Problem 1

Make an observation that out of the 10 solved examples and the 8 exercise problems of this chapter, only 4 problems required the reference to the Mollier Diagram (enthalpy–entropy chart for steam). Locate these four problems. Then, take a Mollier diagram and mark all the four isentropic expansion processes (not actual processes) of the problems, on the same chart, starting from initial state to the final pressure line. These four lines, representing the isentropic expansion, are for reference only.

Now, in the same chart, mark any one of the following isentropic expansion processes:

(a) 12 bar, 300 °C to 0.05 bar; (b) 10 bar, 350 °C to 0.1 bar; (c) 8 bar, 325 °C to 0.05 bar

For the chosen process, determine the number of stages that may be required for the expansion process, with every stage having equal enthalpy drop, about 40 kJ/kg. Because of the integer number of stages, this enthalpy drop may have to be corrected. Also, because you are referring to the Mollier chart, the numerical values may not be exact. Refer to the steam tables to refine your values. Switch over to the actual processes with an isentropic efficiency of 0.83, as you proceed with the expansion process, stage-by-stage. Record the values of enthalpy and specific volume at every point, as was illustrated in the solved Example 5.10.

With this background, now calculate the generation of velocity in each stage. Increase this velocity by about 20%. Let this be the entry velocity of steam in a given stage (refer to comment no. 4 at the end of Example 5.10).

Now, let this turbine be made up of Parsons stages, for which the velocity triangles are symmetrical. Choose a convenient speed and a diameter of the rotor, so that the velocities of steam and blades are in agreement with a speed ratio of your choice. Now follow the steps of choosing/designing the other parameters of the turbine such as inlet angle of steam, blade angles, and blade heights (mass flow rate = 10 kg/s). Check, whether the exit velocity, together with generated velocity, gives rise to the entry velocity; and if not, by how much it differs.

Complete your project by calculating the efficiencies and the reheat factor.

Solution:

The problems, involving the use of Mollier diagram (enthalpy–entropy chart for steam) are: Solved Examples 5.7, 5.9, 5.10 and the exercise Problem 4. The isentropic expansion processes of these four examples are marked on a Mollier chart, which is reproduced in Fig. 5.1. Out of the three processes stated in the present problem, two processes are also marked in the same figure. One of them has the expansion commencing at 12 bar, 300 °C, and another commencing at 8 bar, 325 °C. Only isentropic processes are shown (not actual processes).
Figure 5.1 Mollier diagram.
Now, the process starting at 12 bar, 300 °C is chosen for further investigation. An isentropic enthalpy drop of 50 kJ/kg is chosen per row of blades. The isentropic efficiency is assumed as 0.84. This means that the actual enthalpy drop is \(50 \times 0.84 = 42\) kJ/kg.

At the starting, the state point 1 is at 12 bar, 300 °C; the enthalpy is 3040 kJ/kg. First expansion is in a stator-blade-row with isentropic end-point 2' and actual end-point 2. Enthalpy at 2' is \(3040 - 50 = 2990\) kJ/kg; enthalpy at 2 is \(3040 - 42 = 2998\) kJ/kg. The pressure, at 2' and 2, is 9.5 bar, as seen on the Mollier chart. In continuation, the second expansion is in a rotor-blade-row with isentropic end-point at 3' and actual end-point at 3. Enthalpy at 3' is \(2998 - 50 = 2948\) kJ/kg; enthalpy at 3 is \(2998 - 42 = 2956\) kJ/kg. One stage, that is, one stator-blade-row and one rotor-blade-row, together, has an actual enthalpy drop of 84 kJ/kg, 50% of which is in stator and 50% is in rotor. The process is shown in Fig. 5.2.

![Figure 5.2](image)

**Figure 5.2** Steam expansion in first stage (50% expansion in stator, 50% expansion in rotor, Parsons stage).

The other important information is about temperature and specific volume. All these numerical values are now recorded in Table 5.1, for the complete expansion process, down to 0.05 bar. Somewhere in the process, the steam passes from superheated to saturation region. Once it enters the saturation zone, record of temperature is immaterial; instead, the dryness fraction is recorded. To track the expansion, a schematic enthalpy–entropy diagram is also shown, Fig. 5.3, in which the expansion is up to the lower pressure of 0.05 bar.

The next step is the designing of the blades. At this juncture, the speed of the rotor and its diameter are required to be assumed.

Let the mean diameter of rotor (at mid-height of blades) be 1.2 m.

Let the speed be 3000 rpm.

Let the entry angle of steam be 20°.
Thus, \( D = 1.2 \text{ m}, \quad N = 3000 \text{ rpm}, \quad \alpha_1 = 20^\circ. \)

Consider this as a Parson’s stage.

