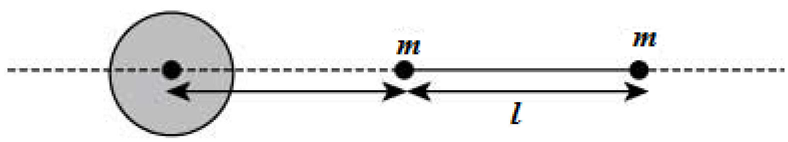
JEE ADVANCED 2015
PAPER 2
PHYSICS

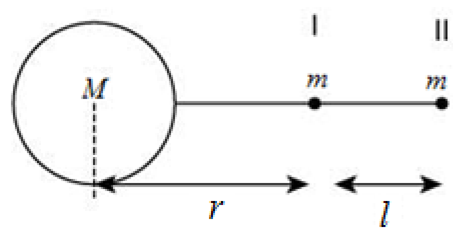
Integer Answer Type

This section contains **EIGHT** questions. The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

1. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3l$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is ______.



Solution



Since tension in the rod is zero, for the first mass m FBD gives

$$\frac{GM_m}{gl_2} - \frac{Gm^2}{l^2} = ma \qquad (1)$$

For the second mass m, FBD gives

$$\frac{GM_m}{16l^2} + \frac{Gm^2}{l^2} = ma \qquad (2)$$

From Eqs. (1) and (2)

$$\frac{GM_m}{g l^2} - \frac{Gm^2}{l^2} = \frac{GM_m}{16 l^2} + \frac{Gm^2}{l^2}$$

$$\frac{M}{9} - \frac{M}{16} = 2m$$

$$\frac{7M}{144} = 2m$$

$$m = 7\left(\frac{M}{288}\right) = k\left(\frac{M}{288}\right)$$

Therefore,

$$k = 7$$

2. The energy of a system as a function of time t is given as $E(l) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2\ \text{s}^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of $E(t)$ at $t = 5$ s is _____.

Solution

$$E = A^2 e^{-\alpha t}$$

$$dE = A^2(-\alpha e^{-\alpha t}\, dt) + 2A(dA)e^{-\alpha t}$$

Relative error $\dfrac{dE}{E} = -\alpha dt + 2\left(\dfrac{dA}{A}\right)$

$$\frac{dE}{E} = -\alpha t\left(\frac{dt}{t}\right) + 2\left(\frac{dA}{A}\right)$$

Therefore, % error (at $t = 5$ s) = $(0.2 \times 5 \times 1.5) + (2 \times 1.25)$

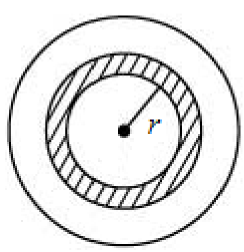
Considering the worst case, all errors are added

% error = 1.5 + 2.5 = 4

3. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\dfrac{r}{R}\right)$ and $\rho_B(r) = k\left(\dfrac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\dfrac{I_B}{I_A} = \dfrac{n}{10}$, the value of n is _____.

Solution

For 1st shell



$$dI_1 = \frac{2}{3}(dm)r^2$$

where *dm* is the mass of the elemental shell shown shaded $dm = \rho_1(4\pi r^2 dr)$

Therefore,

$$dI_1 = \frac{2}{3}\left[\left(\frac{kr}{R}\right)4\pi r^2 dr\right]r^2$$

$$I_1 = \frac{8\pi}{3}\frac{k}{R}\int_0^R r^5 dr = \left(\frac{8\pi k}{18}\right)R^5$$

For the 2nd shell,

$$dI_2 = \frac{2}{3}\left[k\left(\frac{r}{R}\right)^5 4\pi r^2\right]r^2$$

$$= \left(\frac{8\pi k}{30}\right)R^5$$

Therefore,

$$\frac{I_2}{I_1} = \frac{18}{30} = \frac{6}{10} = \frac{n}{10} \Rightarrow n = 6$$

4. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\pi/3$, $2\pi/3$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is _____.

Solution

Let $y_1 = a \sin \omega t$

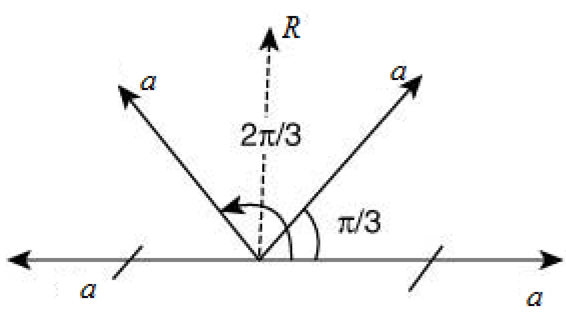
$$y_2 = a \sin(\omega t + \pi/3)$$

$$y_3 = a \sin(\omega t + 2\pi/3)$$

$$y_4 = a \sin(\omega t + \pi)$$

$$y = y_1 + y_2 + y_3 + y_4 = R \sin(\omega t \pm \phi)$$

where R is the amplitude of the resultant wave using phasor method



$$R = \sqrt{a^2 + a^2 + 2a^2 \cos\pi/3} = \sqrt{3}a$$

$$I = R^2 = 3a^2 = 3I_0 = nI_0$$

$$n = 3$$

5. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where $N(t)$ is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at t = 0. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is _____.

