JEE ADVANCED 2017

PAPER 1 – PHYSICS

One or More Than One Option Correct Type

This section contains 7 questions. Each has 4 choices (A), (B), (C) and (D) out of which **ONE OR MORE** is(are) correct.

1. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true?

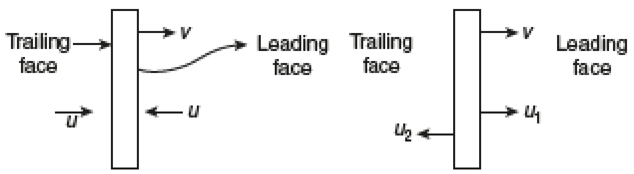
(A) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

(B) The resistive force experienced by the plate is proportional to v.

(C) The plate will continue to move with constant non-zero acceleration, at all times.

(D) At a later time, the external force F balances the resistive force.

1. **(A), (B), (D)** It is given that the average speed of gas molecules is u; the speed of the plate is v; the force on the plate is F.



(a) Before collision (b) After collision

- **Just before the collision:** The gas molecules are approaching the leading and trailing faces of the plate with speed u.
- **Just after the collision:** The gas molecules bounce back with speed u_1 and u_2.
- **At trailing face**

The speed before collision is $u - v$.

The speed after collision is $u_2 + v$.

From law of conservation of linear momentum, we have

$$u_2 + v = u - v$$

That is, $u_2 = u - 2v$ (1)

and $\Delta u_2 = 2u - 2v$ (2)

- **At leading face**

The speed before collision is $u + v$.

The speed after collision is $u_1 - v$.

From law of conservation of linear momentum, we have

$$u_1 - v = u + v$$

That is, $u_1 = u + 2v$ (3)

and $\Delta u_1 = 2u + 2v$ (4)

Now, from Newton's second law of motion, we have

$$F = \frac{dp}{dt}$$

- **For leading face:** $F_1 = \dfrac{dp_1}{dt}$.

The volume swept by the plate is $A(u + v)$. Therefore,

$$F_1 = \frac{dp_1}{dt} = \rho A(u+v)\Delta u_1$$

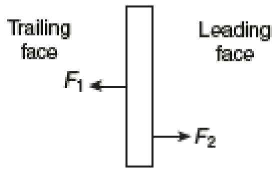
$$F_1 = \rho A(u+v)(2u+2v) \tag{5}$$

- **For trailing face:** $F_2 = \dfrac{dp_2}{dt}$.

  The volume swept by the plate is $A(u - v)$. Therefore,

$$F_2 = \frac{dp_2}{dt} = \rho A(u-v)\Delta u_2$$

$$F_2 = \rho A(u-v)(2u-2v) \tag{6}$$



The net force on the plate is

$$\begin{aligned} F_{\text{net}} &= F_1 - F_2 \\ &= \rho A(u+v)(2u+2v) - \rho A(u-v)(2u-2v) \\ &= 2\rho A(u+v)(u+v) - 2\rho A(u-v)(u-v) \\ &= 2\rho A(u+v)^2 - 2\rho A(u-v)^2 \\ &= 2\rho A[(u+v)^2 - (u-v)^2] \\ &= 2\rho A[u^2+v^2+2uv-(u^2+v^2-2uv)] \\ &= 2\rho A(u^2+v^2+2uv-u^2-v^2+2uv) \\ &= 2\rho A(4uv) \\ &= 8\rho Auv \end{aligned}$$

Therefore, the pressure difference is

$$\frac{F_{\text{net}}}{A} = \frac{8\rho Auv}{A} = 8\rho uv$$

Thus, the pressure difference between the leading and trailing faces of the plate is proportional to uv. Hence, option (A) is correct.

Now $F_{\text{net}} = 8\rho Auv$

The net resistive force is

$$F - (8\rho Au)v = \frac{mdv}{dt}$$

where F is the constant force applied by the gas molecules on the plate.

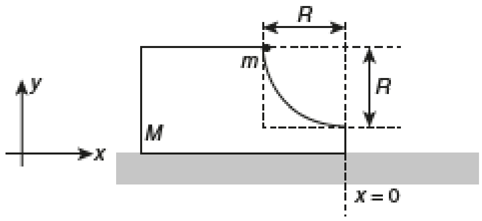
The resistive force experienced by the plate is proportional to v.

Hence, option (B) is also correct.

At a later time, the velocity v becomes sufficient and the external force F (= $8\rho Auv$) balances the resistive force.

Hence, option (D) is also correct.

2. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$, in a *coordinate system fixed to the table*. A point mass m is released from rest at the top most point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct?



(A) The position of the point mass *m* is $x = -\sqrt{2}\dfrac{mR}{M+m}$.

(B) The velocity of the point mass *m* is $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$.

(C) The *x* component of displacement of the centre of mass of the block *M* is $-\dfrac{mR}{M+m}$.

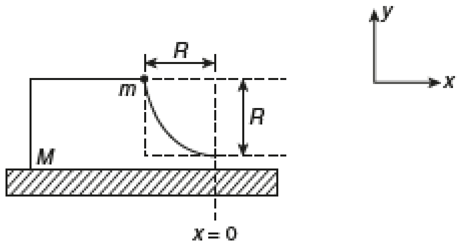
(D) The velocity of the block *M* is $V = -\dfrac{m}{M}\sqrt{2gR}$.

2. **(B), (C)** We discuss the options as follows:

- **For option (B):** From law of conservation of momentum, we have

$$mv = MV \quad (1)$$

where *v* is the velocity of point mass *m* and *V* is velocity of block of mass *M*.



Now, by using law of conservation of energy, after the point mass is released, we have the following:

Loss in P.E. of mass *m* = Gain in K.E. of both mass *M* and the mass *m*.

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Substituting $V = \dfrac{mu}{M}$ [from Eq. (1)], we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}M\frac{m^2v^2}{M^2}$$

$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}\frac{m^2v^2}{M}$$

$$mgR = \frac{1}{2}mv^2\left(1+\frac{m}{M}\right)$$

$$gR = \frac{1}{2}v^2\left(1+\frac{m}{M}\right)$$

Rearranging this equation, we get

$$2gR = v^2\left(1+\frac{m}{M}\right)$$

$$v^2 = \frac{2gR}{\left(1+\frac{m}{M}\right)}$$

$$\Rightarrow v = \sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}}$$

Hence, option (B) is correct.