There is an enthalpy drop in stator, equal to 42 kJ/kg (Process 1–2).

There is an enthalpy drop in rotor, equal to 42 kJ/kg (Process 2–3).

Total enthalpy drop per stage = 84 kJ/kg.
Figure 5.3  Mollier diagram.
Velocity of steam generated due to expansion in stator ($\Delta h_{act} = 42 \text{ kJ/kg}$) = $\sqrt{2(\Delta h_{act})} = 289.83 \text{ m/s}$

Let $V_1$ be equal to 330 m/s, inclusive of the effect of approach velocity

\[
\text{Blade velocity} = \frac{\pi DN}{60} = \frac{\pi \times 1.2 \times 3000}{60} = 188.5 \text{ m/s}.
\]

Combined velocity triangles are drawn, as shown in Fig. 5.4, with symmetrical triangles.

**Figure 5.4** Velocity triangles.

In the triangles,

AB = Blade velocity, $U = 188.5 \text{ m/s}$

AC = Steam velocity at inlet, $V_1 = 330 \text{ m/s}$

Angle CAB = $\alpha_1 = 20^\circ$.

Whirl component, at inlet,

\[
V_{u1} = AE = AC \cos \alpha_1
= 330 \cos 20
= 310.1 \text{ m/s}
\]
Flow component, at inlet,

\[ V_{f1} = CE = AC \sin \alpha_1 \]
\[ = 330 \sin 20 \]
\[ = 112.87 \text{ m/s} \]

Blade angle at inlet,

\[ \theta = \angle CBE = \tan^{-1} \left( \frac{CE}{BE} \right) \]
\[ = \tan^{-1} \left( \frac{V_{f1}}{V_{u1} - U} \right) \]
\[ = \tan^{-1} \left( \frac{112.87}{310.1 - 188.5} \right) \]
\[ = 42.87^\circ \]

Outlet velocity triangle is ABD; AD is exit velocity, \( V_2 \).

Because of symmetry,

\[ V_2 = AD = BC \]
\[ = \frac{V_{f1}}{\sin 42.87^\circ} \]
\[ = 165.81 \text{ m/s} \]

Whirl component at outlet,
$V_{u2} = AF$ (opposite to $AB$)  
\[= BE \text{ (in magnitude)}\]
\[= 310.1 - 188.5\]
\[= 121.6 \text{ m/s}\]

Specific work,

\[W = U(\Delta V_u) = U(V_{u1} - V_{u2})\]
\[= 188.5 \times (310.1 + 121.6)\]
\[= 81375 \text{ J/kg}\]
\[= 81.375 \text{ kJ/kg}\]

Now, the exit velocity, $V_2$, from the rotor is the “approach velocity” of the next stator blade with a “carry-over efficiency”. Let carry-over efficiency be 0.95.

We have

\[V_1 = \sqrt{(V_2 \times \eta_{co})^2 + 2(\Delta h_{sc})}\]
\[= \sqrt{(165.91 \times 0.95)^2 + 2 \times (42000)}\]
\[= 329.9 \text{ m/s}\]
Hence, the assumption of $V_1 = 330$ m/s checks very well. Actually, this is not a coincidence; some trials with various values of (a) rotor diameter, (b) speed of rotor, and (c) inlet velocity of steam are required before arriving at these values. Further, the last assumption of value of carry-over efficiency, although looking arbitrary, is one of the factual values (with usual values of flow-losses and radiation losses from stator, etc.).

Height of the blades:

Area of flow $\times$ velocity of flow $=$ Volume flow rate

$$\pi \times D \times h \times V_t = \dot{m} \nu, \text{ (} \dot{m} \text{ = mass flow rate of steam = 20 kg/s)}$$

($\nu$ = specific volume of steam)

Height of blade,

$$h = \frac{\dot{m} \nu}{(\pi DV_t)}$$

$$= \frac{20 \times 0.2139}{\pi \times 1.2 \times 112.87}$$

$$= 10.54 \text{mm}$$

This height is at inlet to stator, 1st row.

