JEE Main – 2018 (Offline)

Physics

1. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it. Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.)

(1) 50 μm (2) 75 μm

(3) 100 μm (4) 25 μm

Solution

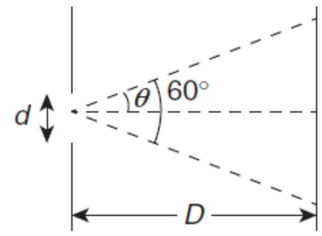
(4) If wave travels at an angle θ from the normal to the slit and if d is the path difference between waves, then

$$\lambda = d\sin\theta$$

$$\Rightarrow \lambda = d\sin 30^\circ = \frac{d}{2}$$

Given: $d = 1 \times 10^{-6}$ m; therefore,

$$\lambda = \frac{1\times10^{-6}}{2} = 5\times10^{-7}\,\text{m} = 5000\ \text{Å}$$



Now, the fringe width is

$$\beta = \frac{\lambda D}{d'}$$

where D is the distance between the slit and the screen; d' is the separation between two slits.

Given: β = 1 cm = 1 × 10^{-2} m;

Also, λ = 5000 Å and D = 50 cm = 50 × 10^{-2} m; therefore,

$$d' = \frac{\lambda D}{\beta} = \frac{5000\times10^{-10}\times50\times10^{-2}}{1\times10^{-2}} = 25\times10^{-6}\,\text{m}$$

Therefore, $d' = 25\,\mu\text{m}$.

2. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the nth state and the ground state, respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the nth state to the ground state. For large n, (A and B are constants)

(1) $\Lambda_n \approx A + B\lambda_n$ (2) $\Lambda_n^2 \approx A + B\lambda_n^2$

(3) $\Lambda_n^2 \approx \lambda$ (4) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

Solution

(4) Given: λ_n and λ_g are de Broglie wavelength of electron in *n*th state and ground state, respectively.

We know that the momentum is

$$p = \frac{h}{\lambda}$$

Therefore, the momentum of *n*th state is

$$P_n = \frac{h}{\lambda_n}$$

The momentum of ground state is

$$P_g = \frac{h}{\lambda_g}$$

Also, we know that

$$E = \frac{P^2}{2m}; \quad E = -k$$

Therefore,

$$E_n = \frac{-h^2}{2m\lambda_n^2} \text{ and } E_g = \frac{-h^2}{2m\lambda_g^2}$$

Now, for emitted photon, we have $E_n - E_g$; therefore,

$$\frac{-h^2}{2m\lambda_n^2} - \left(\frac{-h^2}{2m\lambda_g^2}\right) = \frac{h^2}{2m}\left[\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right]$$

That is,

$$E_n - E_g = \frac{h^2}{2m}\left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2\lambda_n^2}\right)$$

Now, we know that Λ_n be the wavelength of the emitted photon in transition from *n*th state to the ground state. Therefore,

$$E_n - E_g = \frac{h_c}{\Lambda_n}$$

$$\frac{h^2}{2m}\left(\frac{\lambda_n^2 - {}_g^2}{\lambda_g^2\lambda_n^2}\right) = \frac{h_c}{\Lambda_n}$$

$$\Lambda_n = \frac{2mc}{h}\left(\frac{\lambda_g^2\lambda_n^2}{\lambda_n^2 - \lambda_g^2}\right) = \frac{2mc}{h}\lambda_g^2 \frac{\lambda_n^2}{\lambda_n^2\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)}$$

Therefore,

$$\Lambda_n = \frac{2mc\lambda_g^2}{h}\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)^{-1} = \frac{2mc\lambda_g^2}{h}\left(1 + \frac{\lambda_g^2}{\lambda_n^2}\right)$$

That is,

$$\Lambda_n = \frac{2mc\lambda_g^2}{h} + \frac{2mc\lambda_g^4}{h\ \lambda_n^2}$$

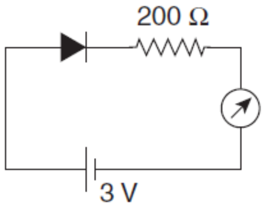
Now, let us consider that

$$A = \frac{2mc\lambda_g^2}{h} \text{ and } B = \frac{2mc\lambda_g^4}{h}$$

Therefore, for large n:

$$\Lambda_n = A + \frac{B}{\lambda_n^2}$$

3. The reading of the ammeter for a silicon diode in the given circuit is

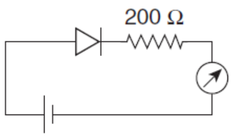


(1) 15 mA (2) 11.5 mA

(3) 13.5 mA (4) 0

Solution

(2) The given diode is forward biased. It offers zero resistance and behaves as a perfect conductor.



The current in the circuit is

$$I = \frac{V - V_{\text{diode}}}{R}$$

Given: $R = 200\ \Omega$; $V = 3$ V. Also, $V_{\text{diode}} = 0.7$ V.