- **For option (C):** Now, from Eq. (1), we have

$$V = \frac{m}{M}v = \frac{m}{M}\sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}}$$

$$V = \sqrt{\frac{m^2 2gR}{M^2\left(1+\frac{m}{M}\right)}} = \sqrt{\frac{m^2 2gR}{M^2\left(\frac{M+m}{M}\right)}}$$

$$V = \sqrt{\frac{m^2 2gR}{M(m+M)}} \Rightarrow V = m\sqrt{\frac{2gR}{M(m+M)}}$$

Since there is no external force acting on the system, the centre of mass does not change. Now, if the change in position of mass m is Δx and the change in position of mass M is ΔX, then

$$m\Delta x + M\Delta x = 0 \qquad \text{(since centre of mass will not change)}$$

Now, we know that the change in position of point mass m is

$$\Delta x = R - x$$

and the change in position of block of mass M is

$$\Delta X = -x$$

Therefore, $m(R - x) - Mx = 0$

$$m(R - x) = Mx$$

$$mR - mx = Mx$$

$$mR = Mx + mx$$

$$mR = (M + m)x$$

$$\Rightarrow x = \frac{mR}{M+m}$$

The change in position of block of mass M is

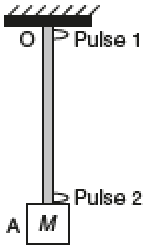
$$\Delta x = -x = \frac{-mR}{M+m}$$

Thus, x-component of the displacement of the centre of mass of the block is

$$M = \frac{-mR}{M+m}$$

Hence, option (C) is also correct.

3. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



(A) The time $T_{AO} = T_{OA}$.

(B) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope.

(C) The wavelength of Pulse 1 becomes longer when it reaches point A.

(D) The velocity of any pulse along the rope is independent of its frequency and wavelength.

3. **(A), (D)** We discuss the options as follows:

- **For option (A):** Since the medium in which both pulses are travelling is the same, the time taken by pulse 1 to reach from point O to A (i.e. T_{OA}) is the same as the time taken by Pulse 2 to reach from Point A to Point O (i.e. T_{AO}). That is,

$$T_{AO} = T_{OA}$$

Hence, option (A) is correct.

- **For option (D):** The velocity of the pulse depends on the property of the medium, that is, the tension (T) in the rope and mass per unit length, m.

$$V = \sqrt{\frac{T}{m}}$$

Thus, the velocity of any pulse along the rope is independent of its frequency and wavelength.

Hence, Option (D) is correct.

Note: Since the tension in the rope is same at the midpoint, the magnitude of the velocity of Pulse 1 and Pulse 2 is the same but not their directions. Thus, if only magnitude is considered, then option (B) is also correct; otherwise, it is incorrect.

4. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. T_0 = 300 K, the value of $\sigma T_0^4 = 460\,\text{Wm}^{-2}$ (where σ is the Stefan–Boltzmann constant). Which of the following options is/are correct?

(A) The amount of energy radiated by the body in 1 s is close to 60 J.

(B) If the surrounding temperature reduces by a small amount $\Delta T_0 \ll T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time.

(C) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation.

(D) If the body temperature rises significantly, then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths.

4. **(A), (B), (C)*** We discuss the options as follows:

- **For option (A)*:**The net power radiated by the human body is given as

$$W = W_{\text{emitted}} - W_{\text{absorbed}}$$

According to Stefan's–Boltzmann's law, we get

$$W = A\sigma e(T^4 - T_0^4)$$

where A is surface area of body, σ is the Stefan–Boltzmann constant, e is emissivity, T is temperature of body and T_0 is surrounding temperature. Assuming e = 1, we get

$$W = A\sigma(T^4 - T_0^4) \quad (1)$$

It is given that A = 1; therefore, from Eq. (1), we get

$$W = \sigma \times 1 \times (T^4 - T_0^4)$$

It is also given that $\sigma T_0^4 = 460\,\text{Wm}^{-2}$. Therefore,

$$W = \sigma T_0^4 \left(\frac{T^4}{T_0^4} - 1 \right)$$

It is given that the normal body temperature is 10 K above the surrounding room temperature, which implies that for T_0 = 300 K, we get T = 310 K. Therefore,

$$W = 460\left[\left(\frac{310}{300} \right)^4 - 1 \right]$$

$$W = 460[(1.033)^4 - 1]$$

$$W = 460(1.1386 - 1)$$

$$W = 460 \times 0.1386$$

$$W = 63.75\text{ W} \simeq 60\text{ W}$$

Therefore, the net energy radiated by the body in 1 s is close to 60 J.

Hence, option (A) is correct.

- **For option (B)*:** We know that

$$W = \sigma A(T^4 - T_0^4)$$

If the surrounding temperature changes by a small amount ΔT_0 and differentiating the above equation, we get

$$\Delta W = \sigma A(0 - 4T_0^3 \Delta T_0)$$

Since the temperature has reduced, we get

$$\Delta T_0 = -\Delta T_0$$

Therefore,

$$\Delta \omega = \sigma A(0 - 4T_0^3(-\Delta T_0))$$

$$\Delta \omega = \sigma A(4T_0^3 \Delta T_0)$$

$$\Delta \omega = 4\sigma A T_0^3 \Delta T_0$$

Hence, option (B) is also correct.

- **For option (C)*:** From $W = \sigma A(T^4 - T_0^4)$, we conclude that to maintain the same body temperature, we have

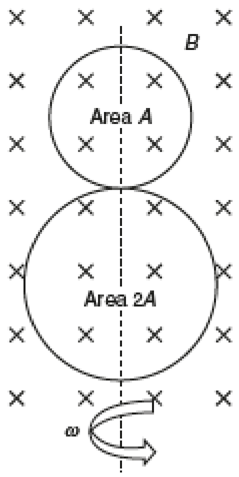
$$W \propto A$$

If the area of exposure reduces, the energy radiated also reduces.

Hence, option (C) is also correct.

**Due to insufficient data in the question, for options (A) and (B), we assume that the "energy radiated" here refers to the "net radiation". However, if the "energy radiated" here in this question essentially discusses only about the concept "emission", then options (A) and (B) are not considered as correct options and only option (C) stands as correct one.*

5. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. *At the point of crossing the wires remain electrically insulated from each other*. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(A) The *emf* induced in the loop is proportional to the sum of the areas of the two loops.

(B) The amplitude of the maximum net *emf* induced due to both the loops is equal to the amplitude of maximum *emf* induced in the smaller loop alone.

(C) The net *emf* induced due to both the loops is proportional to $\cos\omega t$.

(D) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper.

5. **(B), (D)** We discuss the options as follows:

- **For option (B):** The area vector $\vec{A}$ makes an angle θ with the magnetic field $\vec{B}$ at time t such that

$$\theta = \omega t$$

The magnetic flux exists through the loop of area A is

$$\phi_1 = BA\cos(\omega t)$$

The magnetic flux exists through the loop of area $2A$ is

$$\phi_2 = B(2A)\cos(\omega t)$$

Then *emf* induced in loop of area A is

$$\varepsilon_1 = \frac{-d\phi_1}{dt} = \frac{-d}{dt}[BA\cos(\omega t)]$$

$$\varepsilon_1 = \omega BA\sin(\omega t) \qquad \left(\because \frac{d}{dt}\cos\theta = -\sin\theta\right) \qquad (1)$$

and the *emf* induced in loop of area $2A$ is

$$\varepsilon_2 = \frac{-d\phi_2}{dt} = -\frac{d}{dt}[2BA\cos(\omega t)]$$

$$\varepsilon_2 = 2\omega BA\sin(\omega t) \qquad \left(\because \frac{d}{dt}\cos\theta = -\sin\theta\right) \qquad (2)$$

