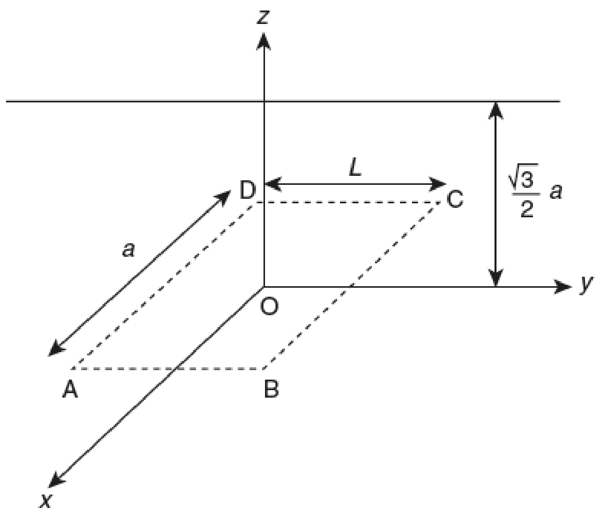
JEE ADVANCED 2015
PAPER 1
PHYSICS

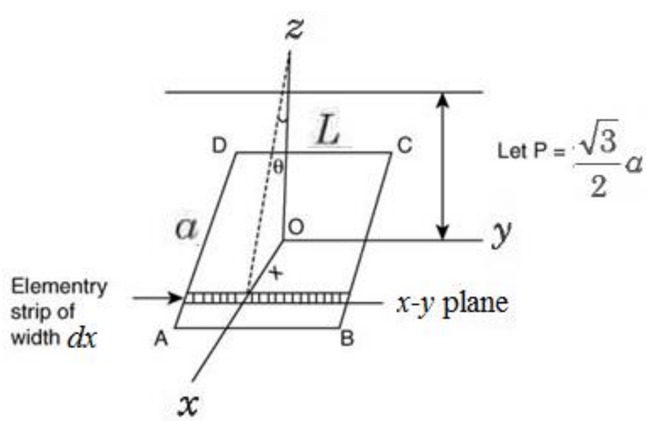
Integer Answer Type

This section contains **EIGHT** questions. The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

1. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the *y*-axis in the *y*-*z* plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface *ABCD* lying in the *x*-*y* plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of *n* is ______.



Solution



Let $d\phi_E$ be the flux through the strip

$$d\phi_E = \vec{E}\cdot\overrightarrow{ds} = (E\cos\theta)ds \text{ where } \cos\theta = \frac{\phi}{\sqrt{p^2+x^2}}$$

Where $ds = Ldx$ = area of the strip

Therefore,

$$d\phi_E = \frac{\lambda}{2\pi\varepsilon_0\sqrt{p^2+x^2}}\left(\frac{p}{\sqrt{p^2+x^2}}\right)Ldx$$

$$\phi_E = \frac{\lambda pL}{2\pi\varepsilon_0}\int_{-a/2}^{a/2}\frac{dx}{p^2+x^2}$$

$$= \frac{\lambda pL}{2\pi\varepsilon_0}\frac{2}{p}\tan^{-1}\left(\frac{x}{p}\right)_0^{a/2}$$

$$= \frac{\lambda L}{\pi\varepsilon_0}\tan^{-1}\frac{a}{2p}$$

Putting for $p = \frac{\sqrt{3}a}{2}$

$$\phi_E = \frac{\lambda L}{\pi\varepsilon_0}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\lambda L}{6\varepsilon_0}$$

2. Consider a hydrogen atom with its electron in the *n*th orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of *n* is _____.

(hc = 1242 eV nm)

Solution

Energy of incident photon $= h\nu = \frac{hc}{\lambda} = \frac{1242}{90} = 13.8$ eV. Since after ionisation, electron is ejected with some kinetic energy, by energy conservation we get,

Energy (photon) = kinetic energy (electron) + ΔE (transition energy from *n*th orbit to $n \to \infty$)

Therefore,

$$13.8 = 10.4 + \Delta E$$

Therefore,

$$\Delta E = 3.4 \text{ eV}$$

From Bohr’s theory, $E_n = \frac{-13.6}{n^2} = -3.4$

Therefore, $n = 2$

3. A bullet is fired vertically upwards with velocity *v* from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet’s gravity is $1/4^{th}$ of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of *N* is _____.

