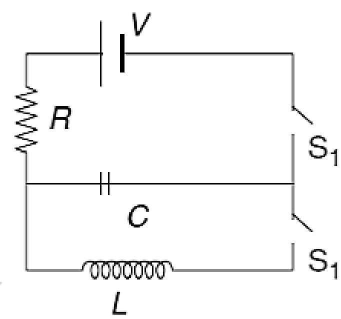
JEE MAIN | 2013
Physics

General Instructions

1. This paper contains 30 Multiple Choice Questions with single correct answer. Each question has four choices (1), (2), (3) & (4), out of which **only one** is Correct.
2. Each question is awarded 4 (four) marks for correct response.
3. One-fourth (1/4) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.

1. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is capacitive time constant). Which of the following statement is correct?



(1) At $t = \tau$, $q = CV/2$
(2) At $t = 2\tau$, $q = CV(1 - e^{-2})$
(3) At $t = \tau/2$, $q = CV(1 - e^{-1})$
(4) Work done by the battery is half of the energy dissipated in the resistor.

Solution:

When switch S_1 is closed and switch S_2 kept open then the given circuit diagram reduces to a series RC circuit having time constant CR. If C is the capacitance and V is the potential difference then the growth of charge on the capacitor with time t is

$$q = CV(1 - e^{-t/\tau})$$

Substituting $t = 2\tau$, where $\tau = RC$ in the above expression, we get $q = CV(1 - e^{-2\tau/\tau})$

$= CV(1 - e^{-2})$

Hence, the correct option is (2).

2. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.

(1) 10.62 kHz (2) 5.31 MHz
(3) 5.31 kHz (4) 10.62 MHz

Solution:

$$\tau = RC = 100 \times 10^{3} \times 250 \times 10^{-12} = 2.5 \times 10^{-5}\ \text{s}$$

The higher frequency that can be detected with tolerable distortion is

$$f = \frac{1}{2\pi\mu\tau}$$

Where, μ is the modulation index

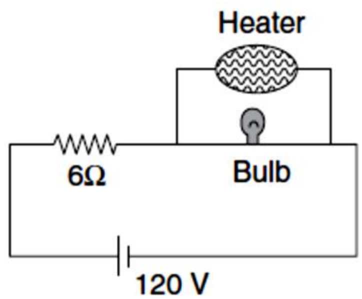
$$f = \frac{1}{2\pi\mu\tau} = \frac{1}{2\times 3.14\times \frac{60}{100}\times 2.5\times 10^{-5}} = 10.62 \text{ kHz}$$

Hence, the correct option is (1).

3. The supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?

(1) 2.9 Volt (2) 13.3 Volt
(3) 10.04 Volt (4) 0 Volt

Solution:



Resistance of bulb $= \frac{120\times 120}{60} = 240\ \Omega$

Resistance of heater $= \frac{120\times 120}{240} = 60\ \Omega$

Voltage across bulb before heater is switched on, with total resistance (240 Ω + 6Ω) = 246 Ω is

$$V_1 = \frac{120}{246}\times 240 = 117.07\text{ V}$$

Voltage across bulb after heater is switched on, with parallel resistance $\frac{240\times 60}{240+60} = 48\ \Omega$

And total resistance (48 Ω + 6 Ω) = 54 Ω is

$$V_2 = \frac{120}{54}\times 48 = 106.66\text{ V}$$

Decrease in the voltage is $V_1 - V_2$ = 10.04 (approximately)

Note: Hence supply voltage is taken as rated voltage.

Hence, the correct option is (3).

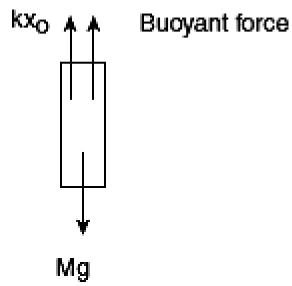
4.A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is:

(1) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (2) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$

(3) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$ (4) $\frac{Mg}{k}$

(Here k is spring constant)

Solution:



At equilibrium sum of all the forces is zero. Tension kx_0 and up thrust which is equal to weight of liquid displaced $AL\sigma g/2$ is directed upwards while weight Mg is directed downwards. Therefore

$$\Sigma F = 0 \Rightarrow kx_0 + \frac{AL\sigma g}{2} - Mg = 0 \Rightarrow kx_0 = Mg - \frac{AL\sigma g}{2} \Rightarrow x_0 = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

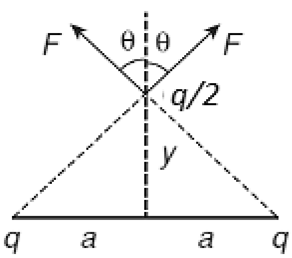
Hence, the correct option is (2).

5. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to:

(1) $-y$ (2) $\frac{1}{y}$

(3) $-\frac{1}{y}$ (4) y

Solution:



From the above given figure the net force will be

$$F_{\text{net}} = 2F\cos\theta = 2 \times \frac{k \times q \times (q/2)}{(\sqrt{a^2 + y^2})^2} \times \frac{y}{\sqrt{a^2 + y^2}} = \frac{kq^2 y}{a^3} \; (y << a)$$

From the above relation we see that net force is directly proportional to y.
Hence, the correct option is (4).

6. A beam of unpolarised light of intensity I_0 is passed through a Polaroid A and then through another Polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is:

(1) $I_0/2$ (2) $I_0/4$
(3) $I_0/8$ (4) I_0

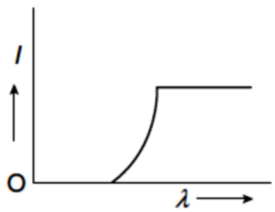
Solution:
After passing through Polaroid A intensity is reduced to half, that is, $I_0/2$ and then before passing through Polaroid B it will be polarized and Malus law is used.
The intensity of the emergent light is

$$I = \frac{I_0}{2}\cos^2 45^\circ = \frac{I_0}{4}$$

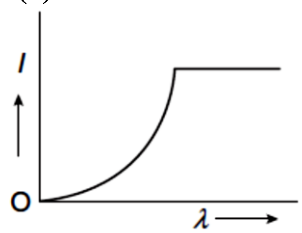
Hence, the correct option is (2).

7. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows:

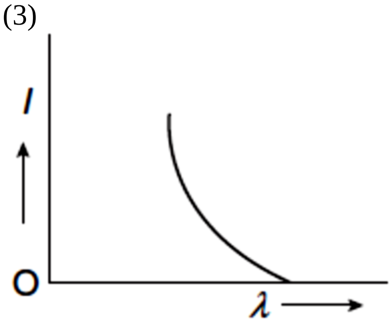
(1)



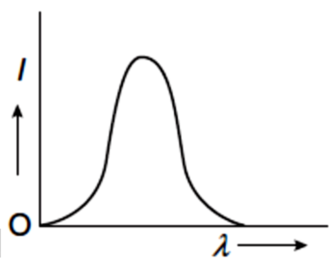
(2)



(3)



(4)

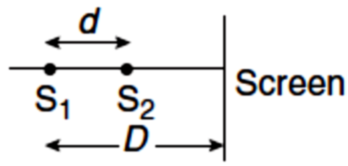


Solution:
Energy of incident photon is given by

$$E = \frac{hc}{\lambda}$$

when λ crosses threshold wavelength, photoemission will stop and current I falls to zero.
Hence, the correct option is (3).

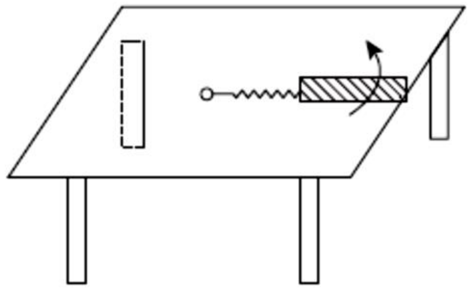
8. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be:



(1) straight lines
(2) semi-circles
(3) concentric circles
(4) points

Solution:
The fringes will be in circular shape as in case of Newton rings.
Hence, the correct option is (3).

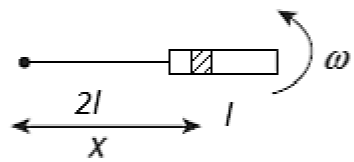
9. A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the e.m.f. induced across the ends of the rod is:



(1) $\frac{3B\omega l^2}{2}$ (2) $\frac{4B\omega l^2}{2}$

(3) $\frac{5B\omega l^2}{2}$ (4) $\frac{2B\omega l^2}{2}$

Solution:



The e.m.f. will only be induced in rod as it is metallic. The limits of integration will be from 2l to 3l.

$$de = B(\omega x)dx \Rightarrow e = B\omega\int_{2l}^{3l} x dx = \frac{5B\omega l^2}{2}$$

Hence, the correct option is (3).

10. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number (n – 1). If $n >> 1$, the frequency of radiation emitted is proportional to

(1) $\frac{1}{n^2}$ (2) $\frac{1}{n^{3/2}}$

(3) $\frac{1}{n^3}$ (4) $\frac{1}{n}$

Solution:

The frequency can be written as, $\nu \propto \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] \propto \frac{(2n-1)}{n^2(n-1)^2}$

Since $n >> 1$. $(n-1)$ can be taken as n and ($2n-1$) as $2n$.

$$\nu \propto \frac{1}{n^3} (\text{since } n >> 1)$$

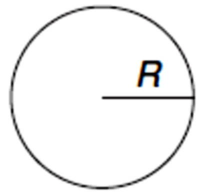
Hence, the correct option is (3).

11. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

(1) $\sqrt{T / \rho L}$ (2) $T / \rho L$

(3) $2T / \rho L$ (4) $\rho L / T$

Solution:



Mass of drop is m = Volume $\times$ Density = $4\pi R^2(\Delta R)\rho$
Heat can be expressed as

$$H = mL = T\Delta S$$

where ΔS is change in surface area. Substituting for m and ΔS we get

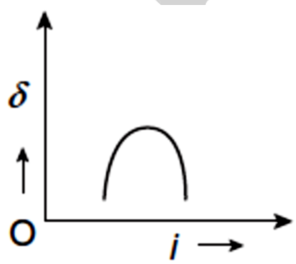
$$4\pi R^2 \Delta R\rho \times L = T \times 4\pi[R^2 - (R-\Delta R)^2] \Rightarrow \quad \rho R^2 \Delta RL = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$$

$$\rho R^2 \Delta RL = 2R\Delta RT \Rightarrow R = \frac{2T}{\rho L} \qquad \text{(neglect } \Delta R^2 \text{ because } \Delta R \text{ is very small)}$$

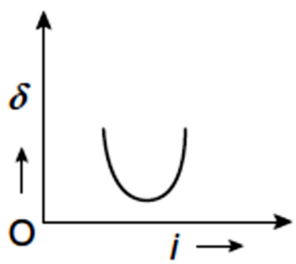
Hence, the correct option is (3).

12. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by:

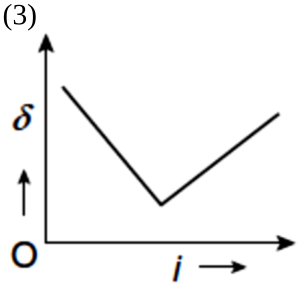
(1)



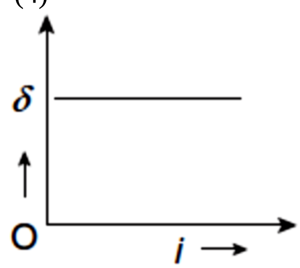
(2)



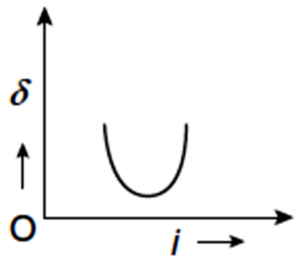
(3)



(4)



Solution:
The variation of between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by



Hence, the correct option is (2).

13. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then:

(1) $[\varepsilon_0]=[\mathrm{M^{-1}L^{-3}T^4A^2}]$ (2) $[\varepsilon_0]=[\mathrm{M^{-1}L^{2}T^{-1}A^{-2}}]$

(3) $[\varepsilon_0]=[\mathrm{M^{-1}L^{2}T^{-1}A}]$ (4) $[\varepsilon_0]=[\mathrm{M^{-1}L^{-3}T^{2}A}]$

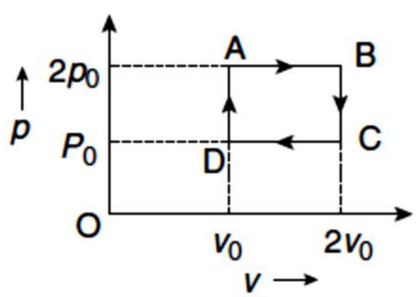
Solution:

According to Coulomb's law the magnitude of force between two charges of magnitude q and separated by distance r is given by relation $F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\Rightarrow \varepsilon_0=\frac{1}{4\pi}\frac{q^2}{Fr^2}$

Substituting the dimensions of $q=[\mathrm{AT}]$, $F=[\mathrm{MLT^{-2}}]$, and $r=[\mathrm{L}]$ we get the dimensional formula of the permittivity of vacuum as $\varepsilon_0=\frac{[\mathrm{A^2T^2}]}{[\mathrm{MLT^{-2}}][\mathrm{L^2}]}=[\mathrm{M^{-1}L^{-3}A^2T^4}]$

Hence, the correct option is (1).

14.



The above *p*-*v* diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat extracted from the source in a single cycle is

(1) $\left(\frac{13}{2}\right)p_0v_0$ (2) $\left(\frac{11}{2}\right)p_0v_0$

(3) $4p_0v_0$ (4) p_0v_0

Solution:

Heat is extracted from the source in path DA and AB.

For path DA, the heat extracted is, $\Delta Q_{\mathrm{DA}}=C_{\mathrm{v}}dT=\frac{3}{2}R\times v_0\times\frac{(2p_0-p_0)}{R}=\frac{3}{2}p_0v_0$

For path AB, the heat extracted is, $\Delta Q_{\mathrm{AB}}=C_{\mathrm{p}}dT=\frac{5}{2}R\times 2p_0\times\frac{(2v_0-v_0)}{R}=5p_0v_0$

Total amount of heat extracted from the source in a single cycle is

$$\Delta Q = \Delta Q_{\text{DA}} + \Delta Q_{\text{AB}} = \frac{3}{2} p_0 v_0 + 5 p_0 v_0 = \frac{13}{2} p_0 v_0$$

Hence, the correct option is (1).

15. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1 %. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively?

(1) 178.2 Hz (2) 200.5 Hz
(3) 770 Hz (4) 188.5 Hz

Solution:
Length of wire l = 1.5 m.
Fundamental frequency

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \qquad (1)$$

Mass per unit length: $\mu = \frac{m}{l} = \frac{Al\rho}{l} = A\rho$

And stress can be calculated as

$$\text{stress} = \frac{T}{A} = Y \times \text{strain} = 2.2 \times 10^{11} \text{N/m}^2 \times 1\% = 2.2 \times 10^{11} \times 10^{-2} \text{ N/m}^2$$

Substituting values in (1) we get

$$f = \frac{1}{2l}\sqrt{\frac{T}{A\rho}} = \frac{1}{2l}\sqrt{\frac{\text{stress}}{\rho}} = \frac{1}{2 \times 1.5}\sqrt{\frac{2.2 \times 10^{11} \times 10^{-2}}{7.7 \times 10^3}} = 178.2\,\text{Hz}$$

Hence, the correct option is (1).

16. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement- I: Higher the range, greater is the resistance of ammeter.
Statement- II: To increase the range of ammeter, additional shunt needs to be used across it.

(1) Statement – I is true, Statement – II is true, Statement – II is not the correct explanation of Statement – I.
(2) Statement – I is true, Statement – II is false.
(3) Statement – I is false, Statement – II is true
(4) Statement – I is true, Statement – II is true, Statement – II is the correct explanation of Statement – I.

Solution:
For ammeter,

$$S = \frac{I_g G}{I - I_g}$$

So, for I to increase, S should decrease, so additional S can be connected across it in parallel.
Hence, the correct option is (3).

17. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$?

(1) $\frac{2GmM}{3R}$ (2) $\frac{GmM}{2R}$

(3) $\frac{GmM}{3R}$ (4) $\frac{5GmM}{6R}$

Solution:

At height $2R$, distance from centre of earth is $(R + 2R) = 3R$.

Total energy is half of gravitational potential energy, that is

$$= \frac{1}{2}\left(\frac{-GMm}{3R}\right) = \frac{-GMm}{6R}$$

Final total energy of satellite at height $2R$ is $\text{T.E.}_f = -\frac{GMm}{6R}$

Initial total energy of satellite is $\text{T.E.}_i = -\frac{GMm}{R}$

Minimum energy required to launch a satellite is $\Delta W = \text{T.E.}_f - \text{T.E.}_i = \frac{5GMm}{6R}$

Hence, the correct option is (4).

18. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10$ m/s^2, the equation of its trajectory is:

(1) $y = 2x - 5x^2$ (2) $4y = 2x - 5x^2$

(3) $4y = 2x - 25x^2$ (4) $y = x - 5x^2$

Solution:

$$x = u_x t = 1t = t$$
$$y = u_y t + \frac{1}{2}(-g)t^2 = 2t - 5t^2$$

Substituting $x = t$ in above relation we get the equation of trajectory as

$$y = 2x - 5x^2$$

Hence, the correct option is (1).

19. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then:

(1) $3C_1 = 5C_2$ (2) $3C_1 + 5C_2 = 0$

(3) $9C_1 = 4C_2$ (4) $5C_1 = 3C_2$

Solution:

For potential to be zero on each capacitor

$$C_1 \times 120 = C_2 \times 200 \Rightarrow C_1 = \frac{200}{120}C_2 \Rightarrow C_1 = \frac{5}{3}C_2 \Rightarrow 3C_1 = 5C_2$$

Hence, the correct option is (1).

20. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(1) $\frac{r\omega_0}{3}$ (2) $\frac{r\omega_0}{2}$

(3) $r\omega_0$ (4) $\frac{r\omega_0}{4}$

Solution:

Moment of inertia about central axis is $I_C = mr^2$

while about rim using parallel axis theorem the moment of inertia is

$I = I_C + mr^2 = mr^2 + mr^2 = 2mr^2$

From conservation of angular momentum: $mr^2\omega_0 = 2mr^2\omega \Rightarrow \omega = \frac{\omega_0}{2}$

The velocity of the centre of the hoop when it ceases to slip is: $v_{CM} = \omega r = \frac{\omega_0 r}{2}$

Hence, the correct option is (2).

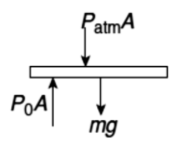
21. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass *M*. The piston and cylinder have equal cross sectional area *A*. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency:

(1) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (2) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$ (4) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$

Solution:

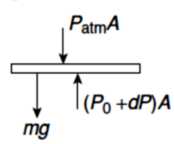
(a) Free body diagram of piston at equilibrium is shown below



(b)

(c) From above figure we get: $P_{atm}A + mg = P_0 A$ (1)

(d) Free body diagram of piston when piston is pushed down a distance *x* is shown below



(e)

(f) Upward pressure of gas increases by *dP* therefore

(g) $$P_{atm}A + mg - (P_0 + dP)A = m\frac{d^2x}{dt^2}$$ (2)

(h) Process is adiabatic, therefore $PV^\gamma = C$

(i) $$\frac{dP}{dV} = -\gamma\frac{P_0}{V_0} \Rightarrow dP = -\gamma\frac{P_0}{V_0}dV$$

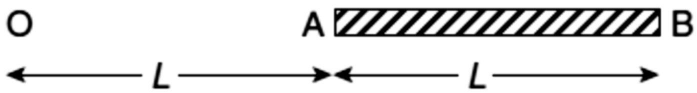
(j) Substituting $dV = Ax$ we get, $dP = -\frac{\gamma P_0 A}{V_0}x$

(k) From (1) and (2) we get, $m\frac{d^2x}{dt^2} = -\frac{\gamma P_0 A^2}{V_0}x$

(l) with $k = \frac{\gamma P_0 A^2}{V_0}$ we get frequency as, $f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

Hence, the correct option is (2).

22. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is:



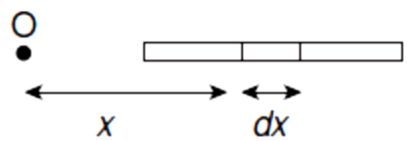
(1) $\frac{3Q}{4\pi\varepsilon_0 L}$

(2) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$

(3) $\frac{Q\ln 2}{4\pi\varepsilon_0 L}$

(4) $\frac{Q}{8\pi\varepsilon_0 L}$

Solution:



The electric potential at the point O lying at a distance L from the end A can be calculated as

$$V = \int_{x=L}^{x=2L} \frac{k}{x}\left(\frac{Q}{L}\right)dx = \frac{kQ}{L}\int_{x=L}^{x=2L}\frac{dx}{x} = \frac{Q\ln 2}{4\pi\varepsilon_0 L}$$

Hence, the correct option is (3).

23. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

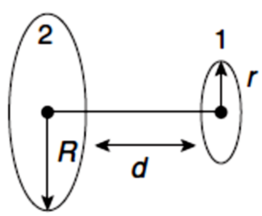
(1) 6×10^{-11} weber

(2) 3.3×10^{-11} weber

(3) 6.6×10^{-9} weber

(4) 9.1×10^{-11} weber

Solution:



Let M_{12} be the coefficient of mutual induction between loops.
The flux linked with smaller loop will be

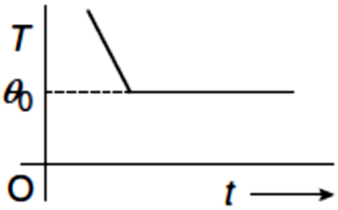
$$\phi_1 = M_{12}i_2 \Rightarrow \frac{\mu_0 i_2 R^2}{2(d^2+R^2)^{3/2}} \times \pi r^2 = M_{12}i_2 \Rightarrow M_{12} = \frac{\mu_0 R^2 \pi r^2}{2(d^2+R^2)^{3/2}}$$

The flux linked with bigger loop will be: $\phi_2 = M_{12}i_1 \Rightarrow \phi_2 = \frac{\mu_0 R^2 \pi r^2 i_1}{2(d^2+R^2)^{3/2}}$

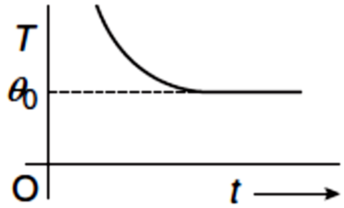
Substituting the values we get: $\phi_2 = 9.1 \times 10^{-11}$ weber

Hence, the correct option is (4).

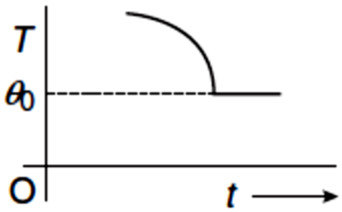
24. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 the graph between the temperature T of the metal and time t will be closest to



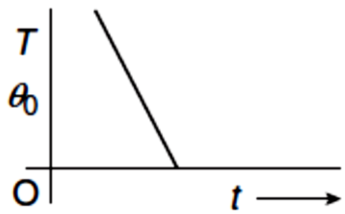
(1)



(2)



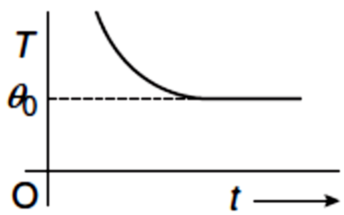
(3)



(4)

Solution:

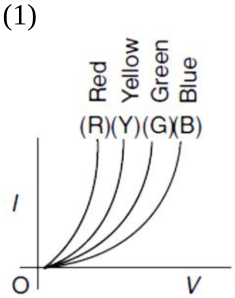
According to Newton's Law of cooling, the temperature goes on decreasing with time (non-linearly) .The rate of decrease of temperature will be more initially which is depicted by slope of the second graph, that is,



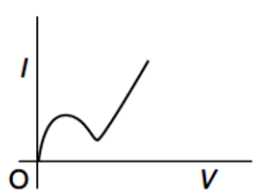
Hence, the correct option is (2).

25. The *I*–*V* characteristic of an LED is

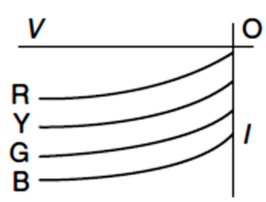
(1)



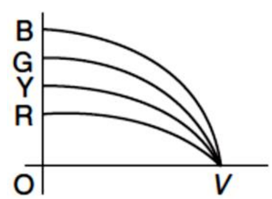
(2)



(3)

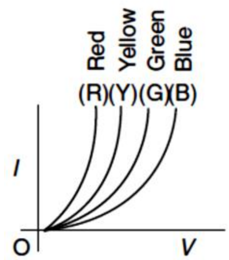


(4)



Solution:

For LED, in forward bias, intensity increases with voltage. Therefore, the *I*–*V* characteristic of an LED is



Hence, the correct option is (1).

26. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements.
Statement – I: A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f=\left(\frac{m}{M+m}\right)$.

