# AIEEE 2011

## Important Instructions

1. *Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of Pencil is strictly prohibited.*
2. *The Answer Sheet is kept inside the Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.*
3. *The test is of 3 hours duration.*
4. *Each question is allotted 4(four) marks for each correct response.*
5. *Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
6. *There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.*
7. *Use Blue/Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet. Use of pencil is strictly prohibited.*
8. *No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.*
9. *On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.*
10. *The CODE for this Booklet is Q. Make sure that the CODE printed on Side-2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.*
11. *Do not fold or make any stray marks on the Answer Sheet.*

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**1.** The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x,t)=e^{-(ax^2+bt^2+2\sqrt{ab}\,xt)}$. This represents a

(a) Wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$

(b) Standing wave of frequency $\sqrt{b}$

(c) Standing wave of frequency $\frac{1}{\sqrt{b}}$

(d) Wave moving in $+x$ direction with $\sqrt{\frac{a}{b}}$

**Solution**

**(a)** The general equation of a wave is

$$y(x,t)=f(x+vt)$$

Therefore

$$y(x,t)=e^{-\left(\sqrt{ax}+\sqrt{bt}\right)^2}$$

or

$$v=\sqrt{\frac{b}{a}}.$$

Therefore the wave travels in the negative $x$-direction with velocity $v=\sqrt{\frac{b}{a}}$.

**2.** A screw gauge gives the following reading when used to measure the diameter of a wire. Main scale reading: 0 mm Circular scale reading: 52 divisions. Given that 1 mm on main scale corresponds to 100 divisions of circular scale. The diameter of the wire is:

(a) 0.052 cm (b) 0.026 cm
(c) 0.005 cm (d) 0.52 cm

**Solution**

**(a)** Least count = one main scale division / no of divisions in head scale.

$$\text{L.C} = \frac{1}{100}\text{mm}$$

The reading of the screw gauge = (main scale reading )+ (head scale reading × L.C)

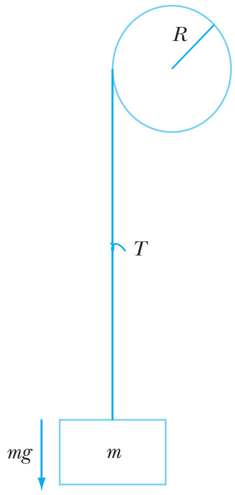
$$d = 0\text{ mm} + \left(\frac{52\times 1}{100}\right)\text{mm} = 0.52\text{mm} = 0.052\text{cm}$$

**3.** A mass $m$ hangs with help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass $m$ and radius $R$. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass $m$, if the string does not slip on the pulley, is

(a) $g$ (b) $\frac{2}{3}g$ (c) $\frac{g}{3}$ (d) $\frac{3}{2}g$

**Solution**

**(b)**




The net force acting on the body is $ma = mg - T$

and the net torque on the pulley.

$$\tau = R \times T = I\alpha$$

But

$$\alpha = \frac{a}{R}.$$

Then

$$TR = \frac{mR^2}{2}\frac{a}{R} \text{ or } T = \frac{ma}{2}$$

Therefore

$$mg = \frac{3}{2}ma \text{ or } a = \frac{2}{3}g.$$

**4.** Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (surface tension of soap solution = 0.03 $\text{Nm}^{-1}$):

(a) $0.2\pi$ mJ (b) $2\pi$ mJ (c) $0.4\pi$ mJ (d) $4\pi$ mJ

**Solution**

**(c)** The work done to change the radius of soap bubble is

$$E = 8\pi T\left(r_f^2 - r_i^2\right)$$

where $r_i$ = 3cm and $r_f$ = 5cm. Then

$$E = 8\pi T(25\times10^{-4} - 9\times10^{-4})$$
$$= 0.4\pi \text{ mJ}.$$

**5.** A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc:

(a) continuously decreases
(b) continuously increases
(c) first increases and then decreases
(d) remains unchanged

**Solution**

**(c)** While the insect approaching the centre the moment of inertia decreases and while it is moving away from the centre the moment of inertia increases. But angular momentum of the disc is conserved. Therefore the angular speed of the disc first increases and then decreases.