The net *emf* induced due to both loops is

$$\varepsilon_{\text{net}} = \varepsilon_2 - \varepsilon_1 = 2\omega BA\sin(\omega t) - \omega BA\sin(\omega t)$$

$$\varepsilon_{\text{net}} = \omega BA\sin(\omega t) \qquad (3)$$

Here, ε_{net} is maximum when $\sin(\omega t) = 1$, that is, $\omega t = 90°$. Therefore,

$$\varepsilon_{\text{maximum}} = \omega BA \qquad (4)$$

Also, from Eq. (1), the maximum *emf* induced in smaller loop is

$$\varepsilon_1 = \omega BA\sin(\omega t)$$

$$\varepsilon_1\big|_{\text{maximum}} = \omega BA \qquad [\text{as } \sin(\omega t) = 1] \qquad (5)$$

From Eqs. (4) and (5), we get

$$\varepsilon_{net}\big|_{maximum} = \varepsilon_1\big|_{maximum}$$

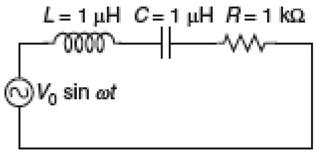
That is, the amplitude of the maximum net *emf* induced due to both the loops is equal to the amplitude of maximum *emf* induced in the smaller loop alone.

Hence, option (B) is correct.

- **For option (D):** Also, we know that ε_{net} is maximum when $\sin(\omega t) = 1$ or $\omega t = 90°$ – this implies that the area vector is perpendicular to the magnetic field. It is given that the magnetic field points into the plane of paper. Thus, the plane of the loops is perpendicular to the plane of the paper.

  Hence, option (D) is correct.

6. The circuit shown, $L = 1$ μH, $C = 1$ μF and $R = 1$ kΩ. They are connected in series with an ac source $V = V_0\sin(\omega t)$ as shown. Which of the following options is/are correct?



(A) The current will be in phase with the voltage if $\omega = 10^4$ rad s^{-1}.

(B) The frequency at which the current will be in phase with the voltage is independent of R.

(C) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero.

(D) At $\omega >> 10^6$ rad s^{-1}, the circuit behaves like a capacitor.

6. **(B), (C)** We discuss the options as follows:

- **For option (B):** At resonant frequency, the current is in phase with the voltage. The resonant frequency is given as

$$\omega = \frac{1}{\sqrt{LC}}$$

  This frequency is independent of the resistor of resistance R.

  Hence, option (B) is correct.

- **For option (C):** impedance of *RLC* series circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

  where $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

  At $\omega \sim 0$: $X_C \sim \infty$ and $X_L \sim 0$; therefore, the current in the circuit is

$$I = \frac{V}{Z} \sim 0$$

  Hence, option (C) is correct.

- **For option (A):** At resonant frequency, the current is in phase with the voltage. We know that at resonance,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times 10^{-6} \times 1\times 10^{-6}}} = 10^6 \text{ rad/sec}$$

  Thus, the current is in phase with the voltage if $\omega = 10^6$ rad/s

  Hence, option (A) is incorrect.

- **For option (D):** For $\omega \gg 10^6$ rad/s, we get $X_L \gg 1$ and $X_C \to 0$. Thus, the circuit does not behave like a capacitor rather it behaves like an inductor.
  Hence, option (D) is incorrect.

7. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

(A) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism.

(B) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$.

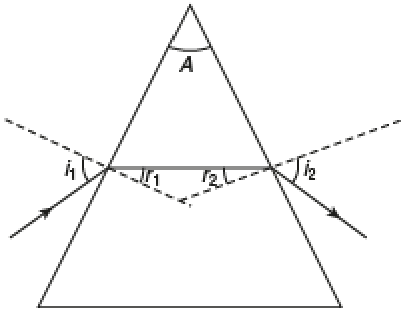
(C) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$.

(D) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$.

7. **(A), (C), (D)** We discuss the options as follows:

- **For option (A):** For the angle of incidence $i_1 = A$, we have minimum deviation and the ray inside the prism is parallel to the base of the prism.

  Hence, option (A) is correct.



- **For option (C):** The angle of deviation is given as

$$\delta = i_1 + i_2 - A$$

At minimum deviation, $i_1 = i_2$ and $r_1 = r_2$. Therefore,

$$i_1 = A \quad (1)$$

Also, we know that

$$r_1 + r_2 = A$$

As $r_1 = r_2$, we get

$$2r_1 = A \Rightarrow r_1 = \frac{A}{2}$$

From Eq. (1), we get

$$r_1 = \frac{i_1}{2}$$

Hence, option (C) is correct.

According to Snell's law, we have

$$\frac{\sin i_2}{\sin r_2} = \mu$$

For emergent ray to be tangential to the surface, we have

$$i_2 = 90°$$

$$\Rightarrow \sin 90° = \mu \sin r_2 \Rightarrow \mu \sin r_2 = 1$$

$$\Rightarrow \sin r_2 = \frac{1}{\mu} \Rightarrow r_2 = \sin^{-1}\left(\frac{1}{\mu}\right) \quad (2)$$

From the relation $r_1 + r_2 = A$, we get

$$r_1 = A - r_2$$

$$\Rightarrow \sin r_1 = \sin(A - r_2)$$

Using $\sin(a - b) = \sin a \cos b - \sin b \cos a$, we have

$$\Rightarrow \sin r_1 = \sin A \cos r_2 - \sin r_2 \cos A \quad (3)$$

Using Eq. (2), we have

$$\sin r_2 = \frac{1}{\mu}$$

Squaring it, we get $\sin^2 r_2 = \frac{1}{\mu^2}$

Using the relation $\sin^2 x + \cos^2 x = 1$, we get

$$\sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow 1 - \cos^2 r_2 = \frac{1}{\mu^2}$$

$$\Rightarrow \cos^2 r_2 = 1 - \frac{1}{\mu^2}$$

That is, $\cos r_2 = \sqrt{1 - \frac{1}{\mu^2}}$

Substituting the values of $\sin r_2$ and $\cos r_2$ in Eq. (3), we get

$$\sin r_1 = \sin A\sqrt{1 - \frac{1}{\mu^2}} - \frac{1}{\mu}\cos A$$

According to Snell's law, we have

$$\frac{\sin i_1}{\sin r_1} = \mu$$

That is, $\sin i_1 = \mu \sin r_1 = \mu\left(\sin A\sqrt{1 - \frac{1}{\mu^2}} - \frac{1}{\mu}\cos A\right)$

$$\sin i_1 = \mu\left(\sin A\frac{\sqrt{\mu^2 - 1}}{\mu} - \frac{\cos A}{\mu}\right)$$

$$\sin i_1 = \sin A\sqrt{\mu^2 - 1} - \cos A \quad (4)$$

Now, at minimum deviation, we have $A = i_1$ and $r_1 = \frac{A}{2}$. Thus, from Snell's law, we get

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin A}{\sin(A/2)}$$

Using $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$, we get

$$\mu = \frac{2\sin A/2 \cos A/2}{\sin A/2} \Rightarrow \mu = 2\cos\frac{A}{2} \quad (5)$$

Now, Eq. (4) becomes

$$\sin i_1 = \sin A\sqrt{\left(2\cos\frac{A}{2}\right)^2 - 1} - \cos A$$

$$= \sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A$$

$$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$$

Hence, option (D) is correct.

