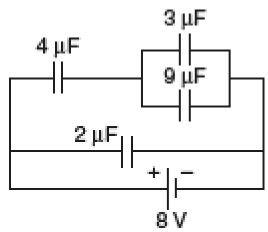
JEE MAIN (OFFLINE) 2016

PHYSICS

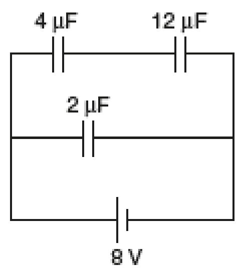
1. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point 30 m from it, would equal



(1) 480 N/C
(2) 240 N/C
(3) 360 N/C
(4) 420 N/C

Solution

The given circuit can be simplified as shown in the following figure:



$$V_4 = 8 \times \frac{12}{16} = 6 \text{ V}$$

$$V_{12} = V_3 = V_9 = 2 \text{ V}$$

Therefore,

$$q_4 = 4V_4 = 24 \text{ μC}$$
$$q_9 = 9V_{12} = 18 \text{ μC}$$
$$Q = q_4 + q_9 = 42 \text{ μC}$$

Therefore, the magnitude of the electric filed is

$$E = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{9 \times 10^2} = 420 \text{ N/C}$$

Hence, the correct option is (4).

2. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer, the tree appears:

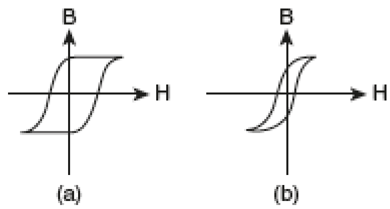
(1) 20 times nearer
(2) 10 times taller
(3) 10 times nearer
(4) 20 times taller

Solution

Telescope is an optical instrument which makes the distant objects to appear as if they are nearer to us by magnifying the object's image. Hence, with a magnifying power of 20, a telescope creates the image of a distant tree, which appears 20 times taller.

Hence, the correct option is (4).

3. Hysteresis loops for two magnetic materials A and B are given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use:

(1) B for electromagnets and transformers.

(2) A for electric generators and transformers.

(3) A for electromagnets and B for electric transformers.

(4) A for transformers and B for electric generators.

Solution

The area enclosed by a hysteresis loop gives loss per unit volume during one cycle of magnetization. Since in electromagnets and transformers, the magnetic flux vary with time, it is necessary to keep the losses to the minimum. Hence, a hysteresis loop with the minimum area is preferred.

Hence, the correct option is (1).

4. Half-lives of two radioactive elements A and B are 20 min and 40 min, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be:

(1) 5 : 4 (2) 1 : 16

(3) 4 : 1 (4) 1 : 4

Solution

We have the following half-lives:

$$t_{1/2}\big|_{\mathrm{A}} = 20 \text{ min}$$

$$t_{1/2}\big|_{\mathrm{B}} = 40 \text{ min}$$

It is given that at $t = 0$ min:

$$N_{0\mathrm{A}} = N_{0\mathrm{B}} = N_0$$

At $t = 80$ min, we have

$$4\,t_{1/2}\big|_{\mathrm{A}} = 2\,t_{1/2}\big|_{\mathrm{B}}$$

Therefore, after 80 min, we get

$$N_{\mathrm{A}}(\text{remaining}) = \frac{N_0}{16}$$

$$N_B(\text{remaining}) = \frac{N_0}{4}$$

Thus,

$$N'_A \text{ (decayed) } = N_0 - \frac{N_0}{16} = \frac{15N_0}{16}$$

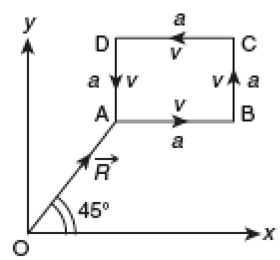
$$N'_B \text{ (decayed) } = N_0 - \frac{N_0}{4} = \frac{3N_0}{4}$$

That is,

$$\frac{N'_A}{N'_B} = \frac{15N_0}{16} \times \frac{4}{3N_0} = 5:4$$

Hence, the correct option is (1).

