JEE MAIN | 2014 (Offline)
Physics

General Instructions

1. This paper contains 30 Multiple Choice Questions with single correct answer. Each question has four choices (1), (2), (3) & (4), out of which **only one** is Correct.
2. Each question is awarded 4 (four) marks for correct response.
3. One-fourth (1/4) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.

1. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied V is in volts and the temperature T is in degree kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?

(1) 0.2 mA (2) 0.02 mA
(3) 0.5 mA (4) 0.05 mA

Solution:

The given current voltage relation is

$$I = (e^{1000V/T} - 1) \text{ mA.} \qquad (1)$$

Now at I = 5 mA, Eq. (1) becomes

$$5 = (e^{1000V/T} - 1)$$

or

$$e^{1000V/T} = 6 \text{ mA}$$

Differentiating Eq. (1) w.r.t V we get,

$$\frac{dI}{dV} = e^{1000V/T} \times \frac{1000}{T}$$

$$dI = e^{1000V/T} \times \frac{1000}{T} \times dV$$

$$= 6\,\text{mA} \times \frac{1000}{300} \times 0.01$$

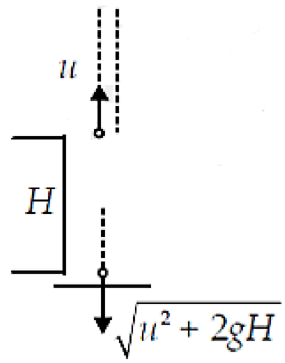
$$= 0.2\,\text{mA}$$

Hence, the correct option is (1).

2. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is:

(1) $2gH = n^2u^2$ (2) $gH = (n-2)^2u^2$
(3) $2gH = nu^2(n-2)$ (4) $gH = (n-2)u^2$

Solution:



Time taken by the particle to reach highest point is $t_1 = \frac{u}{g}$

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$-H = ut - \frac{1}{2} g t^2$$

$$t = \frac{u + \sqrt{u^2 + 2gH}}{g}$$

Using second kinematics relation we get $\frac{nu}{g} = \frac{u + \sqrt{u^2 + 2gH}}{g}$

$$nu = u + \sqrt{u^2 + 2gH} \Rightarrow nu - u = \sqrt{u^2 + 2gH}$$

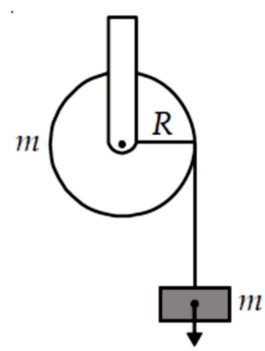
$$\left[u(n-1)\right]^2 = u^2 + 2gH \Rightarrow u^2(n-1)^2 - u^2 = 2gH$$

$$u^2(n^2 + 1 - 2n - 1) = 2gH \Rightarrow u^2(n^2 - 2n) = 2gH$$

$$2gH = n(n-2)u^2$$

Hence, the correct option is (3).

3. A mass *m* is supported by a massless string wound around a uniform hollow cylinder of mass *m* and radius *R*. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



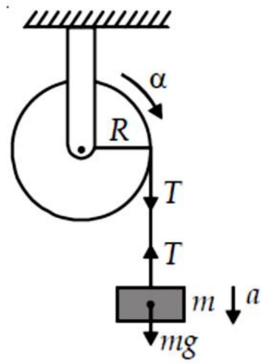
(1) $\frac{2g}{3}$

(2) $\frac{g}{2}$

(3) $\frac{5g}{6}$

(4) g

Solution:
The various forces acting on the mass *m* are shown below in free body diagram



Let *a* be the acceleration with which the mass *m* will move downwards and α be the angular acceleration produced in uniform hollow cylinder. Therefore,

$a = \alpha R$ (1)

Now applying condition of translational motion for mass *m* we have

$mg - T = ma$ (2)

Applying the condition for rotational motion for uniform hollow cylinder we get

$$TR = mR^2\alpha$$

$$T = mR\alpha \Rightarrow T = mR\frac{a}{R}$$

$$T = ma \quad (3)$$

From (2) and (3) we get

$$mg - ma = ma$$

$$mg = 2ma \Rightarrow a = \frac{g}{2}$$

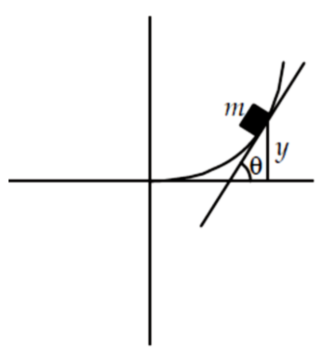
Hence, the correct option is (2).