Therefore, the reading of the ammeter for a silicon diode in the given circuit is

$$I = \frac{3 - 0.7}{200} = 0.0115 = 11.5 \text{ mA}$$

4. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are, respectively, 1.5% and 1%, the maximum error in determining the density is

(1) 3.5% (2) 4.5%

(3) 6% (4) 2.5%

Solution

(2) We know that

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

That is,

$$\rho = \frac{M}{l^3}$$

$$\frac{d\rho}{\rho} = \frac{dm}{m} + \frac{3dl}{l}$$

Given : $\frac{dm}{m} = 1.5\%$ and $\frac{dl}{l} = 1\%$.

$$\frac{d\rho}{\rho} = 1.5 + 3 \times 1 = 1.5 + 3 = 4.5\%$$

Therefore, the maximum error in determining the density is 4.5%.

5. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α, respectively, in a uniform magnetic field B. The relation between r_e, r_p, r_α is

(1) $r_e < r_p = r_\alpha$ (2) $r_e < r_p < r_\alpha$

(3) $r_e < r_\alpha < r_p$ (4) $r_e > r_p = r_\alpha$

Solution

(1) The radius of the circular path in magnetic field is given by

$$R = \frac{\sqrt{2km}}{qB}$$

where k is kinetic energy of particle, m is the mass of the particle, q is the charge on the particle, B is the magnetic field intensity and R is the radius of the path.

- For electron: $r_e = \frac{\sqrt{2km_e}}{eB}$.
- For proton: $r_p = \frac{\sqrt{2km_p}}{q_p B} = \frac{\sqrt{2km_p}}{eB}$ (as $q_p = e$)
- For α particle,: $r_\alpha = \frac{\sqrt{2km_\alpha}}{q_\alpha B}$

Now, we know that $m_\alpha = 4m_p$ and $q_\alpha = 2e$. Therefore,

$$r_\alpha = \frac{\sqrt{2k4m_p}}{2eB} = \frac{\sqrt{2km_p}}{eB}$$

Now we know that $m_e < m_p$. Therefore, the correct relationship between r_e, r_p and r_α is

$$r_e < r_p = r_\alpha$$

6. Three concentric metals shells A, B and C of respective radii a, b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$, respectively. The potential of shell B is

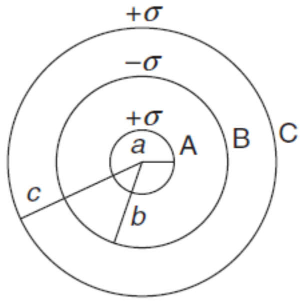
(1) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$ (2) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{b} + a\right]$

(3) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{c} + a\right]$ (4) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{a} + c\right]$

Solution

(1) It is given that three concentric metal shells A, B and C of radii a, b and c, respectively. These have the surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$, respectively.

- Charge on shell A: $q_A = \sigma \times 4\pi a^2$
- Charge on shell B: $q_B = -\sigma \times 4\pi b^2$
- Charge on shell C: $q_C = \sigma \times 4\pi c^2$



Potential of the metal shell B is

$$\frac{q_A}{4\pi\varepsilon_0 b}+\frac{q_B}{4\pi\varepsilon_0 b}+\frac{q_C}{4\pi\varepsilon_0 c}$$

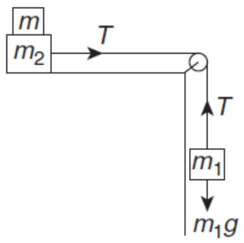
Substituting values of q_A, q_B and q_C, we get

$$V_B=\frac{\sigma a^2}{\varepsilon_0 b}-\frac{\sigma b^2}{\varepsilon_0 b}+\frac{\sigma c^2}{\varepsilon_0 c}$$

$$V_B=\frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b}-\frac{b^2}{b}+\frac{c^2}{c}\right)$$

$$V_B=\frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2}{b}+c\right)$$

7. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is



(1) 27.3 kg (2) 43.3 kg

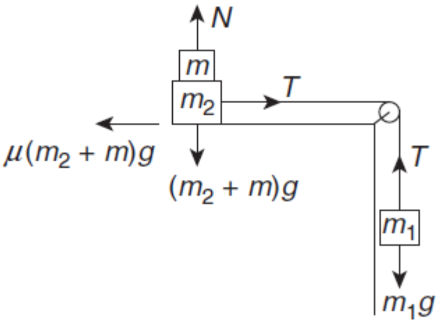
(3) 10.3 kg (4) 18.3 kg

Solution

(1) In order to stop the moving block of mass m_2 acceleration of block should be opposite to its velocity. Therefore,

$$m_1 g < \mu(m_2 + m)g$$

$$\Rightarrow\; m_1 < \mu(m_2 + m)$$



Given: $m_1 = 5$ kg; $m_2 = 10$ kg and $\mu = 0.15$.

$$5 < 0.15\,(10 + m)$$

$$\frac{5}{0.15} - 10 < m \text{ or } m > 23.33 \text{ kg}$$

Therefore, the minimum weight m that should be put on top of m_2 to stop the motion is 23.33 kg.

8. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is

(1) $\frac{k}{2a^2}$ (2) zero

(3) $\frac{3}{2}\frac{k}{a^2}$ (4) $-\frac{k}{4a^2}$

Solution

(2) It is given that the particle is moving in circular path of radius a under the action of attractive potential, which is given by

$$U = \frac{-k}{2r^2}$$

Now, the force acting on particle is

$$F = \frac{-dU}{dr} \Rightarrow F = \frac{-d}{dr}\left(\frac{-k}{2r^2}\right) \Rightarrow F = k \cdot \left(\frac{1}{r^3}\right)$$

Now, this force is the centripetal force. Therefore,

$$F = \frac{mv^2}{r}$$

$$\Rightarrow \frac{mv^2}{r} = \frac{k}{r^3} \Rightarrow mv^2 = \frac{k}{r^2}$$

Thus, the kinetic energy is

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{k}{r^2}$$

and the potential energy is

$$\text{P.E.} = \frac{-k}{2r^2}$$

Now, the total energy is given by

Total energy = Kinetic energy + Potential energy

$$\text{T.E.} = \frac{1}{2}\frac{k}{r^2} - \frac{1}{2}\frac{k}{r^2} = 0$$

Thus, the total energy is zero.

9. A parallel-plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be

(1) 0.3 nC (2) 2.4 nC

(3) 0.9 nC (4) 1.2 nC

Solution

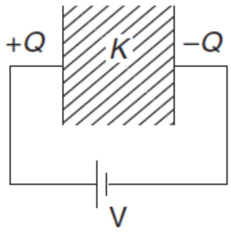
(4) Charge in the capacitor is

$$Q = K \times C \times V$$

Given: $K = 5/3$; $C = 90$ pF $= 90 \times 10^{-12}$; $V = 20$ V

Therefore, the charge in the capacitor is calculated as follows:

$$Q = \frac{5}{3} \times 90 \times 10^{-12} \times 20 = 3000 \times 10^{-12}\,\text{C}$$



Now, the induced charge is

$$Q_{\text{induced}} = Q\left(1 - \frac{1}{k}\right) = 3000 \times 10^{-12}\left(1 - \frac{1}{5/3}\right)$$

$$= 3000 \times 10^{-12}\left(1 - \frac{3}{5}\right) = 3000 \times 10^{-12} \times \frac{2}{5}$$

$$= 1200 \times 10^{-12} = 1.2 \times 10^{-9}\,\text{C}$$

Therefore, the magnitude of the induced charge is

$$Q_{\text{induced}} = 1.2\text{ nC}$$

10. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/s. What is the force constant of the bonds connecting one atom with the other? (Mole weight of silver = 108) and Avogadro number = 6.02×10^{23} g mole^{-1})

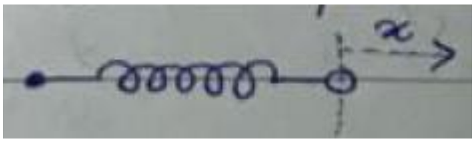
(1) 7.1 N/m (2) 2.2 N/m

(3) 5.5 N/m (4) 6.4 N/m

Solution

(1) The time period of simple harmonic motion is

$$T = 2\pi\sqrt{\frac{m}{k}}$$



Therefore, the frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Given: Frequency, $f = 10^{12}$. Therefore,

$$\frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12} \Rightarrow \sqrt{\frac{k}{m}} = 2\pi \times 10^{12}$$

$$\Rightarrow \frac{k}{m} = (2\pi \times 10^{12})^2$$

$$\Rightarrow k = m(2\pi \times 10^{12})^2$$

Given: Molecular weight of silver is 108.

Also, Avagadro number is 6.02×10^{23} g/mole. Therefore,

$$m = \frac{108}{6.023 \times 10^{23}} \times 10^{-3}$$

Thus, the force constant of the bonds connecting one atom with the other is

$$k = \frac{108}{6.023 \times 10^{23}} \times 10^{-3} \times (2\pi \times 10^{12})^2 = 7.1 \text{ N/m}$$

11. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_e. The values of p_d and p_c are, respectively,

(1) (0.28, 0.89) (2) (0, 0)

(3) (0, 1) (4) (0.89, 0.28)

Solution

(4) Let the initial velocity of neutron be V_i and its kinetic energy be E.

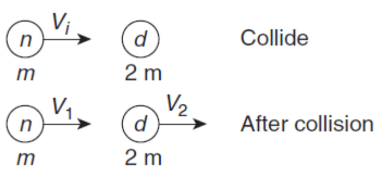
Case 1: From conservation of momentum, we have

$$mv_i = mv_1 + 2mv_2$$

$$\Rightarrow v_i = v_1 + 2v_2 \quad (1)$$

Now for elastic collision:

$$v_i = v_2 - v_1 \quad (2)$$



Solving Eqs. (1) and (2), we get

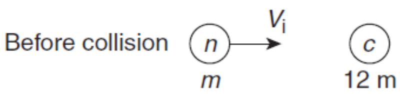
$$2v_i = 3v_2 \Rightarrow v_i = \frac{3v_2}{2}$$

or $v_2 = \frac{2v_i}{3}$ and $v_i = \frac{-v_i}{3}$. Thus, the fractional loss of deuterium's energy is