5. A particle of mass *m* is moving along the side of a square of side *a*, with a uniform speed *v* in the *xy*-plane as shown in the figure:



Which of the following statement is false for the angular momentum $\vec{L}$ about the origin?

(1) $\vec{L} = \frac{mv}{\sqrt{2}} R\hat{k}$ when the particle is moving from D to A.

(2) $\vec{L} = -\frac{mv}{\sqrt{2}} R\hat{k}$ when the particle is moving from A to B.

(3) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

(4) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

Solution

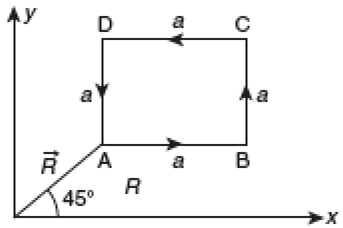
The angular momentum is

$$\vec{L} = m(\bar{r} \times \bar{v})$$

For the particle moving from D→A, we have

$$\vec{L} = m\left[\left(\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right) \times V(-\hat{j})\right]$$

$$= \frac{mvR}{\sqrt{2}}(-\hat{k})$$



For the particle moving from A→B, we have

$$\vec{L} = m\left[\left(\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right)\times v\hat{i}\right]$$

$$= \frac{mvR}{\sqrt{2}}(-\hat{k})$$

For the particle moving from C→D, we have

$$\vec{L} = mv \times (\perp \text{ distance}) \times (\hat{k})$$

$$= mv\left(\frac{R}{\sqrt{2}} - a\right)(\hat{k})$$

For the particle moving from B→C, we get

$$\vec{L} = mv \times (\perp \text{ distance}) \times (\hat{k})$$

$$= mv\left(\frac{R}{\sqrt{2}} + a\right)(\hat{k})$$

Hence, the correct options are (1) and (3).

6. Choose the correct statement:

(1) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

(2) In amplitude modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(3) In amplitude modulation, the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(4) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

Solution

If message (modulating) signal is $m(t) = A_m \sin\omega_m t$ and carrier signal is $c(t) = A_c \sin\omega_c t$, the modulated signal is

$$c_M(t) = A_c(1 + \mu\sin\omega_M t)\sin\omega_c t$$

where μ = modulation index $= A_M / A_c$. The amplitude of the carrier wave varies in accordance with the modulating signal where the amplitude of the modulated wave is given by

$$A_c(1 + \mu\sin\omega_M t)$$

Hence, the correct option is (2).

7. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case, which of the following is closest to the maximum possible value of the refractive index?

(1) 1.8
(2) 1.5
(3) 1.6
(4) 1.7

Solution

Given that i = 35°, δ = 40° and e = 79°.

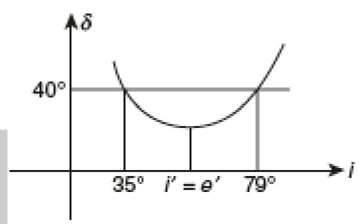
$$i + e = A + \delta$$

Therefore,

$$\delta = i + e - A$$
$$40° = 35° + 79° - A$$
$$\Rightarrow A = 74°$$

For minimum deviation, we have

$$i' = e' = \frac{A+\delta_M}{2}$$



From ($i - \delta$) plot as depicted in the following figure, we get

$$i' = e' = \frac{35+79}{2} = \frac{114}{2} = 57°$$

Therefore,

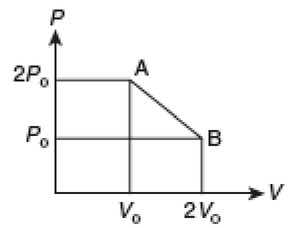
$$\delta_M = 2i - A = 2(57°) - 74°$$
$$= 114° - 74°$$
$$= 40°$$

Therefore,

$$\mu = \frac{\sin[(A+\delta_M)/2]}{\sin(A/2)}$$
$$= \frac{\sin[(74°+40°)/2]}{\sin(A/2)}$$
$$= \frac{\sin 57°}{\sin 37°}$$
$$\approx \frac{\sin 60°}{\sin 37°}$$
$$= \frac{\sqrt{3}}{2} \times \frac{5}{3} = \frac{2.5}{1.73} \approx 1.5$$

Hence, the correct option is (2).