**6.** Two particles are executing simple harmonic motion of the same amplitude $A$ and frequency $\omega$ along the $x$-axis. Their mean position is separated by distance $X_0$ ($X_0 > A$). If the maximum separation between them is ($X_0 + A$), the phase difference between their motion is:

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

**Solution**

**(d)** Assume the maximum separation happens when the particle one is at equilibrium position, then for maximum displacement the particle two is at its maximum displaced position or the phase difference is $\pi/2$.

**7.** Two bodies of masses $m$ and 4 $m$ are placed at a distance $r$. The gravitational potential at a point on the line joining them where the gravitational field is zero is:

(a) $-\frac{4Gm}{r}$ (b) $-\frac{6Gm}{r}$

(c) $-\frac{9Gm}{r}$ (d) zero

**Solution**

**(c)** Let $m$ be sitting at the origin then the gravitational field is zero at

$$\frac{Gm}{l^2} = \frac{4Gm}{(r-l)^2}$$

or

$$\frac{(r-l)^2}{l^2} = 4$$

$$r - l = 4 \text{ for } l = \frac{r}{3}.$$

Therefore the potential at that point is

$$V = -\left(\frac{Gm}{l} + \frac{4Gm}{(r-l)}\right)$$

$$V = -\left(\frac{Gm}{\frac{r}{3}} + \frac{4Gm}{\left(r - \frac{r}{3}\right)}\right)$$

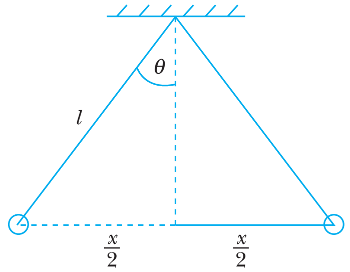
$$V = \frac{-9Gm}{r}.$$

**8.** Two identical charged spheres suspended from a common point by two massless strings of length $l$ are initially a distance $d$ ($d$ << 1) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity $v$. Then as a function of distance $x$ between them,

(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$
(c) $v \propto x$ (d) $v \propto x^{-1/2}$

**Solution**

**(d)**




As $x << l$;  $\tan\theta \simeq \sin\theta = \dfrac{x}{2l}$

$$\Rightarrow \frac{x}{2l} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{x^2 mg}$$

$$\Rightarrow \frac{x^3}{2l}(4\pi\varepsilon_0)mg = q_2$$

$$\Rightarrow 4\pi\varepsilon_0 \frac{mg}{2l} 3x^2 \frac{dx}{dt} = 2q\frac{dq}{dt}$$

$$\Rightarrow \frac{dx}{dt} \propto \frac{q}{x^2}$$

But $q^2 \propto x^3$

Hence $\dfrac{dx}{dt} \propto x^{-1/2}$

$$\Rightarrow v \propto x^{-1/2}.$$

**9.** A boat is moving due east in a region where the earth's magnetic field is $5.0\times10^{-5}$ NA$^{-1}$m$^{-1}$ due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms$^{-1}$, the magnitude of the induced emf in the wire of aerial is:

(a) 0.75 mV  (b) 0.50 mV  (c) 0.15 mV  (d) 1 mV

**Solution**

**(c)** The induced emf

$$\varepsilon = Blv$$
$$= 5.0\ 10^{-5} \times 2 \times 1.5 = 0.15 \text{ mV}.$$

**10.** An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by: $\dfrac{dv}{dt} = -2.5\sqrt{v}$ where $v$ is the instantaneous speed. The time taken by the object, to come to rest, would be:

(a) 2 s  (b) 4 s  (c) 8 s  (d) 1 s

**Solution**

**(a)** It is given that

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

or  $$dt = -\frac{dv}{2.5\sqrt{v}}$$

$$t = \int_{6.25}^{0} -\frac{dv}{2.5\sqrt{v}}$$

$$t = -\left|\frac{2\sqrt{v}}{2.5}\right|_{6.25}^{0} = 2 \text{ s}$$

**11.** A fully charged capacitor $C$ with initial charge $q_0$ is connected to a coil of self inductance $L$ at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic field is:

(a) $\dfrac{\pi}{4}\sqrt{LC}$  (b) $2\pi\sqrt{LC}$  (c) $\sqrt{LC}$  (d) $\pi\sqrt{LC}$