(ignore energy loss due to atmosphere)

Solution

We know that, g varies with altitude h, above the planet's surface as $g_n = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{4}$

Therefore,

$$\frac{R}{R+h} = \frac{1}{2}$$

$h = R$ (here $v_f = 0$ for the bullet)

given $v_{\text{esc}} = v\sqrt{N}$

$$v_{\text{esc}} = \sqrt{2gR} = \sqrt{\frac{2GM}{R}} = v\sqrt{N}$$

from energy conservation $E_1 = E_2$

E_1 (at the surface) $= \frac{-GM_m}{2R} + \frac{1}{2}mv^2$

E_2 (at $h = R$) $= \frac{-GM_m}{2R} + 0$

Therefore, on putting $E_1 = E_2$

$$\frac{1}{2}mv^2 = \frac{GM_m}{2R}$$

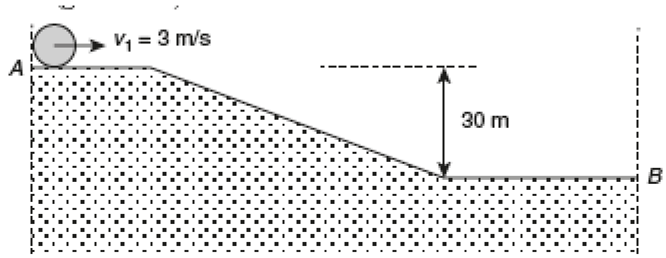
$$v = \sqrt{\frac{GM}{R}}$$

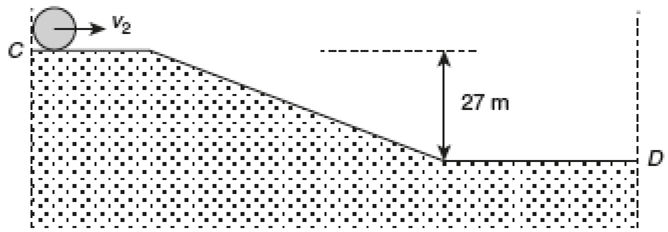
Therefore,

$$\sqrt{\frac{2GM}{R}} = \sqrt{N} \times \sqrt{\frac{GM}{R}}$$

Therefore, $N = 2$

4. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surface. If they reach B and D with the same linear speed and $v_1 = 3$ m/s then v_2 in m/s is _____.
($g = 10$ m/s^2)





Solution

Total K.E. of a body in pure rolling $= \frac{1}{2}mv^2\left(\frac{k^2}{R^2}+1\right)$ where k = radius of gyration

for the disc $\frac{k^2}{R^2}=\frac{1}{2}$. By energy conservation

Therefore,

Case 1:

$$\frac{1}{2}\left(\frac{3}{2}mr^3\right)(\omega_i^2)+mgh_1=\text{KE}_f \qquad (1)$$

Case 2:

$$\frac{1}{2}\left(\frac{3}{2}mr^3\right)(\omega_2^2)+mgh_2=\text{KE}_f \qquad (2)$$

$$\omega_1=\frac{v_1}{R}=\frac{3}{R}$$

$$\omega_2=\frac{v_2}{R}$$

From Eqs. (1) and (2)

$$\frac{1}{2}\left(\frac{3}{2}\cancel{m}\cancel{R}^2\right)\left(\frac{3}{\cancel{R}}\right)^2+\cancel{m}g(30)=\frac{1}{2}\left(\frac{3}{2}\cancel{m}\cancel{R}^2\right)\left(\frac{V_2}{\cancel{R}}\right)^2+\cancel{m}g(27)$$

$$3g=\frac{3}{4}(v_2^2-9)$$

$$v_2^2=49$$

$$v_2=7$$

5. Two spherical stars *A* and *B* emit blackbody radiation. The radius of *A* is 400 times that of *B*, and *A* emits 10^4 times the power emitted from *B*. The ratio $\frac{\lambda_A}{\lambda_B}$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is _____.

Solution

$$R_A = 400\, R_B$$

$$P_A = 10^4\, P_B$$

$$\frac{\lambda_A}{\lambda_B} = ?$$

From Stefan's law $P = \sigma T^4 A_s$ where $A_s = 4\pi R^2$

$$\frac{P_A}{P_B} = \frac{T_A^4 R_A^2}{T_B^2 R_B^2}$$

$$10^4 = \frac{T_A^4}{T_B^4} \times 16 \times 10^4$$

$$\frac{T_A}{T_B} = \frac{1}{2}$$

From Wien's law $\lambda_A T_A = \lambda_B T_B$

$$\frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} = 2$$

6. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life *T* years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of *nT* years, then the value of *n* is _____.

Solution

Initial requirement of power 12.5%, that is $\frac{1}{8}$ th of power available.