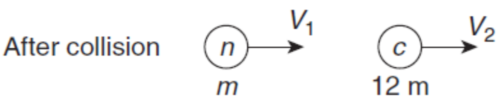
$$\frac{\left(\frac{\Delta E}{E} = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_1^2\right)}{\left(\frac{1}{2}mv_i^2\right)} = p_d$$

Therefore, $$p_d = \frac{v_i^2 - v_1^2}{v_i^2} = \frac{v_i^2 - (-v_i/3)^2}{v_i^2} = 1 - \frac{1}{9} = \frac{8}{9} = 0.89$$

Case 2: Before collision:



After collision:



From conservation of momentum: $mv_i = mv_1 + 12mv_2$

$$v_i = v_1 + 12v_2 \quad (3)$$

For elastic collision: $$v_i = v_2 - v_1 \quad (4)$$

Solving Eqns. (3) and (4), we get

$$2v_i = 13v_2 \Rightarrow v_2 = \frac{2v_i}{13} \text{ and } v_1 = \frac{-11v_i}{13}$$

Thus, the fractional loss of carbon's energy is

$$\frac{\Delta E}{E} = \frac{\left(\frac{1}{2}mv_i^2 - \frac{1}{2}mv_i^2\right)}{\left(\frac{1}{2}mv_i^2\right)} = p_c$$

Therefore, $$p_c = \frac{v_i^2 - v_i^2}{v_i^2} = \frac{\left[v_i^2 - \left(\frac{-11v_i^2}{13}\right)^2\right]}{v_i^2} = 1 - \left(\frac{11}{13}\right)^2$$

$$p_c = 1 - \frac{121}{169} = \frac{48}{169} = 0.28$$

Hence, the value of p_d is 0.89 and that of p_c is 0.28.

12. The dipole moment of a circular loop carrying a current I is m and the magnitude field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is

(1) $\sqrt{3}$ (2) $\sqrt{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) 2

Solution

(2) The dipole moment of a circular loop is

$$m = I \times \text{Area} = I(\pi R^2) \quad (1)$$

The magnetic field is

$$B_1 = \frac{\mu_0}{2}\frac{I}{R}$$

where R is the radius of the circular loop when the dipole moment is doubled keeping current constant. We have

$$m' = I(\pi R'^2)$$

$$\Rightarrow 2m = I(\pi R'^2) \qquad (2)$$

From Eqs. (1) and (2), we get

$$R' = \sqrt{2}R$$

Therefore, the magnetic field the magnetic field at the centre of the loop is

$$B_2 = \frac{\mu_0}{2}\frac{I}{R'} = \frac{\mu_0}{2}\frac{I}{\sqrt{2}R}$$

Therefore, the ratio of the two magnetic fields is

$$\frac{B_1}{B_2} = \frac{\left(\frac{\mu_0}{2}\frac{I}{R}\right)}{\left(\frac{\mu_0}{2}\frac{I}{\sqrt{2}R}\right)} = \sqrt{2}$$

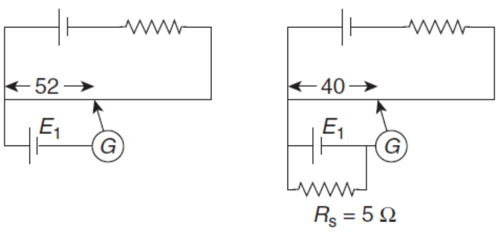
13. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 1.5 Ω (2) 2 Ω

(3) 2.5 Ω (4) 1 Ω

Solution

(1) The given arrangements are depicted in the following figures:



We know that $E_1 \propto l_1$.

If the cell is shunted by 5 Ω resistor, then $E_2 \propto l_2$ or

$$E_1 - I_r \propto l_2$$

where r is the internal resistance of the cell; therefore,

$$\frac{E_1}{E_1 - I_r} = \frac{l_1}{l_2}$$

Now, $$I = \frac{V}{R} = \frac{E_1}{r + R_s} = \frac{E_1}{r+5}$$

Therefore, $$\frac{E_1}{E_1-\left(\frac{E_1}{r+5}\right)r}=\frac{l_1}{l_2} \Rightarrow \frac{1}{1-\left(\frac{r}{r+5}\right)}=\frac{l_1}{l_2}$$

Given: l_1 = 52 and l_2 = 40; therefore,

$$\frac{1}{\left(\frac{4+5-r}{r+5}\right)}=\frac{52}{40} \Rightarrow \frac{r+5}{5}=\frac{52}{40} \Rightarrow r = 1.5\ \Omega$$

14. A telephonic communication service is working at the carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) 2×10^4 (2) 2×10^5

(3) 2×10^6 (4) 2×10^3

Solution

(2) It is given that the carrier frequency is

$$10\text{ GHz} = 10 \times 10^9\text{ Hz}$$

The frequency utilized for transmission = 10% of carrier frequency.

$$=\frac{10}{100}\times 10\times 10^9 = 10^9$$

Therefore, the available bandwidth is 10^9 Hz.