Solution

Law of radioactivity $N = N_0 e^{-\lambda t}$

where λ = decay constant

Activity $|A| = \left|\frac{-dN}{dt}\right| = N_0 \lambda e^{-\lambda t}$

Rate of activity $R = \frac{d|A|}{dt} = N_0 \lambda^2 e^{-\lambda t}$

At t = 0, $A_1 = A_2$

$$N_{OP}\lambda_P = N_{OQ}\lambda_Q$$

At $t = 2\tau$; $\frac{R_P}{R_Q} = \left(\frac{\lambda_P}{\lambda_Q}\right)^2 \left(\frac{N_{OP}}{N_{OQ}}\right) \frac{e^{-\lambda P(25)}}{e^{-\lambda Q(25)}} = \frac{\lambda_P}{\lambda_Q} e^{(\lambda_Q - \lambda_P)25}$

Mean life $\tau = \frac{1}{\lambda}$

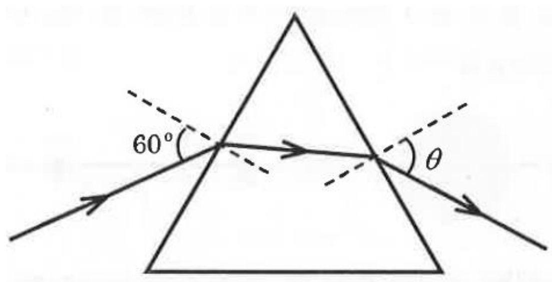
Therefore,

$$\frac{R_P}{R_Q} = \frac{\lambda_P}{\lambda_Q} e^{\left(\frac{1}{25} - \frac{1}{5}\right)^{25}} = \frac{\lambda_P}{\lambda_Q} e^{-1}$$

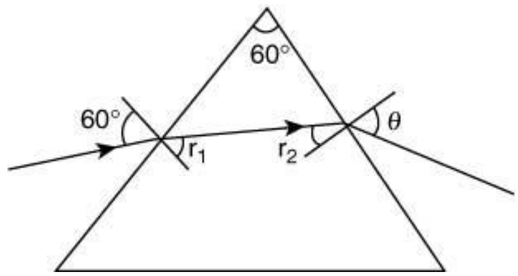
$$\frac{R_P}{R_Q} = \frac{\lambda_P}{\lambda_Q} \frac{1}{e} = \frac{n}{e}$$

$$n = \frac{\lambda_P}{\lambda_Q} = \frac{2\tau}{\tau} = 2$$

6. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$, the value of θ is 60° and $\frac{d\theta}{dn} = m.$ The value of m is _____.



Solution



Using Snell's law

$$\sin 60° = n \sin r_1 \qquad (1)$$

$$\sin r_1 = \frac{\sqrt{3}}{2\times\sqrt{3}} = \frac{1}{2}$$

$$r_1 = 30°$$

Also

$$n \sin r_2 = 1 \sin \theta$$

Also

$$r_1 + r_2 = A = 60°$$

Therefore,

$$n \sin (60° - r_1) = 1 \sin \theta \qquad (2)$$

Differentiating on both sides, we get

$$\sin(60° - r_1) - n\cos(60° - r_1)\frac{dr_1}{dn} = \cos\theta\frac{d\theta}{dn}$$

Differentiating Eq. (1) on both sides, we get

$$0 = \sin r_1 + n\cos r_1\frac{dr_1}{dn}$$

$$0 = \frac{1}{2} + \sqrt{3}\cdot\frac{\sqrt{3}}{2}\frac{dr_1}{dn}$$

Therefore,

$$\frac{dr_1}{dn} = \frac{-1}{3}$$

Hence putting $r_1 = 30°$

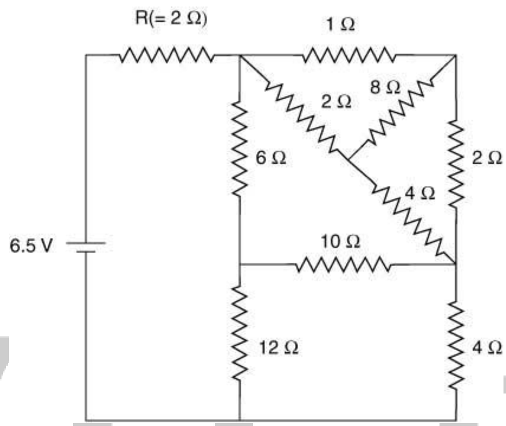
$$\frac{dr_1}{dn} = \frac{-1}{3}$$

$$\sin 30^\circ - \sqrt{3}\cos 30^\circ\left(-\frac{1}{3}\right) = \cos 60^\circ \frac{d\theta}{dn}$$