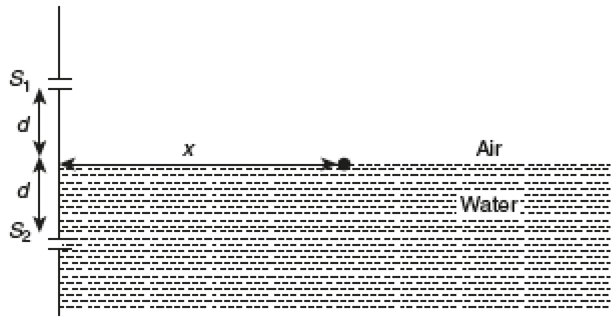
$$P = P_0 \left(\frac{1}{2}\right)^x$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^x$$

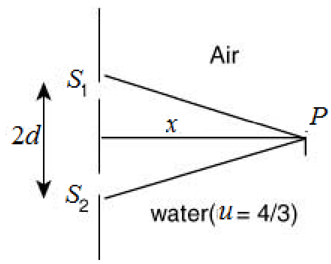
$$x = 3$$

After 3 half-lives, $P = \frac{P_0}{8}$, that is the nuclear power plant can serve the village for $t = 3T$

7. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2 m^2 \lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), $2d$ is the separation between the slits and *m* is an integer. The value of *p* is _____.



Solution



Path difference at point P

$$\Delta x = S_2P - S_1P$$

$$= \frac{4}{3}\sqrt{x^2 + d^2} - \sqrt{x^2 + d^2} = \frac{1}{3}\sqrt{x^2 + d^2}$$

For maxima $\Delta x = m\lambda$

Therefore,

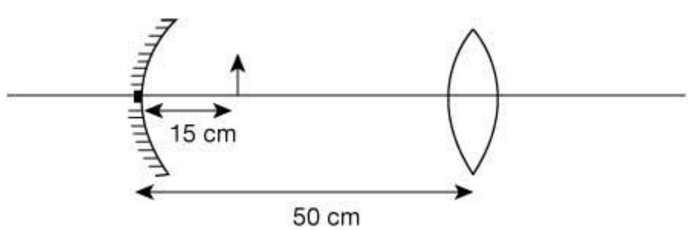
$$\frac{1}{3}\sqrt{x^2 + d^2} = m\lambda$$

$$x^2 + d^2 = (3)^2 m^2 \lambda^2$$

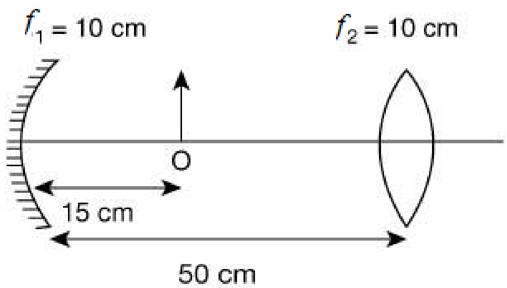
$$x^2 = (3)^2 m^2 \lambda^2 - d^2$$

Comparing this with $x^2 = p^2 m^2 \lambda^2 - d^2$ we get $p = 3$.

8. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is _____.



Solution



For concave mirror

Mirror formula gives

$$\frac{1}{v_1}+\frac{1}{u_1}=\frac{1}{f_1}$$

$$\frac{1}{v_1}-\frac{1}{15}=\frac{1}{-10}$$

Therefore,

$$\frac{1}{v_1}=-\frac{1}{30}$$

$$v_1 = -30 \text{ cm}$$

Magnification $M_1=-\frac{v_1}{u_1}=-\left(\frac{-30}{-15}\right)=-2$

For convex lens

Lens formula gives

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f_2}$$

$$\frac{1}{v_2}-\frac{1}{-20}=\frac{1}{10}$$

$$\frac{1}{v_2}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$$

$$v_2 = 20 \text{ cm}$$

Magnification $M_2=\frac{v_2}{v_2}=\frac{20}{-20}=-1$

Total magnification $M = M_1M_2 = (-2)(-1) = 2$

On increasing the entire set up in the liquid, f of the concave mirror remains unchanged while that of convex lens becomes

$$f_e = f_2 \frac{\mu - 1}{\frac{\mu}{\mu_e} - 1}$$

where f_2 = focal length of the lens in air

f_e = focal length of the lens in liquid

$$\mu = 1.5,\ \mu_e = \frac{7}{6}$$

Therefore

$$f_e = 10 \times \frac{1.5 - 1}{\frac{1.5}{7/6} - 1} = \frac{5}{\frac{9}{7} - 1} = \frac{35}{2}$$

Hence again $\frac{1}{v_3} - \frac{1}{-20} = \frac{2}{35}$

$$\frac{1}{v_3} = \frac{2}{35} - \frac{1}{20} = \frac{8-7}{140} = \frac{1}{140}$$

Magnification $M_3 = \frac{v_3}{v_2} = \frac{140}{-20} = -7$

So total magnification here is $M' = M_1 M_3$ = (–2) (–7) = 14

Ratio $\left|\frac{M'}{M}\right| = \frac{14}{2} = 7$

<H2>One or More than One Options Correct Type

This section contains **TEN** questions. Each has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.

9. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then:

(A) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(B) If the pitch of the screw gauge is twice the least count of the Vernier callipers the least count of the screw gauge is 0.005 mm.

(C) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(D) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

Solution

On Vernier callipers, $1\,\text{MSD} = \frac{1}{8}\,\text{cm}$

Given that 5 VSD coincides with 4 MSD

Therefore,

$$1\,\text{VSD} = \frac{4}{5}\text{MSD}$$

Least count = 1 MSD − 1 VSD $= \frac{1}{40}\text{cm}$

Now if the pitch of the screw gauge is twice the least count of the Vernier callipers, that is

$$\text{Pitch} = 2\times\frac{1}{40} = \frac{1}{20}\text{cm}$$

$$\text{Least count} = \frac{\text{Pitch}}{\text{Total divisions of circular scale}} = \frac{1/20}{100} = 0.5\times10^{-3}\text{cm}$$

$$= 0.0005 \text{ cm} = 0.005 \text{ mm}$$

Now least count of the linear scale of screw gauge is twice the least count of the Vernier callipers, that is

$2\times\frac{1}{40} = \frac{1}{20}\text{cm}$

Therefore, 2 divisions on the linear scale is equivalent to $2\times\frac{1}{20} - \frac{1}{10}\text{cm} = 1\,\text{mm}$

Therefore, least count of screw gauge $= \frac{\text{Pitch}}{\text{Total divisions of circular scale}}$

$$= \frac{1\,\text{mm}}{100} = 0.01 \text{ mm}$$

Hence, the correct options are (B) and (C).

10. Planck's constant *h*, speed of light *c* and gravitational constant *G* are used to form a unit of length *L* and a unit of mass *M*. Then the correct option(s) is (are)

(A) $M \propto \sqrt{c}$

(B) $M \propto \sqrt{G}$

(C) $L \propto \sqrt{h}$

(D) $L \propto \sqrt{G}$

Solution

Dimensions of

Planck's constant $h = [M^1L^2T^{-1}]$

Speed of light $C = [L^1T^{-1}]$

Gravitational constant $G = [M^{-1}L^3T^{-2}]$

Let $L \propto h^x c^y G^z$

$[L] = [M^1L^2T^{-1}]^x\,[L^1T^{-1}]^y\,[M^{-1}L^3T^{-2}]^z$

Comparing we get

$$\left.\begin{aligned} x-z&=0\\ 2x+y+3z&=1\\ -x-y-2z&=0\end{aligned}\right\}$$

Solving we get $x = z$

$$y + 5x = 1$$

$$-–y - 3x = 0$$

or

$$2x = 1$$

$$x = 1/2 = z$$

$$y = -3/2$$

Therefore,

$$L \propto h^{1/2}C^{-3/2}G^{1/2}$$

$$L \propto \sqrt{h}$$

$$L \propto \sqrt{G}$$

Let $M \propto h^a C^b G^c$

$$[M] = [M^1L^2T^{-1}]^a\ [L^1T^{-1}]^b\ [M^{-1}L^3T^{-2}]^c$$

Comparing we get $a - c = 1$

$$\left.\begin{aligned} 2a+b+3c&=0\\ -a-b-2c&=0\end{aligned}\right\} a+c=0$$

Therefore,

$$2a = 1$$

$$a = 1/2$$

$$c = -1/2$$

$$b = 1/2$$

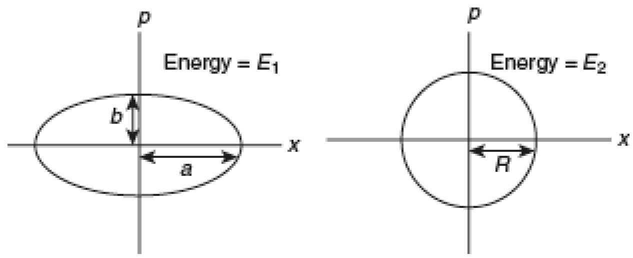
Therefore,

$$M \propto h^{1/2}c^{1/2}G^{-1/2}$$

$$M \propto \sqrt{c}$$

Hence, the correct options are (A), (C) and (D).

11. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n,$ then the correct equation(s) is (are)



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$

(C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

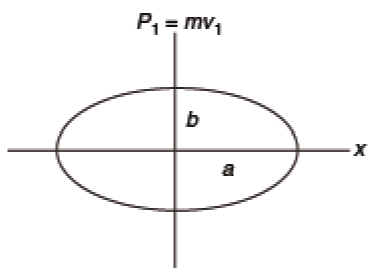
Solution

<u>For case 1:</u>

We know that

$$v_1 = \omega_1\sqrt{A_1^2 - x^2}$$

$$\frac{v_1^2}{(A_1\omega_1)^2} + \frac{x^2}{A_1^2} = 1 \Rightarrow \frac{p_1^2}{(A_1\omega_1 m)^2} + \frac{x^2}{A_1^2} = 1$$



Comparing we get

$$a = A_1$$

$$b = A_1\omega_1 m$$

<u>For case 2:</u>

$$a_1 = b_1 = R$$

That is, $A_2 = A_2\omega_2 m \Rightarrow \omega_2 m = 1 \Rightarrow \omega_2 = \frac{1}{m}$

given $\frac{a}{b} = n^2 = \frac{1}{\omega_1 m} \Rightarrow \omega_1 = \frac{1}{mn^2}$

$$\frac{a}{R} = n = \frac{A_1}{A_2} = \frac{A_1}{A_2\omega_2 m}$$

$$\frac{\omega_2}{\omega_1} = n^2$$

Also $E_1 = \frac{1}{2}m\omega_1^2 A_1^2$

$$E_2 = \frac{1}{2} m\omega_2^2 A_2^2$$

$$\frac{E_1}{\omega_1} = \frac{1}{2} m\omega_1 A_1^2$$

$$\frac{E_2}{\omega_2} = \frac{1}{2} m\omega_2 A_2^2$$

If $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

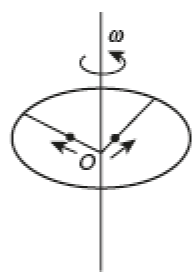
$$\omega_1 A_1^2 = \omega_2 A_2^2$$

or

$$\frac{\omega_2}{\omega_1} = \frac{A_1^2}{A_2^2} = n^2$$

Hence, the correct options are (B) and (D).

12. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the below figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At the instant the distance of the other mass from O is,



(A) $\frac{2}{3}R$

(B) $\frac{1}{3}R$

(C) $\frac{3}{5}R$

(D) $\frac{4}{5}R$

Solution

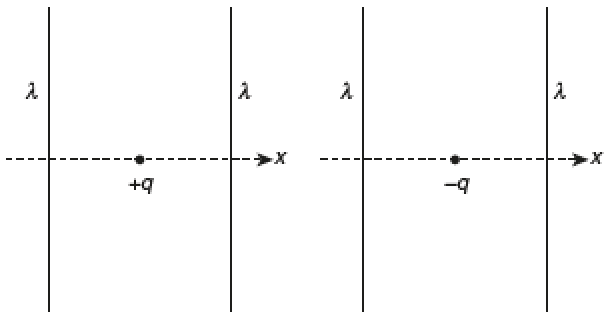
Applying conservation of angular momentum

$$M\omega R^2 = MR^2\left(\frac{8}{9}\omega\right) + \frac{M}{8}\left(\frac{9}{25} \times R^2\right)\left(\frac{8}{9}\omega\right) + \frac{M}{8}(x^2)\left(\frac{8}{9}\omega\right)$$

Solving we get $x = \frac{4R}{5}$

Hence, the correct option is (D).

13. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is (are)



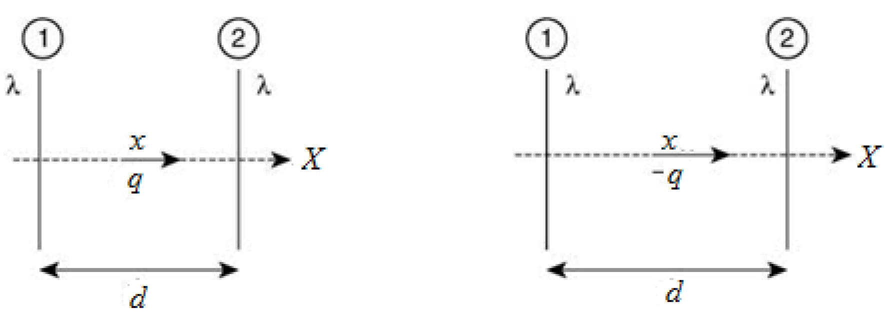
(A) Both charges execute simple harmonic motion.