Height at outlet of stator, inlet to rotor, is at 2.

$$\text{Height} = \frac{20 \times 0.25}{\pi \times 1.2 \times 112.87}$$

$$= 12.32 \text{mm}$$

Height at exit of rotor, inlet to next stator = 15.82 mm

The specific volumes and the corresponding heights of the blades are also entered in Table 5.1.
Table 5.1  Expansion of steam from 12 bar, 300 °C

<table>
<thead>
<tr>
<th>State Point</th>
<th>Enthalpy (kJ/kg)</th>
<th>Entropy (kJ/kg)</th>
<th>Pressure (bar)</th>
<th>Temperature (°C)</th>
<th>Specific Volume (m³/kg)</th>
<th>Height of Blade (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3040</td>
<td>7.01</td>
<td>12</td>
<td>300</td>
<td>0.2139</td>
<td>10.54</td>
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<td>2</td>
<td>2990</td>
<td>7.01</td>
<td>9.5</td>
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<td>2'</td>
<td>2998</td>
<td>7.03</td>
<td>9.5</td>
<td>281</td>
<td>0.25</td>
<td>12.32</td>
</tr>
<tr>
<td>3</td>
<td>2948</td>
<td>7.03</td>
<td>7.5</td>
<td>251</td>
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<td></td>
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<td>3'</td>
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<td>7.5</td>
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<td>0.321</td>
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<td>4</td>
<td>2906</td>
<td>7.05</td>
<td>6.0</td>
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<td></td>
</tr>
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<td>4'</td>
<td>2914</td>
<td>7.07</td>
<td>6.0</td>
<td>251</td>
<td>0.382</td>
<td>18.823</td>
</tr>
<tr>
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<td>2864</td>
<td>7.07</td>
<td>4.8</td>
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<td></td>
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<td>4.8</td>
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</tr>
<tr>
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<td>182</td>
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<td></td>
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<td>6'</td>
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<td>3.8</td>
<td>186</td>
<td>0.628</td>
<td>30.945</td>
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<td>7</td>
<td>2780</td>
<td>7.11</td>
<td>2.92</td>
<td>160</td>
<td></td>
<td></td>
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<tr>
<td>7'</td>
<td>2788</td>
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<td>2.92</td>
<td>165</td>
<td>0.746</td>
<td>36.76</td>
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<td>8</td>
<td>2738</td>
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<td>2.25</td>
<td>138</td>
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<td></td>
</tr>
<tr>
<td>8'</td>
<td>2746</td>
<td>7.18</td>
<td>2.25</td>
<td>142</td>
<td>0.985</td>
<td>48.54</td>
</tr>
<tr>
<td>9</td>
<td>2696</td>
<td>7.18</td>
<td>1.6</td>
<td>115</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9'</td>
<td>2704</td>
<td>7.21</td>
<td>1.6</td>
<td>122</td>
<td>1.098</td>
<td>54.1</td>
</tr>
<tr>
<td>10</td>
<td>2654</td>
<td>7.21</td>
<td>1.1</td>
<td>122</td>
<td>1.098</td>
<td>54.1</td>
</tr>
<tr>
<td>10'</td>
<td>2662</td>
<td>7.24</td>
<td>1.1</td>
<td>Dryness fraction</td>
<td>0.995</td>
<td>1.5415</td>
</tr>
<tr>
<td>11</td>
<td>2612</td>
<td>7.24</td>
<td>0.8</td>
<td></td>
<td></td>
<td>75.96</td>
</tr>
<tr>
<td>11'</td>
<td>2620</td>
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<td>0.8</td>
<td>0.985</td>
<td>2.0543</td>
<td>101.226</td>
</tr>
<tr>
<td>12</td>
<td>2570</td>
<td>7.28</td>
<td>0.6</td>
<td>0.975</td>
<td>2.6624</td>
<td>131.19</td>
</tr>
<tr>
<td>12'</td>
<td>2578</td>
<td>7.33</td>
<td>0.6</td>
<td></td>
<td></td>
<td>131.19</td>
</tr>
<tr>
<td>13</td>
<td>2528</td>
<td>7.33</td>
<td>0.4</td>
<td></td>
<td></td>
<td>131.19</td>
</tr>
<tr>
<td>13'</td>
<td>2536</td>
<td>7.37</td>
<td>0.4</td>
<td>0.96</td>
<td>3.8327</td>
<td>188.86</td>
</tr>
<tr>
<td>14</td>
<td>2486</td>
<td>7.37</td>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14'</td>
<td>2494</td>
<td>7.40</td>
<td>0.28</td>
<td>0.945</td>
<td>5.2701</td>
<td>259.69</td>
</tr>
<tr>
<td>15</td>
<td>2444</td>
<td>7.40</td>
<td>0.19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15'</td>
<td>2452</td>
<td>7.45</td>
<td>0.19</td>
<td>0.932</td>
<td>7.4807</td>
<td>–</td>
</tr>
<tr>
<td>16</td>
<td>2402</td>
<td>7.45</td>
<td>0.19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16'</td>
<td>2410</td>
<td>7.51</td>
<td>0.108</td>
<td>0.912</td>
<td>16.51</td>
<td>–</td>
</tr>
<tr>
<td>17</td>
<td>2360</td>
<td>7.51</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17'</td>
<td>2368</td>
<td>7.57</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>2318</td>
<td>7.57</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18'</td>
<td>2326</td>
<td>7.60</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is observed that at the end of the sixth ring of rotor blades, that is, when the pressure is 0.4 bar, point 13, the height of blades has reached 188.86 or 189 mm. At the next step, at the exit of stator blades (7th row), the height of blade works out as 260 mm.
Instead of this, the mean diameter of the rotor is revised as 1.4 m. (The speed cannot be different.) Also, the enthalpy drops are modified: isentropic enthalpy drops are taken as 70 kJ/kg; and with 0.84 efficiency, the actual enthalpy drops are now \(70 \times 0.84 = 58.8\) kJ/kg. The velocity generated is different; blade velocity is different; the velocity triangles are drawn again. The expansion processes are continued from the point 13, and the new levels of pressure, enthalpy, entropy, and specific volumes are recorded, as shown in Table 5.2.

