JEE ADVANCED 2013
PAPER 1
PHYSICS

Single Option Correct Type

This section contains **10 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

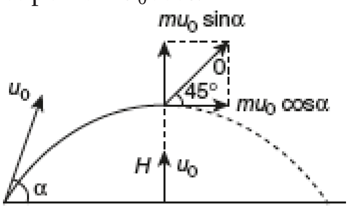
1. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}+\alpha$

(C) $\frac{\pi}{2}-\alpha$ (D) $\frac{\pi}{2}$

Solution

At the highest point, the vertical component $u_0\sin\alpha$ becomes zero.
Speed of the first particle at the highest point = $u_0\cos\alpha$



Speed of the second particle at the highest point $=\sqrt{u_0^2-2gH}$

where, $H=\dfrac{u_0^2\sin^2\alpha}{2g}$

$\Rightarrow$ Speed of the second particle $=\sqrt{u_0^2-2g\dfrac{u_0^2\sin^2\alpha}{2g}}=\sqrt{u_0^2-u_0^2\sin^2\alpha}=\sqrt{u_0^2(1-\sin^2\alpha)}=u_0\cos\alpha$

Therefore, final momentum $=mu_0\cos\alpha\hat{i}+mu_0\cos\alpha\hat{j}$

$$\Rightarrow \tan\alpha=\frac{mu_0\cos\alpha}{mu_0\cos\alpha}=1 \text{ or } \alpha=45^\circ=\frac{\pi}{4}$$

The two particles have equal momentum directed at right angles to each other. The resultant will be equally inclined to both, that is, at an angle of 45° with the horizontal.
Hence, the correct option is (A).

2. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is

(A) 1 m (B) 2 m
(C) 3 m (D) 6 m

Solution

The refractive index will be $\mu=\dfrac{c}{v}=\dfrac{f\lambda_{\text{air}}}{f\lambda_{\text{med}}}=\dfrac{\lambda_{\text{air}}}{\lambda_{\text{med}}}=\dfrac{3}{2}$

Now, $v=+8$ m

As image is real, $m=\dfrac{-1}{3}\Rightarrow\dfrac{v}{u}=\dfrac{-1}{3}\Rightarrow\dfrac{8}{u}=\dfrac{-1}{3}\Rightarrow u=\dfrac{8}{-(1/3)}=-24$ m

Using the lens formula, we get

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \Rightarrow \frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

We have $u=-24$ m and $v=+8$ m. Hence,

$$\frac{1}{f}=\frac{1}{8}+\frac{1}{24}=\frac{4}{24}$$

$$\Rightarrow f=6 \text{ m}$$

Using lens maker's formula ,

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R}-\frac{1}{\infty}\right]$$

$$f=\frac{R}{(\mu-1)}=\frac{R}{1.5-1}$$

$$6=\frac{R}{0.5}$$

$$\Rightarrow R=3 \text{ m}$$

Hence, the correct option is (C).

3. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(A) 5.112 cm (B) 5.124 cm

(C) 5.136 cm (D) 5.148 cm

Solution

We know that 50 Vernier divisions are equivalent to 2.45 cm or 24.5 mm or 49 scale divisions of 0.5 mm each.

$$50 \text{ Vernier scale divisions} = 49 \text{ main scale divisions}$$

$$1 \text{ V.S.D.}=\frac{49}{50}\text{M.S.D.}$$

$$\text{Vernier Constant} = 1 \text{ M.S.D.} - 1 \text{ V.S.D.} = \frac{1}{50}\text{M.S.D. where 1M.S.D.}= 0.5\,\text{mm}$$

$$\text{V.C.}=\frac{1}{50}\times\frac{1}{2}\text{mm}=\frac{1}{100}\text{mm}=0.001\,\text{cm}$$

$$\text{Reading} = 5.10+\text{V.C.}\times 24 = 5.10+0.024=5.124\,\text{cm}$$

Hence, the correct option is (B).