- **For option (B):** From Eq. (5), we have

$$\mu = 2\cos\frac{A}{2}$$

$$\cos\frac{A}{2} = \frac{\mu}{2}$$

$$\Rightarrow \frac{A}{2} = \cos^{-1}\left(\frac{\mu}{2}\right)$$

$$\Rightarrow A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

Hence, option (B) is incorrect.

Integer Answer Type

This section contains 5 questions. The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

8. A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \frac{0.1}{4\pi}\,\text{N m}^{-1}$ divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$J. If $K = 10^{\alpha}$ then the value of α is _______.

8. From law of conservation of mass, we have

$$\frac{4}{3}\pi R^3 = \left(\frac{4}{3}\pi r^3\right)\cdot K$$

where R is radius of liquid initially, r is the radius of the liquid drop after division.

$$R^3 = r^3 \cdot K \qquad (1)$$

It is given that the change in the surface energy is 10^{-3} J. The surface energy is given by

$$U_1 = S4\pi R^2$$

where S is surface tension and

$$U_2 = SK(4\pi r^2)$$

Then, the change in surface energy is

$$\Delta U = SK(4\pi r^2) - S(4\pi R^2) \qquad (2)$$

$$= 4\pi S(kr^2 - R^2)$$

From Eq. (1), we substitute $r^3 = \frac{R^3}{K}$; therefore,

$$r = \left(\frac{R^3}{K}\right)^{1/3}$$

Substituting the value of r in Eq. (2), we get

$$\Delta U = 4\pi S\left[K\left(\frac{R^3}{K}\right)^{2/3} - R^2\right]$$

$$= 4\pi S\left[K\cdot\frac{R^2}{K^{2/3}} - R^2\right]$$

$$= 4\pi S[K^{1/3}\cdot R^2 - R^2] = 4\pi R^2 S(K^{1/3} - 1)$$

Now, substituting the given values, $\Delta U = 10^{-3}$ J; $S = \frac{0.1}{4\pi}\,\text{N m}^{-1}$; $K = 10^{\alpha}$ and $R = 10^{-2}$

$$\Rightarrow\ 10^{-3} = 4\pi \times \frac{0.1}{4\pi} \times (10^{-2})^2((10^{\alpha})^{1/3} - 1)$$

$$\Rightarrow\ 10^{-3} = 10^{-5}(10^{\alpha/3} - 1)$$

$$\Rightarrow\ 10^{\alpha/3} - 1 = 10^2$$

$$\Rightarrow\ 10^{\alpha/3} \simeq 10^2$$

$$\Rightarrow\ \frac{2}{3} = 2 \Rightarrow\ \alpha = 6$$

9. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_t and V_f are, respectively, the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the *smallest possible* n_f is ______.

9. The potential energy of electron in hydrogen atom is inversely proportional to its quantum number n, that is,

$$V \propto \frac{1}{n^2}$$

The ratio of the initial and the final potential energies is given by

$$\frac{V_i}{V_f} = \frac{1/n_i^2}{1/n_f^2} = \frac{n_f^2}{n_i^2}$$

It is given that

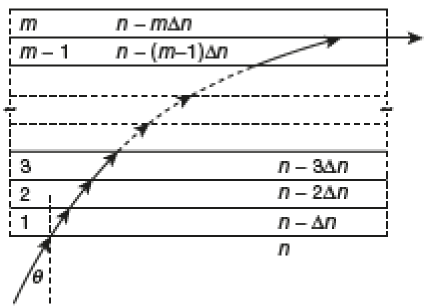
$$\frac{V_i}{V_f} = 6.25$$

$$\Rightarrow\ \frac{n_f^2}{n_i^2} = 6.25\ \Rightarrow\ \frac{n_f}{n_i} = \sqrt{6.25} = 2.5$$

$$\Rightarrow\ \frac{n_f}{n_i} = \frac{25}{10} = \frac{5}{2}$$

$$\Rightarrow\ n_i = 2 \text{ and } n_f = 5$$

10. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle θ = 30°. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and Δn = 0.1 (sec the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m?



10. Applying Snell's law, we have $n\sin\theta = (n - m\Delta n)\sin 90°$ it is given that n = 1.6, θ = 30°, Δn = 0.1. The above equation becomes

$$1.6\sin 30 = [1.6 - m(0.1)]\sin 90$$

$$1.6 \times \frac{1}{2} = (1.6 - 0.1m) \times 1$$

$$0.8 = (1.6 - 0.1m)$$

$$0.1m = 1.6 - 0.8$$

$$0.1m = 0.8$$

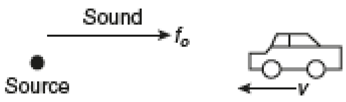
$$m = 8$$

11. A stationary source emits sound of frequency f_0 = 492 Hz. The sound is *reflected* by a large car *approaching* the source with a speed of 2 m s^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 m s^{-1} and the car reflects the sound at the frequency *it* has received).

11. It is given that the source emits sound of frequency f_0 = 492 Hz.

The car is approaching the source and the speed of car is v = 2 m/s.

Also, the speed of sound in air, v_s = 330 m/s.



The frequency of sound received by car is given as

$$f_1 = \left(\frac{V_s + V}{V_s}\right) f_0 = \left(\frac{330+2}{330}\right) 492$$

Here, f_1 = 494.98 Hz, which is the frequency reflected by the car towards the source.

Therefore, now, the car acts as the source. The frequency of sound received by the source is

$$f_2 = \left(\frac{V_s}{V_s - v}\right) f_1 = \left(\frac{330}{330-2}\right) 494.98$$

Here, f_2 = 498 Hz. Therefore, the beat frequency of the resulting signal is

$$|f_0 - f_2| = |492 - 498| = 6 \text{ Hz}$$

12. ^{131}I is an isotope of Iodine that β-decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4 × 10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 h, 2.5 mL of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in litres, is approximately (you may use $e^x \approx 1 + x$ for $|x| << 1$ and ln2 ≈ 0.7).

12. We know that the radioactivity is given by

$$A = A_0 e^{-\lambda t} \qquad (1)$$

Given: Half-life is $t_{1/2}$ = 8 days = 8 × 24 h.

Now, at t = 11.5 h, we have A = 115 Bq.

Rearranging Eq. (1), we get

$$A_0 = Ae^{\lambda t}$$

Using $e^x \approx 1 + x$ for $|x| \ll 1$, we get

$$A_0 = A(1 + \lambda t)$$

We know that $\lambda = \frac{\ln 2}{t_{1/2}}$; therefore,

$$A_0 = A\left[1 + \frac{\ln^2 t}{t_{1/2}}\right]$$

Substituting all values, we get

$$A_0 = 115\left[1 + \frac{0.7}{8 \times 24} \times 11.5\right] = 115(1 + 0.0419)$$

$$A_0 = 115 \times 1.0419 = 119.8185$$

$$A_0 \sim 120 \text{ Bq}$$

Thus, 2.5 mL of blood has 120 Bq activity.