### Table 5.2 Expansion of steam from 0.4 bar, \(x = 0.96\)

<table>
<thead>
<tr>
<th>State Point</th>
<th>Enthalpy (kJ/kg)</th>
<th>Entropy (kJ/kg)</th>
<th>Pressure (bar)</th>
<th>Dryness Fraction</th>
<th>Specific Volume (m³/kg)</th>
<th>Height of Blade (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>2536</td>
<td>7.39</td>
<td>0.4</td>
<td>0.96</td>
<td>3.8327</td>
<td>128.7</td>
</tr>
<tr>
<td>14'</td>
<td>2466</td>
<td>7.39</td>
<td>0.24</td>
<td>0.935</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>2477</td>
<td>7.42</td>
<td>0.24</td>
<td>0.941</td>
<td>0.0616</td>
<td>203.5</td>
</tr>
<tr>
<td>15'</td>
<td>2407</td>
<td>7.42</td>
<td>0.15</td>
<td>0.918</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>2418</td>
<td>7.46</td>
<td>0.15</td>
<td>0.922</td>
<td>9.24</td>
<td>310.27</td>
</tr>
<tr>
<td>16'</td>
<td>2348</td>
<td>7.46</td>
<td>0.08</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>2360</td>
<td>7.52</td>
<td>0.08</td>
<td>0.908</td>
<td>16.438</td>
<td>552.0</td>
</tr>
<tr>
<td>17'</td>
<td>2290</td>
<td>7.52</td>
<td>0.05</td>
<td>0.892</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>2301</td>
<td>7.56</td>
<td>0.05</td>
<td>0.898</td>
<td>25.32</td>
<td>850</td>
</tr>
</tbody>
</table>

For the new velocity triangles (Fig. 5.5)

\[
\text{Blade velocity} = \frac{\pi DN}{60} = \frac{\pi \times 1.4 \times 3000}{60} = 220 \text{m/s}.
\]

Velocity of steam generated = \(\sqrt{2 \times 58800} = 343 \text{ m/s}.

Let inlet velocity, including effect of approach velocity, be equal to 396 m/s.

\[
\Delta V_u = 524.24
\]

**Figure 5.5** Velocity triangles.

As earlier,
\[ V_{ui} = 396 \cos 20 = 372.12 \text{ m/s} \]
\[ V_{ri} = 396 \sin 20 = 135.44 \text{ m/s} \]

Combined velocity triangles are shown in Fig. 5.5.

In the velocity triangles,
\[ AB = U = 220, \quad AC = V_1 = 396, \quad \text{Angle CAB} = 20^\circ \]
\[ AE = V_1 \cos \alpha_1 = 396 \cos 20 = 372.12 \]
\[ CE = V_1 \sin \alpha_1 = 396 \sin 20 = 135.44 \]
\[ BE = 372.12 - 220 = 152.12 \]
\[ \beta_1 = \tan^{-1} \left( \frac{135.44}{152.12} \right) = 41.68^\circ \]
\[ V_2 = BC = 203.68 \]

Again for checking,
\[ V_1 = \sqrt{(203.68 \times 0.95)^2 + (58800 \times 2)} \]
\[ = 395 \]

(Once again, the revision of rotor diameter to 1.4 m and of other parameters is after quite a few trials, so that the new assumed inlet velocity, 396 m/s, matches with the calculated value of 395 m/s, including the effect of approach velocity.)

With the new flow velocity of 135.44 m/s,
Height of blades, \[ h = \frac{20 \times 3.8327}{\pi \times 1.4 \times 135.44} = 128.7 \text{ mm} \]

These calculations are continued until the end of next two rows of rotor blades, together with two rows of stator blades, when the pressure reaches to 0.05 bar, the specified exhaust pressure. Heights of blades, so obtained, are also recorded in Table 5.2.