The bandwidth of each telephonic channel is

$$5\text{ kHz} = 5 \times 10^3\text{ Hz}$$

Thus, the number of channels transmitted simultaneously is

$$\frac{\text{Available bandwidth}}{\text{Bandwidth of each channel}} = \frac{10^9}{5\times 10^3} = 2\times 10^5$$

15. Unpolarized light of intensity *I* passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{1}{2}$. Now, another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{1}{8}$. The angle between polarizer A and C is

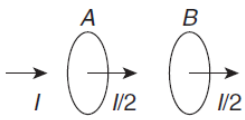
(1) 30° (2) 45°

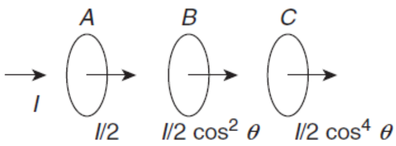
(3) 60° (4) 0°

Solution

When an unpolarized light of intensity *I* passes through an ideal polarizer, the intensity of emerging light is $\frac{I}{2}$.

It is given that when another polarizer B is placed behind an ideal polarizer A, then intensity of light beyond B is $\frac{I}{2}$. This implies that polarizer A and B are placed parallel. When another polarizer C is placed between A and B, then the intensity beyond B is $I/8$.





From Malus law, we have

$$I \propto \cos^2\theta$$

$$I = I_0 \cos^2\theta$$

$$\Rightarrow \frac{I}{2}\cos^4\theta = \frac{I}{8}$$

$$\Rightarrow \cos^4\theta = \frac{I}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

16. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances?

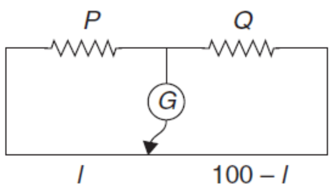
(1) 505 kΩ (2) 550 kΩ

(3) 910 kΩ (4) 990 kΩ

Solution

(2) In the balanced condition, we have

$$\frac{P}{Q} = \frac{l}{100 - l} \quad (1)$$



When the resistances are interchanged, the balance point of a meter bridge shifts to left by 10 cm. Therefore, the balance condition becomes

$$\frac{Q}{P} = \frac{l - 10}{110 - l} \quad (2)$$

Using Eqs. (1) and (2), we get

$$\frac{l}{100 - l} = \frac{110 - l}{l - 10}$$

$$\Rightarrow l^2 - 10l = 11{,}000 - 100l - 100l + l^2$$

$$\Rightarrow 200l = 11{,}000$$

$$\Rightarrow l = 55 \text{ cm}$$

Substituting l in Eq. (1), we get

$$\frac{P}{Q} = \frac{55}{100 - 55}$$

$$\Rightarrow P = Q\left(\frac{55}{45}\right) \qquad (3)$$

It is given that

$$P + Q = 1 \text{ k}\Omega \Rightarrow P + Q = 1000\ \Omega$$

Therefore, using Eq. (3), we get

$$Q\left(\frac{55}{45}\right) + Q = 1000$$

$$55Q + 45Q = 45000$$

$$\Rightarrow 100Q = 45000$$

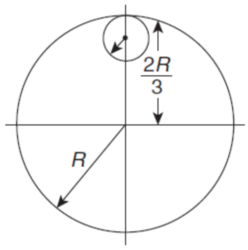
$$\Rightarrow Q = 450\ \Omega$$

Therefore, before interchanging the resistances, the resistance on the left slot should be

$$P + 450 = 1000$$

$$\Rightarrow P = 550\ \Omega$$

17. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is



(1) $\frac{40}{9}MR^2$ (2) $10MR^2$

(3) $\frac{37}{9}MR^2$ (4) $4MR^2$

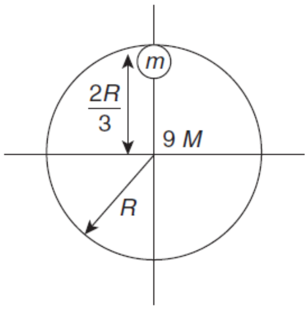
Solution

(4) We have

$$m = \frac{9M}{9} = M$$

$$I_1 = \frac{9M \times R^2}{2}$$

and
$$I_2 = \frac{M \times \left(\frac{R}{3}\right)^2}{2} + M \times \left(\frac{2R}{3}\right)^2 = \frac{MR^3}{2}$$



Therefore, the moment of inertia of the remaining disc is

$$I_{\text{remain}} = I_1 - I_2 = \frac{9}{2}MR^2 - \frac{MR^2}{2} = \frac{8MR^2}{2}$$

$$I_{\text{remain}} = 4mR^2$$

18. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(1) $\sqrt{2}v_0$ (2) $\frac{v_0}{2}$

(3) $\frac{v_0}{\sqrt{2}}$ (4) $\frac{v_0}{4}$

Solution

(1) It is given that the initial speed of particle is v_0.

The final total kinetic energy is 50% greater than the original kinetic energy.