4. The work done on a particle of mass *m* by a force, $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (*K* being a constant of appropriate dimensions), when the particle is taken from the point (*a*, 0) to the point (0, *a*) along a circular path of radius *a* about the origin in the *x*–*y* plane is

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$

(C) $\frac{K\pi}{2a}$ (D) 0

Solution

1. We have

$$F=K\left[\frac{x\hat{i}}{(x^2+y^2)^{3/2}}+\frac{y\hat{i}}{(x^2+y^2)^{3/2}}\right]$$

$$dW = \vec{F}\cdot\vec{dr} \;\; = \vec{F}\cdot(\hat{i}dx + \hat{j}dy)$$

$$dW = K\left[\frac{xdx + ydy}{(x^2+y^2)^{3/2}}\right]$$

Let $x = r\cos\theta$ and $y = r\sin\theta$, then $x^2 + y^2 = r^2$,

$$dW = K\int\frac{xdx+ydy}{(r^2)^{3/2}} = K\int_a^0\frac{x}{(r^2)^{3/2}}dx + K\int_0^a\frac{y}{(r^2)^{3/2}}dy$$

$$= \frac{K}{r^3}\left|\frac{x^2}{2}\right|_a^0 + \frac{K}{r^3}\left|\frac{y^2}{2}\right|_0^a$$

$$= \frac{K}{r^3}\left[\frac{0^2}{2} - \frac{a^2}{2} + \frac{a^2}{2} - \frac{0^2}{2}\right] = 0$$

Hence, the correct option is (D).

5. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(A) 0.25 (B) 0.50
(C) 2.00 (D) 4.00

Solution

Force will be same.

From $Y = \frac{FL}{Al}$, we have $l = \frac{FL}{AY}$

The ratio of the elongation in the thin wire to that in the thick wire will be

$$\frac{l_1}{l_2} = \frac{FL}{\pi R^2\ Y} \times \frac{\pi\ (2R)^2\ Y}{F\ .2L} = 2$$

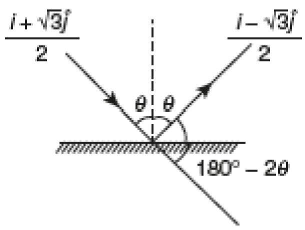
Hence, the correct option is (C).

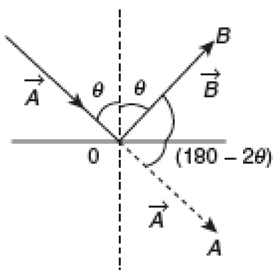
6. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is

(A) 30° (B) 45°
(C) 60° (D) 75°

Solution

1.





Vectors $\vec{A}$ and $\vec{B}$ are originating from O have angle (180° – 2θ) between them. Using $\vec{A}\cdot\vec{B} = AB\cos\theta$, we get

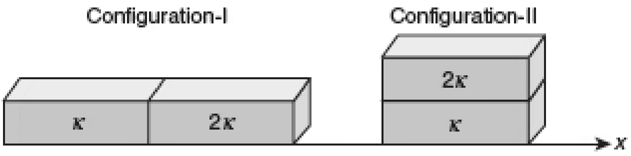
$$\cos(180° - 2\theta) = \frac{\left(\frac{\hat{i}}{2}+\frac{\sqrt{3}}{2}\hat{j}\right)\cdot\left(\frac{\hat{i}}{2}-\frac{\sqrt{3}}{2}\hat{j}\right)}{\left|\frac{\hat{i}}{2}+\frac{\sqrt{3}}{2}\hat{j}\right|\left|\frac{\hat{i}}{2}-\frac{\sqrt{3}}{2}\hat{j}\right|} = \frac{\left(\frac{1}{4}-\frac{3}{4}\right)}{\left(\frac{1}{4}+\frac{3}{4}\right)\left(\frac{1}{4}+\frac{3}{4}\right)} = -\frac{1}{2}$$

$$\cos(180° - 2\theta) = 120°$$

$$\theta = 30°$$

Hence, the correct option is (A).