Now, the specific work,

\[ W = U(\Delta V_u) = U(V_{u1} - V_{u2}) \]
\[ = 220 \times (372.12 + 152.12) \]
\[ = 115332.8 \text{ J/kg} \]
\[ = 115.333 \text{ kJ/kg} \]

There are six rotor-blade-rings up to state-point 13 and two rotor-blade-rings from state point 13 to state-point 17. The total work output is

\[ W_T = (81.375 \times 6) + (115.333 \times 2) \]
\[ = 488.25 + 230.666 \]
\[ = 718.916 \text{ kJ/kg} \]

This total output of 718.916 kJ/kg is against the net input enthalpy drop from 3040 kJ/kg to 2301 kJ/kg, that is, 739 kJ/kg.

It was assumed that the flow rate is 20 kg/s. Hence, the total output power is

\[ P = W_T \times \dot{m} \]
\[ = 718.916 \times 20 \]
\[ = 14378 \text{ kW} \]
Suppose that the overall idea of the expansion process is fully known, along with details such as pressure, temperature, specific volume, and blade height. Further fine-tuning of the details can be undertaken.

For example, a percentage of area of flow, occupied by the thickness of blades, can be assumed; and in effect, the height of the blades can be increased as required.

Another factor is to account for the axial distance of blades. A third factor is to design the structural detail, by considering the strength aspects. Also, the number of blades in a row is not considered so far. The separating distance between the two stretches of expansion is required to be detailed.

A schematic, rough sketch of the layout of the stator/rotor blades is shown in Fig. 5.6, representing the first stretch of expansion, at a mean diameter of 1.2 m.

Finally, the overall rotor efficiency $= \frac{718,916}{739} = 0.9728$

Reheat factor $= \frac{(50 \times 12) + (70 \times 4)}{739} = \frac{880}{739} = 1.19$. 
Figure 5.6  Schematic layout of blades (First part: six rings of rotor blades).

Problem 2
In Problem 1, you have chosen one of the three alternate enthalpy drops. Now, choose one of the remaining two enthalpy drops to design Curtis stages, each stage having two rotors.

For the chosen process, determine the number of stages that may be required for the expansion process, with every stage having equal enthalpy drop, about 100 kJ/kg. Because of the integer number of stages, this enthalpy drop may have to be corrected. Also, because you are referring to the Mollier chart, the numerical values may not be exact. Refer to the steam tables to refine your values. Switch over to the actual processes with an isentropic efficiency of 0.8, as you proceed with the expansion process stage-by-stage. Record the values of enthalpy and specific volume at every point, as was illustrated in the solved Example 5.10.

Recall from your earlier course on thermodynamics that:

1. Any enthalpy drop that generates a velocity more than sonic velocity, requires a convergent-divergent nozzle;
2. A convergent-divergent nozzle can accommodate only a fixed ratio of pressures;
3. This pressure ratio (down-stream to up-stream pressures, $p_2/p_1$) is about 0.53;
4. If this pressure ratio is different, then, the shock-waves occur in the flow with losses.

As a first approximation, for this problem, assume that shock-waves are absent.

An enthalpy drop of about 100 kJ/kg generates higher velocities. Let the exit velocity from each stage be axial. Choose a convenient speed, and a diameter of the rotor, so that the velocities of steam and blades are in agreement with a speed ratio of your choice. Now follow the steps of choosing/designing the other parameters of the turbine such as inlet angle of steam, blade angles, and blade heights (mass flow rate = 10 kg/s). Check, whether the exit velocity, together with generated velocity, gives rise to the entry velocity; and if not, by how much it differs. Complete your project by calculating the efficiencies and the reheat factor.

**Solution:** The process chosen for the design of 2-row Curtis stages is the one commencing at 8 bar, 325 °C. The properties at the starting point are: Pressure 8 bar, temperature 325 °C, enthalpy 3110 kJ/kg, entropy 7.31 kJ/kgK, specific volume 0.339 m³/kg.