8. n moles of an ideal gas undergoes a process A→B as shown in the figure. The maximum temperature of the gas during the process will be



(1) $\frac{9P_0V_0}{nR}$

(2) $\frac{9P_0V_0}{4nR}$

(3) $\frac{3P_0V_0}{2nR}$

(4) $\frac{9P_0V_0}{2nR}$

Solution

We have the equation of the process AB as

$$P - 2P_0 = \frac{P}{-V_0}(V - V_0)$$

$$P = 3P_0 - \frac{P_0}{V_0}V$$

Applying $PV = nRT$, we have

$$\left(3P_0 - \frac{P_0}{V_0}V\right)V = nRT$$

$$T = \frac{1}{nR}\left(3P_0V - \frac{P_0V^2}{V_0}\right)$$

To maximize T, we have $\frac{dT}{dV} = 0.$

$$3P_0 - \frac{2P_0V}{V_0} = 0$$

$$\Rightarrow V = \frac{3V_0}{2}$$

Therefore,

$$T_{\max} = \frac{1}{nR}\left[\frac{9P_0V_0}{2} - \frac{9}{4}P_0V_0\right]$$

$$= \frac{9P_0V_0}{4nR}$$

Hence, the correct option is (2).

9. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of the magnetic field at the centres of the circle and square, respectively, then the ratio B_A / B_B is

(1) $\dfrac{\pi^2}{8\sqrt{2}}$

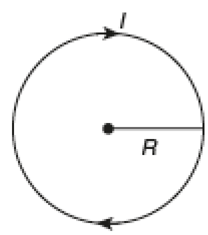
(2) $\dfrac{\pi^2}{8}$

(3) $\dfrac{\pi^2}{16\sqrt{2}}$

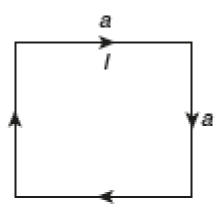
(4) $\dfrac{\pi^2}{16}$

Solution

When the wire is bent into a circle: $l = 2R$ and $R = l/2\pi$.



When the wire is bent into a square: $l = 4a$ and $a = l/4$.



$$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2l} \times 2\pi = \frac{\mu_0 \pi I}{l}$$

$$B_B = 2\sqrt{2}\frac{\mu_0 I}{\pi a} = 2\sqrt{2}\frac{\mu_0 I}{\pi l} \times 4 = \frac{8\sqrt{2}\mu_0 I}{\pi l}$$

Therefore,

$$\frac{B_A}{B_B} = \frac{\mu_0 \pi I}{I} \times \frac{\pi l}{8\sqrt{2}\mu_0 I} = \frac{\pi^2}{8\sqrt{2}}$$

Hence, the correct option is (1).

10. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line?

(1) 0.50 mm

(2) 0.75 mm

(3) 0.80 mm

(4) 0.70 mm

Solution

For a screw gauge, we have

$$\text{Least count} = \frac{\text{Pitch}}{\text{Total no. of divisions on the circular scale}}$$

$$= \frac{0.5}{50} = 0.01 \text{ mm}$$

Since 45th division of the screw is considered with the main scale line, it is a case of negative zero error of magnitude 5 × (L.C.) = 0.05 mm. Therefore,

$$\text{Measured value} = \text{M.S.R.} + [\text{C.S.D} \times \text{L.C.} + \text{Zero error}]$$

$$= 0.5 + [(25 \times 0.01) + 0.05]$$

$$= 0.5 + 0.30$$

$$= 0.80 \text{ mm}$$

Hence, the correct option is (3).

11. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is

(1) $\alpha = \frac{\beta^2}{1+\beta^2}$

(2) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

(3) $\alpha = \frac{\beta}{1-\beta}$

(4) $\alpha = \frac{\beta}{1+\beta}$

Solution

For a common-emitter configuration, we have

$$\alpha = \frac{I_C}{I_E}, \quad \beta = \frac{I_C}{I_B}, \; I_E = I_C + I_B$$

From the above we get

$$\alpha = \frac{\beta}{1+\beta}$$

$$\beta = \frac{\alpha}{1-\alpha}$$

So the incorrect relation are given in options (1) and (3).

Hence, the correct options are (1) and (3).

12. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say, $b_{\min}$) when

(1) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \sqrt{4\lambda L}$

(2) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

(3) $a = \sqrt{\lambda L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

(4) $a = \sqrt{\lambda L}$ and $b_{\min} = 4\lambda L$

Solution

The difference angle λ / a (due to the radius of the hole, a) causes a spreading of $\lambda L / a$ in the size of the spot. When a is small, this becomes large; therefore, we get the following spot size:

$$a + \frac{\lambda L}{a}$$

The spot would have its minimum size when

$$1 - \frac{\lambda L}{a^2} = 0$$

$$\Rightarrow a = \sqrt{\lambda L}$$

The geometric spread and the diffraction spread are equal; therefore, the minimum spot size is

$$b_{\min} = \sqrt{4\lambda L}$$

Hence, the correct option is (4).

13. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8$ ms^{-2}:

(1) 12.89×10^{-3} kg
(2) 2.45×10^{-3} kg
(3) 6.45×10^{-3} kg
(4) 9.89×10^{-3} kg

Solution

The work done by the person against gravity during lifting of weight 1000 times is

$$W = 1000 \times mgh$$
$$= 1000 \times 10 \times 9.8 \times 1$$
$$= 9.8 \times 10^4 \text{ J}$$

The energy needed is obtained by burning fat. Let the mass of fat that is burnt be m. Then

$$\frac{20}{100} \times 3.8 \times 10^7 \times m = 9.8 \times 10^4$$

$$\Rightarrow m = 12.89 \times 10^{-3} \text{ kg}$$

Hence, the correct option is (1).

14. Arrange the following electromagnetic radiations per quantum in the order of increasing energy:

A: Blue light

B: Yellow light

C: X-ray

D: Radio waves

(1) B, A, D, C
(2) D, B, A, C
(3) A, B, D, C
(4) C, A, B, D

Solution

Energy is

In terms of increasing wavelength, we have

$$\lambda_{\text{X-Rays}} < \lambda_{\text{Blue light}} << \lambda_{\text{Yellow light}} < \lambda_{\text{Radio waves}}$$

$$E = \frac{hc}{\lambda}$$

Therefore, order of increasing energy is

$$E_{\text{Radio waves}} < E_{\text{Yellow light}} < E_{\text{Blue light}} < E_{\text{X-Rays}}$$

Hence, the correct option is (2).

15. An ideal gas undergoes a quasi-static, reversible process in which its molar heat capacity C remains constant. If during this process, the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here, C_P and C_V are molar specific heat at constant pressure and constant volume, respectively)

(1) $n = \frac{C - C_V}{C - C_P}$

(2) $n = \frac{C_P}{C_V}$

(3) $n = \frac{C - C_P}{C - C_V}$

(4) $n = \frac{C_P - C}{C - C_V}$

Solution

For a polytropic process, PV^n = Constant. The molar heat capacity is given by

$$C = C_V + \frac{R}{1-n}$$

$$C = C_V + \frac{C_P - C_V}{1-n} \qquad (\because R = C_P - C_V)$$

$$1 - n = \frac{C_P - C_V}{C - C_V}$$

$$n = 1 - \frac{C_P - C_V}{C - C_V}$$

$$n = \frac{C - C_P}{C - C_V}$$

Hence, the correct option is (3).

16. A satellite is revolving in a circular orbit at a height h from the Earth's surface (radius of Earth R: $h << R$). The maximum increase in its orbital velocity required, so that the satellite could escape from the Earth's gravitational field, is close to: (Neglect the effect of atmosphere).