It is given that the activity of amount of ^{131}I injected is 2.4×10^5 Bq.

Hence, the total volume of the blood in the person's body is

$$\frac{2.4\times10^5\times2.5\times10^{-3}}{120} = \frac{600}{120} = 5\text{ L}$$

Matrix-Match Type

This section contains 6 questions of matching type. There are two tables (each having 3 columns and 4 rows). Based on each table, there are 3 questions. Each question has 4 options (A), (B), (C) and (D), **ONLY ONE** of these four options is correct.

Directions for Qns. 13, 14 and 15: Answer the questions by appropriately matching the information given in the three columns of the following table:

A charged particle (electron or proton) is introduced at the origin ($x = 0, y = 0, z = 0$) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Column 1	Column 2	Column 3
(I) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV) Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

13. In which case will the particle move in a straight line with *constant* velocity?

(A) (III) (ii) (R) (B) (IV) (i) (S)
(C) (III) (iii) (P) (D) (II) (iii) (S)

13. **(D)** We know that the Lorentz's force (in vector form) is given by

$$\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B}) \quad (1)$$

For a particle to move in a straight line with constant velocity, $\vec{F} = 0$. Therefore, from Eq. (1), we get

$$q\vec{E} + q(\vec{V}\times\vec{B}) = 0$$

$$\Rightarrow q\vec{E} - q(\vec{v}\times\vec{B})$$

$$\Rightarrow \vec{E} = -(\vec{v}\times\vec{B})$$

For $\vec{v} = \frac{E_0}{B_0}\hat{y}$ and $\vec{B} = B_0\hat{z}$, we get

$$\vec{v}\times\vec{B} = \left(\frac{E_0}{B}\hat{y}\right)\times(B_0\hat{z}) = \frac{E_0}{B_0}\times B_0(\hat{y}\times z)$$

Therefore, $\vec{v}\times\vec{B} = E_0\hat{x}$ (as $\hat{y}\times\hat{z} = \hat{x}$)

and $\vec{E} = -E_0\hat{x}$

Therefore, $v = \frac{E_0}{B_0}\hat{y};\ \vec{E} = -E_0\hat{x};\ \vec{B} = B_0\hat{z}$

Hence, option (D) is correct.

14. In which case will the particle describe a helical path with axis along the positive x direction?

(A) (IV) (i) (S) (B) (II) (ii) (R)

(C) (III) (iii) (P) (D) (IV) (ii) (R)

14. **(A)** For a helical path along +z-direction, the magnetic field should also be in the same direction, that is, +z-direction.

Hence, the option (A) correct:

$$\vec{v} = \frac{2E_0}{B_0}\hat{x}\,;\ \vec{E} = E_0\hat{z}\,;\ \vec{B} = B_0\hat{z}$$

15. In which case would the particle move in a straight line along the negative direction of y-axis (i.e. move along $-\hat{y}$)?

(A) (II) (iii) (Q) (B) (III) (ii) (R)
(C) (IV) (ii) (S) (D) (III) (ii) (P)

15. **(B)** For particle to move in –y-direction, its

(i) velocity must be in –y-direction (however, the velocity of the particle is not in –y-direction in any of the given options)

(ii) net force $\vec{F}$ on the particle is along –y-direction and $\vec{v} = 0$.

Therefore,

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

$$\vec{v} = 0 \;\Rightarrow\; \vec{F} = q\vec{E}$$

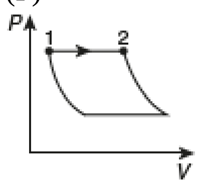
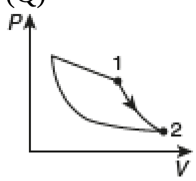
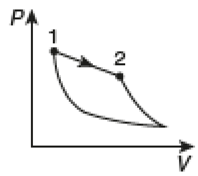
For F to be in –y-direction, $\vec{E}$ should also be in –y-direction. That is,

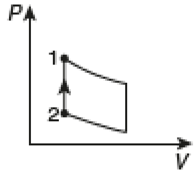
$$\vec{E} = -E_0\hat{y}$$

Therefore, option (B) is correct.

$$\vec{V} = 0;\ \vec{E} = -E_0\hat{y};\ \vec{B} = B_0\hat{y}$$

Directions for Qns. 16, 17 and 18: Answer the questions by appropriately matching the information given in the three columns of the following table:

<table>
<tr><td colspan="3">An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding $P – V$ diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equation and plots in the table have standard notations as used in thermodynamic process. Here, γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.</td></tr>
<tr><th>Column 1</th><th>Column 2</th><th>Column 3</th></tr>
<tr><td>(I) $W_{1\to 2} = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$</td><td>(i) Isothermal</td><td>(P)

</td></tr>
<tr><td>(II) $W_{1\to 2} = -P_2V_2 + PV_1$</td><td>(ii) Isochoric</td><td>(Q)

</td></tr>
<tr><td>(III) $W_{1\to 2} = 0$</td><td>(iii) Isobaric</td><td>(R)

</td></tr>
</table>

(IV) $W_{1\to 2} = -nRT\ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

16. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(A) (II) (iv) (R) (B) (III) (iii) (P)

(C) (II) (iii) (S) (D) (II) (iii) (P)

16. **(D)** According to first law of thermodynamics, we have the change in the internal energy of the system as

$$\Delta U = \Delta Q - P\Delta V$$

From all possible combinations from the given columns, the following cases satisfy this equation.

- (I) $\omega_{1\to 2} = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$

  (iv) Adiabatic ⟶ (Q)

  This refers to (i) (iv) (Q), which does not exist among the given options.
- (ii) $\omega_{1\to 2} = -PV_2 + PV_1$

  (iii) Isobaric ⟶ (P)

  This refers to (ii) (iii) (P), which is option (D).
- (iii) $\omega_{1\to 2} = 0$

  (ii) Isochoric ⟶ (S)

  That is, (iii) (ii) (S) ⟶ which does not exist among the given options.
- (iv) $\omega_{1\to 2} = -nRT\ln\left(\frac{V_2}{V_1}\right)$

  (i) Isothermal ⟶ (R)

  This reefers to (iv) (i) (R) ⟶ which does not exist among the given options.

Hence, the option (D) is correct.

17. Which one of the following options is the correct combination?

(A) (IV) (ii) (S) (B) (III) (ii) (S)

(C) (II) (iv) (P) (D) (II) (iv) (R)

17. **(B)** Following the solution of Qn. 16, the combination (iii) (ii) (S) is correct.

Therefore, option (B) is correct.

18. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

(A) (I) (ii) (Q) (B) (IV) (ii) (R)

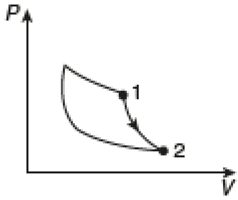
(C) (III) (iv) (R) (D) (I) (iv) (Q)

18. **(D)** For determination of the speed of sound in an ideal gas, Laplace correction is used:

$$C = \sqrt{\frac{\gamma P}{\delta}}$$

In this, the process assumed is adiabatic. Therefore,

$$\omega = \frac{P_2V_2 - P_1V_1}{\gamma - 1} \Rightarrow \text{(i)(iv)(Q)}$$



Therefore, the correct option is (D).

PAPER 2 – PHYSICS

Single Option Correct Type

This section contains 7 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the *instantaneous* density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to

(A) R (B) R^3

(C) $\frac{1}{R}$ (D) $R^{2/3}$

1. **(A)** We know that the density is expressed as

$$\rho = \frac{\text{Mass } (m)}{\text{Volume } (V)}$$

Also, we have

$$\text{Volume of sphere} = \frac{4}{3}\pi R^3$$

Using this, we have the density as

$$\rho = \frac{m}{(4/3)\pi R^3} = \frac{3m}{4\pi R^3}$$

Rearranging this equation, we get

$$\rho R^3 = \frac{3}{4\pi}\text{m}$$

It is given that the mass m remains constant and let the constant be k. Therefore.

$$\rho R^3 = k$$

Differentiating it w.r.t. time, we get

$$\frac{d}{dt}(\rho R^3) = 0 \qquad \text{(as differentiation of constant is zero)}$$

$$R^3\frac{d\rho}{dt} + 3R^2\rho\frac{dR}{dt} = 0$$

Now, $\frac{dR}{dt}$ (i.e. rate of change of radius) equals the velocity v; therefore,

$$R^3\frac{d\rho}{dt} + 3R^2\rho V = 0$$

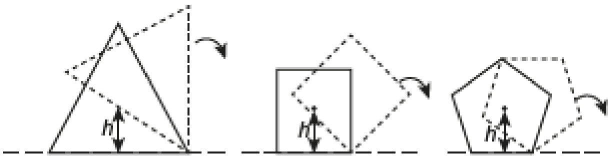
$$\Rightarrow\ 3R^2\rho V = -R^3\frac{d\rho}{dt}$$

$$\Rightarrow V = \frac{-1}{3}\frac{R^3}{R^2}\frac{1}{\rho}\frac{d\rho}{dt} = \frac{-1R}{3}\frac{1}{\rho}\frac{d\rho}{dt}$$

It is given that the rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant; hence, $V \propto R$.

Hence, the velocity *v* of any point on the surface of the expanding sphere is proportional to *R*.

2. Consider regular polygons with number of sides *n* = 3, 4, 5, … as shown in the figure. The centre of mass of all the polygons is at height *h* from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the centre of mass for each polygon is Δ. Then, Δ depends on *n* and *h* as



(A) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (B) $\Delta = h\left[\frac{1}{\cos(\pi/n)} - 1\right]$

(C) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ (D) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

2. **(B)** The centre of mass of all polygons is at height *h* from the ground.

For a polygon of side *n*, we have

$$\cos\left(\frac{\pi}{n}\right) = \frac{h}{H}$$

where *H* is the maximum height of the centre of mass. Then,

$$H = \frac{h}{\cos(\pi/n)}$$

The maximum increase in height of locus of centre of mass is

$$\Delta = H - h = \frac{h}{\cos(\pi/n)} - h = h\left[\frac{1}{\cos(\pi/n)} - 1\right] = h\left[\frac{1}{\cos(\pi/n)} - 1\right]$$

3. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda\left(\lambda < \frac{hc}{\phi_0}\right)$.

The fastest photoelectron has a de Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then, the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

(A) λ_d/λ (B) λ_d^2/λ^2

(C) λ_d^3/λ (D) λ_d^3/λ^2

3. **(D)** According to photoelectric effect, we have

$$\frac{hc}{\lambda} = \phi_0 + \text{K.E.}$$

We know that K.E. of electron is

$$\frac{1}{2}mV^2 = \frac{p^2}{2m}$$

where *m* is the mass of electron. For the fastest photoelectron, the de Broglie wavelength is λ_d. Therefore,

$$\text{K.E.} = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{\lambda_d}\right)^2 \qquad \left(p = \frac{h}{\lambda}\right)$$

Therefore, $$\frac{hc}{\lambda} = \phi_0 + \frac{h^2}{2m\lambda_d^2}$$

Differentiating the above equation, we get

$$d\left(\frac{hc}{\lambda}\right) = d(\phi_0) + d\left(\frac{h^2}{2m\lambda_d^2}\right)$$

$$\Rightarrow \; hc\left(\frac{-1}{\lambda^2}d\lambda\right) = 0 + \frac{h^2}{2m}\left(\frac{-2}{\lambda d^3}\right)d\lambda_d$$

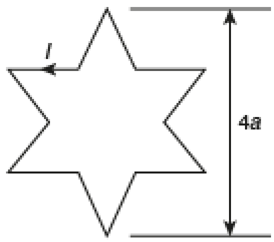
$$\Rightarrow \; \frac{hc}{\lambda^2}d\lambda = \frac{h^2}{m\,\lambda^3 d}d\lambda_d$$

Rearranging this equation, we get

$$\frac{d\lambda_d}{d\lambda} = \frac{m\lambda d_d^3}{h^2} \times \frac{hc}{\lambda^2} = \frac{mc\lambda_d^3}{h\lambda^2}$$

$$\Rightarrow \frac{d\lambda_d}{d\lambda} \propto \frac{\lambda_d^3}{\lambda^2} \Rightarrow \frac{\Delta\lambda_d}{\Delta\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$$

4. A symmetric star-shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the centre of the loop is

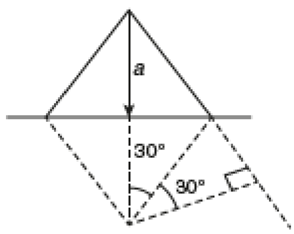


(A) $\frac{\mu_0 I}{4\pi a}6(\sqrt{3}-1)$ (B) $\frac{\mu_0 I}{4\pi a}6(\sqrt{3}+1)$

(C) $\frac{\mu_0 I}{4\pi a}3(\sqrt{3}-1)$ (D) $\frac{\mu_0 I}{4\pi a}3(2-\sqrt{3})$

4. **(A)** The star shape is composed of 12 wires. Thus, the total magnetic field at centre is 12 times of magnetic field due to one wire.

