JEE ADVANCED 2014
PAPER 1
PHYSICS

One or More than One Options Correct Type

This section contains TEN questions. Each has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct.

1. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(A) $Q = 4\sigma\pi r_0^2$

(B) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$

(D) $E_2(r_0/2) = 4E_3(r_0/2)$

Solution

$E_1(r_0) = E_2(r_0) = E_3(r_0)$

$$\frac{Q}{4\pi E_0 r_0^2} = \frac{\lambda}{2\pi E_0 r_0} = \frac{\sigma}{2E_0}$$

On comparison

$$Q = 2\lambda r_0$$

$$E_1\left[\frac{r_0}{2}\right] = \frac{Q}{4\pi E_0 \frac{r_0^2}{4}}$$

$$E_1\left(\frac{r_0}{2}\right) = \frac{Q}{\pi E_0 r_0^2} = \frac{2\lambda r_0}{\pi E_0 r_0^2} = \frac{2\lambda}{\pi E_0 r_0} = 2\left[\frac{\lambda}{2\pi E_0\left(\frac{r_0}{2}\right)}\right]$$

$$\Rightarrow E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right) \qquad \left[\text{Since } E_2\ \ r_0/2 = \frac{\lambda}{2\pi E_0\ \ r_0/2}\right]$$

Hence, the correct option is (C).

2. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K?

(A) 4 if wires are in parallel

(B) 2 if wires are in series

(C) 1 if wires are in series

(D) 0.5 if wires are in parallel

Solution

From $H = \frac{V^2}{R} \cdot t$ we get $R = \frac{V^2}{H} \cdot t$; $R_1 = \frac{V^2}{H} \cdot t_1$

When diameter is doubled, resist once decreases 4 times, $R_2 = \frac{V^2}{H} \cdot t^2$

$$\frac{V^2}{H} t = \frac{V^2}{H} t_1 + \frac{V^2}{H} t_2$$

$$t = \frac{4}{4} = 1\,\text{min}$$

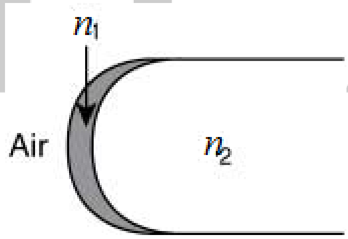
$$t = t_1 + t_2 = 1 + 1 = 2\text{ min}$$

In parallel $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{H}{V^2 t} = \frac{H}{V^2 t_1} + \frac{H}{V^2 t_2}$

$$t = \frac{t_1 t_2}{t_1 + t_2} = \frac{1 \times 1}{1+1} = \frac{1}{2} = 0.5\,\text{min}$$

Hence, the correct options are (B) and (D).

3. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the below figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



(A) $|f_1| = 3R$

(B) $|f_1| = 2.8R$

(C) $|f_2| = 2R$

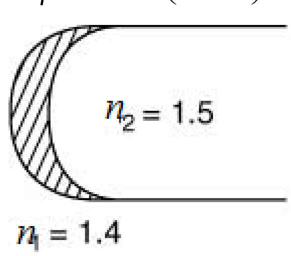
(D) $|f_2| = 1.4R$

Solution

As film has $R_1 = R_2 = R$ (say)

Its focal length

$$\frac{1}{f} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{R}\right)$$



$\frac{1}{f} = 0$ or $f = \infty$. It will not cause refraction.

Refraction will take place at air and glass cylinder

Given $u = \infty$

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

$$-\frac{\mu_1}{\infty} + \frac{1.5}{f_1} = \frac{1.5 - 1}{R} \text{ or } f_1 = 3R$$

For glass–air refraction

$$-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

$$-\frac{\mu_2}{\infty} + \frac{1}{b_2} = \frac{1 - 1.5}{R}$$

or $f_2 = -2R$ or $|b_2| = 2R$

Hence, the correct options are (A) and (C).

4. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(**Useful information:** $\sqrt{167RT} = 640\ J^{1/2}\ mol^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}\ mol^{-1/2}$. The molar mass M in grams is given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

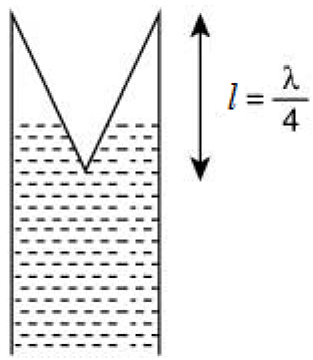
(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

Solution

At first resonance position $\lambda = 4l$, $v = n\lambda$ becomes $v = 4ne$

For $l = (0.350 \pm 0.005)$



v lies between 336.7 m s^{-1} and 346.5 m s^{-1}

Also

$$v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} = \sqrt{\frac{100\gamma RT}{M} \times 10}$$

$v = \sqrt{(100 \times 1.67RT)}\sqrt{\frac{10}{M}}$ for monoatomic gas

$$v_{\text{Ne}} = 640\sqrt{\frac{10}{M}} = 640 \times \frac{7}{10} = 448 \text{ m s}^{-1}$$

$$v_{\text{Ar}} = 640 \times \frac{17}{32} = 340 \text{ m s}^{-1}$$

For diatomic gases $\gamma = 1.4$

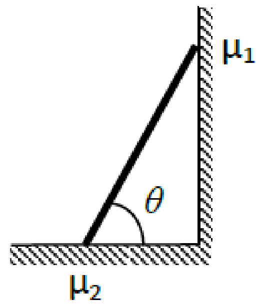
$$v = \sqrt{140RT}\sqrt{\frac{10}{M}} \Rightarrow 590\sqrt{\frac{10}{M}}$$

$V_{\text{O}_2} = 590 \times \frac{9}{18} = 331.8 \text{ m s}^{-1}$

$V_{\text{N}_2} = 590 \times \frac{3}{5} = 354 \text{ m s}^{-1}$

Hence, the correct option is (D).

5. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(A) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan\theta = \dfrac{mg}{2}$

(B) $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

(C) $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

Solution

As the rod is about to slip, wall and floor exert limiting friction on the ladder.

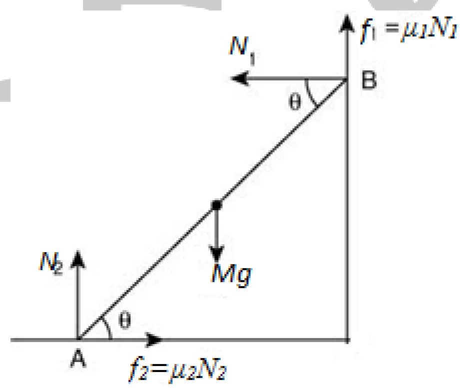
<u>Case 1:</u> If $\mu_1 = 0$ $\Sigma \vec{J}_A = \vec{0}$

$$mg\left(\frac{l}{2}\cos\theta\right) = N_1(l\sin\theta)$$

$$N_1 = \frac{mg\cot\theta}{2}$$

or $N_1 \tan\theta = \dfrac{mg}{2}$

and N_2 = mg



<u>Case 2:</u> If $\mu_2 = 0$

N_1 remain unbalanced and rod can never be in equilibrium.

<u>Case 3:</u> If $\mu_1 \neq 0$, $\mu_2 \neq 0$

$N_1 = f_2 = \mu_2 N_2$

$N_2 + f_1$ = mg

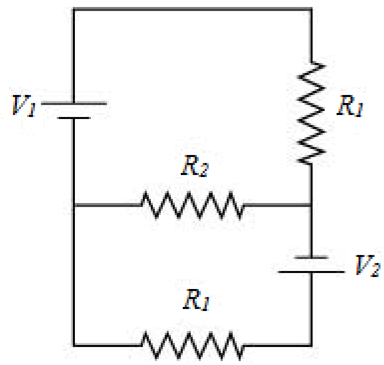
or $N_2 + \mu_1 N_1$ = mg

or $N_2 + \mu_1(\mu_2 N_2)$ = mg

or $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

Hence, the correct options are (C) amnd (D).

6. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the below figure. The current in resistance R_2 would be zero if



(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$

(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$

(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$

(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

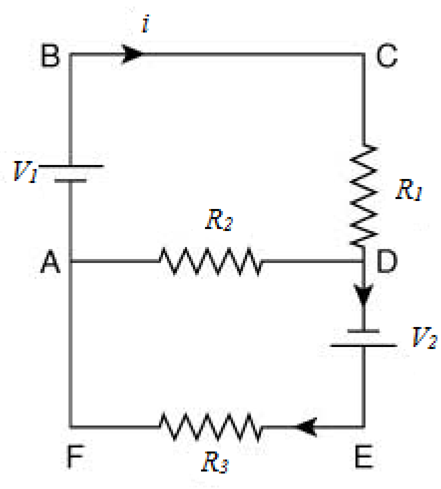
Solution

In mesh, ABCDA

$$iR_1 + 0 \times R_2 = V_1$$

or

$$i = \frac{V_1}{R_1} \qquad (1)$$



In mesh ABCDEFA

$$iR_1 + iR_3 = V_1 + V_2$$

$$i = \frac{V_1 + V_2}{R_1 + R_3} \qquad (2)$$

From Eqs. (1) and (2)

$$\frac{V_1}{R_1} = \frac{V_1 + V_2}{R_1 + R_3}$$

$$\cancel{V_1 R_1} + V_1 R_3 = \cancel{V_1 R_1} + V_2 R_1$$

$$\frac{V_1}{V_2} = \frac{R_1}{R_3}$$

This relation is satisfied in options (A), (B) and (D).

Hence, the correct options are (A), (B) and (D).

7. A light source, which emits two wavelength λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) The angular separation of fringes for λ_1 is greater than λ_2

Solution

Fringe width $\beta = \frac{\lambda D}{d}$

λ_1 = 400 nm; λ_2 = 400 nm

Since $\lambda_2 > \lambda_1$, $\beta_2 > \beta_1$

Number of fringes within a distance y is given by

$$m_1 = \frac{y}{\beta_1}$$

$$m_2 = \frac{y}{\beta_2}$$

Since $\beta_2 > \beta_1$

$m_2 < m_1$

Position of 3rd maxima of λ_2; $y' = \frac{D}{d}(3\lambda_2) = 1800\frac{D}{d}$

Position of 5th minima of λ_1; $y'' = \frac{D}{d}\left(\frac{9\lambda_1}{2}\right) = 1800\frac{D}{d}$

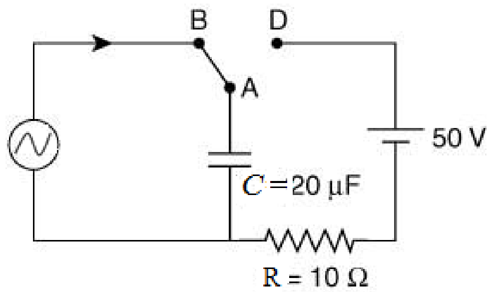
Hence $y' = y''$

Angular fringe width $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

Since, $\beta_2 > \beta_1 \Rightarrow \theta_2 > \theta_1$

Hence, the correct options are (A), (B) and (C).

8. At time $t = 0$, terminal A in the circuit shown in the below figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$ A and $\omega = 500$ rad/s starts flowing in it with the initial direction shown in the below figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20$ μF, $R = 10$ Ω and the battery is ideal with emf of 50 V, identify the correct statement(s).



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C.

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10 A.

(D) $Q = 2 \times 10^{-3}$ C.

Solution

For an AC, i in the capacitor leads the voltage by $\pi/2$.

Voltage across it is given by $V_C = \frac{1}{C}\int i\,dt = \frac{i_0}{\omega C}\sin\omega t$

$$V_C = 100 \sin \omega t$$

$$q = C \cdot v_C = 2 \times 10^{-3} \sin \omega t$$

Therefore, $q_{mx} = 2 \times 10^{-3}$ C

At $t = \frac{7\pi}{6\omega}$; $i = i_0 \cos\omega.\frac{7\pi}{6\omega} = i_0 \cos\frac{7\pi}{6} = i_0 \cos\left(\pi + \frac{\pi}{6}\right) = -\frac{\sqrt{3}i_0}{2}$ i.e. anticlockwise

Immediately after $t = \frac{7\pi}{6\omega}$, $i = \frac{50 + V_C}{R}$

Now $V_C = 100 \sin \omega t$

At $t = \frac{7\pi}{6\omega}$; $V_C = 100\sin\frac{7\pi}{6} = -50$

i.e. lower plate of the capacitor is positive and upper is negative.