(1) $\sqrt{gR}(\sqrt{2} - 1)$

(2) $\sqrt{2gR}$

(3) $\sqrt{gR}$

(4) $\sqrt{gR/2}$

Solution

For low-orbit satellites, we have $v_0 \simeq \sqrt{gR}$

Escape speed is $v_e = \sqrt{2gR}$

Therefore, $\Delta v = v_e - v_0 = \sqrt{gR}(\sqrt{2} - 1)$

Hence, the correct option is (1).

17. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is

(1) 3Ω

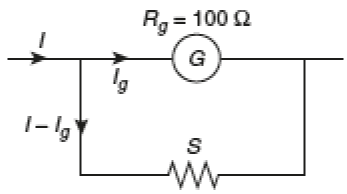
(2) 0.01Ω

(3) 2Ω

(4) 0.1Ω

Solution

To convert a galvanometer into an ammeter, a shunt resistance is connected across it as shown in the following figure:



Then, $R_g I_g = S(I - I_g)$

$$\Rightarrow S = \frac{R_g I_g}{I - I_g} = \frac{100 \times 0.001}{10 - 0.001} = \frac{0.10}{9.99} = 0.01\,\Omega$$

Hence, the correct option is (2).

18. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength of changed to $3\lambda/4$ the speed of the faster emitted electron will be

(1) $= v\left(\frac{3}{5}\right)^{1/2}$

(2) $> v\left(\frac{4}{3}\right)^{1/2}$

(3) $< v\left(\frac{4}{3}\right)^{1/2}$

(4) $= v\left(\frac{4}{3}\right)^{1/2}$

Solution

From Einstein's photoelectric equation, we get

$$\frac{hc}{\lambda} = \phi + \left(\frac{1}{2}mv^2\right)_{\max} \qquad (1)$$

where ϕ in the work function of the emitter. On changing the wavelength to $3\lambda/4$, we get

$$\frac{hc}{3\lambda/4} = \phi + \left[\frac{1}{2}m(v')^2\right]_{\max} \qquad (2)$$

Dividing Eq. (1) by Eq. (2), we get

$$\frac{3}{4} = \frac{\phi + (1/2)mv^2}{\phi + (1/2)mv^{12}}$$

$$\Rightarrow 4\phi + 4\left[\frac{1}{2}m(v')^2\right] = 3\phi + 3\left[\frac{1}{2}m(v')^2\right]$$

$$\Rightarrow \phi + 2mv^2 = \frac{3}{2}m(v')^2$$

Therefore,

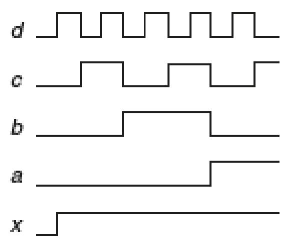
$$(v')^2 = \frac{2}{3}\frac{\phi}{m} + \frac{4}{3}v^2$$

Since $\frac{2}{3}\frac{\phi}{m} > 0$, we get

$$v' > v\left(\frac{4}{3}\right)^{1/2}$$

Hence, the correct option is (2).

19. If *a*, *b*, *c*, *d* are inputs to a gate and *x* is its output, then as per the following time graph, the gate is



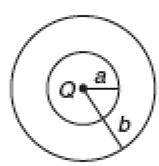
(1) NAND
(2) NOT
(3) AND
(4) OR

Solution

According to the truth table of OR gate, the output is 1 when any of the inputs is 1 and output is 0 when all inputs are zero.

Hence, the correct option is (4).

20. The region between two concentric sphere of radii *a* and *b*, respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where *A* is a constant and *r* is the distance from the centre. At the centre of the sphere is a point charge Q. The value of *A* such that the electric field in the region between the sphere will be constant, is