4. A block of mass *m* is placed on a surface with a vertical cross-section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

(1) $\frac{1}{6}$m (2) $\frac{2}{3}$m

(3) $\frac{1}{3}$m (4) $\frac{1}{2}$m

Solution:



At limiting equilibrium

$$\mu = \tan\theta \qquad (1)$$

Now

$$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^3}{6}\right)$$

$$= \frac{x^2}{2} \qquad (2)$$

Substituting Eq. (2) in Eq. (1) we get

$$\mu = \frac{x^2}{2}$$

$$0.5 = \frac{x^2}{2}$$

$$x^2 = 1$$

$$x = \pm 1$$

Now the maximum height above the ground at which the block can be placed without slipping is

$$y = \frac{x^3}{6} = \frac{1^3}{6} = \frac{1}{6}\,\text{m}$$

Hence, the correct option is (1).

5. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the un-stretched rubber-band by L is:

(1) $aL^2 + bL^3$

(2) $\frac{1}{2}\left(aL^2 + bL^3\right)$

(3) $\frac{aL^2}{2} + \frac{bL^3}{3}$

(4) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

Solution:

The restoring force acting on the rubber band is

$$F = ax + bx^2$$

Small amount of work done can be expressed as

$$dW = Fdx$$

The work done in stretching the un-stretched rubber-band by L will be

$$W = \int_0^L (ax + bx^2)dx$$

$$= \left|\frac{ax^2}{2} + \frac{bx^3}{3}\right|_0^L$$

$$= \frac{aL^2}{2} + \frac{bL^3}{3}$$

Hence, the correct option is (3).

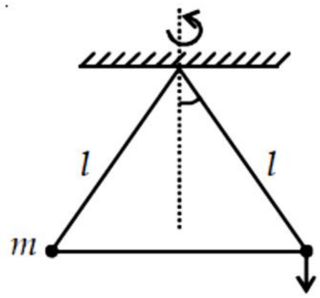
6. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension

(1) Angular momentum is conserved.

(2) Angular momentum changes in magnitude but not in direction.
(3) Angular momentum changes in direction but not in magnitude.
(4) Angular momentum changes both in direction and magnitude.
Solution:
The torque is given by relation

$$\tau = mg \times l \sin\theta .$$

Direction of the torque is perpendicular to the axis of rotation.



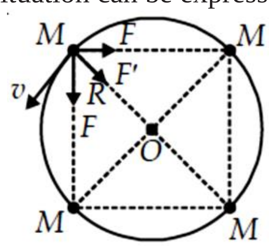
As τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.
Hence, the correct option is (3).

7. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(1) $\sqrt{\frac{GM}{R}}$ (2) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(3) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ (4) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

Solution:
Let the speed of each particle be v. The situation can be express with help of diagram shown below



$$\frac{F}{\sqrt{2}}+\frac{F}{\sqrt{2}}+F'=\frac{Mv^2}{R}$$

Substituting $F=\frac{GM^2}{\left(R\sqrt{2}\right)^2}$ and $F'=\frac{GM^2}{4R^2}$, we get

$$\frac{GM^2}{\left(R\sqrt{2}\right)^2}+\frac{GM^2}{\left(R\sqrt{2}\right)^2}+\frac{GM^2}{4R^2}=\frac{Mv^2}{R}$$

$$\frac{2\times GM^2}{\sqrt{2}\left(R\sqrt{2}\right)^2}+\frac{GM^2}{4R^2}=\frac{Mv^2}{R}$$

$$\frac{GM^2}{R^2}\left[\frac{1}{\sqrt{2}}+\frac{1}{4}\right]=\frac{Mv^2}{R}$$

$$\frac{GM^2}{R}\left[\frac{1}{\sqrt{2}}+\frac{1}{4}\right]=Mv^2$$

$$v=\sqrt{\frac{GM}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)}=\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$$

Hence, the correct option is (4).

8. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is : (For steel Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

(1) 2.2×10^8 Pa (2) 2.2×10^9 Pa
(3) 2.2×10^7 Pa (4) 2.2×10^6 Pa

Solution:

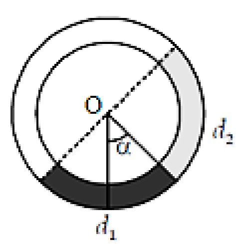
As length is constant,

$$\text{Strain}=\frac{\Delta L}{L}=\alpha\Delta T$$

Now pressure = stress = Y × strain
$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$
$= 2.2 \times 10^8$ Pa

Hence, the correct option is (1).