For elastic collision:

$$mv_0 = mv_1 + mv_2$$

$$\Rightarrow v_0 = v_1 + v_2 \qquad (1)$$

Thus, the kinetic energy is

$$\frac{3}{2} \times \frac{1}{2}mv_0^2 = \frac{1}{2}m(v_1^2 + v_2^2)$$

$$\Rightarrow \frac{3}{2}v_0^2 = v_1^2 + v_2^2$$

Adding and subtracting $2v_1v_2$ on right-hand side of this equation, we get

$$v_1^2 + v_2^2 + 2v_1v_2 - 2v_1v_2 = \frac{3}{2}v_0^2$$

$$(v_1 + v_2)^2 - 2v_1v_2 = \frac{3}{2}v_0^2$$

Using Eq. (1), we get

$$v_0^2 - 2v_1v_2 = \frac{3}{2}v_0^2$$

$$\Rightarrow \frac{3}{2}v_0^2 - v_0^2 = -2v_1v_2$$

$$\frac{v_0^2}{2} = -2v_1v_2$$

$$\Rightarrow \; v_1v_2 = \frac{-v_0^2}{4} \qquad (2)$$

Now, $$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2$$

Adding and subtracting $2v_1v_2$ on right-hand side of this equation, we get

$$(v_1 - v_2)^2 = v_1^2 + v_2^2 + 2v_1v_2 - 2v_1v_2 - 2v_1v_2$$

$$(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$$

From Eqs. (1) and (2), we get

$$(v_1 - v_2)^2 = v_0^2 + v_0^2 = 2v_0^2$$

$$\Rightarrow \; v_1 - v_2 = \sqrt{2}v_0$$

Therefore, the relative velocity between the two particles after collision is $\sqrt{2}v_0$.

19. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c} - t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z - ct)]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is non-magnetic. If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$ (2) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

(3) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$ (4) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$

Solution

(2) It is given that the electric field in air is

$$E_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c} - t\right)\right] \qquad \ldots(1)$$

The electric field in mediumis

$$E_2 = E_{02}\hat{x}\cos[(2z - ct)] \qquad \ldots(2)$$

After refraction, frequency remains unchanged whereas wavelength changes. Thus, from equation Eqs. (1) and (2), we have

$$k' = 2k$$

We know that

$$k = \frac{2\pi}{\lambda} \;\Rightarrow\; \frac{2\pi}{\lambda'} = 2\left(\frac{2\pi}{\lambda}\right) \;\Rightarrow\; \lambda' = \frac{\lambda}{2}$$

Also, $$\frac{v}{c} = \frac{1}{\lambda}$$

- For medium 1: Velocity = c

- For medium 2: Velocity = $c/2$

Now, we know that

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon}}$$

$$\Rightarrow \frac{1}{\sqrt{\mu_0 \varepsilon_{r_2}}} = \frac{1}{2}\frac{1}{\sqrt{\mu_0 \varepsilon_{r_1}}}$$

$$\Rightarrow \sqrt{\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}} = \frac{1}{2} \Rightarrow \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$$

20. For an *RLC* circuit driven with voltage of amplitude V_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the current exhibits resonance. The quality factor, Q is given by

(1) $\frac{\omega_0 R}{L}$ (2) $\frac{R}{(\omega_0 C)}$

(3) $\frac{CR}{\omega_0}$ (4) $\frac{\omega_0 L}{R}$

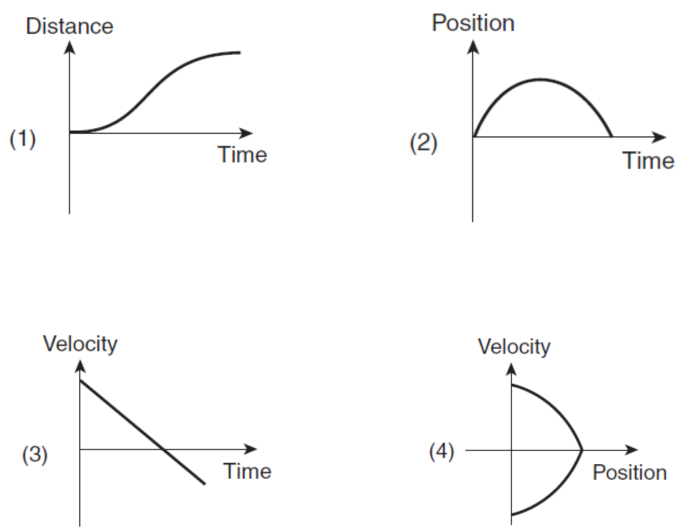
Solution

(4) The quality factor is

$$Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$

Quality factor is a dimensionless parameter and it indicates energy losses within a resonant element.

21. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.



Solution

(1) Graphs provided in options (2), (3) and (4) correspond to uniformly accelerated motion in a straight line with positive initial velocity and constant negative acceleration. Therefore, graph provided in option (1) does not represent same motion.

22. Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω, respectively. The voltage across the load lies between

(1) 11.5 V and 11.6 V.

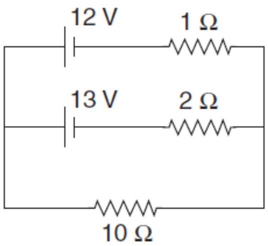
(2) 11.4 V and 11.5 V.

(3) 11.7 V and 11.8 V.