7. Two rectangular blocks, having identical dimensions, can be arranged in either configuration-I or configuration-II as shown in the figure. One of the blocks has thermal conductivity κ and the other 2κ. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in configuration-I. The time to transport the same amount of heat in configuration-II is



(A) 2.0 s
(B) 3.0 s
(C) 4.5 s
(D) 6.0 s

Solution

In configuration-I: If κ and 2κ be values of thermal conductivity, then $\frac{1}{\kappa}$ and $\frac{1}{2\kappa}$ will be thermal resistance. Net thermal resistance in series is

$$\left[\frac{1}{\kappa}+\frac{1}{2\kappa}\right] = \frac{2+1}{2\kappa} = \frac{3}{2\kappa}$$

and net thermal conductivity is $\frac{2\kappa}{3}$.

$$Q = \frac{[A\ (T_1 - T_2)t]/l}{(1/\kappa)+(1/2\kappa)} = \frac{[A\ (T_1-T_2)]/l}{(2+1)/2\kappa} = \frac{2\kappa A\ (T_1-T_2)t}{3l} \qquad (1)$$

In configuration-II: Equivalent thermal resistance is

$$\frac{(1/\kappa)\times(1/2\kappa)}{(1/\kappa)+(1/2\kappa)} = \frac{1}{2\kappa^2}\times\frac{2\kappa}{3} = \frac{1}{3\kappa}$$

and thermal conductivity is 3κ.

$$Q = \frac{3\kappa A(T_1-T_2)t'}{l} \qquad (2)$$

Dividing Eq.(2) by Eq. (1), we get

$$\frac{[3kA(T_1-T_2)t']/l}{[2kA(T_1-T_2)t]/3l} = 1 \Rightarrow \frac{9t'}{2t} = 1 \Rightarrow \frac{t'}{t} = \frac{2}{9} \Rightarrow t' = \frac{2}{9}\times 9 = 2\text{ s}$$

Hence, the correct option is (A).

8. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is

(A) 0.3×10^{-17} kg-ms^{-1}
(B) 1.0×10^{-17} kg-ms^{-1}
(C) 3.0×10^{-17} kg-ms^{-1}
(D) 9.0×10^{-17} kg-ms^{-1}

Solution

Duration of pulse, t = 100 ns = 100×10^{-9} s

Power of the pulse, P = 30 mW $= 30 \times 10^{-3}$ W
Speed of light, $c = 3 \times 10^8$ ms^{-1}
The final momentum of the object will be

$$p = \frac{E}{c} = \frac{P \times t}{c} = \frac{30 \times 10^{-3} \times 100 \times 10^{-9}}{3 \times 10^8} = 10^{-17} \text{ kg-ms}^{-1}$$

Hence, the correct option is (B).

9. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(A) $(2n+1)\frac{\lambda}{2}$
(B) $(2n+1)\frac{\lambda}{4}$
(B) $(2n+1)\frac{\lambda}{8}$
(D) $(2n+1)\frac{\lambda}{16}$

Solution

Wavelength of light = λ

Now, using the relation for intensity, $I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$

$$\frac{1}{2} = \cos^2\left(\frac{\phi}{2}\right) \Rightarrow \cos\frac{\phi}{2} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\phi}{2} = \frac{\pi}{4} \Rightarrow \phi = \frac{\pi}{2} \Rightarrow \phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$\phi = \frac{2\pi}{\lambda}\Delta x \Rightarrow \frac{\pi}{2} = \frac{2\pi}{\lambda}\Delta x \Rightarrow \Delta x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \Rightarrow \Delta x = (2n+1)\frac{\lambda}{4}$$

Hence, the correct option is (B).

10. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2:3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4:3. The ratio of their densities is

(A) 1:4
(B) 1:2
(C) 6:9
(D) 8:9

Solution

The ratio of atomic masses of two non-reactive monoatomic ideal gases is M_1:M_2 = 2:3.

$$PV = RT \Rightarrow P \times \frac{M}{\rho} = RT \Rightarrow P = \frac{\rho RT}{M}$$

Now for two different gases, the above relation can be written as

$$\frac{P_1}{P_2} = \frac{\rho_1}{M_1} \times \frac{M_2}{\rho_2} \Rightarrow \frac{4}{3} = \frac{\rho_1}{2} \times \frac{3}{\rho_2} \Rightarrow \frac{\rho_1}{\rho_2} = \frac{8}{9}$$

Hence, the correct option is (D).