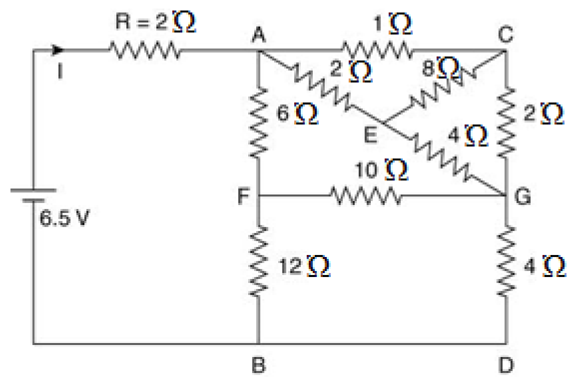
$$\frac{1}{2} + \frac{\cancel{3}}{2\times\cancel{3}} = \frac{1}{2}\frac{d\theta}{dn}$$

$$\frac{d\theta}{dn} = 2$$

7. In the following circuit, the current through the resistor R (= 2 Ω) is I amperes. The value of I is _____.



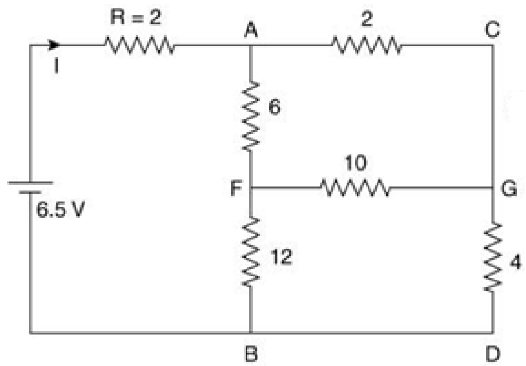
Solution



ACGA constitutes a Wheatstone bridge, hence 8 Ω is redundant and hence can be removed

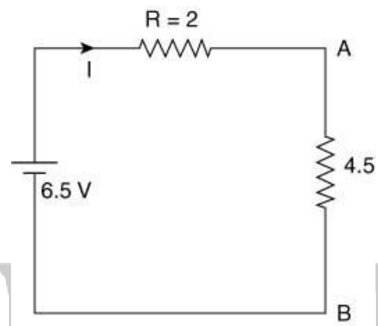
Therefore,

$$R_{\text{AG}} = \frac{3\times 6}{9} = 2\Omega$$



AGDFA again constitutes a Wheatstone bridge 10 Ω which is redundant and hence can be removed.

$$R_{AB} = \frac{6\times 18}{24} = 4.5\,\Omega$$



$$I = \frac{6.5}{6.5} = 1\text{ A}$$

8. An electron in an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is _____.

Solution

Angular momentum $mvr = \frac{nh}{2\pi}$ where $r = 3a_0$ where $n = 3$, that is, electron in Li^{2+} is in second excited state

$$\lambda = \frac{h}{mv} = p\pi a_0$$

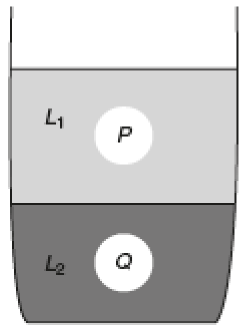
$$\Rightarrow n = p\pi(mva_0) = p\pi\left(\frac{mvr}{3}\right) = \frac{p\not{\pi}}{\not{3}}\left(\frac{\not{3}h}{2\not{\pi}}\right) = \frac{ph}{2}$$

Therefore, $p = 2$

One or More than One Options Correct Type

This section contains **EIGHT** questions. Each has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.

9. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_Q$, then



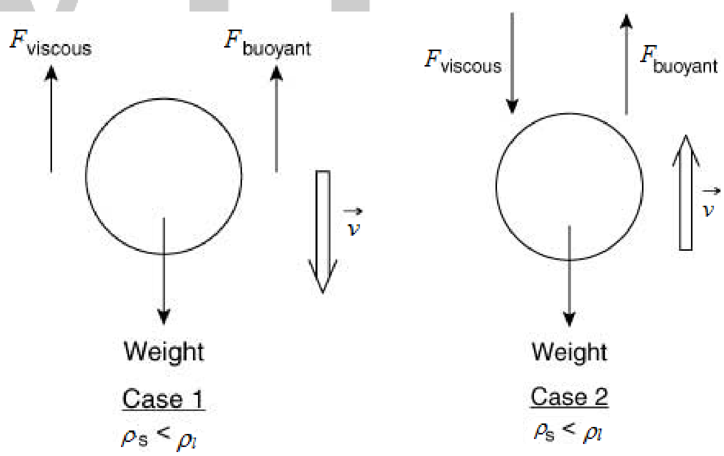
(A) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_1}{\eta_2}$