Statement – II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(1) Statement – I is true, Statement – II is true, Statement – II is not a correct explanation of Statement – I.
(2) Statement – I is true, Statement – II is false.
(3) Statement – I is false, Statement – II is true
(4) Statement – I is true, Statement – II is true, Statement – II is a correct explanation of Statement – I.

Solution:
Loss of energy is maximum when collision is inelastic as in an inelastic collision there will be maximum deformation.

Kinetic energy in COM frame is = $\frac{1}{2}\left(\frac{Mm}{M+m}\right)v_{\text{rel}}^2$

Initial kinetic energy is $\text{KE}_i=\frac{1}{2}\left(\frac{Mm}{M+m}\right)v^2$ and final kinetic energy is $\text{KE}_f=0$ $(\because v_{\text{rel}}=0)$

Hence loss in energy is $=\frac{1}{2}\left(\frac{Mm}{M+m}\right)v^2 \Rightarrow f=\frac{M}{M+m}$

Hence, the correct option is (3).

27. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5 s. In another 10 s it will decrease to α times its original magnitude, where α equals.

(1) 0.81
(2) 0.729
(3) 0.6
(4) 0.7

Solution:
$A=A_0e^{-t/\tau}$
where, τ = time constant

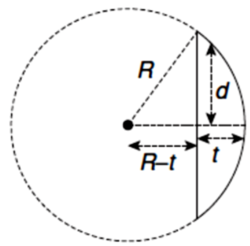
At t = 5 s; $0.9=e^{-5/\tau}\Rightarrow \frac{5}{\tau}=\ln(10/9)$

At t =15 s; $\alpha=e^{-15/\tau}\Rightarrow \frac{15}{\tau}=\ln(1/\alpha) \Rightarrow \frac{5}{15}=\frac{\ln(10/9)}{\ln(1/\alpha)} \Rightarrow \alpha^{1/3}=\frac{9}{10} \Rightarrow \alpha=\frac{729}{1000}=0.729$

Hence, the correct option is (2).

28. Diameter of plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is:
(1) 20 cm (2) 30 cm
(3) 10 cm (4) 15 cm

Solution:



Refractive index of material of lens will be $\mu = \frac{c}{v} = \frac{3}{2}$

Using figure we get

$$R^2 = d^2 + (R-t)^2 \Rightarrow R^2 - d^2 = R^2\left(1-\frac{t}{R}\right)^2 \Rightarrow 1-\frac{d^2}{R^2} = 1-\frac{2t}{R} \Rightarrow R = \frac{(3)^2}{2\times(0.3)} = \frac{90}{6} = 15\,\text{cm}$$

The focal length of the lens can be calculated using relation

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{15}\right) \Rightarrow f = 30\,\text{cm}$$

Hence, the correct option is (2).

29. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is:
(1) 6 V/m (2) 9 V/m
(3) 12 V/m (4) 3 V/m

Solution:

Speed of electromagnetic wave $c = 3\times10^8\,\text{m/s}$.

Peak value of magnetic field $B_0 = 20\times10^{-9}\,\text{T}$. $E_0 = cB_0 = 3\times10^8\,\text{m/s}\times20\times10^{-9}\,\text{T} = 6\,\text{V/m}$

Hence, the correct option is (1).

30. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of Earth's magnetic induction is $3.6 \times 10^{-5}\,\text{Wb/m}^2$)

(1) $2.56 \times 10^{-4}\,\text{Wb/m}^2$ (2) $3.50 \times 10^{-4}\,\text{Wb/m}^2$
(3) $5.80 \times 10^{-4}\,\text{Wb/m}^2$ (4) $3.6 \times 10^{-5}\,\text{Wb/m}^2$

Solution:

The value of the resultant horizontal magnetic induction at the mid - point O of the line joining their centres is

$$B_{\text{net}} = B_{M_1} + B_{M_2} + B_H = \frac{\mu_0 M_1}{4\pi x^3} + \frac{\mu_0 M_2}{4\pi x^3} + B_H = \frac{\mu_0}{4\pi x^3}(M_1 + M_2) + B_H$$

$$= \frac{10^{-7}}{(0.1)^3} \times (1.2 + 1) + 3.6 \times 10^{-5} = \frac{10^{-7}}{10^{-3}} \times 2.2 + 3.6 \times 10^{-5} = 2.56 \times 10^{-4} \text{ Wb/m}^2$$

Hence, the correct option is (1).