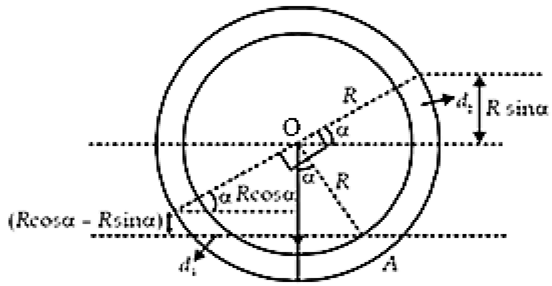
9. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio d_1/d_2 is



(1) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (2) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(3) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (4) $\frac{1+\sin\alpha}{1-\cos\alpha}$

Solution:



Now equating pressure at A we get

$$\left(R\cos\alpha + R\sin\alpha\right)d_2 g = \left(R\cos\alpha - R\sin\alpha\right)d_1 g$$

$$\left(R\cos\alpha + R\sin\alpha\right)d_2 = \left(R\cos\alpha - R\sin\alpha\right)d_1$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{\left(R\cos\alpha + R\sin\alpha\right)}{\left(R\cos\alpha - R\sin\alpha\right)}$$

$$= \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha}$$

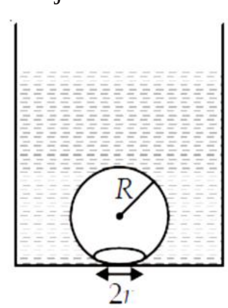
Dividing numerator and denominator by $\cos\alpha$ we get

$$\frac{d_1}{d_2} = \frac{\left(\frac{\cos\alpha + \sin\alpha}{\cos\alpha}\right)}{\left(\frac{\cos\alpha - \sin\alpha}{\cos\alpha}\right)}$$

$$= \frac{1+\tan\alpha}{1-\tan\alpha}$$

Hence, the correct option is (3).

10. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius *R* and making a circular contact of radius *r* with the bottom of the vessel. If *r* << *R*, and the surface tension of water is *T*, value of *r* just before bubbles detach is (Density of water is ρ_w)

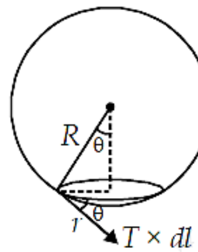


(1) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (2) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(3) $R^2\sqrt{\frac{\rho_w g}{T}}$ (4) $R^2\sqrt{\frac{3\rho_w g}{T}}$

Solution:

When the bubble gets detached, then the surface tension will be equal to the buoyant force acting on the bubble.



Buoyant force is

$$F_B = \int T \times dl \sin\theta \qquad (1)$$

Force due to surface tension is

$$F_S = \frac{4}{3}\pi R^3 \rho_w g \qquad (2)$$

Equating (1) and (2) we get

$$\int T \times dl \sin\theta = \frac{4}{3}\pi R^3 \rho_w g$$

$$T \sin\theta \int dl = \frac{4}{3}\pi R^3 \rho_w g$$

$$T \times 2\pi r \times \frac{r}{R} = \frac{4}{3}\pi R^3 \rho_w g$$

$$\frac{T \times 2\pi r^2}{R} = \frac{4}{3}\pi R^3 \rho_w g$$

$$\Rightarrow r^2 = \frac{2R^4 \rho_w g}{3T} \Rightarrow r = R^2 \sqrt{\frac{2\rho_w g}{3T}}$$

Hence, no option is correct.

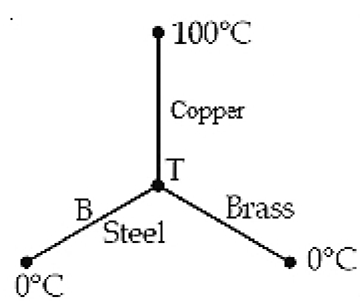
11. Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100°C whereas ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46 cm, 13 cm and 12 cm respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is

(1) 1.2 cal/s (2) 2.4 cal/s

(3) 4.8 cal/s (4) 6.0 cal/s

Solution:

Consider the figure given below



Now we know that rate of flow of heat through copper rod will be equal to sum of the rate of flow of heat through brass rod and steel rod. Therefore, we have

$$\frac{dQ_c}{dt} = \frac{dQ_b}{dt} + \frac{dQ_s}{dt}$$

$$\frac{0.92 \times 4(100 - T)}{46} = \frac{0.26 \times 4 \times (T - 0)}{13} + \frac{0.12 \times 4 \times (T - 0)}{12}$$

$$\frac{3.68(100 - T)}{46} = \frac{1.04T}{13} + \frac{0.48T}{12}$$

$$\Rightarrow T = 40\,^{\circ}\text{C}$$

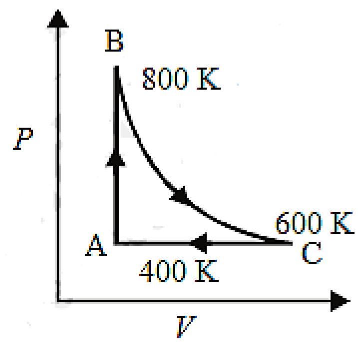
Rate of heat flow through copper rod will be

$$Q_c = \frac{0.92 \times 4(100 - 40)}{46}$$

$$= \frac{0.92 \times 4 \times 60}{46} = 4.8\,\text{cal/s}$$

Hence, the correct option is (3).

12. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure.



The process BC is adiabatic. The temperatures at A, B, and C are 400 K, 800 K and 600 K respectively. Choose the correct statement

(1) The change in internal energy in whole cyclic process is 250*R*.
(2) The change in internal energy in the process CA is 700*R*.
(3) The change in internal energy in the process AB is –350*R*.
(4) The change in internal energy in the process BC is – 500*R*.

Solution:

The change in internal energy is given by relation

$$\Delta U_{\text{CA}} = nC_V \Delta T$$

$$= nC_V (T_{\text{A}} - T_{\text{C}}) \quad (1)$$

Now temperature at point A is T_A = 400 K and temperature at point C is T_C= 600 K. Substituting the values in Eq. (1) we get

$$\Delta U_{\text{CA}} = 1 \times \frac{5R}{2}(400 - 600)$$

$$= \frac{5R}{2} \times (-200) = -500R$$

Hence, the correct option is (4).

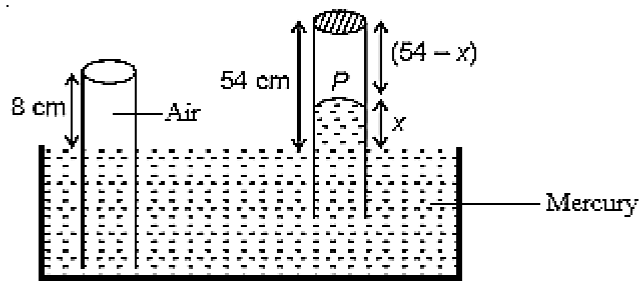
13. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)

(1) 16 cm (2) 22 cm

(3) 38 cm (4) 6 cm

Solution:

The figure given below shows an open tube immersed in mercury



Now we have

$$P + x = P_0$$
$$P = P_0 - x \qquad \text{[where } P_0 = 76 \text{ cm of Hg]}$$
$$P = (76 - x)$$

Again we can write

$$8 \times A \times 76 = (76 - x) \times A \times (54 - x)$$
$$8 \times 76 = (76 - x)(54 - x)$$
$$608 = 4104 - 76x - 54x + x^2$$
$$x^2 - 130x + 3496 = 0$$
$$x = 38 \qquad \text{(neglecting the negative value of quadratic equation)}$$

Length of air column will, be

$$L = 54 \text{ cm} - x$$
$$= 54 \text{ cm} - 38 \text{ cm}$$
$$= 16 \text{ cm}$$

Hence, the correct option is (1).

14. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels $2a$ in same direction then

(1) Amplitude of motion is $3a$

(2) Time period of oscillations is 8τ

(3) Amplitude of motion is $4a$

(4) Time period of oscillations is 6τ

Solution:

As it starts from rest, we have $x = A \cos \omega t$.