(B) Both charges will continue moving in the direction of their displacement.

(C) Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement.

(D) Charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement.

Solution



$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

Case 1: If q is shifted towards right by x

$$F = F_2 - F_1 = \frac{\lambda q}{2\pi\varepsilon_0}\left(\frac{1}{\frac{d}{2} - x} - \frac{1}{\frac{d}{2} + x}\right) \text{ towards left}$$

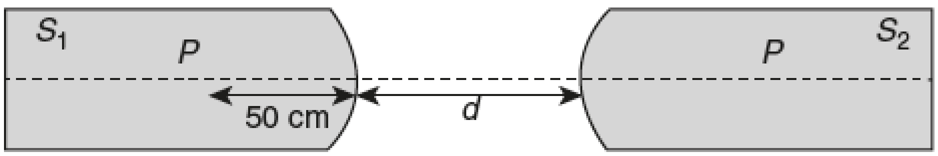
Case 2: If $-q$ is shifted towards right by x

$$F = F_2 - F_1 = \frac{\lambda q}{2\pi\varepsilon_0}\left(\frac{1}{\frac{d}{2} - x} - \frac{1}{\frac{d}{2} + x}\right) \text{ towards right}$$

Thus $+q$ exhibits SHM while $-q$ continues to move towards rightward

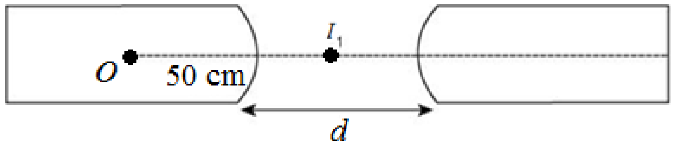
Hence, the correct option is (C).

14. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is



(A) 60 cm
(B) 70 cm
(C) 80 cm
(D) 90 cm

Solution



Applying Gaussian formula $\frac{\mu_2}{v} - \frac{\mu_1}{v} = \frac{\mu_2 - \mu_1}{R}$

For first rod: $\frac{1}{v_1} - \frac{1.5}{-50} = \frac{1-1.5}{-10}$

Solving $v_1 = 50$ cm

For second rod $\frac{1.5}{\infty} - \frac{1}{-(d-50)} = \frac{1.5-1}{10}$

$v_2 \to \infty$ as rays in the second rod are parallel to the axis

Solving we get $d = 70$ cm

Hence, the correct option is (B).

15. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is (are)

(A) If $\vec{B}$ is along $\hat{z}$, $F \propto (L+R)$

(B) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(C) If $\vec{B}$ is along $\hat{y}$, $F \propto (L+R)$

(D) If $\vec{B}$ is along $\hat{z}$, $F = 0$

Solution

$$\vec{F} = \hat{i}(\vec{l} \times \vec{b})$$

$\vec{l} =$ vector length (displacement)

Therefore,

$$\vec{F} = i[2L + 2R]\hat{i} + \vec{B}$$

If $\vec{B}$ is along x-axis, $\vec{F} = 0$ as $\hat{i} \times \hat{i} = \vec{0}$

If $\vec{B}$ is along y-axis, $\vec{F} = 2i(L+R)B(\hat{k})$

If $\vec{B}$ is along z-axis, $\vec{F} = 2i(L+R)B(-\hat{j})$

That is, for $\vec{B}$ along y or z-axis, $|\vec{F}| \propto (L+R)$

Hence, the correct options are (A), (B) and (C).

16. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is(are)

(A) The average energy per mole of the gas mixture is $2RT$.

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

Solution

Average energy (K.E.) per unit mole $= \frac{1}{2} fRT$

where f is degrees of freedom

$$f_{\max} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2} = \frac{1(5) + 1(3)}{1+1} = 4$$

Therefore, average energy per unit mole = $2RT$

Also $\dfrac{n_1 + n_2}{r_{\text{mix}} - 1} = \dfrac{n_1}{r_1 - 1} + \dfrac{n_2}{r_2 - 1}$

$$\frac{2}{r_{\text{mix}} - 1} = \frac{1}{\frac{7}{5} - 1} + \frac{1}{\frac{5}{3} - 1} = \frac{5}{2} + \frac{3}{2} = 4$$

Therefore,

$$4r_{\text{mix}} - 4 = 2$$

$$r_{\text{mix}} = 3/2$$

Also molar mass $M_{\text{mix}} = \dfrac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \dfrac{1(2) + 1(4)}{2} = 3$

