate speed of 2880 rpm, it generates a head of only 10.2 m of water. (10.2 m is approximately the difference between the underground sump level and an overhead tank in a single-storey building (G + F).

Problem 12

Axial flow machines are of high specific speeds. The average specific work is 112.42 J/kg, equivalent to a head of 11.46 m of water. At this head and flow rate of $Q = 1.21$ m^3/s, the specific speed is 141. This type of pump is employed to create some flow, rather than to cause head of water as though the unit is installed in a pipeline.

Problem 2

In Problem 3.11, an impeller of a radial flow pump with inlet diameter of 3.5 cm and outlet diameter of 8.0 cm has been mentioned. Imagine that you are actually fabricating an impeller of these diameters; and that you are equipped with a motor that can be set to run at three different speeds (750, 1000, and 1500 rpm). Design three blade profiles for the impellers with a blade-inlet angle of 70° in each case and blade-outlet angles of 55°, 65°, and75°. Calculate the theoretical flow rates when each impeller is run at the three different speeds. Plot the results in a suitable set of coordinates. Draw up your conclusions. Frame at least one more such possibility of conducting experiment to prove a chosen theoretical result.

Solution: The pump of Problem 11 has been referred here. It has an inlet diameter of 3.5 cm and outlet diameter of 8 cm. It is now required to retain the inlet blade angle β_1 of 70° and the outlet blade angle β_2 is required to be varied as 55°, 65°, and 75°. The speeds are required to be 750, 1000, and 1500 rpm, in each case. While working out, the values of angle β_2 are started with 45° and continued as 50°, 55°, 65°, and 75°. In order to calculate the flow rates, the blade widths are required as additional data. It is assumed that the blade width at inlet, B_1, is equal to 1 cm and flow components remain constant ($V_{f2} = V_{f1}$).

Now,

$$D_1 = 0.035\,\text{m},\ D_2 = 0.08\,\text{m},\ B1 = 0.01\,\text{m},$$
$$N = 750, 1000, \text{and } 1500\,\text{rpm. Choose } 750\,\text{rpm}$$

We have

$$U_1 = \frac{\pi D_1 N}{60} = 1.37445\,\text{m/s}$$
$$U_2 = \frac{\pi D_2 N}{60} = 3.1416\,\text{m/s}$$
$$\tan 70 = \frac{V_{f1}}{U_1}$$

Therefore,

$$\begin{aligned} V_{f1} &= U_1 \tan 70 \\ &= 1.37445 \times \tan 70 \\ &= 3.77627\,\text{m/s} \end{aligned}$$

Flow rate of the pump,

$$
\begin{aligned}
Q &= \pi D_1 B_1 V_{f1} \\
&= \pi \times 0.035 \times 0.01 \times 3.77627 \\
&= 0.00415\,\text{m}^3/\text{s} = 4.15\,\text{lps}
\end{aligned}
$$

The velocity triangles are shown in Fig. 3.1.