At $t = 0$, Eq. (1) becomes $x = A \cos 0 = A$

when $t = \tau$, Eq. (1) becomes

$$x = A \cos\omega\tau$$
$$\Rightarrow A - a = A \cos\omega\tau$$
$$\frac{A - a}{A} = \cos\omega\tau$$

when $t = 2\tau$, Eq. (1) becomes

$$\Rightarrow x = A\cos 2\omega\tau$$
$$A - 3a = A\cos 2\omega\tau$$

$$\frac{A-3a}{A}=\cos 2\omega\tau$$

Now we know that $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$

$$\frac{A-3a}{A}=2\left(\frac{A-a}{A}\right)^2-1$$

$$\frac{A-3a}{A}=\frac{2A^2+2a^2-4Aa-A^2}{A^2}$$

$$A^2-3aA=A^2+2a^2-4Aa$$

$$2a^2=aA \Rightarrow A=2a$$

Now

$$A-a=A\cos\omega\tau \Rightarrow 2a-a=2a\cos\omega\tau$$

$$\cos\omega\tau=\frac{a}{2a} \Rightarrow \cos\omega\tau=\frac{1}{2}$$

$$\cos\frac{2\pi}{T}\tau=\cos\frac{\pi}{3}$$

$$\Rightarrow \frac{2\pi}{T}\tau=\frac{\pi}{3} \Rightarrow T=6\tau$$

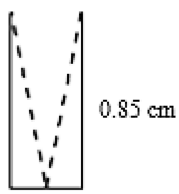
Hence, the correct option is (4).

15. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.

(1) 12 (2) 8
(3) 6 (4) 4

Solution:

The diagram showing a pipe closed from one end is shown below



Now the length of pipe $l = 0.85\text{cm}$.

Speed of sound in air v = 340 m/s.

As we know in fundamental mode

$$\frac{\lambda}{4}=l$$

$$\lambda=4l$$

$$\lambda=4\times 0.85$$

$$\lambda=3.4\text{cm}$$

Now the frequency can be calculated using

$$v = \nu\lambda$$

$$\nu = \frac{v}{\lambda}$$

$$= \frac{340}{3.4}$$

$$= 100\,\text{Hz}$$

Therefore, the possible frequencies are 100 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz, and 1100 Hz. The number of possible natural oscillations of air column in the pipe is 6.

Hence, the correct option is (3).

16. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is

(1) 120 J (2) –120 J

(3) – 80 J (4) 80 J

Solution:

We know that relation between electric field and potential is

$$E = \frac{-dV}{dx}$$

$$dV = -Edx$$

Taking definite integral both sides we get

$$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2 dx$$

$$V_A - V_O = -\left[30 \times \frac{x^3}{3}\right]_0^2$$

$$= -\left[10x^3\right]_0^2$$

$$= -80\,\text{J}$$

Hence, the correct option is (3).

17. A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to

(1) 6×10^{-7} C/m^2 (2) 3×10^{-7} C/m^2

(3) 3×10^4 C/m^2 (4) 6×10^4 C/m^2

Solution:

Dielectric constant between the two plates of parallel plate capacitor $K = 2.2$.

Electric field in the dielectric $E = 3 \times 10^4$ V/m.

Let the charge density of the positive plate be σ =?

Now using the relation for electric field

$$E = \frac{\sigma}{K\varepsilon_0}$$

$$\sigma = K\varepsilon_0 E$$

$$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$

$$\approx 6 \times 10^{-7}\,\text{C/m}^2$$

Hence, the correct option is (1).

18. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be:

(1) 8 A	(2) 10 A
(3) 12 A	(4) 14 A

Solution:

Power consumed by 15 bulbs of 40 W each = (15 × 40) W

Power consumed by 5 bulbs of 100 W each = (5 × 100) W

Power consumed by 5 fans of 80 W each = (5 × 80) W

Power consumed by heater = 1 kW = 1000 W

Voltage of electric mains V = 220 V.

Now the total power consumed can be expressed as

$$P_{\text{total}} = V \times I$$

$$15 \times 40 + 5 \times 100 + 5 \times 80 + 1000 = V \times I$$

$$600 + 500 + 400 + 1000 = 220\, I$$

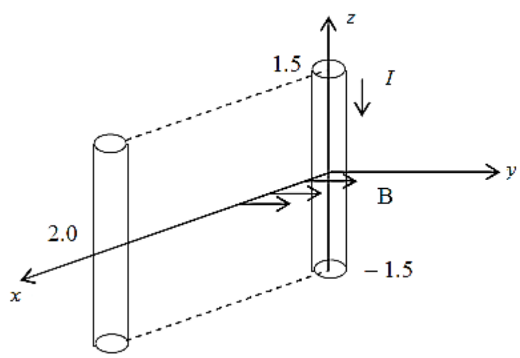
$$I = \frac{2500}{220}$$

$$= 11.36\,\text{A}$$

$$\approx 12\,\text{A}$$

Hence, the correct option is (3).