The first expansion is in a nozzle ring. The isentropic enthalpy drop is taken as 100 kJ/kg; and the actual enthalpy drop, at 0.9 efficiency, is 90 kJ/kg. Thus at the outlet of the nozzle-ring, the isentropic state is 2' with enthalpy of 3110 – 100 = 3010 kJ/kg; and the actual state is 2 with enthalpy of 3110 – 90 = 3020 kJ/kg. The nozzle ring is followed by the two rows of rotor blades with a row of stator blades between them (pressure and specific volume do not change when the steam flows over these rotor and stator blade rings). This completes one Curtis stage. Such stages follow one after other, until the pressure reaches the exhaust pressure. The entire expansion process is tabulated in Table 5.3 and the components of the expansion process are traced, as shown in Fig. 5.7.
Figure 5.7  Expansion of steam.
Table 5.3  Expansion of steam from 8 bar, 325 °C

<table>
<thead>
<tr>
<th>State Point</th>
<th>Enthalpy (kJ/kg)</th>
<th>Entropy (kJ/kg)</th>
<th>Pressure (bar)</th>
<th>Temperature (°C)</th>
<th>Specific Volume (m³/kg)</th>
<th>Height of Blade (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3110</td>
<td>7.31</td>
<td>8.0</td>
<td>325</td>
<td>0.339</td>
<td>25.94</td>
</tr>
<tr>
<td>2'</td>
<td>3010</td>
<td>7.31</td>
<td>5.5</td>
<td>280</td>
<td>0.464</td>
<td>35.5</td>
</tr>
<tr>
<td>2</td>
<td>3020</td>
<td>7.34</td>
<td>5.5</td>
<td>285</td>
<td>0.464</td>
<td>35.5</td>
</tr>
<tr>
<td>3'</td>
<td>2920</td>
<td>7.34</td>
<td>3.6</td>
<td>235</td>
<td>0.650</td>
<td>49.74</td>
</tr>
<tr>
<td>3</td>
<td>2930</td>
<td>7.36</td>
<td>3.6</td>
<td>240</td>
<td>0.650</td>
<td>49.74</td>
</tr>
<tr>
<td>4'</td>
<td>2830</td>
<td>7.36</td>
<td>2.25</td>
<td>180</td>
<td>0.992</td>
<td>75.9</td>
</tr>
<tr>
<td>4</td>
<td>2840</td>
<td>7.38</td>
<td>2.25</td>
<td>185</td>
<td>0.992</td>
<td>75.9</td>
</tr>
<tr>
<td>5'</td>
<td>2740</td>
<td>7.38</td>
<td>1.4</td>
<td>135</td>
<td>1.375</td>
<td>105.2</td>
</tr>
<tr>
<td>5</td>
<td>2750</td>
<td>7.4</td>
<td>1.4</td>
<td>140</td>
<td>1.375</td>
<td>105.2</td>
</tr>
<tr>
<td>6'</td>
<td>2650</td>
<td>7.4</td>
<td>0.75</td>
<td>Dryness fraction 0.995</td>
<td>2.211</td>
<td>169.18</td>
</tr>
<tr>
<td>6</td>
<td>2660</td>
<td>7.43</td>
<td>0.75</td>
<td>0.998</td>
<td>2.211</td>
<td>169.18</td>
</tr>
<tr>
<td>7'</td>
<td>2560</td>
<td>7.43</td>
<td>0.41</td>
<td>0.969</td>
<td>2.211</td>
<td>169.18</td>
</tr>
<tr>
<td>7</td>
<td>2570</td>
<td>7.45</td>
<td>0.41</td>
<td>0.972</td>
<td>3.79</td>
<td>290</td>
</tr>
<tr>
<td>8'</td>
<td>2470</td>
<td>7.45</td>
<td>0.2</td>
<td>0.938</td>
<td>2.211</td>
<td>169.18</td>
</tr>
<tr>
<td>8</td>
<td>2480</td>
<td>7.48</td>
<td>0.2</td>
<td>0.943</td>
<td>7.21</td>
<td>551.7</td>
</tr>
<tr>
<td>9'</td>
<td>2380</td>
<td>7.48</td>
<td>0.1</td>
<td>0.911</td>
<td>13.43</td>
<td>1027.6</td>
</tr>
<tr>
<td>9</td>
<td>2390</td>
<td>7.51</td>
<td>0.1</td>
<td>0.915</td>
<td>13.43</td>
<td>1027.6</td>
</tr>
<tr>
<td>10'</td>
<td>2290</td>
<td>7.51</td>
<td>0.05</td>
<td>0.891</td>
<td>25.23</td>
<td>1.93 m</td>
</tr>
<tr>
<td>10</td>
<td>2300</td>
<td>7.54</td>
<td>0.05</td>
<td>0.895</td>
<td>25.23</td>
<td>1.93 m</td>
</tr>
</tbody>
</table>

The velocity generated due to actual enthalpy drop is given by

\[ V = \sqrt{2(\Delta h)} = \sqrt{2 \times 90000} = 424.26 \text{ m/s} \]

However, including the effect of approach velocity, the velocity at the outlet of nozzles is taken as 430 m/s. It is now required to construct velocity triangles for a two-row Curtis stage; the following assumptions are made:

1. Mean diameter of rotor at mid-height of blades, \( D = 1.23 \text{ m} \)
2. Speed of the rotor, \( N = 1500 \text{ rpm} \)
3. Blade angle at outlet of second row, \( \beta_{22} = 35^\circ \)
4. Velocity at outlet from second rotor blade row is axial, that is, no whirl component at outlet, \( \alpha_{22} = 90^\circ \)
5. Coefficient of blade friction, \( c_b = 0.95 \) for all three rows
6. The flow components of velocity, \( V_{f11}, V_{f12}, V_{f21}, V_{f22} \), are all equal to one another (i.e. axial thrust is zero).