**Solution**

**(a)** The charge in the capacitor at time $t$ is

$$Q = Q_0 \cos \omega t \qquad (1)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

The energy in the circuit is

$$E = \frac{Q_0^2}{2C}$$

The energy becomes the half when the charge in the capacitor is

$$\frac{Q_0^{\,2}}{2C} = 2\frac{Q^2}{2C}$$

$$\Rightarrow Q = \frac{Q_0}{\sqrt{2}} \Rightarrow \frac{Q_0}{\sqrt{2}} = Q_0 \cos\omega t$$

$$\Rightarrow \omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega}$$

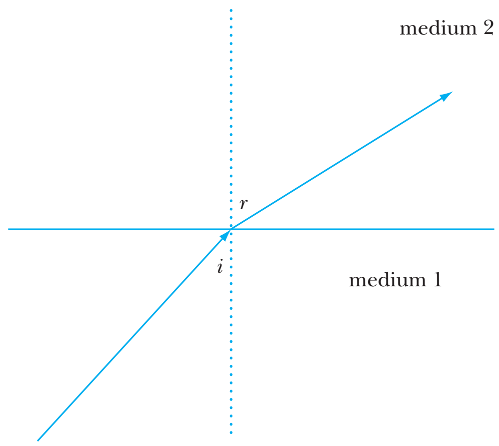
So,  $$t = \frac{\pi}{4}\sqrt{LC}.$$

**12.** Let the $x$–$z$ plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is

(a) 45°  (b) 60°  (c) 75°  (d) 30°

**Solution**

**(d)**




We have from Snell's Law, $\mu_1 \sin i = \mu_2 \sin r$

$$\mu_1(\hat{n}_1 \times \vec{k}_1) - \mu_2(\hat{n}_2 \times \vec{k}_2) = 0$$

$$\mu_1(-\hat{z} \times (6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k})) = \mu_1 |k_1| \sin i$$

$$\mu_1 |k_1| \sin i = \mu_1(-8\sqrt{3}\hat{i} - 6\sqrt{3}\hat{j})$$

or $$\mu_1 \sin i = \frac{|-8\sqrt{3}\hat{i} - 6\sqrt{3}\hat{j}|}{|k_1|} = \frac{\sqrt{3}}{2}$$

$$\sin r = \frac{\mu_1 \sin i}{\mu_2} = \frac{1}{2} \text{ or } r = 30°.$$

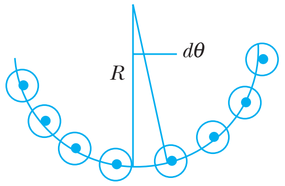
**13.** A current $I$ flows in an infinitely long wire with cross section in the form of a semicircular ring of radius $R$. The magnitude of the magnetic induction along its axis is

(a) $\frac{\mu_0 I}{2\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi R}$

(c) $\frac{\mu_0 I}{4\pi^2 R}$ (d) $\frac{\mu_0 I}{\pi^2 R}$

**Solution**

**(d)**




The magnetic field is

$$dB = \frac{\frac{1}{\pi R} R d\theta \mu_0}{2\pi R} \sin\theta$$

Therefore the net magnetic field is

$$B = \int_0^{\frac{\pi}{2}} \frac{\mu_0 I}{2\pi R^2} \sin\theta \, d\theta$$

$$= \frac{\mu_0 I}{\pi R^2}.$$

**14.** A thermally insulated vessel contains an ideal gas of molecular mass $M$ and ratio of specific heats $\gamma$. It is moving with speed $v$ and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by:

(a) $\frac{(\gamma-1)}{2\gamma R} Mv^2 K$ (b) $\frac{\gamma M v^2}{2R} K$

(c) $\frac{(\gamma-1)}{2R} Mv^2 K$ (d) $\frac{(\gamma-1)}{2(\gamma+1)R} Mv^2 K$

**Solution**

**(c)** The heat gained by the gas

$$\Delta E = Q = \frac{1}{2} mv^2$$

Since the process is isochoric

$$C_v = C_p - R$$

But

$$\frac{C_p}{C_v} = \gamma$$

Therefore

$$C_v = \frac{R}{\gamma - 1}$$

The heat gained is

$$Q = nC_v dT$$

$$\frac{1}{2} Mv^2 = \frac{R}{\gamma - 1} dT$$

$$dT = \frac{(\gamma-1)Mv^2}{2R} K.$$

**15.** A mass $M$, attached to a horizontal spring, executes S.H.M. with amplitude $A_1$. When the $M$ passes through its mean position then a smaller mass $m$ is placed over it and both of them move together with amplitude $A_2$. The ratio of $\left(\frac{A_1}{A_2}\right)$ is:

(a) $\frac{M+m}{M}$ (b) $\left(\frac{M}{M+m}\right)^{1/2}$

(c) $\left(\frac{M+m}{M}\right)^{1/2}$ (d) $\frac{M}{M+m}$

**Solution**

**(c)** From law conservation of momentum

$$Mv_1 = (M+m)\, v_2$$

or $$\frac{v_1}{v_2} = \frac{M+m}{M}$$

The energy of vibrating spring is

$$E = \frac{1}{2} kA^2.$$

But since no energy is supplied from outside, the energy before adding the mass $m$ and after adding the mass $m$ are same.

$$\frac{1}{2} Mv_1^2 = \frac{1}{2} kA_1^2$$

and $$\frac{1}{2}(M+m)v_2^2 = \frac{1}{2} kA_2^2.$$

Therefore

$$\frac{A_1}{A_2} = \sqrt{\frac{M}{m+M}} \left(\frac{v_1}{v_2}\right) = \left(\frac{M+m}{M}\right)^{1/2}$$

**16.** Water is flowing continuously from a tap having an internal diameter $8 \times 10^{-3}$ m. The water velocity as it leaves the tap is 0.4 ms$^{-1}$. The diameter of the water stream at a distance $2 \times 10^{-1}$ m below the lap is close to:

(a) $7.5 \times 10^{-3}$ m (b) $9.6 \times 10^{-3}$ m
(c) $3.6 \times 10^{-3}$ m (d) $5.0 \times 10^{-3}$ m

**Solution**

**(c)** From

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_0 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$2gh + v_1^2 = v_2^2$$

or $$v_2 = \sqrt{2\times 10\times 2\times 10^{-1} + (0.4)^2} = 2.03\text{m/s}.$$

The volume of the water flowing in the both cross-sections is same.

$$A_1 v_1 = A_2 v_2$$

$$\frac{r_1}{r_2} = \sqrt{\frac{v_2}{v_1}} = \sqrt{\frac{2.03}{0.4}} = 2.25$$

or $$r_2 = \frac{r_1}{2.25} = \frac{(4\times 10^{-3})}{2.25}\text{m} = 1.77\times 10^{-3}\text{m}$$

Or the diameter is $d_2 = 2r_2 = 3.6\times 10^{-3}$ m.

**17.** This question has Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements.

**Statement-1:** Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

**Statement-2:** The state of ionosphere varies from hour to hour, day to day and season to season.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is false.

**Solution**

**(a)**

**18.** Three perfect gases at absolute temperatures $T_1$, $T_2$ and $T_3$ are mixed. The masses of molecules are $m_1$, $m_2$ and $m_3$ and the number of molecules are $n_1$, $n_2$ and $n_3$ respectively. Assuming no loss of energy, the final temperature of the mixture is:

(a) $\dfrac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$
(b) $\dfrac{n_1T_1 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$
(c) $\dfrac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$
(d) $\dfrac{(T_1 + T_2 + T_3)}{3}$

**Solution**

**(a)** The energy of each of the gases before mixing is

$$E_1 = \frac{3}{2}n_1kT_1$$

$$E_2 = \frac{3}{2}n_2kT_2$$

$$E_3 = \frac{3}{2}n_3kT_3$$

The total energy of the gas after mixing is

$$E = \frac{3}{2}kT(n_1 + n_2 + n_3)$$

But since there is no loss of energy the total energy is

$$E = E_1 + E_2 + E_3$$

$$\frac{3}{2}kT(n_1 + n_2 + n_3) = \frac{3}{2}k(n_1T_1 + n_2T_2 + n_3T_3)$$

or $$T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}.$$

**19.** A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where $t$ is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kgm$^{-2}$, the number of rotations make by the pulley before its direction of motion if reversed, is:

(a) more than 3 but less than 6
(b) more than 6 but less than 9
(c) more than 9
(d) less than 3

**Solution**

**(a)**

$$F = 20\,t - 5\,t^2$$

$$\tau = 40\,t - 10\,t^2$$

$$\alpha = \frac{\tau}{I} = 4t - t^2$$

$$\int_0^W dW = 4\int_0^t t\,dt - \int_0^t t^2\,dt$$

$$\Rightarrow W = 2t^2 - \frac{t^3}{3}.$$

When $W = 0$ then $t = 6$ s

$$\int_0^\theta d\theta = \int_0^6 W\,dt$$

$$\Rightarrow \theta = 2\int_0^6 t^2\,dt - \frac{1}{3}\int_0^6 t^3\,dt = 36$$

$$N = \frac{36}{2\pi} < 6 \text{ revolutions.}$$

**20.** A resistor '$R$' and 2 $\mu$F capacitor in series are connected through a switch to 200V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of $R$ to make the bulb light up 5 s after the switch has been closed. ($\log_{10} 2.5 = 0.4$)

(a) $1.7\times 10^5\ \Omega$
(b) $2.7\times 10^6\ \Omega$
(c) $3.3\times 10^7\ \Omega$
(d) $1.3\times 10^4\ \Omega$

**Solution**

**(b)** The voltage in the circuit after the switch has been closed

$$V = V_0\left(1 - e^{-t/RC}\right)$$

The bulb lights up at V = 120 volts and $V_0$ = 200 volts.

$$\frac{120}{200} = 1 - e^{-t/RC}$$

$$e^{-t/RC} = 1 - \frac{3}{5} = \frac{2}{5}$$

$$\frac{t}{RC} = \log_{10} 2.5 \times 2.303$$

or $$R = \frac{5}{2\times10^{-6}\times0.4\times2.330} = 2.7\times10^{6}\,\Omega.$$

**21.** A Carnot engine operating between temperatures $T_1$ and $T_2$ has efficiency $\frac{1}{6}$. When $T_2$ is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then $T_1$ and $T_2$ are, respectively:

(a) 372 K and 330 K (b) 330 K and 268 K
(c) 310 K and 248 K (d) 372 K and 310 K

**Solution**

**(d)**

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{6} = 1 - \frac{T_2}{T_1} \text{ or } \frac{T_2}{T_1} = \frac{5}{6} \quad (1)$$

After lowering $T_2$ by 62 K,

$$\frac{1}{3} = 1 - \frac{T_2 - 62}{T_1}$$

$$\frac{T_2 - 62}{T_1} = \frac{2}{3} \quad (2)$$

Dividing (2) by (1),

$$\frac{T_2 - 62}{T_2} = \frac{4}{5}$$

or $$T_2 = 310\text{ K}.$$

By substituting $T_2$ in (1)

$$T_1 = 372\text{ K}.$$

**22.** If a wire is stretched to make it 0.1% longer, its resistance will:

(a) increase by 0.2% (b) decrease by 0.2%
(c) decrease by 0.05% (d) increases by 0.05%

**Solution**

**(a)** The volume of the wire is constant

Therefore

$$\pi r_1^2 l_1 = \pi r_2^2 (l + 0.001l)$$

or

change in $$r_2^2 = \frac{r_1^2}{1.001}$$

The resistance

$$R \propto \frac{l}{A}$$

$$\frac{R_2}{R_1} = \frac{1.001l}{l} \times \frac{1.001r_1^2}{r_1^2} = 1.002.$$

The resistance increases by 0.2%.

**23.** Direction:

The question has a paragraph followed by two statements, Statement–1 and statement–2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane–convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

**Statement–1:** When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of $\pi$.
**Statement–2:** The centre of the interference pattern is dark.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is false.

**Solution**

**(b)** When the light is reflected at the interface of two media, if the light is reflected back to the rarer medium, it light suffers a phase change of $\pi$.