<H2>One or More Options Correct Type

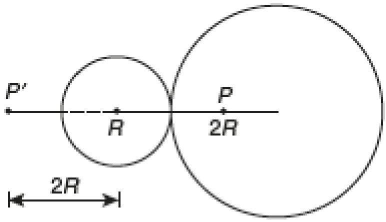
This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** are correct.

11. Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2, respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be

(A) -4
(B) $-\frac{32}{25}$
(C) $\frac{32}{25}$
(D) 4

Solution

It is given that, $E_P = 0$.



Now, $E_1 = \dfrac{q_1}{4\pi\varepsilon_0(2R)^2} = \dfrac{\rho_1(4/3)\pi R^3}{4\pi\varepsilon_0(2R)^2} = \dfrac{\rho_1 R}{12\varepsilon_0}$ and $E_2 = \dfrac{\rho_2 R}{3\varepsilon_0}$

$E_1 = E_2 \Rightarrow \dfrac{\rho_1 R}{12\varepsilon_0} = \dfrac{\rho_2 R}{3\varepsilon_0} \Rightarrow \dfrac{\rho_1}{\rho_2} = 4$

Also, $E_{P'} = 0$

$E_1 = \dfrac{\rho_1(4/3)\pi R^3}{4\pi\varepsilon_0(2R)^2} = \dfrac{\rho_1 R}{12\varepsilon_0}$ and $E_2' = \dfrac{q_2}{4\pi\varepsilon_0(5R)^2} = \dfrac{\rho_2 \times (4/3)\pi(2R)^3}{4\pi\varepsilon_0(5R)^2} = \dfrac{8\rho_2 \times R}{25\times 3\varepsilon_0}$

Now, $E_1 + E_2' = 0$

$\Rightarrow \dfrac{\rho_1 R}{12\varepsilon_0} + \dfrac{8\rho_2 R}{25\times 3\varepsilon_0} = 0 \Rightarrow \dfrac{\rho_1 R}{12\varepsilon_0} = -\dfrac{8\rho_2 R}{25\times 3\varepsilon_0} \Rightarrow \dfrac{\rho_1}{\rho_2} = -\dfrac{32}{25}$

Hence, the correct options are (B) and (D).

12. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01\text{ m}) \sin[(62.8\text{ m}^{-1})x] \cos[(628\text{ s}^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is(are)
(A) The number of nodes is 5.
(B) The length of the string is 0.25 m.
(C) The maximum displacement of the mid-point of the string, from its equilibrium position is 0.01 m.
(D) The fundamental frequency is 100 Hz.

Solution

A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic as shown in the following figure:

Number of nodes $= 6$

Wavelength $\lambda = \dfrac{2\pi}{k} = \dfrac{2\times 3.14}{62.8} = 0.1\text{ m}$

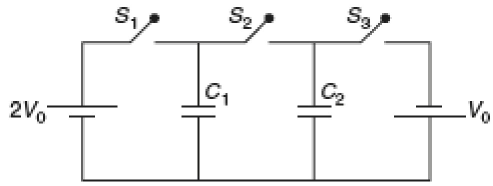
Length $l = \dfrac{5\lambda}{2} = 0.25\text{ m}$

The mid-point is an antinode. Its maximum displacement or amplitude = 0.01 m.
The fundamental frequency will be

$$f = \frac{v}{2l} = \frac{\omega}{k \times 2l} = 20\text{ Hz}$$

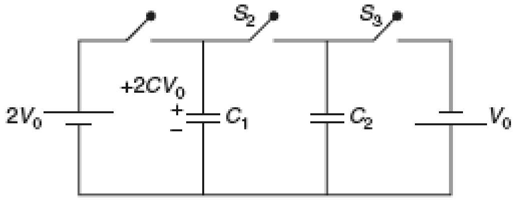
Hence, the correct options are (B) and (C).

13. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time,
(A) the charge on the upper plate of C_1 is $2CV_0$.
(B) the charge on the upper plate of C_1 is CV_0.
(C) the charge on the upper plate of C_2 is 0.
(D) the charge on the upper plate of C_2 is $-CV_0$.

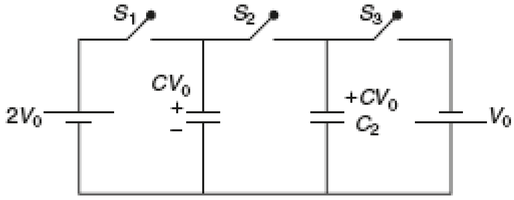


Solution

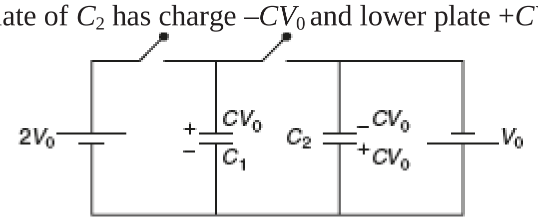
When S_1 is pressed and released, charge on C_1 is $2CV_0$, as battery V_0 is out of circuit.



When S_2 is pressed and released, charge is shared and is same in each,



When S_3 is pressed, upper plate of C_2 has charge $-CV_0$ and lower plate $+CV_0$



Hence, the correct options are (B) and (D).

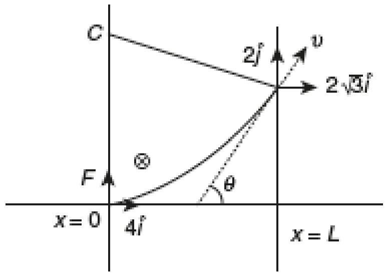
14. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}\ \text{ms}^{-1}$ enters a region of uniform static magnetic field, normal to the x–y plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 ms with a velocity $\vec{u}_2 = 2(\sqrt{3}\hat{i} + \hat{j})\ \text{ms}^{-1}$. The correct statement(s) is (are)

(A) The direction of the magnetic field is $-z$ direction.
(B) The direction of the magnetic field is $+z$ direction.
(C) The magnitude of the magnetic field is $\dfrac{50\pi M}{3Q}$ units.
(D) The magnitude of the magnetic field is $\dfrac{100\pi M}{3Q}$ units.

Solution

We have

$$t = \frac{\theta}{\omega} = \frac{M\theta}{qB}$$



Clearly,

$$\tan\theta = \frac{\text{Coefficient of } \hat{j}}{\text{Coefficient of } \hat{i}} = \frac{2}{2\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ = \frac{\pi}{6}$$

$$B = \frac{M\pi}{6q\times 10\times 10^{-3}} = \frac{100\pi M}{6q} = \frac{50\pi M}{3Q}$$

From $\vec{F} = q(\vec{v}\times\vec{B})$ $\vec{B}$ must be in $-z$ direction.

Hence, the correct options are (A) and (C).

15. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is(are)

(A) the net elongation of the spring is $\frac{4\pi R^3\rho g}{3k}$.

(B) the net elongation of the spring is $\frac{8\pi R^3\rho g}{3k}$.

(C) the light sphere is partially submerged.

(D) the light sphere is completely submerged.

Solution

At equilibrium, for the smaller sphere, we have

$$kx + \frac{4}{3}\pi R^3\rho g = \frac{4}{3}\pi R^3(2\rho)g \Rightarrow x = \frac{4\pi R^3\rho g}{3k}$$

For the bigger sphere, we have

$$kx + \frac{4}{3}\pi R^3 3\rho g = \frac{4}{3}\pi R^3(2\rho)g \Rightarrow x = -\frac{4\pi R^3\rho g}{3k}$$

For the entire system, the total buoyant force $= \frac{8}{3}\pi R^3(2\rho)g = \frac{16}{3}\pi R^3\rho g$

And the net weight $= \frac{16}{3}\pi R^3\rho y$

Hence, the system is completely submerged as total weight = total Buoyant force.