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With the above assumptions, the velocity triangles are drawn as shown in Fig. 5.8. The construction of velocity triangles is in the reverse direction (last triangle first), because the absolute velocity at the exit of second row of rotor blades is required to be axial. The steps are mentioned as follows:

1. Blade velocity, \( U = \frac{\pi DN}{60} = \frac{\pi \times 1.23 \times 1500}{60} = 96.61 \text{ m/s} \).

2. Assume a scale, 1 cm = 25 m/s.

3. Draw a horizontal line, mark \( U = 96.61 = AB \) on it.
Figure 5.8 Velocity triangles for 2-row Curtis stage (project oriented question 2, ch.5)
Figure 5.8  Velocity triangles for 2-row Curtis stage.

4. Erect a perpendicular at A \( (\alpha_{22} = 90^\circ) \), a line at \( \beta_{22} = 35^\circ \) to BA at B, to locate D_2. ABD_2 is the outlet velocity triangle of second row of blades.

5. \( AD_2 = V_{f22} = U \tan \beta_{22} = 96.61 \tan 35 = 67.64 \text{ m/s} \).

6. BD_2 is \( V_{r22} \). Transfer \( (V_{r22}/0.95) \), with centre at B, and locate C_2 on a line, parallel to AB, from D_2. Join AC_3 and BC_2. The triangle ABC_2 is the inlet velocity triangle of second row of rotor blades. BC_2 is \( V_{r21} \), AC_2 is \( V_{21} \).

7. Transfer \( (AC_2/0.95) \), that is, \( (V_{21}/0.95) \), with centre at A, to locate D_1, on a line from D_2 parallel to AB. Join AD_1, BD_1. ABD_1 is outlet velocity triangle of first row of blades. AD_1 = V_{12}, BD_1 = V_{112}.

8. Transfer \( (BD_1/0.95) \), that is, \( (V_{112}/0.95) \), with centre at B, to locate C_1, on a line from D_2, parallel to AB. Join AC_1, BC_1. ABC_1 is the inlet velocity triangle of the first row of rotor blades. AC_1 is \( V_{11} \), BC_1 = \( V_{r11} \).

9. Check for values of \( V_{11} \) as obtained. This \( V_{11} \) is the outlet velocity of steam from the nozzles to the inlet of the first row of rotor blades.

We have

\[
V_{11} = \sqrt{V_{22}^2 + 2 \times \Delta h}
\]

\[
= \sqrt{67.64^2 + 2 \times 90000}
\]

\[
= 429.6 \text{ m/s}
\]

The value checks very well with the value obtained from the velocity triangles.

This is not a coincidence. After quite a few trials with trial values of the diameter and speed of rotor, angles at inlet and outlet of blades, steam angles, etc., the above values are arrived at.

Finally, the results are as follows (inclusive of initial assumptions):

1. Mean diameter of rotor, \( D = 1.23 \text{ m} \).
2. Speed of the rotor, \( N = 1500 \text{ rpm} \).
3. Steam inlet velocity to the first rotor blade ring, \( V_{11} = 429.6 \text{ m/s} \).
4. Angle if inclination of nozzles, \( \alpha_{11} = 9.5^\circ \).
5. Blade inlet angle, first rotor, \( \beta_{11} = 12^\circ \).
6. Blade outlet angle, first rotor, \( \beta_{12} = 13^\circ \).
7. Stator blade inlet angle, $\alpha_{12} = 18^\circ$.
8. Stator blade outlet angle, $\alpha_{21} = 19^\circ$.
9. Blade inlet angle, second rotor, $\beta_{21} = 34.5^\circ$.
10. Blade outlet angle, second rotor, $\beta_{22} = 35^\circ$.
11. Outlet angle of steam, second rotor, $\alpha_{22} = 90^\circ$.
12. Flow component of steam velocities at inlet and outlet of the two rotors (all are equal), 67.64 m/s.
13. Effective change in whirl component, $(\Delta V_u)_1 = 627.5$ m/s in the first ring and $(\Delta V_u)_2 = 196.25$ m/s in the second ring.
14. Specific work in the first ring, $W_1 = U \cdot (\Delta V_u)_1$
   
   $= 96.61 \times 627.5$
   
   $= 60622.775$ J/kg

15. Specific work in the second ring, $W_2 = U \cdot (\Delta V_u)_2$

   $= 96.61 \times 196.25$

   $= 18959.7$ J/kg

16. Stage work output $= W_1 + W_2$

   $= 60.623 + 18.96$

   $= 79.583$ kJ/kg

The velocity triangles hold good for the successive stages (unless any parameter is altered), because the steam velocity is again 429.6 m/s at the entry of the next rotor ring, after the nozzle ring. The blade geometries are retained, except the height of the blades.