**24.** A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is:

(a) $\frac{1}{15}$ m/s (b) 10 m/s
(c) 15 m/s (d) $\frac{1}{10}$ m/s

**Solution**

**(a)** We have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Therefore

$$v = \frac{2.8}{15}.$$

The magnification is

$$\frac{v}{u} = \frac{1}{15}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v^2}\left(\frac{dv}{dt}\right) + \frac{1}{u^2}\left(\frac{du}{dt}\right) = 0.$$

or $$\frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right) = \left(\frac{1}{15}\right)^2 \times 15 = \frac{1}{15}\text{m/s}.$$

**25.** Energy required for the electron excitation in $Li^{++}$ from the first to the third Bohr orbit is:

(a) 36.3 eV (b) 108.8 eV
(c) 122.4 eV (d) 12.1 eV

**Solution**

**(b)** The energy levels of the a hydrogen-like atom is given by.

$$E_n = \frac{-13.6Z^2\text{eV}}{n^2}$$

Therefore the energy required to excite the electron in $Li^{++}$ is

$$E_3 - E_1 = -13.6 \times 3^2\left(\frac{1}{3^2} - 1\right) = 108.8\,\text{eV}.$$

**26.** The electrostatic potential inside a charged spherical ball is given by $\phi = a\rho^2 + b$ where $r$ is the distance from the centre; $a$, $b$ are constants. Then the charge density inside ball is

(a) $-6a\varepsilon_0 r$ (b) $-24\pi a\varepsilon_0 r$
(c) $-6a\varepsilon_0$ (d) $-24\pi a\varepsilon_0 r$

**Solution**

**(c)** The potential is

$$V = ar^2 + b$$

Therefore the electric field is

$$\vec{E} = -\frac{dV}{dr} = 2ar$$

According to Gauss's law

$$\vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_0}$$

or $$Q = -4\pi\varepsilon_0 r^2 \times 2ar$$

$$\rho = \frac{Q}{V} = \frac{-4\pi\varepsilon_0 r^2}{\frac{4}{3}\pi r^3} = -6\varepsilon_0 a.$$

**27.** A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is $v$, the total area around the fountain that gets wet is:

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$ (c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^4}{g}$

**Solution**

**(c)** The maximum radius of the water column is

$$R_{max} = \frac{v^2}{g}.$$

Therefore the area is

$$A = \pi R_{max}^2 = \frac{\pi v^2}{g}.$$

**28.** 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4148 J/kg/K):

(a) 8.4 kJ (b) 84 kJ (c) 2.1 kJ (d) 4.2 kJ

**Solution**

**(a)** The change in internal energy is given by

$$\begin{aligned}\Delta U &= Q\\ &= MC_v dT\\ &= 0.1\text{ kg} \times 4148 \times 20 = 8.4\text{ kJ}.\end{aligned}$$

**29.** The half life of a radioactive substance is 20 minutes. The approximate time interval ($t_2 - t_1$) between the time $t_2$ when $\frac{2}{3}$ of it has decayed and time $t_1$ when $\frac{1}{3}$ of it had decayed is:

(a) 14 min (b) 20 min (c) 28 min (d) 7 min

**Solution**

**(b)** The half life is given by

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

Therefore

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

At time $t_2$, 2/3 of the nuclei have decayed. This means 1/3 remains. That is,

From $$N_t = N_o e^{-\lambda t}$$

$$\frac{N_0}{3} = N_0 e^{-\lambda t_2}$$

$$t_2 = \left(\frac{\log 3}{\log 2}\right)T_{1/2}$$

Similarly

$$t_1 = \left(\frac{\log 3/2}{\log 2}\right)T_{1/2}$$

Therefore

$$t_2 - t_1 = 20\left(\frac{\log 3 - \log 3/2}{\log 2}\right)$$

$$t_2 - t_1 = 20\text{ minutes}.$$

**30.** This question has Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements.

**Statement-1:** A metallic surface is irradiated by a monochromatic light of frequency $\nu > \nu_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are $K_{max}$ and $V_0$ respectively. If the frequency incident on the surface doubled, both $K_{max}$ and $V_0$ are also doubled.

**Statement-2:** The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is false.

**Solution**

**(c)**

$$\text{K.E}_{max} = eV_0 = h\nu - \Phi$$

or $$h\nu = eV_0 + \Phi = \text{K.E}_{max} + \Phi$$

If the frequency is doubled then the energy is

$$\text{K.E}'_{max} = 2h\nu - \Phi \neq 2\text{K.E}_{max}$$