Considering one section of the star-shaped wire, the magnetic field due to this section is as depicted in the following figure:



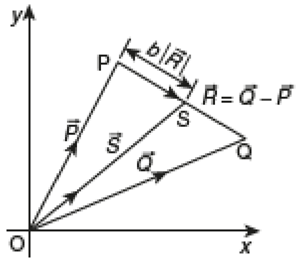
Now, the magnetic field is

$$B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin 60° - \sin 30°) = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{\sqrt{3}}{9} - \frac{1}{2}\right) = \frac{\mu_0}{4\pi}\frac{I}{2a}(\sqrt{3}-1)$$

Therefore, the total magnetic field at the centre of the loop is

$$12B = 12\left[\frac{\mu_0}{4\pi}\frac{I}{2a}(\sqrt{3}-1)\right] = \frac{\mu_0}{4\pi}\frac{I}{a}6(\sqrt{3}-1)$$

5. Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is



(A) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$ (B) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

(C) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$ (D) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$

5. **(A)** From the given figure in the question, we can write as

$$\vec{P} = b\vec{R} = \vec{S}$$

It is given that $\vec{R} = \vec{Q} - \vec{P}$ and substituting this in the above equation, we get

$$\vec{P} + b(\vec{Q} - \vec{P}) = \vec{S}$$

$$\Rightarrow \vec{P} + b\vec{Q} - b\vec{P} = \vec{S}$$

$$\Rightarrow \vec{S} = \vec{P}(1-\vec{b}) + b\vec{Q}$$

6. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the *Sun–Earth system* is closest to (*Ignore the rotation and revolution of the Earth and the presence of any other planet*)

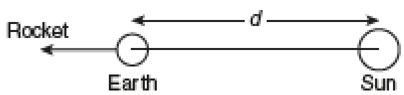
(A) $v_s = 22$ km s^{-1} (B) $v_s = 42$ km s^{-1}

(C) $v_s = 62$ km s^{-1} (D) $v_s = 72$ km s^{-1}

6. **(B)** Let m be the mass of Earth.

Then mass of Sum is 3×10^5 m.

Let R be radius of Earth, then $d = (2.5 \times 10^4)R$.



From law of conservation of energy, we have the following:

Loss in K.E. = Gain in P.E.

$$\Rightarrow \frac{1}{2}MV_s^2 = \frac{GmM}{R} + \frac{G(3\times10^5 m)M}{(2.5\times10^4)R}$$

where M is the mass of the rocket; therefore,

$$\frac{1}{2}mv_s^2 = \frac{GmM}{R}\left(1+\frac{3\times10^5}{2.5\times10^4}\right) = \frac{GmM}{R}(1+12)$$

$$\Rightarrow \frac{1}{2}V_s^2 = \frac{Gm}{R}13 \Rightarrow V_s^2 = 2\times13\frac{Gm}{R}$$

$$\Rightarrow V_s = \sqrt{2\times13\frac{Gm}{R}}$$

It is given that the escape velocity is $v_e = 11.2$ km/s.

We know that $v_e = \sqrt{\frac{2Gm}{R}}$. Therefore,

$$v_{ss} = \sqrt{13}v_e = \sqrt{13} \times 11.2 \text{ km/s} = 40.38 \text{ km/s} \simeq 42 \text{ km/s}$$

7. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be $L = 20$ meters. Take the acceleration due to gravity $g = 10$ ms^2 and the velocity of sound is 300 ms^{-1}. Then, the fractional error in the measurement, $\delta L/L$, is closest to

(A) 0.2% (B) 1%
(C) 3% (D) 5%

7. **(B)** Let the time taken by the stone to reach the bottom of well be t_1 and the time taken by sound to reach the observer be t_2, then

$$t_1 = \sqrt{\frac{2L}{g}} \quad (1)$$

and

$$t_2 = \frac{L}{v_s} \quad (2)$$

where L is the depth of well, g is acceleration due to gravity and v_s is the velocity of sound.

The total time taken is

$$T = t_1 + t_2$$

$$T = \frac{\sqrt{2L}}{g} + \frac{L}{v_s} \quad (3)$$

If δL is error in measurement of the depth of the well and δT is the error in the measurement of time, then

$$T + \delta T = \sqrt{\frac{2(L+\delta L)}{g}} + \frac{(L+\delta L)}{v_s}$$

$$= \left(1 + \frac{\delta L}{L}\right)^{1/2} + \frac{L}{v_s}\left(1 + \frac{\delta L}{L}\right)$$

Expanding $\left(1 + \frac{\delta L}{L}\right)^{1/2}$ using binomial approximation, we get

$$T + \delta T = \sqrt{\frac{2L}{g}}\left(1 + \frac{1}{2}\frac{\delta L}{L}\right) + \frac{L}{v_s}\left(1 + \frac{\delta L}{L}\right) = \sqrt{\frac{2L}{g}} + \sqrt{\frac{2L}{g}}\left(\frac{1}{2}\frac{\delta L}{L}\right) + \frac{L}{v_s} + \frac{L}{v_s}\frac{\delta L}{L}$$

$$T + \delta T = \sqrt{\frac{2L}{g}} + \frac{L}{v_s} + \left(\frac{1}{2}\sqrt{\frac{2L}{g}} + \frac{L}{v_s}\right)\frac{\delta L}{L} \quad (4)$$

Given: $L = 20$ m; $g = 10$ m/s; $v_s = 300$ m/s.

Using Eq. (3) in Eq. (4), we get

$$T + \delta T = T + \left(\frac{1}{2}\sqrt{\frac{2 \times 20}{10}} + \frac{20}{300}\right)\frac{\delta L}{L}$$

$$\Rightarrow \delta T = \left(\frac{1}{2}\sqrt{4} + \frac{1}{15}\right)\frac{\delta L}{L} = \left(1 + \frac{1}{15}\right)\frac{\delta L}{L} = \left(\frac{16}{15}\right)\frac{\delta L}{L}$$

$$\Rightarrow \frac{\delta L}{L} = \delta T\left(\frac{15}{16}\right)$$

It is given that $\delta T = 0.01$ s.

$$\left(\frac{\delta L}{L}\right)=\frac{15}{16}\times 0.01=\frac{15}{16}\times\frac{1}{100}$$

Therefore,

$$\left(\frac{\delta L}{L}\right)\times 100\%=\frac{15}{16}\times\frac{1}{100}\times 100\%=\frac{15}{16}\%$$

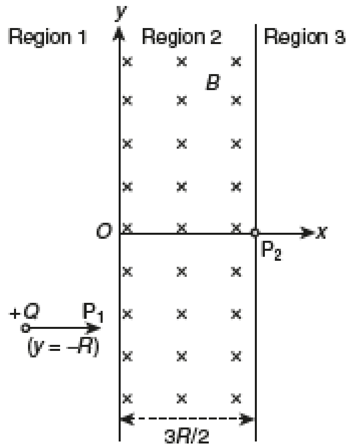
Hence, the fractional error in the measurement is

$$\frac{\delta L}{L}=1\%$$

One or More Than One Option Correct Type

This section contains 7 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) out of which **ONE OR MORE** is(are) correct.

8. A uniform magnetic field B exists in the region between $x = 0$ and $x=\frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along x-axis enters region 2 from region 1 at point $P_1(y = -R)$. Which of the following option(s) is/are correct?



(A) For $B>\frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1.

(B) For $B=\frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis.

(C) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest, point from y-axis is $p/\sqrt{2}$.

(D) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.