19. In the given figure, for a field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x} a_y$ T, find the power required to move the conductor at constant speed to x = 2.0 m, y = 0 m in 5×10^{-3} s. Assume parallel motion along the x-axis



(1) 1.57 W	(2) 2.97 W
(3) 14.85 W	(4) 29.7 W

Solution:

Current I = 10.0 A.

Time $t = 5 \times 10^{-3}$ s.

Length of the conductor l = 3.0 m.

Magnetic field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x} a_y$ T.

The work done can be calculated as

$$W = \int_0^2 F.dx$$
$$= \int_0^2 IlB.dx = \int_0^2 10 \times 3 \times 3.0 \times 10^{-4} e^{-0.2x} dx$$
$$= 9 \times 10^{-3} \int_0^2 e^{-0.2x} dx = 9 \times 10^{-3} \left[\frac{-e^{-0.2x}}{0.2} \right]_0^2$$
$$= \frac{9 \times 10^{-3}}{0.2} \left[-e^{-0.2 \times 2} + 1 \right] = \frac{9 \times 10^{-3}}{0.2} \left[1 - e^{-0.4} \right]$$
$$= 9 \times 10^{-3} \times 0.33$$
$$= 2.97 \times 10^{-3}\,\text{J}$$

Now average power can be calculated using

$$P = \frac{\text{Work}}{\text{Time}}$$
$$= \frac{W}{t} = \frac{\left(\frac{2.97 \times 10^{-3}}{0.2} \right)}{5 \times 10^{-3}}$$
$$= 2.97\,\text{W}$$

Hence, the correct option is (2).

20. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is

(1) 30 mA (2) 60 mA
(3) 3 A (4) 6 A

Solution:

Coercivity of small magnet $\frac{B}{\mu_0} = 3 \times 10^3\,\text{Am}^{-1}$.

Number of turns of solenoid N = 100.

Length of solenoid L = 10 cm = $10 \times 10^{-2}\,\text{m}$.

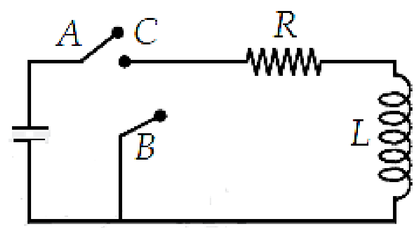
Let the required current be I =?

Now we know that magnetic field of solenoid is given by relation

$$B = \mu_0 n I \qquad \text{[where } n \text{ is the number of turns per unit length]}$$
$$\frac{B}{\mu_0} = \frac{N}{L} \times I$$
$$3 \times 10^3 = \frac{NI}{L} = \frac{100i}{10 \times 10^{-2}}$$
$$I = 3\text{A}$$

Hence, the correct option is (3).

21. In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly, point C is disconnected from point A and connected to point B at time t = 0. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to



(1) $\frac{e}{1-e}$ (2) 1

(3) –1 (4) $\frac{1-e}{e}$

Solution:
Let the voltage drop across resistor be V_R and voltage drop across inductor be V_L.
Applying Kirchhoff's law in closed loop, we get

$$V_R + V_L = 0$$
$$V_R = -V_L$$
$$\frac{V_R}{V_L} = -1$$

Thus the ratio of the voltage across resistance and the inductor at $t = L/R$ will be 1:1.
Hence, the correct option is (3).

22. During the propagation of electromagnetic waves in a medium
(1) Electric energy density is double of the magnetic energy density
(2) Electric energy density is half of the magnetic energy density
(3) Electric energy density is equal to the magnetic energy density
(4) Both electric and magnetic energy densities are zero

Solution:
During the propagation of electromagnetic waves in a medium electric energy density is equal to the magnetic energy density because energy is equally divided between electric and magnetic field, that is,

$$\frac{B^2}{2\mu_0} = \frac{1}{2}\varepsilon_0 E^2$$

Hence, the correct option is (3).

23. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices 4/3 and 5/3, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is
(1) $f_1 = f_2 < f$
(2) $f_1 > f$ and f_2 becomes negative
(3) $f_2 > f$ and f_1 becomes negative
(4) f_1 and f_2 both become negative

Solution:

Refractive index of crown glass $\mu = \frac{3}{2}$.

Refractive index of first liquid μ_1 = 4/3.