(1) $\frac{2Q}{\pi a^2}$

(2) $\frac{Q}{2\pi a^2}$

(3) $\frac{Q}{2\pi(b^2 - a^2)}$

(4) $\frac{2Q}{\pi(a^2 - b^2)}$

Solution

Applying Gauss's law, we get

$$E(4\times r^2) = \frac{Q+q'}{\varepsilon_0}$$

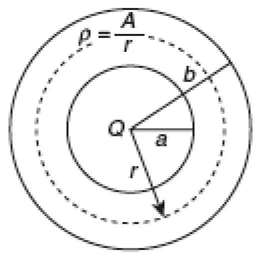
$$q' = \int_a^r \rho 4\pi x^2 dx \qquad \text{(where } \rho = \frac{A}{x} \text{ for } a < x < N\text{)}$$

$$= A4\pi\int_a^r x\,dx = 2\pi A(r^2 - a^2)$$

Therefore,

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r^2} + 2\pi A\left(1 - \frac{a^2}{r^2}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r^2}(Q - 2\pi A a^2) + 2\pi A\right]$$



If E is constant for $a < r < b$, then E should be independent of r. That is,

$$Q - 2\pi A a^2 = 0$$

$$\Rightarrow A = \frac{Q}{2\pi a^2}$$

Hence, the correct option is (2).

21. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be

(1) 92 ± 3 s
(2) 92 ± 2 s
(3) 92 ± 5.0 s
(4) 92 ± 1.8 s

Solution

We have

$$\bar{T} = \frac{90 + 91 + 95 + 92}{4} = 92 \text{ s}$$

$$\Delta T_1 = |T_1 - \bar{T}| = 2 \text{ s}$$

$$\Delta T_2 = |T_2 - \bar{T}| = 2 \text{ s}$$

$$\Delta T_3 = |T_3 - \bar{T}| = 3 \text{ s}$$

$$\Delta T_4 = |T_4 - \bar{T}| = 0 \text{ s}$$

Therefore,

$$|\Delta T|_{\text{av}} = \frac{\Delta T_1 + \Delta T_2 + \Delta T_3 + \Delta T_4}{4} = 1.5 \text{ s}$$

Since the minimum division in the measuring clock is 1 s, the reported mean time is 92 ± 2 s.

Hence, the correct option is (2).

22. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by

(1) Linear decrease for Cu, linear decrease for Si.

(2) Linear increase for Cu, linear increase for Si.

(3) Linear increase for Cu, exponential increase for Si

(4) Linear increase for Cu, exponential decrease for Si

Solution

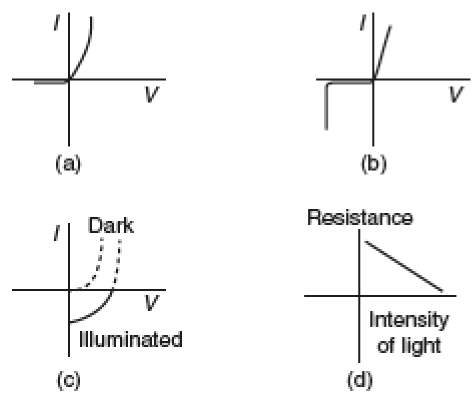
Copper (Cu) is a conductor. Its resistance varies with temperature since

$$R = k_0 (1 + \alpha\Delta T)$$

Silicon (Si) being a semiconductor, the resistance shows an exponential decrease with temperature.

Hence, the correct option is (4).

23. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d).



(1) Zener diode, Solar cell, Simple diode, Light dependent resistance.

(2) Simple diode, Zener diode, Solar cell, Light dependent resistance.

(3) Zener diode, Simple diode, Light dependent resistance, Solar cell.

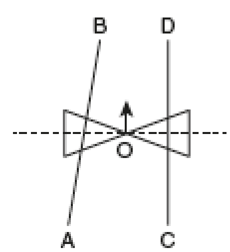
(4) Solar cell, Light dependent resistance, Zener diode, Simple diode.

Solution

All the given graphs are factual from the theoretical meanings.

Hence, the correct option is (2).

24. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to



(1) turn left and right alternately.

(2) turn left.

(3) turn right.

(4) go straight.

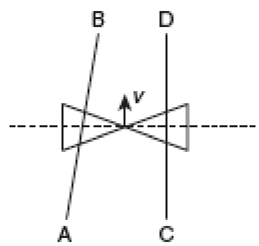
Solution

As the roller rolls forward, v' (left) becomes smaller than v (right). Therefore,

$$\omega r' < \omega r$$

$$v' < v$$

Thus, the roller tends to turn towards left.