(B) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_2}{\eta_1}$

(C) $\vec{V}_P \cdot \vec{V}_Q > 0$

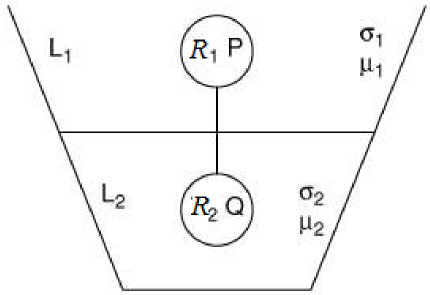
(D) $\vec{V}_P \cdot \vec{V}_Q < 0$

Solution



where ρ_s = density of solid

ρ_l = density of liquid



For sphere Q in L_2: T + $F_{bf} = mg$

$$T = \frac{4}{3}\pi r^3 \sigma_2 g = \frac{4}{3}\pi r^2 \rho_2 g$$

$$T = \frac{4}{3}\pi r^3 (\rho_2 - \sigma_2) g \text{ , that is } \rho_2 > \sigma_2$$

For sphere P in L_1: $T + m'g = F'_{bf}$

$$T = \frac{4}{3}\pi r^3 (\sigma_1 - \rho_2) g \text{ , that is } \rho_1 < \sigma_1$$

Given liquids are immiscible, $\sigma_2 > \sigma_1$

From above we get $\rho_2 > \sigma_2 > \sigma_1 > \rho_1$

We know viscous force $F_V = 6\pi\mu r v$

Terminal velocity, $V \propto \frac{1}{\mu}$, for a given viscous force. If P is put in L_2, $|\vec{V}_\rho| \propto \frac{1}{\mu_2}$, $\rho_1 < \sigma_2$, $\vec{F}_{vf}$ decreases, that is $\vec{V}_P$ increases.

If Q is put in L_1, $|\vec{V}_Q| \propto \frac{1}{\mu_1}$, $\rho_2 > \sigma_1$, $\vec{F}_{vf}$ increases, that is $\vec{V}_Q$ decreases.

Therefore, $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\mu_1}{\mu_2}$

$$\vec{V}_P \cdot \vec{V}_Q \;\; V_P V_Q \cos 180° = -V_P V_Q$$

that is, $\vec{V}_P \cdot \vec{V}_Q < 0$

Hence, the correct options are (A) and (D).

10. In terms of potential difference *V*, electric current *I*, permittivity ε_0, permeability μ_0 and speed of light *c*, the dimensionally correct equation(s) is (are)

(A) $\mu_0 I^2 = \varepsilon_0 V^2$

(B) $\varepsilon_0 I = \mu_0 V$

(C) $I = \varepsilon_0 c V$

(D) $\mu_0 c I = \varepsilon_0 V$

Solution

Dimensions of [V] = $[M^1L^2T^{-3}A^{-1}]$

$[I] = [A]$

$[E_0] = [M^{-1}L^{-3}T^4A^2]$

$[\mu_0] = [M^1L^1T^{-2}A^{-2}]$

$[C] = [L^1T^{-1}]$

(A) $\mu_0 I^2 = E_0 V^2$

LHS $\rightarrow [M^1L^1T^{-2}]$;

RHS $\rightarrow [M^1L^1T^{-2}]$ dimensionally correct

(B) $E_0 I = \mu_0 V$

LHS $\rightarrow [M^{-1}L^{-3}T^4A^3]$

RHS $\rightarrow [M^2L^3T^{-5}A^{-3}]$ dimensionally incorrect

(C) $I = E_0 CV$

LHS $\rightarrow \theta[A]$;

RHS $\rightarrow [A]$

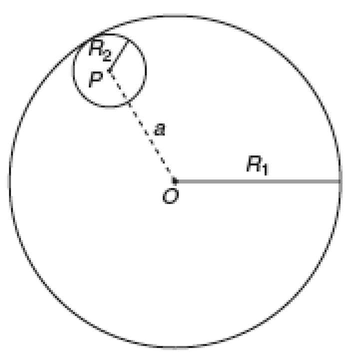
(D) $\mu_0 CI = E_0 V$

LHS $\rightarrow [M^1L^2T^{-3}A^{-1}]$,

RHS $\rightarrow [L^{-1}T^1A^1]$ dimensionally incorrect

Hence, the correct options are (A) and (C).

11. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance OP = $a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)



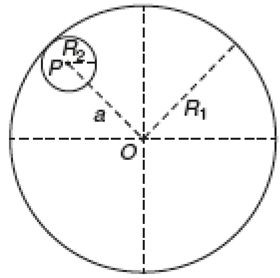
(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(C) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(D) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

Solution

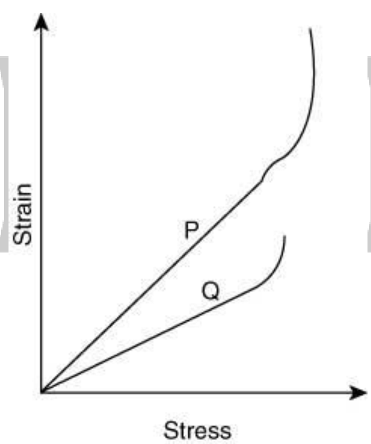


For a uniformly charged sphere, at $r < R_1$, $\vec{E} = \dfrac{e\vec{r}}{3E_0}$ where $\vec{r}$ is the position vector of the point from the centre.

Considering a point P' distant r_1 from O and r_2 from P: $\vec{E} = \vec{E}_1 - \vec{E}_2 = \dfrac{\rho}{3E_0}(\vec{r}_1 - \vec{r}_2) = \dfrac{\rho\vec{a}}{3E_0} = \text{constant}$

Hence, the correct option is (D).

12. In plotting stress versus strain curves for two materials *P* and *Q*, a student by mistake puts strain on the *y*-axis and stress on the *x*-axis as shown in the figure. Then the correct statement(s) is(are)



(A) *P* has more tensile strength than *Q*

(B) *P* is more ductile than *Q*

(C) *P* is more brittle than *Q*

(D) The Young's modulus of *P* is more than that of *Q*

Solution

Young's modulus of elasticity $y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{1}{\text{slope}}$

Slope of *Q* < slope of *P*

$y_Q > y_P$

For a given stress, strain *P* > strain *Q*

Therefore, ρ is more ductile.

Breakpoint of P > breakpoint of Q

Therefore, P has more tensile strength.

Hence, the correct options are (A) and (B).

13. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct option(s) is(are)

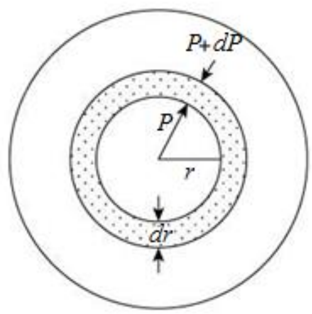
(A) $P(r = 0) = 0$

(B) $\frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{63}{80}$

(C) $\frac{P(r = 3R/5)}{P(r = 2R/5)} = \frac{16}{21}$

(D) $\frac{P(r = R/2)}{P(r = R/3)} = \frac{20}{27}$

Solution



$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

Mass of elemental strip or spherical shell of radius r and thickness dr is, $dm = \rho 4\pi r^2\, dr$

Under equilibrium,

$$\frac{Gm(dm)}{r^2} = -(dp)A$$

Therefore,

$$\rho = \int_0^p dp = \int_r^R \frac{G\left(\rho \frac{4}{3}\pi \cancel{r^2}\right)\rho \cancel{4\pi r^2}\, dr}{\cancel{r^2}\ \cancel{(4\pi r^2)}}$$

$$= G\rho^2 \frac{4}{3}\int_r^R r\, dr = G\rho^2 \frac{4\pi}{3}\left(\frac{R^2 - r^2}{2}\right)$$

Therefore, $P = \beta(R^2 - r^2)$ where $\beta = G \cdot \rho^2 \frac{4\pi}{6}$

$$P(r=0)=\beta R^2 \neq 0$$

$$P\left(r=\frac{3R}{4}\right)=\beta\left(R^2-\frac{9R^2}{16}\right)=\beta\frac{7R^3}{16}$$

$$P\left(r=\frac{2R}{3}\right)=\beta\left(R^2-\frac{4R^2}{9}\right)=\beta\frac{5R^2}{9}$$

Therefore, $\dfrac{P\left(r=\frac{3R}{4}\right)}{P\left(r=\frac{2R}{3}\right)}=\dfrac{63}{80}$ (option (B))

$$P\left(r=\frac{3R}{5}\right)=\beta\left(R^2-\frac{9R^2}{25}\right)=\beta\frac{16R^2}{25}$$

$$P\left(r=\frac{2R}{5}\right)=\beta\left(R^2-\frac{4R^2}{25}\right)=\beta\frac{21R^2}{25}$$

Therefore, $\dfrac{P\left(r=\frac{3R}{5}\right)}{P\left(r=\frac{2R}{5}\right)}=\dfrac{16}{21}$ (option (C))

$$P\left(r=\frac{R}{2}\right)=\beta\left(R^2-\frac{R^2}{4}\right)=\beta\frac{3R^2}{4}$$

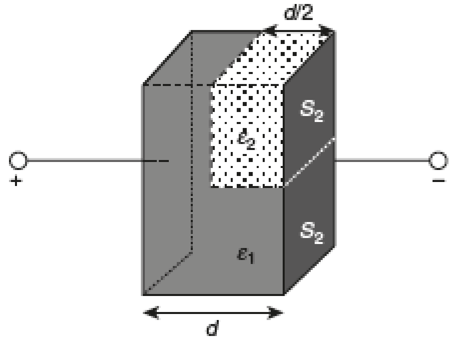
$$P\left(r=\frac{R}{3}\right)=\beta\left(R^2-\frac{R^2}{9}\right)=\beta\frac{8R^2}{9}$$

Therefore,

$$\frac{P\left(r=\frac{R}{2}\right)}{P\left(r=\frac{R}{3}\right)}=\frac{27}{32}$$

Hence, the correct options are (B) and (C).