Refractive index of second liquid $\mu_2 = 5/3$.
Applying Lens maker's formula we get

$$\frac{1}{f_1} = \left(\frac{\mu}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots (1)$$

Similarly from lens maker formula, we get $\frac{1}{f_2} = \left(\frac{\mu}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots.. (2)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots. (3)$$

Now dividing Eq. (1) and Eq. (3) we get

$$\frac{\left(\frac{1}{f_1}\right)}{\left(\frac{1}{f}\right)} = \frac{\left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \Rightarrow \frac{f}{f_1} = \frac{\left(\frac{9}{8} - 1\right)}{\left(\frac{3}{2} - 1\right)} \Rightarrow \frac{f}{f_1} = \frac{1}{4} \Rightarrow f_1 = 4f$$

Again dividing Eq. (2) and Eq. (3) we get

$$\frac{\left(\frac{1}{f_2}\right)}{\left(\frac{1}{f}\right)} = \frac{\left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \Rightarrow \frac{f}{f_2} = \frac{\left(\frac{9}{10} - 1\right)}{\left(\frac{3}{2} - 1\right)} \Rightarrow \frac{f}{f_2} = \frac{-1}{5} \Rightarrow f_2 = -5f$$

Hence, the correct option is (2).

24. A green light is incident from the water to the air–water interface at the critical angle (θ). Select the **correct** statement

(1) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.
(2) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.
(3) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.
(4) The entire spectrum of visible light will come out of the water at various angles to the normal.

Solution:

We know that as the frequency of visible light increases refractive index increases. The critical angle are refractive index are related by relation given below

$$\sin\theta_c = \frac{1}{\mu}$$

where θ_c is the critical angle and μ is the refractive index. From the above relation we conclude that refractive index is inversely related to critical angle. Therefore as the refractive index increases, the critical angle decreases. Now light having frequency greater than green light will suffer total internal reflection and light having frequency less than green light will suffer refraction and pass to air.

Hence, the correct option is (2).

25. Two beams, *A* and *B*, of plane polarized light with mutually perpendicular planes of polarization are seen through a Polaroid. From the position when the beam *A* has maximum intensity (and beam *B* has zero intensity), a rotation of Polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_{A} and I_{B} respectively, then $I_{\text{A}} / I_{\text{B}}$ equals

(1) 3 (2) 3/2
(3) 1 (4) 1/3

Solution:

Rotation of Polaroid is $\theta = 30^{\text{o}}$.
Now according to law of Malus

$$I = I_0 \cos^2 \theta$$

For beam A, initial intensity is I_{A}, the by Malus law the final intensity will be

$$I_{\text{A}'} = I_{\text{A}} \cos^2 30$$

$$= I_{\text{A}} \times \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{3I_{\text{A}}}{4} \qquad (1)$$

For beam B, initial intensity is I_{B}, the by Malus law the final intensity will be

$$I_{\text{B}} = I_{\text{B}} \cos^2 60 = I_{\text{B}} \times \left(\frac{1}{2}\right)^2$$

$$= \frac{I_{\text{B}}}{4} \qquad (2)$$

Now as $I_{\text{A}'} = I_{\text{B}'}$ from Eq. (1) and Eq. (2) we get

$$\frac{3I_{\text{A}}}{4} = \frac{I_{\text{B}}}{4} \Rightarrow 3I_{\text{A}} = I_{\text{B}} \Rightarrow \frac{I_{\text{A}}}{I_{\text{B}}} = \frac{1}{3}$$

Hence, the correct option is (4).

26. The radiation corresponding to 3→2 transition of hydrogen atoms falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to

(1) 1.8 eV (2) 1.1 eV
(3) 0.8 eV (4) 1.6 eV

Solution:

Magnetic field $B = 3 \times 10^{-4}$ T.
Radius of the largest circular wire r = 10.0 mm $= 10.0 \times 10^{-3}$ m.
Charge of electron $e = 1.6 \times 10^{-19}$ C.
Mass of electron $m = 9.1 \times 10^{-31}$.
Now the stopping potential can be calculates using relation

$$r = \frac{mv}{qB}$$

$$= \frac{\sqrt{2meV}}{eB} = \frac{1}{B}\sqrt{\frac{2m}{e}V}$$

Squaring both sides we get

$$r^2 = \frac{2mV}{B^2 e}$$

$$\Rightarrow V = \frac{B^2 r^2 e}{2m} = \frac{\left(3\times10^{-4}\right)^2\left(10.0\times10^{-3}\right)^2\left(1.6\times10^{-19}\right)}{2\times9.1\times10^{-31}}\,\text{J}$$

$$= 0.8\ \text{eV}$$

Energy corresponding to 3→2 transition of hydrogen atoms will be

$$E = 13.6\left(\frac{1}{2^2}-\frac{1}{3^2}\right)\text{eV} = 13.6\left(\frac{1}{4}-\frac{1}{9}\right)\text{eV} = \left(\frac{13.6\times5}{36}\right)\text{eV}$$

$$= 1.88\,\text{eV}$$

Therefore the work function will be $= 1.88\text{eV} - 0.8\text{eV} = 1.08\text{eV} \approx 1.1\text{eV}$

Hence, the correct option is (2).

27. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionized Helium $(_2He^4)^+$ and doubly ionized lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from n = 2 to n = 1. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct?

(1) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(2) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(3) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

(4) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

Solution:

The wavelength can be calculates using relation

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right]$$

$$\frac{1}{\lambda} = RZ^2\left(1-\frac{1}{4}\right) \Rightarrow \frac{1}{\lambda} = \frac{3RZ^2}{4}$$

$$\lambda = \frac{4}{3RZ^2} \qquad (1)$$

For Hydrogen ($_1H^1$), Z =1 therefore the wavelength corresponding to hydrogen can be calculated using Eq. (1) as

$$\lambda_1 = \frac{4}{3R\times1^2} = \frac{4}{3R} \qquad (2)$$

For Deuterium ($_1H^2$), Z =1 therefore the wavelength corresponding to hydrogen can be calculated using Eq. (1) as

$$\lambda_2 = \frac{4}{3R\times1^2} = \frac{4}{3R} \qquad (3)$$

For singly ionized Helium $(_2He^4)^+$, Z =2 therefore the wavelength corresponding to hydrogen can be calculated using Eq. (1) as

$$\lambda_3 = \frac{4}{3R \times 2^2} = \frac{4}{12R} \qquad (4)$$

For doubly ionized lithium $({}_3Li^6)^{++}$, Z =3 therefore the wavelength corresponding to hydrogen can be calculated using Eq. (1) as

$$\lambda_4 = \frac{4}{3R \times 3^2} = \frac{4}{27R} \qquad (5)$$

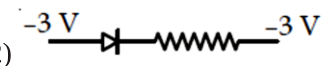
Now from Eq. (2), Eq. (3), Eq. (4), and Eq. (5) we get

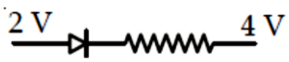
$$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$$

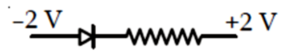
Hence, the correct option is (3).

28. The forward biased diode connection is

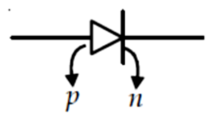
(1) 

(2) 

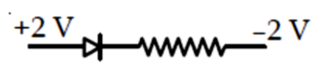
(3) 

(4) 

Solution:



In forward bias, the p-side of diode should be connected to positive terminal of battery or the terminal at higher potential and n-side should be connected to negative terminal of battery (terminal to lower potential). Therefore the configuration in which diode is forward biased is



Hence, the correct option is (1).

29. Match **List-I** (Electromagnetic wave type) with **List - II** (Its association/application) and select the correct option from the choices given below the lists:

List-I		List-II	
(a)	Infrared waves	(i)	To treat muscular strain
(b)	Radio waves	(ii)	For broadcasting
(c)	X-rays	(iii)	To detect fracture of bones
(d)	Ultraviolet rays	(iv)	Absorbed by the ozone layer of the atmosphere

	(a)	**(b)**	**(c)**	**(d)**
(1)	(iv)	(iii)	(ii)	(i)
(2)	(i)	(ii)	(iv)	(iii)
(3)	(iii)	(ii)	(i)	(iv)
(4)	(i)	(ii)	(iii)	(iv)

Solution:
Infrared rays are used to treat muscular strain. Thus the correct mapping is $(a) \rightarrow (i)$.
Radio waves are used for broadcasting. Thus the correct mapping is $(b) \rightarrow (ii)$.
Fractures in bones are detected using X-rays. Thus the correct mapping is $(c) \rightarrow (iii)$.
Ultraviolet rays are absorbed by ozone layer. Thus the correct mapping is $(d) \rightarrow (iv)$.
Hence, the correct option is (4).

30. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?
(1) A meter scale
(2) A Vernier calliper where the 10 divisions in Vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm
(3) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(4) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

Solution:
Measured value = 3.50 cm therefore least count of measuring instrument must be 0.01 cm = 0.1 mm.
For Vernier caliper the least count is = 0.01 cm = 0.1 mm.

Thus the student used Vernier caliper to measure the length of the rod.
Hence, the correct option is (2).