(4) 11.6 V and 11.7 V.

Solution

(1) Here, $R_1 = 1\ \Omega$ and $R_2 = 2\ \Omega$ are parallel; therefore, the equivalent resistance is



Therefore,
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{2 \times 1}{2+1} = \frac{2}{3}\Omega$$

The equivalent emf is

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{R_1} + \frac{\varepsilon_2}{R_2} \Rightarrow \varepsilon_{eq} = r_{eq}\left(\frac{\varepsilon_1}{R_1} + \frac{\varepsilon_2}{R_2}\right)$$

$$\varepsilon_{eq} = \frac{2}{3}\left(\frac{12}{1} + \frac{13}{2}\right) = \frac{2}{3} \times \frac{37}{2} = \frac{37}{3}\text{ V}$$

Now,
$$V = \varepsilon_{eq}\left(\frac{R}{R + R_{eq}}\right) = \frac{37}{3} \times \frac{10}{\left(10 + \frac{2}{3}\right)} = \frac{37}{3} \times \frac{10}{32} \times 3$$

Therefore, $V = 11.5625$ V.

Therefore, the voltage across the load lies between 11.5 V and 11.6 V.

23. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then

(1) $T \propto R^{\frac{n}{2}+1}$ (2) $T \propto R^{(n+1)/2}$

(3) $T \propto R^{n/2}$ (4) $T \propto R^{3/2}$ for any n

Solution

(2) Given: The particle is moving with uniform speed in a circular orbit of radius R in a central force inversely proportional to nth power of R. That is,

$$\text{Force} \propto \frac{1}{R^n}$$

Also, force is $m\omega^2 R$. Therefore,

$$m\omega^2 R \propto \frac{1}{R^n}$$

$$\Rightarrow \omega^2 \propto \frac{1}{R^{n+1}}$$

$$\Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$$

We know that the time period is

$$T = \frac{2\pi}{\omega}$$

Therefore, $$T \propto R^{\frac{n+1}{2}}$$

24. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is

(1) $16\nu_L$ (2) $\nu_L/16$

(3) $\nu_L/25$ (4) $25\nu_L$

Solution

(3) We know that

$$h\nu = E_0\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For Lyman series: $n_1 = 1$ and $n_2 = \infty$.

Therefore, $$h\nu_L = E\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] \Rightarrow h\nu_L = E \quad \ldots(1)$$

For Pfund series: $n_1 = 5$ and $n_2 = \infty$. Therefore,

$$h\gamma_P = E\left[\frac{1}{5^2} - \frac{1}{\infty^2}\right] = \frac{E}{25}$$

From Eq. (1), we have

$$h\nu_P = \frac{h\nu_L}{25}$$

Therefore, the series limit frequency of Pfund series is

$$\gamma_P = \frac{\gamma_L}{25}$$

25. In an AC circuit, the instantaneous emf and current are given by $\varepsilon = 100\sin 30t$ and $i = 20\left(30t - \frac{\pi}{4}\right)$, respectively. In one cycle of AC, the average power consumed by the circuit and the wattles current are, respectively,

(1) $\frac{1000}{\sqrt{2}}, 10$ (2) $\frac{50}{\sqrt{2}}, 0$

(3) 50, 0 (4) 50, 10

Solution

(1) The emf is given by

$$100 \sin 30t \quad \ldots(1)$$

and $$i = 20\sin\left(30t - \frac{\pi}{4}\right) \quad \ldots(2)$$

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos\theta = \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\theta$$

From Eqs. (1) and (2), we get

$$V_0 = 100; \quad I_0 = 20; \quad \theta = 45°$$

Therefore,

$$P_{\text{avg}} = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right)\cos 45°$$

$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \left(\frac{1000}{\sqrt{2}}\right) \text{ W}$$

Now, the wattles current is

$$I_{\text{rms}} \cos\theta = \frac{I_0}{\sqrt{2}}\cos\theta = \frac{20}{\sqrt{2}}\cos 45°$$

$$= \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 10 \text{ A}$$

Therefore, the average power consumed by the circuit is

$$P_{\text{avg}} = \left(\frac{1000}{\sqrt{2}}\right) \text{W}$$

26. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 195 K (b) −2.7 kJ

(2) (a) 189 K (b) −2.7 kJ

(3) (a) 195 K (b) 2.7 kJ

(4) (a) 189 K (b) 2.7 kJ

Solution

(2) For adiabatic process: PV^{γ} = constant

(a) we have

$$TV^{\gamma-1} = \text{constant} \qquad (\text{as } PV = nRT)$$

It is given that

$$T_i = 27°\text{C} = 27 + 273 = 300 \text{ K}$$

$$V_f = 2V_i$$

For monatomic gas: $\gamma = \frac{5}{3}$.