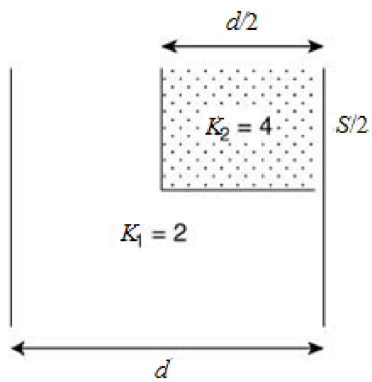
14. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\dfrac{C_2}{C_1}$ is



(A) 6/5 (B) 5/3

(C) 7/5 (D) 7/3

Solution



Originally $C_1 = \frac{Gs}{d}$

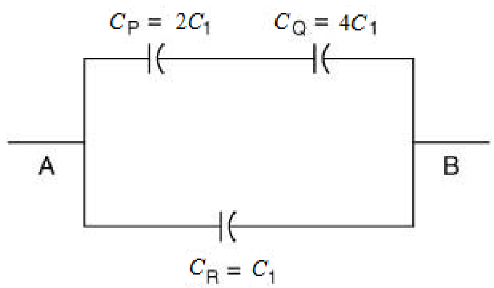
Let the given arrangement be divided into three capacitances

$$C_p = \frac{K_1 E_0 \frac{S}{2}}{d/2} = K_1 \frac{E_0 S}{d} = K_1 C_1 = 2C_1$$

$$C_Q = \frac{K_2 E_0 \frac{S}{2}}{d/2} = K_2 \frac{E_0 S}{d} = K_2 C_1 = 4C_1$$

$$C_R = \frac{K_1 E_0 \frac{S}{2}}{d} = \frac{K_1}{2} \frac{E_0 S}{d} = \frac{K_1}{2} C_1 = C_1$$

Equivalent arrangement would be

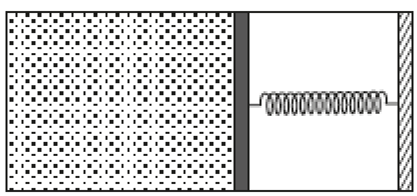


$$C_{AB} = C_2 = \frac{4}{3}C_1 + C_1 = \frac{7C_1}{3}$$

Therefore, $\frac{C_2}{C_1} = \frac{7}{3}$

Hence, the correct option is (D).

15. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are)



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

Solution

$$P = P_1 + \frac{kx}{A}$$

$$W = \int P\,dV = P_1(V_2 - V_1) + \frac{1}{2}kx^2$$

$$\Delta U = nC_V\Delta T = \frac{3}{2}(nR\Delta T) = \frac{3}{2}(P_2V_2 - P_1V_1)$$

For case 1: If $V_2 = 2V_1$, $T_2 = 3T_1$

$$\Delta U = 3P_1V_1, \quad W = \frac{5P_1V_1}{4}$$

$$Q = \Delta U + W = \frac{17P_1V_1}{4}, \; U_{\text{spring}} = \frac{P_1V_1}{4}$$

For case 2: If $V_2 = 3V_1$ and $T_2 = 4T_1$

$$\Delta U = \frac{P_1V_1}{2}, \quad W = \frac{7P_1V_1}{3}$$

$$Q = \Delta U + W = \frac{41P_1V_1}{6}, \; U_{\text{spring}} = \frac{P_1V_1}{3}$$

Using $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

If $V_2 = 2V_1$, $T_2 = 3T_1$

$$P_2 = \frac{3}{2}P_1$$

$$\Delta U = nC_V\Delta T = n\left(\frac{3R}{2}\right)\left(\frac{P_2V_2}{nR} - \frac{P_1V_1}{nR}\right)$$

$$= \frac{3}{2} \; P_2V_2 - P_1V_1$$

$$= 3P_1V_1$$

$$P = P_1 + \frac{kx}{A}$$

$$W = \int PdV = P_1(V_2 - V_1) + \frac{1}{2}kx^2$$

$$\Delta U = nC_V\Delta T = \frac{3}{2}(nR\Delta T) = \frac{3}{2}(P_2V_2 - P_1V_1)$$

For case 1:

$$\Delta U = 3P_1V_1 \qquad W = \frac{5P_1V_1}{4}$$

$$Q = \Delta U + W = \frac{17P_1V_1}{4} \qquad U_{\text{spring}} = \frac{P_1V_1}{4}$$

For case 2:

$$\Delta U = \frac{P_1V_1}{2} \qquad W = \frac{7P_1V_1}{3} \qquad Q = \frac{41P_1V_1}{6}$$

$$\Rightarrow U_{\text{spring}} = \frac{P_1V_1}{3}$$

Hence, the correct options are (A), (B) and (D).