The heights of the successive blade rings keep increasing, due to the increasing specific volumes of the steam, even though the flow velocity remains constant. The specific volumes are recorded in Table 5.3.

We have

\[ \pi Dh \times V_f = \dot{m}v \]

Therefore,
\[ h = \frac{\dot{m}v}{\pi D V_r} \]

Thus, in the first stage, first rotor ring,

**Height of blades**

\[
= \frac{20 \times 0.339}{\pi \times 1.23 \times 67.64} = 25.94 \text{ mm}
\]

The heights of blades are also recorded in Table 5.3.

It is seen that over the range of expansion, the increase of specific volume is so much that the heights of blades also increase very much. At the state point 7, that is, after six rows of nozzles, the height of blades is 290 mm. Because the next height is 551.7 mm (which is not acceptable), the diameter of rotor is to be increased.

Along with increase of diameter, the pressure ratio of expansion is also increased. In fact, only two more expansions are planned, one expansion is from 0.41 bar to 0.16 bar \((\Delta h = 2570 \text{ to } 2444 = 126 \text{ kJ/kg})\) and then from 0.16 bar to 0.05 bar \((\Delta h = 2444 \text{ to } 2300 = 144 \text{ kJ/kg})\). After the initial diameter of 1.23 m, the diameter is fixed as 1.46 m for one stage; and then for the next stage (last stage), the diameter is 1.6 m. Heights of the blades are calculated accordingly. The velocity triangles hold good, with two different scales, for the last two stages. Hence, all angles remain as earlier; the values of pressure, enthalpy, etc. are recorded in Table 5.4. These new expansion processes are shown in Fig. 5.7, in the dotted line form.

**Table 5.4** Expansion of steam from 0.41 bar, \(x = 0.972\)

<table>
<thead>
<tr>
<th>State Point</th>
<th>Enthalpy (kJ/kg)</th>
<th>Entropy (kJ/kg)</th>
<th>Pressure (bar)</th>
<th>Dryness Fraction</th>
<th>Specific Volume (m^3/kg)</th>
<th>Height of Blade (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2570</td>
<td>7.45</td>
<td>0.41</td>
<td>0.972</td>
<td>3.79</td>
<td>290</td>
</tr>
<tr>
<td>8’</td>
<td>2430</td>
<td>7.45</td>
<td>0.16</td>
<td>0.925</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2444</td>
<td>7.49</td>
<td>0.16</td>
<td>0.93</td>
<td>8.77</td>
<td>477</td>
</tr>
<tr>
<td>9’</td>
<td>2284</td>
<td>7.49</td>
<td>0.05</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>2300</td>
<td>7.55</td>
<td>0.05</td>
<td>0.9</td>
<td>25.37</td>
<td>1380</td>
</tr>
</tbody>
</table>

As earlier, the trials are required, to get the correct values of diameter of rotor, etc.

The overall results are recorded in Table 5.5.

**Table 5.5** Overall results.

<table>
<thead>
<tr>
<th>Stages</th>
<th>(U)</th>
<th>(\Delta V_{u1})</th>
<th>(\Delta V_{u2})</th>
<th>(W_1)</th>
<th>(W_2)</th>
<th>(W_T) (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–6</td>
<td>96.61</td>
<td>627.5</td>
<td>196.25</td>
<td>60.623</td>
<td>18.96</td>
<td>477.5</td>
</tr>
</tbody>
</table>
Total work = 724.182 kJ/kg

Total enthalpy drop = 3110 – 2300 = 810 kJ/kg

Rotor efficiency = \( \frac{724.182}{810} = 0.894. \)

The total stagewise isentropic enthalpy drops = \((100 \times 6) + 140 + 154\)

= 894 kJ/kg

The one-stretch isentropic enthalpy drop from 8 bar 325° to 0.05 bar = 3110 – 2240

= 870 kJ/kg

Therefore, reheat factor = \( \frac{894}{870} = 1.0276 \)

Capacity of the plant = 724.182 \( \times \) 20 = 14483 kW.