Hence, the correct option is (2).

25. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the coefficient of linear expansion (a) of the metal of the pendulum shaft are, respectively

(1) 55°C; $\alpha = 1.85 \times 10^{-2}$/°C

(2) 25°C; $\alpha = 1.85 \times 10^{-5}$/°C

(3) 60°C; $\alpha = 1.85 \times 10^{-4}$/°C

(4) 30°C; $\alpha = 1.85 \times 10^{-3}$/°C

Solution

We have

$$T = 2\pi\sqrt{\frac{l}{g}}$$

and

$$l = l_0(1 + \alpha\Delta\theta)$$

$$\frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} = \alpha\Delta\theta$$

Using error analysis, we get

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

Case 1: When the clock gains 12 s, we get

$$\frac{12}{T} = \frac{1}{2}\alpha(40 - \theta_0) \quad (1)$$

where θ_0 is the optimum temperature at which it shows the correct time.

Case 2: When the clock loses 4 s, we get

$$\frac{4}{T} = \frac{1}{2}\alpha(\theta_0 - 20) \quad (2)$$

Dividing Eq. (1) by Eq. (2), we get

$$3 = \frac{40-\theta_0}{\theta_0 - 20}$$

$$3\theta_0 - 60 = 40 - \theta_0$$

$$4\theta_0 = 100$$

$$\theta_0 = 25°$$

Therefore, from Eq. (2), we get

$$\frac{4}{24 \times 3600} = \frac{1}{2}\alpha(25-20)$$

$$\Rightarrow \alpha = \frac{8}{24 \times 3600 \times 5} = 1.85 \times 10^{-5} /°C$$

Hence, the correct option is (2).

26. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (take g = 10 ms^{-2})

(1) $\sqrt{2}$ s

(2) $(2\pi\sqrt{2})$ s

(3) 2s

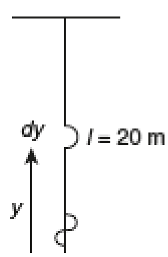
(4) $2\sqrt{2}$ s

Solution

The velocity of the wave in a stretched string is

$$v = \sqrt{\frac{T}{\mu}}$$

where μ is mass/length of the string.



At any y, as shown in the figure,

$$T = \mu y g$$

$$V = \sqrt{\frac{\mu y g}{\mu}} = \sqrt{yg} = \frac{dy}{dt} \Rightarrow \sqrt{g}\, dt = \frac{dy}{\sqrt{y}}$$

On integrating, we get

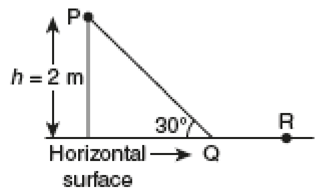
$$\sqrt{g}\int_0^t dt = \int_0^l \frac{dy}{\sqrt{y}}$$

$$\sqrt{g}\, t = 2\sqrt{l}$$

$$t = 2\sqrt{\frac{l}{g}} = 2\sqrt{\frac{20}{10}} = 2\sqrt{2} \text{ s}$$

Hence, the correct option is (4).

27. A point particle of mass *m* moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and PR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x (= QR) are, respectively, close to