Therefore, $i = \frac{50 + 50}{10} = 10\,\text{A}$

Hence, the correct options are (C) and (D).

9. One end of a taut string of length 3 m along the x axis is fixed at $x = 0$. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are)

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t) = A\sin\frac{5\pi x}{6}\cos 250\pi t$

Solution

At the fixed end ($x = 0$), there is a node and at the free end ($x = 3$ m), there is an antinode.

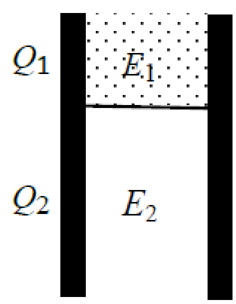
$$v = \frac{\omega}{k} = \frac{50\pi/3}{\pi/6} = 100\ \text{m s}^{-1}$$

$$\therefore\ y = 0 \text{ at } x = 0 \text{ and } y = \pm A \text{ at } x = 3\text{ m}$$

This condition is satisfied only by options (A), (C) and (D).

Hence, the correct options are (A), (C) and (D).

10. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



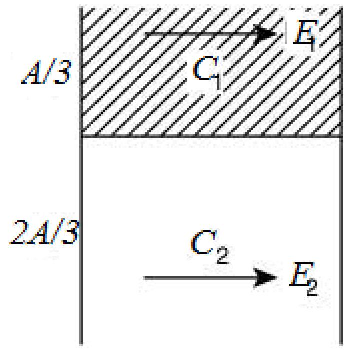
(A) $\frac{E_1}{E_2} = 1$

(B) $\frac{E_1}{E_2} = \frac{1}{K}$

(C) $\frac{Q_1}{Q_2} = \frac{3}{K}$

(D) $\frac{C_1}{C_2} = \frac{2+K}{K}$

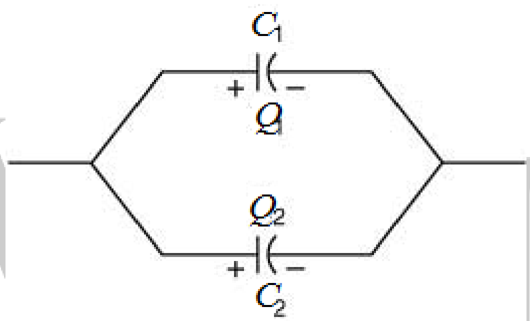
Solution



$$C_1 = \frac{K \cdot E_0 A/3}{d} = \frac{KC_2}{2}$$

$$C_2 = \frac{E_0 \cdot 2A/3}{d}$$

C_1 and C_2 are in parallel. Therefore, $C = C_1 + C_2$.



Since C_1 & C_2 are in parallel, ΔV is some across each $\Delta V = Ed$

Therefore, $E_1 = E_2$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{C_1}{2C_1/K} = \frac{K}{2}$$

$$\frac{C}{C_1} = \frac{C_1 + C_2}{C_1} = \frac{\frac{KC_2}{2} + C_2}{\frac{KC_2}{2}} = \frac{K+2}{K}$$

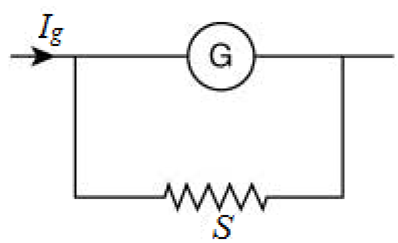
Hence, the correct options are (A) and (D).

<H2>Integer Answer Type

This section contains **TEN** questions. The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

11. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0–30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0–1.5 A. The value of n is _____.

Solution



$$I_g = 0.006\text{ A};$$

$$V = I_g[R + G]$$

$$30 = 0.006\,[4990 + G]$$

$$5000 = 4990 + G \Rightarrow G = 10\ \Omega$$

$$\frac{I_m}{I_g} = m = \frac{1.5}{0.006} = \frac{15}{10}\times\frac{1000}{6}$$

$$\Rightarrow m = 250$$

$$\text{Now, } S = \frac{G}{(m-1)}$$

$$\Rightarrow \frac{2n}{249} = \frac{10}{(250-1)} \Rightarrow \frac{10}{249} \Rightarrow n = 5$$

12. To find the distance d over which a signal can be seen clearly in foggy conditions, a railway engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is _____.