Speed of sound $v = \sqrt{\dfrac{rRT}{M}}$

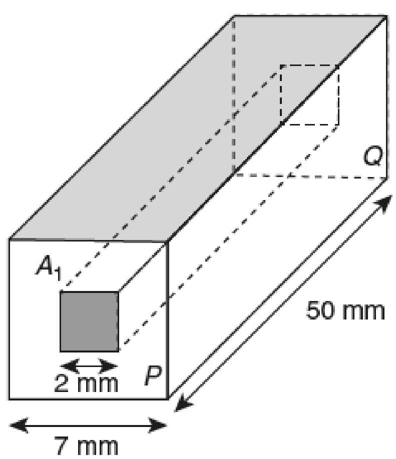
$$\frac{V_{\text{mix}}}{V_{\text{He}}} = \sqrt{\frac{r_{\text{mix}}}{M_{\text{mix}}} \times \frac{M_{\text{He}}}{r_{\text{He}}}} = \sqrt{\frac{\cancel{3}}{2 \times \cancel{3}} \times \frac{4 \times 3}{5}} = \sqrt{\frac{6}{5}}$$

r.m.s. speed $C_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

$$\frac{C_{\text{rms(He)}}}{C_{\text{rms(H2)}}} = \sqrt{\frac{M_{\text{H2}}}{M_{\text{He}}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

Hence, the correct options are (A), (B) and (D).

17. In an aluminum (Al) bar of square cross section, a square hole is called and is filled with iron (Fe) as shown in the below figure. The electrical resistivities of Al and Fe are 2.7×10^{-8} Ω m and 1.0×10^{-7} Ω m, respectively. The electrical resistance between the two faces *P* and *Q* of the composite bar is



(A) $\dfrac{2475}{64}$ μΩ (B) $\dfrac{1875}{64}$ μΩ

(C) $\dfrac{1875}{49}$ μΩ (D) $\dfrac{2475}{132}$ μΩ

Solution

$$R = \rho \frac{l}{A}$$

For Al, $A_1 = 7^2 - 2^2 = 45\ \text{mm}^2$

$$\rho_1 = 2.7 \times 10^{-8}\ \Omega\ \text{m}$$

$$l_1 = 50\ \text{mm}$$

$$R_1 = \frac{\rho_1 l_1}{A_1} = \frac{2.7\times10^{-8}\times 50}{45\times10^{-3}} = 3 \times 10^{-5}\ \Omega = 30\ \mu\Omega$$

For iron, $A_2 = 4\ \text{mm}^2$

$$\rho\rho_2 = 10^{-7}\ \Omega\ \text{m}$$

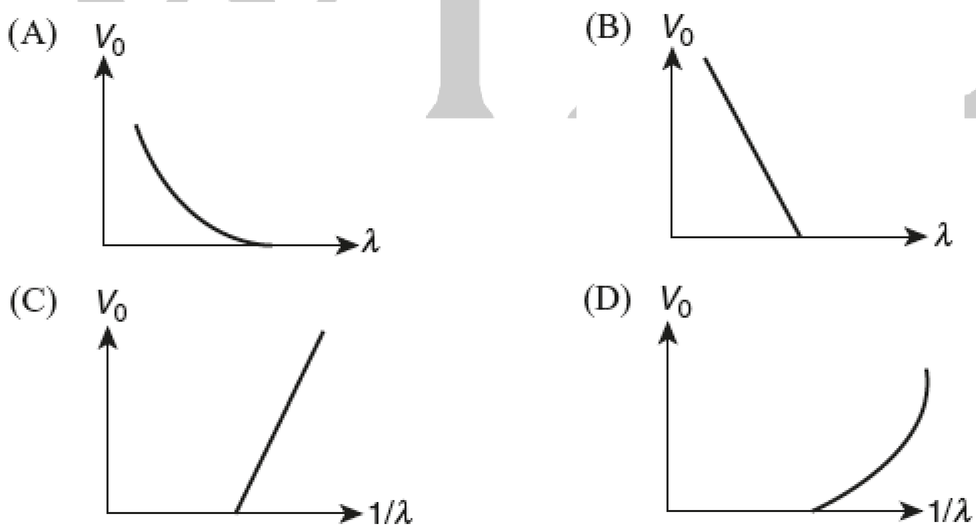
$$l_2 = 50\ \text{mm}$$

$$R_2 = \frac{10^{-7}\times 50}{4\times10^{-3}} = 12.5\times10^{-4} = 1250\ \mu\Omega$$

$$R_{eq} = \frac{R_1 + R_2}{R_1 + R_2} = \frac{\overset{15}{\cancel{30}}\times 125\cancel{0}}{\underset{64}{\cancel{128}}\ \cancel{0}} = \frac{1875}{64}\ \mu\Omega$$

Hence, the correct option is (B).