(1) 0.29 and 6.5 m
(2) 0.2 and 6.5 m
(3) 0.2 and 3.5 m
(4) 0.29 and 3.5 m

Solution

The energy lost against friction for the path PQ is

$$W_{PQ} = [\mu(mg)\cos\theta]\frac{h}{\sin\theta}$$

Similarly,

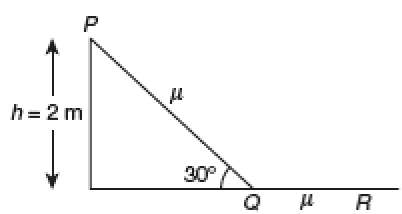
$$W_{QR} = \mu(mg)x$$

It is given that $W_{PQ} = W_{QR}$

$$[\mu(mg)\cos\theta]\frac{h}{\sin\theta} = \mu(mg)x$$

$$\Rightarrow x = h\cot\theta = h\cot 30°$$

$$\Rightarrow x = \sqrt{3}h = 3.5\text{ m}$$



Also, we have

$$W_{PQ} + W_{QR} = mgh$$

That is,

$$[\mu(mg)\cos\theta]\frac{h}{\sin\theta} + \mu(mg)x = mgh$$

$$\Rightarrow \mu h(\cot\theta) + \mu x = h$$

$$\Rightarrow (\mu \times 2 \times \cot 30°) + \mu(\sqrt{3}h) = h$$

$$\Rightarrow 2\sqrt{3}\mu + 2\sqrt{3}\mu = h = 2$$

$$\Rightarrow 4\sqrt{3}\mu = 2$$

$$\Rightarrow \mu = \frac{1}{2\sqrt{3}} \approx 0.29$$

Hence, the correct option is (4).

28. A pipe open at both ends has a fundamental frequency *f* in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now

(1) f

(2) $\frac{f}{2}$

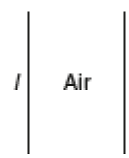
(3) $\frac{3f}{4}$

(4) $2f$

Solution

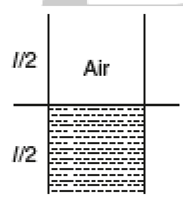
For the open pipe as shown in the following figure, we have

$$v_0 = \frac{v}{2l} = f$$



For the open pipe as shown in the following figure, we have

$$v_0' = \frac{v}{4(l/2)} = \frac{v}{2l} = v_0 = f$$



Here, it acts as a closed pipe with vibrating length, $l/2$.

Hence, the correct option is (1).

29. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $2A/3$ from equilibrium position. The new amplitude of the motion is

(1) $\frac{7A}{3}$

(2) $\frac{A}{3}\sqrt{41}$

(3) $3A$

(4) $A\sqrt{3}$

Solution

Here, we have

$$v = \omega\sqrt{A^2 - x^2}$$

$$\Rightarrow v^2 = \omega^2 (A^2 - x^2)$$

where A is the amplitude of the simple harmonic motion. Initially, at $x = 2a/3$, we have

$$v^2 = \omega^2\left(A^2 - \frac{4A^2}{9}\right) = \frac{5}{9}a^2\omega^2 \qquad (1)$$

Now, at $x = 2a/3$, $v' = 3v$, let the new amplitude be a'. Therefore,

$$(3v)^2 = \omega^2\left[(A')^2 - \frac{4A^2}{9}\right] \qquad (2)$$

Dividing Eq. (1) by Eq. (2), we get

$$\frac{1}{9} = \frac{5}{9}\frac{A^2}{(A')^2 - (4A^2/9)} = \frac{5A^2}{9(A') - 4A^2}$$

Therefore,

$$9(A')^2 - 4A^2 = 45A^2$$
$$9(A')^2 = 49A^2$$
$$\Rightarrow A' = \frac{7}{3}A$$

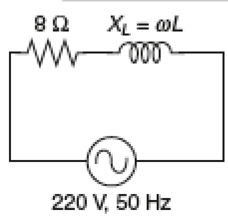
Hence, the correct option is (1).

30. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to

(1) 0.065 H
(2) 80 H
(3) 0.08 H
(4) 0.044 H

Solution

From circuit, we have



$$R = \frac{80}{10} = 8\,\Omega$$

$$\Rightarrow 10 = \frac{220}{\sqrt{R^2 + X_L{}^2}} \Rightarrow \sqrt{64 + X_L^2} = 22$$

$$\Rightarrow X_L^2 = 484 - 64 = 420$$

$$\Rightarrow X_L = \sqrt{420} = 20.5\,\Omega$$

$$\Rightarrow \omega L = 20.5$$

$$L = \frac{20.5}{2\pi \times 50} = \frac{20.5}{314} = 0.065\text{ H}$$

Hence, the correct option is (1).