16. A fission reaction is given by ${}^{236}_{92}\text{U} \rightarrow {}^{160}_{34}\text{Xe} + {}^{94}_{38}\text{Sr} + x + y,$ where x and y are two particles. Considering ${}^{236}_{92}\text{U}$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr} K_x(2 MeV) and K_y (2 MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}\text{U}$, ${}^{140}_{84}\text{Xe}$ and ${}^{94}_{38}\text{Sr}$ be 7.2 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation law the correct option(s) is(are)

(A) $x = n$, $y = n$, $K_{\text{Sr}} = 129$ MeV, $K_{\text{Xe}} = 86$ MeV

(B) $x = p$, $y = e^-$, $K_{\text{Sr}} = 129$ MeV, $K_{\text{Xe}} = 86$ MeV

(C) $x = p$, $y = n$, $K_{\text{Sr}} = 129$ MeV, $K_{\text{Xe}} = 86$ MeV

(D) $x = n$, $y = n$, $K_{\text{Sr}} = 86$ MeV, $K_{\text{Xe}} = 129$ MeV

Solution

${}^{236}_{92}\text{U} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + x + y$

For the given fission reaction

Q value = (140 × 8.5) + (94 × 8.5) – (236 × 7.5) = 219 MeV

Therefore, KE of Xe and Sr combined = Q value – KE_x – KE_y

$$= 219 - 2 - 2 = 215 \text{ MeV}$$

Conservation of charge implies x and y to be neutral

Conservation of mass implies x and y are neutrons ${}^{1}_{0}r$

$$\left.\begin{array}{l}\vec{P}_i = \vec{0} \\ \vec{P}_f = \vec{0}\end{array}\right\}\text{conservation of momentum}$$

$|\vec{P}_{\text{Xe}}| = |P_{\text{Sr}}| = |\vec{P}|$ say

$$\text{KE} = \frac{p^2}{2m}$$

Therefore, $\text{KE} \propto \dfrac{1}{m}$

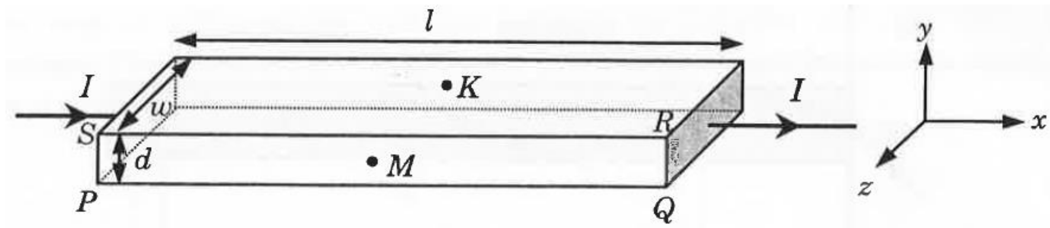
Hence, the correct option is (A).

< H2>Paragraph Type

This section contains **TWO** paragraphs. Based on each paragraph, there will be **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.

Paragraph for Questions 17 and 18: In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the below figure. The length, width and thickness of the strip are l, w and d, respectively.

A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross-section of the strip and carried by electrons.



17. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thickness of d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential difference between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is (are)

(A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$

(B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

(D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

Solution

Given $I_1 = I_2$

$\cancel{ne}A_1v_{d_1} = \cancel{ne}A_2v_{d_2}$ where v_d = drift velocity

$$A_1v_{d_1} = A_2v_{d_2}$$

$$(d_1\omega_1)v_{d_1} = (d_x\omega_2)v_{d_2}$$

Potential difference $V = Bv_d\omega$

Therefore,

$$\frac{V_1}{V_2} = \frac{V_{d_1}\omega_1}{V_{d_2}\omega_2} = \frac{d_2}{d_1}$$

For choice (1) if $\omega_1 = \omega_2$ and $d_1 = 2d_2$; $V_{d_1} = \frac{V_{d_2}}{2}$

Therefore,

$$\frac{V_1}{V_2} = \frac{1}{2} \Rightarrow V_2 = 2V_1$$

For choice (2), if $\omega_1 = 2\omega_2$ and $d_1 = d_2$

$$V_{d_2} = 2V_{d_1}$$

Therefore,

$$\frac{V_1}{V_2} = 1$$

Hence, the correct options are (A) and (D).