18. For photoelectric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $1/\lambda$.



Solution

$$hV = hV_0 + eV_0$$

$$\frac{hc}{\lambda} = +eV_0$$

where V_0 = stopping potential

ϕ = work function

V_0 versus $\frac{1}{\lambda}$ is a straight line with a positive slope $\frac{hc}{e}$

V_0 versus λ is not a straight line

Hence, the correct options are (A) and (C).

Matching List Type

This section contains **TWO** multiple choice questions. Each question contains two columns, **Column I** and **Column II**. **Column I** has **four** entries (A), (B), (C) and (D). **Column II** has **five** entries (P), (Q), (R), (S) and (T). Match the entries in **Column I** with the entries in **Column II**.

19. Match the nuclear processes given in column I with the appropriate option(s) in column II.

Column I	Column II
(A) Nuclear fusion	(P) Absorption of thermal neutrons by $^{235}_{92}\text{U}$
(B) Fission in a nuclear reactor	(Q) $^{60}_{27}\text{Co}$ nucleus
(C) β-decay	(R) Energy production in stars via hydrogen conversion to helium
(D) γ-ray emission	(S) Heavy water
	(T) Neutrino emission

Solution

(A) → (R) or (R, T); (B) → (P, S); (C) → (Q, T); (D) → (R)

The solution is mostly factual. Please refer to the relevant articles in school books.

20. A particle of unit mass is moving along the *x*-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

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Column I	Column II
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) The force acting on the particle is zero at $x = a$.
(B) $U_2(x) - \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero $x = 0$.
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp -\left[-\left(\frac{x}{a}\right)^2\right]$	(R) The force acting on the particle is zero at $x = -a$.
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^2\right]$	(S) The particle experiences an attractive force towards $x = 0$ in the region $\lvert x\rvert < a$.

(T) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$.

Solution

(A) → (P, Q, R, T); (B) → (Q, S); (C) → (P, Q, R, S); (D) → (P, R, T)

$$U_1 = \frac{U_0}{2}\left[1 - \frac{x^2}{a^2}\right]^2$$

$$F = -\frac{dU_1}{dx} = -\frac{v_0}{2} 2\left(1 - \frac{x^2}{a^2}\right)\left(\frac{-2x}{a^2}\right)$$

$$= \frac{2v_0}{a^4}(a^2 - x^2)x$$

$$F = 0 \text{ at } x = 0,\ x = \pm a$$

Now at $x = -a$, $U_1 = 0$

$x = 0$, $U_1 = \frac{U_0}{2}$ (P.E. due to oscillation + some other PE)

$$U_2 = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$$

$$F = -\frac{dU_2}{dx} = -\frac{U_0 x}{a^2}$$

$$F = 0 \text{ at } x = 0$$

$$U_3 = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 e^{-\left(\frac{x}{a}\right)^2}$$

$$F = -\frac{dU_3}{dx} = -\frac{U_0}{2}\left[\frac{2x}{a^2} e^{-\left(\frac{x}{a}\right)^2} + \frac{x^2}{a^2} e^{-\left(\frac{x}{a}\right)^2}\left(\frac{-2x}{a^2}\right)\right]$$

$$= -\frac{U_0}{2}\frac{2x}{a^4}[a^2 - x^2]e^{-\left(\frac{x}{a}\right)^2}$$

$$= \frac{U_0}{a^4} e^{-\left(\frac{x}{a}\right)^2}[x(x+a)(x-a)]$$

$$F = 0 \text{ at } x = 0,\ x = \pm a$$

F is an even function in x; so for $|x| < a$, F is always negative

$$U_4 = \frac{U_0}{2}\left[\frac{x}{a} - \frac{x^3}{3a^3}\right]$$

$$F = -\frac{dU_4}{dx} = \frac{-U_0}{2}\left[\frac{1}{a} - \frac{\cancel{3}x^2}{\cancel{3}a^3}\right]$$

$$= \frac{U_0}{2a^3}[x^2 - a^2]$$

$$F = 0 \text{ at } x = \pm a$$

At $x = -a,\ U_4 = \frac{-U_0}{3}$

$$x = a;\ U_4 = \frac{U_0}{3}$$

Hence can oscillate about $x = -a$ if total energy < $\frac{U_0}{3}$