Therefore, $$\frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} \Rightarrow \frac{T_f}{300} = \left(\frac{V_i}{2V_i}\right)^{5/3-1}$$

That is, the final temperature of the gas is

$$T_f = 300\left(\frac{1}{2}\right)^{2/3} = 189 \text{ K}$$

(b) Now, the change in internal energy is given as

$$\Delta U = nCv\Delta T = 2\times\frac{3}{2}R\times(189-300) = -2.7\text{ kJ}$$

27. A solid sphere of radius r made of a soft material of bulk modulus *K* is surrounded by a liquid in a cylindrical container. A massless piston of area of floats on the surface of the liquid, covering the entire cross-section of the cylindrical container. When a mass *m* is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is

(1) $\frac{Ka}{3mg}$ (2) $\frac{mg}{3Ka}$

(3) $\frac{mg}{Ka}$ (4) $\frac{Ka}{mg}$

Solution

(2) We know that

$$\text{Bulk modulus } = \frac{\text{Volumetric stress}}{\text{Volumetric stress}}$$

$$K = \frac{dP}{dV/V} \quad \ldots \qquad (1)$$

Also, for sphere:

$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dV}{V} = \frac{3dr}{r} \qquad (2)$$

From Eqs. (1) and (2), we have

$$\frac{3dr}{r} = \frac{dP}{k}$$

$$\Rightarrow \frac{dr}{r} = \frac{dp}{3k} = \frac{mg}{3ka} \qquad \left(\text{as volumetric stress } \Delta P = \frac{mg}{a}\right)$$

Therefore, the fractional decrement in the radius of the sphere is

$$\frac{dr}{r} = \frac{mg}{3ka}$$

28. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 2.5 kHz (2) 10 kHz

(3) 7.5 kHz (4) 5 kHz

Solution

(4) The velocity of wave is given as

$$v = \sqrt{\frac{Y}{\rho}} \qquad (1)$$

where Y is Young's modulus: $Y = 9.27 \times 10^{10}$ Pa

and ρ is the density of granite: $\rho = 2.7 \times 10^3$ kg/m^3

Now, the fundamental frequency is

$$f = \frac{v}{\lambda}$$

Also, $\lambda = 2L$. Therefore,

$$f = \frac{v}{2L}$$

From Eq. (1), we get

$$f = \frac{1}{2L}\sqrt{\frac{Y}{\rho}}$$

Given: $L = 60$ cm $= 60 \times 10^{-2}$ m. Therefore,

$$f = \frac{1}{2 \times 60 \times 10^{-2}}\sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 70^3}} = 4.88 \times 10^3 \text{ Hz} \ \sim 5 \text{ kHz}$$

Therefore, the fundamental frequency of longitudinal vibrations is 5 kHz.

29. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly

(1) 4.70×10^3 N/m^2

(2) 2.35×10^2 N/m^2

(3) 4.70×10^2 N/m^2

(4) 2.35×10^3 N/m^2

Solution

(4) In this case, 10^{23} hydrogen molecules strike a fixed wall at angle of 45° to normal.

We know that

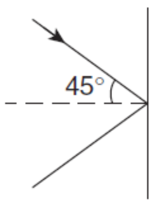
$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

Also,

$$\text{Force} = \text{Change in momentum}$$

That is,

$$\text{Force} = \frac{\Delta P}{\Delta t} \times N$$



Therefore,

$$\text{Force} = 2mv\cos 45 \times N$$

$$\text{Pressure} = \frac{2mv\cos 45 \times N}{Area}$$

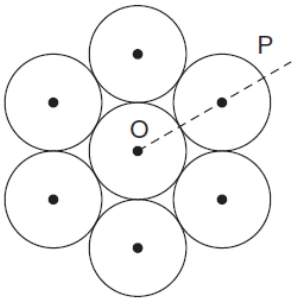
Now, $m = 3.32 \times 10^{-27}$ kg; $N = 10^{23}$; $V = 10^3$ m/s.

Area = 2 cm = 2×10^{-2} m.

Therefore, the pressures on the wall is

$$P = \frac{\left(2 \times 3.32 \times 10^{-27} \times \frac{1}{\sqrt{2}} \times 10^3 \times 10^{23}\right)}{2 \times 10^{-2}} = 2.35 \times 10^3 \text{ N/m}^2$$

30. Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is

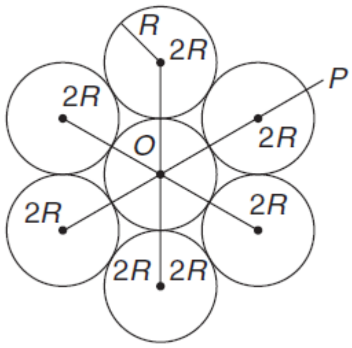


(1) $\frac{55}{2}MR^2$ (2) $\frac{73}{2}MR^2$

(3) $\frac{181}{2}MR^2$ (4) $\frac{19}{2}MR^2$

Solution

(3) Moment of inertia about the axis normal to the plane and passing through point O is depicted in the following figure:



Given, the radius of each disk is R and mass is M.

$$I_0 = I_{CM} + Md^2 = \frac{7mR^2}{2} + 6m(2R)^2 = \frac{55mR^2}{2}$$

Now, the moment of inertia about the axis normal to the plane and passing through point P is

$$I_P = I_0 + Md^2 = \frac{55mR^2}{2} + 7m(3R)^2 = \frac{181}{2}MR^2$$