Hence, the correct options are (A) and (D).

<H2>Integer Answer Type

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (*both inclusive*).

16. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is _____.

Solution

By conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

For ring we use parallel axis theorem

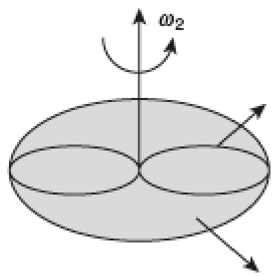
$I = m\ (r^2 + d^2)$, where $d = r$

$$\frac{1}{2}MR^2\omega_1 = \left[\frac{1}{2}MR^2 + 2m(r^2 + r^2)\right]\omega_2$$

$$\frac{1}{2}\times 50\times(0.4)^2\times 10 = \left[\frac{1}{2}\times 50\times(0.4)^2 + 4\times 6.25\times(0.2)^2\right]\omega_2$$

$$40 = [4+1]\ \omega_2$$

$$\omega_2 = \frac{40}{4+1} = 8 \text{ rad/s}$$



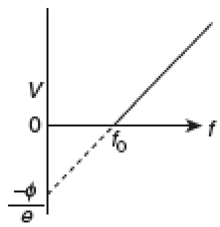
17. The work functions of silver and sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for silver to that of sodium is _______.

Solution

We have

$$V = \frac{hf}{e} - \frac{\phi}{e}$$

$$\text{Slope } = \frac{h}{e}$$



Slope is same for both silver and sodium.

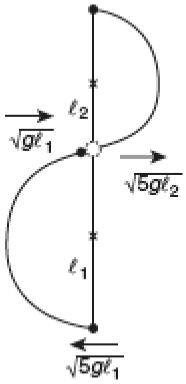
Therefore, the ratio of the slope of the stopping potential versus frequency plot for silver to that of sodium is 1:1.

18. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is _______.

Solution

Speed of the first bob at highest point $= \sqrt{gl_1}$

For elastic collision between objects of same mass, velocities are exchanged $\Rightarrow$ Speed of second bob $= \sqrt{gl_1}$



In order to have a vertical loop, minimum velocity at the lowest point is $\sqrt{5gl}$ and at the highest point is $\sqrt{gl}$.

Therefore, $$\sqrt{gl_1} = \sqrt{5gl_2} \Rightarrow \frac{l_1}{l_2} = 5$$

19. A particle of mass 0.2 kg is moving in one-dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is ______.

Solution

Mass of particle, m = 0.2 kg
Power, P = 0.5 W
Change in kinetic energy = ΔK
As power is constant,

$$P \times t = \Delta K$$

$$\Rightarrow 0.5 \times 5 = \frac{1}{2} \times (0.2)\ (v^2 - 0^2)$$

$$\Rightarrow 2.5 = 0.1v^2$$

$$\Rightarrow v = 5 \text{ m/s}$$

20. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that ln2 = 0.693, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is ______.

Solution

Half-life of radioisotope = 1386 s
Activity of sample = 10^3 disintegrations per second.
Time t = 80 s

$$N = N_0 e^{-\lambda t} \Rightarrow \frac{N}{N_0} = e^{-\lambda t} \Rightarrow \frac{N}{N_0} = e^{-\left(\frac{\ln 2}{1386} \times 80\right)} \Rightarrow \frac{N}{N_0} = e^{-\left(\frac{0.693 \times 80}{1386}\right)}$$

$$\Rightarrow \frac{N}{N_0} = e^{-0.04} \Rightarrow \frac{N}{N_0} = \left(\frac{1}{e}\right)^{0.04}$$

Fraction of nuclei decayed $= 1 - \frac{N}{N_0} = 1 - \left(\frac{1}{e}\right)^{0.04} = 0.04 = 4\%$