Solution

Given $d \propto \rho^a S^b b^C$

$$M^0LT^0 \propto (ML^{-3})^a(MT^{-3})^b(T^{-1})^c$$

$$M^0LT^0 \propto M^{(a+b)}L^{-3a}T^{-3b-c}$$

Equating the coefficients, we get,

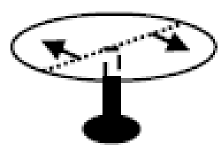
$a + b = 0$ $\quad$ $-3a = 1$ $\quad$ $-3b - c = 0$

$b = -a$ $\quad$ $a = -\frac{1}{3}$ $\quad$ $-c = 3b$

$b = \frac{1}{3}$ $\quad$ $c = -3b \Rightarrow c = 1$

$$\therefore b = \frac{1}{n} = \frac{1}{3} \Rightarrow n = 3$$

13. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is _____.



Solution:

From $Y = \frac{FL}{AC}$ we get

$$\frac{\Delta Y}{Y} \times 100 = \frac{\Delta l}{e} \times 100$$

$$l_2 = 3.20 \times 10^{-2} + 20 \times 1 \times 10^{-5}$$

$$l_1 = 3.20 \times 10^{-2} + 45 \times 1 \times 10^{-5}$$

$$l = (l_2 - l_1) = 25 \times 1 \times 10^{-5} \text{ m}$$

Out of 25×10^{-5} m, the error can be for 1 division, that is 1×10^{-5} m

$$\frac{\Delta Y}{Y} = \frac{1 \times 10^{-5}}{25 \times 10^{-5}} \times 100 = 4\%$$

14. The horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see the below figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 m s^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is _____.

Solution:

From conservation of angular momentum

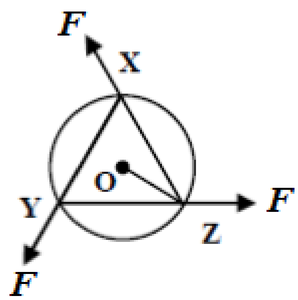
$$2mv\left(\frac{r}{2}\right) = (I\omega) \Rightarrow \frac{1}{2}mr^2\omega$$

$$0.05 \times 9 \times 0.5 = \frac{1}{2} \times 0.45 \times 0.5 \times 0.5\,\omega$$

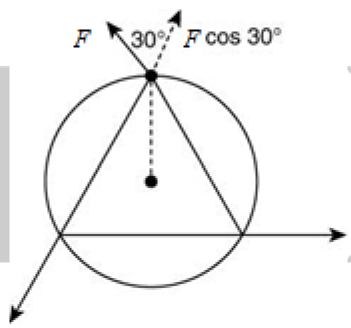
or

$$\omega = 4 \text{ rad s}^{-1}$$

15. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude *F* = 0.5 N are applied simultaneously along the three sides of an equilateral triangle *XYZ* with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is _____.



Solution:



$$T = 3rF \sin 30° = 3 \times 0.5 \times 0.5 \times \frac{1}{2}$$

$$T = \frac{3}{8} \text{Nm}$$

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 1.5 \times 0.5 \times 0.5$$

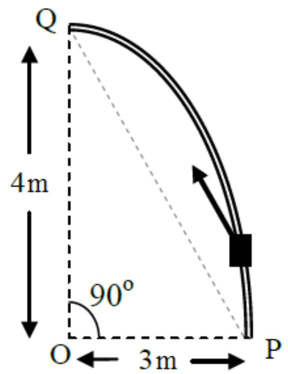
$$I = \frac{3}{16} \text{Kg m}^2$$

$$\alpha = \frac{T}{I} = \frac{3/8}{3/16} = 2 \text{ rad s}^{-2}$$

$\omega = \omega_0 + \alpha t \Rightarrow 0 + 2 \times 1 = 2 \text{ rad s}^{-1}$

16. Consider an elliptically shaped rail *PQ* in the vertical plane with *OP* = 3 m and *OQ* = 4 m. A block of mass 1 kg is pulled along the rail from *P* to *Q* with a force of 18 N, which is always parallel to line *PQ* (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches *Q* is ($n \times 10$) Joules. The value of *n* is _____.