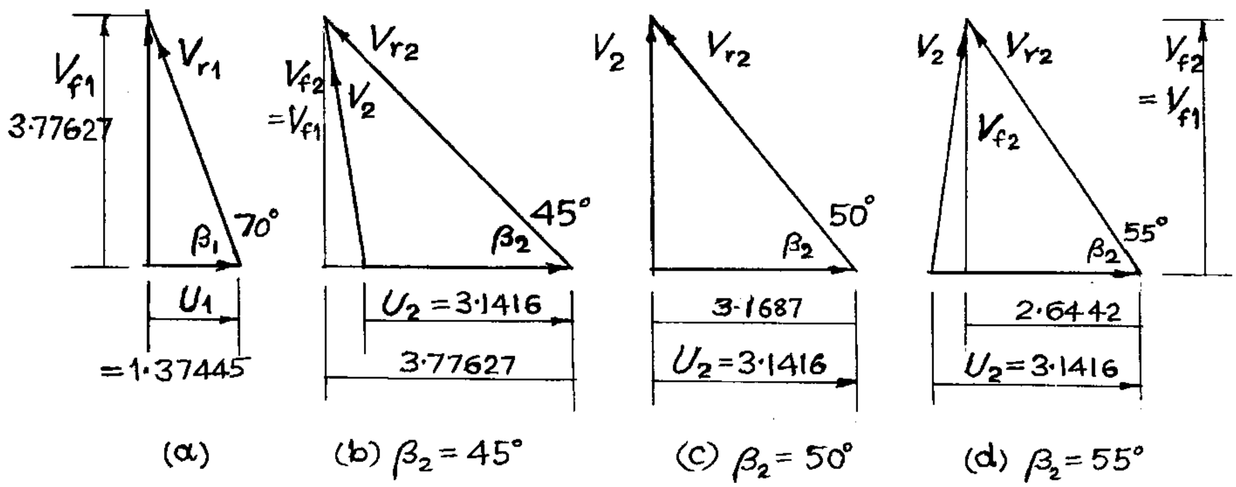

Figure 3.1 Velocity triangles, Problem 2 of project oriented questions, Chapter 3. Part (a) of the figure is the inlet velocity triangle. Parts (b)–(d) are the outlet velocity triangles, when the outlet blade angles (β_2) are, respectively, 45°, 50°, and 55°.

Now,

$$V_{f2} = V_{f1} = 3.77627\,\text{m/s}$$

When β_2 = 45°, we have

$$
\begin{aligned}
V_{u2} &= U_2 - \frac{V_{f2}}{\tan 45} \\
&= 3.1416 - 3.77627 \\
&= -0.63467
\end{aligned}
$$

Specific work,

$$\begin{aligned} W &= U_1 V_{u1} - U_2 V_{u2} \\ &= 0 - (3.1416) \times (-0.63467) \\ &= +1.994 \text{ J/kg} \end{aligned}$$

When β_2 = 50°, we have

$$\begin{aligned} V_{u2} &= U_2 - \frac{V_{f2}}{\tan 50} \\ &= 3.1416 - 3.1687 \\ &= -0.0271 \end{aligned}$$

and specific work,

$$\begin{aligned} W &= -(3.1416) \times (-0.027) \\ &= 0.085 \text{ J/kg} \end{aligned}$$

When β_2 = 55°, we have

$$\begin{aligned} V_{u2} &= 3.1416 - 2.6442 \\ &= 0.4974 \end{aligned}$$

and specific work,

$$\begin{aligned} W &= -(3.1416) \times (-0.4974) \\ &= 1.5627 \text{ J/kg} \end{aligned}$$

Again when β_2 = 65°, we have

$$V_{u2} = 3.1416 - 1.761$$
$$= 1.3807$$

and specific work,

$$W = -(3.1416) \times (1.3807)$$
$$= -4.3376 \text{ J/kg}$$

These results as well as the results of calculations at 1000 rpm and 1500 rpm are recorded in the following table.

RPM, N	***Q* (lps)**	***W***			
		β_2 **= 45°**	β_2 **= 50°**	β_2 **= 55°**	β_2 **= 65°**
750	4.15	+1.994	+0.085	–1.5627	–4.3376
1000	5.5363	+3.5446	+0.1511	–2.7782	–7.7113
1500	8.3044	+7.97534	+0.3400	–6.2509	–17.350

It is very clearly seen that at β_2 = 45°, the specific work of this "pump" is actually positive, as though it works like a turbine. At β_2 = 50°, the specific work is almost zero; and then as β_2 increases, the specific work increases in the negative direction, representing a truly pumping action.

Further it may be verified that the flow rate is proportional to the speed. When the same pump is used, the flow coefficient (Q/ND^3) remains same and, therefore, the flow rate is proportional to the speed.

A further experiment can be verified that the head created is proportional to the (speed)^2, that is,

$$H \,\alpha\, N^2$$

In fact, the same "readings" as above verify this result also (*W* represents *H*).

Again, it may be verified that the power ($W \times \dot{m} = P$) is proportional to the cube of speed ($P \,\alpha\, N^3$).