18. Consider two different metallic strips (1 and 2) of same dimensions (length l, with ω and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2 both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is (are)

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

(D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

Solution

Given $I_1 = I_2$

$$n_1 \cancel{e} \cancel{A_1} V_{d_1} = n_2 \cancel{e} \cancel{A_2} V_{d_2}$$

$$n_1(\cancel{d_1 \omega_1})V_{d_1} = n_2(\cancel{d_2 \omega_2})V_{d_2}$$

$$n_1 V_{d_1} = n_2 V_{d_2}$$

Also $V = BV_d\omega$

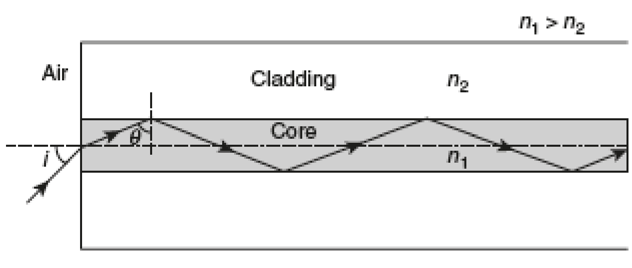
$$\frac{V_1}{V_2} = \frac{B_1 V_{d_1} \cancel{\omega_1}}{B_2 V_{d_2} \cancel{\omega_2}} = \frac{B_1 n_2}{B_2 n_1}$$

If $B_1 = B_2$, $n_1 = 2n_2$, $\frac{V_1}{V_2} = \frac{1}{2}$

If $B_1 = 2B_2$, $n_1 = n_2$, $\frac{V_1}{V_2} = 2$

Hence, the correct options are (A) and (C).

Paragraph for Questions 19 and 20: Light guidance in an optical fibre can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the below figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as sin i_m.



19. For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is (are)

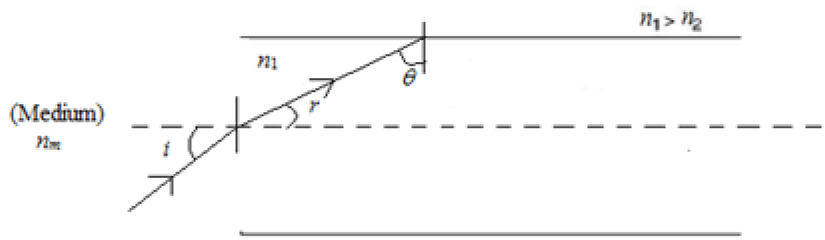
(A) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(B) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(C) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

(D) NA of S_1 placed in air is the same as that of S_2 placed in water

Solution



Total internal reflection occurs at the core cladding interface if $\theta \geq i_C$ where i_C is the critical angle

$\theta \geq i_C$ Also $\frac{\sin i}{\sin r} = \frac{n_1}{n_m}$ (Snell's law)

$$(90 - r) \geq i_C$$

$$\sin (90 - r) \geq \sin i_C$$

$$\cos r \geq \sin i_C = \frac{n_2}{n_1}$$

As I increases, r also increases, so $\theta = (90 - r)$ decreases.

Thus there is a maximum value of i (say i_m) beyond which no total internal reflection occurs at the interface.

i_m = maximum angle of acceptance

Now $n_1 \sin r_m = n_1 \sin (90 - i_C) = n_1 \cos i_C$

$$= n_1\sqrt{1 - \sin^2 i_C}$$

$$= n_1\sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$= \sqrt{n_1^2 - n_2^2}$$

$$(\text{NA})^2 = \sin^2 i_m = \frac{n_1^2 - n_2^2}{n_m^2}$$

If $n_1 = \frac{\sqrt{45}}{4}$, $n_2 = \frac{3}{2}$, $n_m = \frac{4}{3}$

Therefore, $\text{NA} = \sin i_m = \frac{9}{16}$

If $n_1 = \frac{8}{5}$, $n_2 = \frac{7}{5}$, $n_m = \frac{16}{3\sqrt{15}}$; $\text{NA} = \sin i_m = \frac{9}{16}$

If $n_1 = \frac{\sqrt{45}}{4}$, $n_2 = \frac{3}{2}$, $n_m = 1$; $\text{NA} = \sin i_m = \frac{3}{4}$

If $n_1 = \frac{8}{5}$, $n_2 = \frac{7}{5}$, $n_m = \frac{4}{\sqrt{15}}$; $\text{NA} = \sin i_m = \frac{3}{4}$

Hence, the correct options are (A) and (C).

20. If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

(A) $\frac{NA_1 NA_2}{NA_1 + NA_2}$

(B) $NA_1 + NA_2$

(C) NA_1

(D) NA_2

Solution

For total internal reflection to occur in both, NA should be the least of the two constituting the combined structure. Hence it should be NA_2.

Hence, the correct option is (D).