(take acceleration due to gravity = 10 ms^{-2})



Solution:

$PQ = 5\omega = d$

Work done by force from P to $Q = Fd$

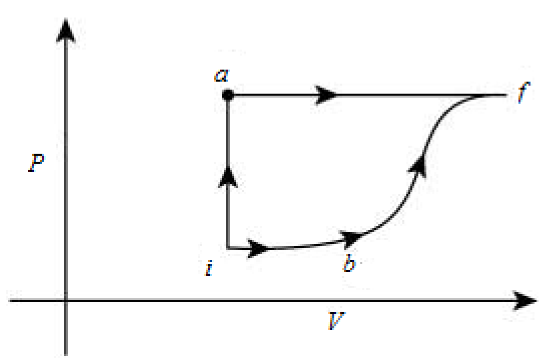
$$Fd = mgh + \frac{1}{2}mv_Q^2$$

$$18 \times 5 = (1 \times 10 \times 4) + \frac{1}{2} \times 1 \times V_Q^2$$

$$\frac{V_Q^2}{2} = KE = 50 = (h \times 10) \text{ Joules}$$

Therefore, $n = 5$

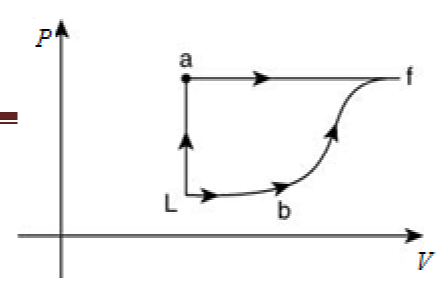
17. A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the below. The work done by the system along the path af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J, respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_{b} = 200 J and Q_{iaf} = 500 J, the ratio $Q_{\text{bf}}/Q_{\text{ib}}$ is ______.



Solution:

Given U_i = 100 J, U_b = 200 J

For the process iaf



Process	W(J)	Q(J)	ΔU(J)
ia	0		
af	200		
Net	200	500	300

For the process *ibf*

Process	W(J)	Q(J)	ΔU(J)
ib	50	150	100
bf	100	300	200
Net	150	450	300

$U_f = U_i = \Delta U$ is same for both the processes *iaf* and *ibf*.

Therefore, $U_f - 100 = 300$

$U_f = 400$ J

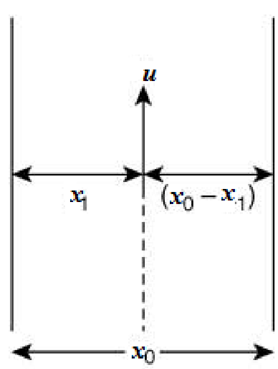
$U_b - U_i = 100$

$U_f - U_b = 200$

Therefore, $\frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$

18. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3,$ the value of $\frac{R_1}{R_2}$ is ______.

Solution:



When current in wires is in same direction, the magnetic fields due to two wires all in opposite direction

From $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$ we get,

$$B_1 = \frac{\mu_0}{4\pi}\cdot 2I\left[\frac{1}{x_1} - \frac{1}{(x_0 - x_1)}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{x_0 - x_1 - x_1}{x_1(x_0 - x_1)}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{x_0 - 2x_1}{x_1(x_0 - x_1)}\right] \qquad (1)$$

When direction of current in two wires is opposite, field will be in the same direction.

$$B_2 = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x_1} + \frac{1}{(x_0 - x_1)}\right]$$

$$B_2 = \frac{\mu_0 I}{2\pi}\left[\frac{x_0 - x_1 + x_1}{x_1(x_0 - x_1)}\right]$$

$$B_2 = \frac{\mu_0 I}{2\pi}\left[\frac{x_0}{x_1(x_0 - x_1)}\right]$$

From $\frac{mv^2}{r} = qvB$ or $v = \frac{qBr}{m}$ or $r = \frac{mv}{qB}$

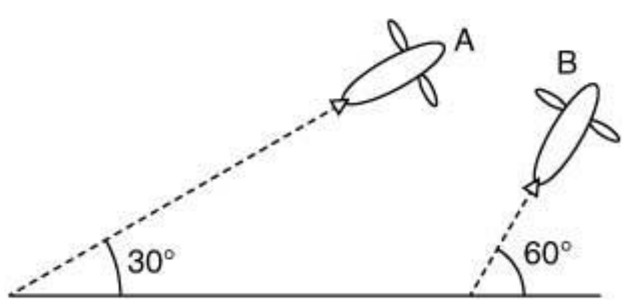
$$B \propto \frac{1}{r}$$

Therefore,

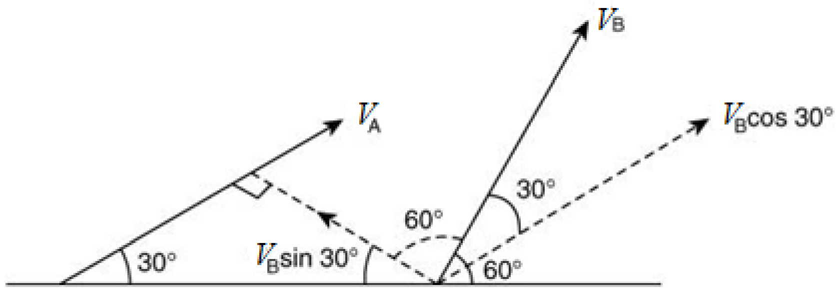
$$\frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{x_0}{(x_0 - 2x_1)}$$

$$\frac{R_1}{R_2} = \frac{\frac{x_0}{x_1}}{\frac{x_0}{x_1} - \frac{2x_1}{x_1}} = \frac{3}{3-2} = 3$$

19. Airplanes *A* and *B* are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of *A* is $100\sqrt{3}\text{ ms}^{-1}$. At time $t = 0$ s, an observer in *A* finds *B* at a distance of 500 m. This observer sees *B* moving with a constant velocity perpendicular to the line of motion of *A*. If at $t = t_0$, *A* just escapes being hit by *B*, t_0 in seconds is _____.



Solution:



<AQ>The perpendicular sign needs to be square in shapte<AQ>

Since *A* observes *B* as moving normal to it

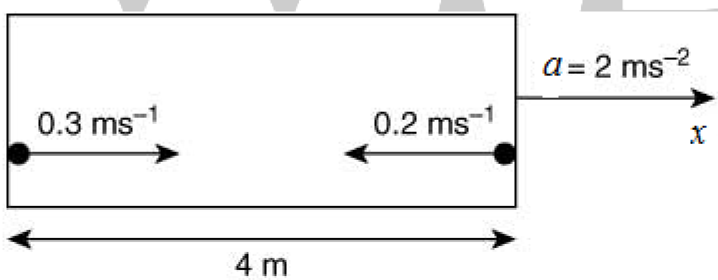
$$V_B \cos 30° = V_A$$

$$V_B \frac{\sqrt{3}}{2} = 100\sqrt{3}$$

$$V_B = 200 \text{ m/s}$$

Therefore, $t_0 = \dfrac{500}{200 \sin 30°} = 5 \text{ s}$

20. A rocket is moving in a gravity free space with a constant acceleration of 2 m s^{-2} along + *x* direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + *x* direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –*x* direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is _____.



Solution:

Distance covered by ball A before stopping

$$0 = (0.3)^2 - 2as_A$$

$$s_A = \frac{(0.3)^2}{2 \times 2} = 2.25 \text{ cm}$$

Therefore, $s_B = 400 - 2.25 = 397.75 \text{ cm} \approx 400 \text{ cm}$

That is, collision occurs very near to the left wall.

Also for ball *B*, $4 = 0.2t + \frac{1}{2}(2)t^2$

$$t^2 + 0.2t - 4 = 0$$

$$t = \frac{-0.2 + \sqrt{0.04 + 16}}{2} = \frac{-0.2 + 4}{2} = 1.9 \text{ s} \approx 2 \text{ s}$$

Alternate solution:

$$S_A = 0.2t + \frac{1}{2} \times 2 \times t^2$$

$$S_B = 0.3t - \frac{1}{2} \times 2 \times t^2$$

$$S_A + S_B = 4$$

$$0.5t = 4 \Rightarrow t = 8 \text{ s}$$