8. **(A), (B)** We discuss the options as follows:

- **For option (A):** For $B>\frac{2}{3}\frac{p}{QR}$, the radius is $\frac{3}{2}R>\frac{p}{QB}$.

  Thus, the particle does not enter in region 3 and the particle re-enters region 1.

- **For option (B):** For $B=\frac{8}{13}\frac{p}{QR}$, the radius is $\frac{13}{8}R=\frac{p}{QB}$.

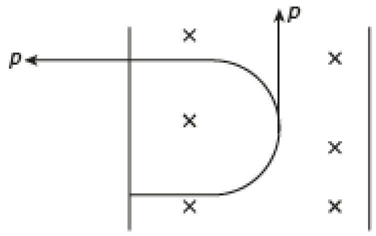
  Using Pythagoras theorem, we get

$$x=\sqrt{\left(\frac{13}{8}R\right)^2-\left(\frac{5}{8}R\right)^2}=R\sqrt{\left(\frac{13}{8}\right)^2-\left(\frac{5}{8}\right)^2}$$

$$= R\sqrt{\left(\frac{13}{8}-\frac{5}{8}\right)\left(\frac{13}{8}+\frac{5}{8}\right)} = R\sqrt{\left(\frac{8}{8}\right)\left(\frac{18}{8}\right)} = R\sqrt{\frac{9}{4}} = \frac{3}{2}R$$

Thus, the particle enters region 3 through point P_2 on x-axis.

- **For option (C):** When particle re-enters region 1 through the longest path in region 2, it re-enters horizontally only.



At the farthest point from y-axis, the momentum p is directed upwards. Therefore, the change in linear momentum is $\sqrt{2}p$.

- **For option (D):** For fixed B, if the particle has same charge Q and same velocity v.
The distance between point p_1 and point of re-entry two region 1 is $2R$.
Therefore, $R = \frac{mv}{QB} \Rightarrow R \propto m.$ Thus, the distance between point P_1 and point of re-entry into region 1 is directly proportional to m.

Hence, only options (A) and (B) are correct.

9. The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_X = V_0 \sin \omega t$$

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and}$$

$$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right).$$

An ideal voltmeter is configured to read *rms* value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

(A) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$.

(B) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$.

(C) $V_{XY}^{rms} = V_0$.

(D) independent of the choice of the two terminals.

9. **(A), (D)** It is given that

$$V_X = V_0 \sin \omega t$$

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

- Potential difference between X and Y is $V_X - V_Y = V_{XY}$
- Potential difference between Y and Z is $V_Y - V_Z = V_{YZ}$
- Potential difference between X and Z is $V_X - V_Z = V_{XZ}$

Now, $$V_{XY} = V_0 \sin \omega t - V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$$

Using $\sin(a + b) = \sin a \cos b + \cos a \sin b$, we get

$$V_{XY} = V_0 \sin\omega t - \cos\frac{2\pi}{3} - V_0 \cos\omega t \sin\frac{2\pi}{3}$$

$$= V_0 \sin\omega t - V_0 \sin\omega t\left(\frac{-1}{2}\right) - V_0 \cos\omega t\left(\frac{\sqrt{3}}{2}\right)$$

$$= V_0 \sin\omega t + \frac{1}{2}V_0 \sin\omega t - \frac{\sqrt{3}}{2}V_0 \cos\omega t$$

$$= \frac{3}{2}V_0 \sin\omega t - \frac{\sqrt{3}}{2}V_0 \cos\omega t = \sqrt{3}V_0\left(\frac{\sqrt{3}}{2}\sin\omega t - \frac{1}{2}\cos\omega t\right)$$

$$= \sqrt{3}V_0\left(\sin\omega t \cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\omega t\right)$$

$$= \sqrt{3}V_0 \sin\left(\omega t - \frac{\pi}{6}\right)$$

Therefore, $$V_{XY}^{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}} = \sqrt{\frac{3}{2}}V_0$$

Hence, option (A) is correct and option (C) is incorrect.

Similarly, $$V_{YZ} = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) - V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

$$= V_0\left(\sin\omega t \cos\frac{2\pi}{3} + \cos\omega t \sin\frac{2\pi}{3}\right) - V_0\left(\sin\omega t \cos\frac{4\pi}{3} + \cos\omega t \sin\frac{4\pi}{3}\right)$$

$$= V_0\left[\sin\omega t\left(\frac{-1}{2}\right) + \cos\omega t\left(\frac{\sqrt{3}}{2}\right) - \sin\omega t\left(\frac{-1}{2}\right) - \cos\omega t\left(\frac{-\sqrt{3}}{2}\right)\right]$$

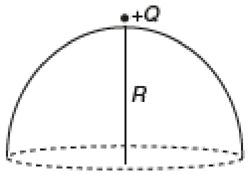
$$= V_0\left[\frac{-1}{2}\sin\omega t + \frac{\sqrt{3}}{2}\cos\omega t + \frac{1}{2}\sin\omega t + \frac{\sqrt{3}}{2}\cos\omega t\right]$$

$$= \sqrt{3}V_0 \cos\omega t$$

Therefore, $$V_{YZ}^{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}} = \sqrt{\frac{3}{2}}V_0$$

Hence, option (B) is incorrect.
Therefore, the voltmeter reading is the same and it is independent of the choice of the two terminals.
Hence, option (D) is correct.

10. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statement is/are correct?

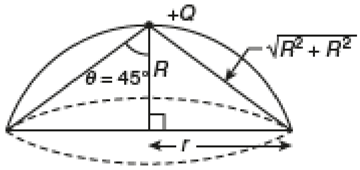


(A) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$.

(B) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$.

(C) The component of the electric field normal to the flat surface is constant over the surface.
(D) The circumference of the flat surface is an equipotential.

10. **(A), (D)** Since charge Q is outside the hemispherical surface, the net flux passing through the curved surface of hemispherical surface and flat surface is zero.



Therefore, $$\phi_{\text{curved}} + \phi_{\text{flat}} = 0 \quad (1)$$

Hence, option (B) is incorrect.

Now, $$\phi_{\text{flat}} = \int \vec{E}\cdot d\vec{A} = \int E dA\cos\theta$$

and $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(\sqrt{R^2+r^2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(R^2+r^2)}$$

Also, $$\cos\theta = \frac{R}{\sqrt{R^2+r^2}}$$

and $$A = 2\pi r \Rightarrow dA = 2\pi r dr$$

Therefore, $$\phi_{\text{flat}} = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2+r^2}2\pi r dr\frac{R}{\sqrt{R^2+r^2}}$$

$$\phi_{\text{flat}} = \frac{QR}{2\varepsilon_0}\int_0^R \frac{rdr}{(R^2+r^2)^{3/2}}$$

Substituting $R^2 + r^2 = t$, we get

$$2rdr = dt$$

$$\Rightarrow \phi_{\text{flat}} = \frac{QR}{2\varepsilon_0}\int_0^R \frac{dt}{2}\frac{1}{t^{3/2}} = \frac{QR}{2\varepsilon_0}\left[\frac{1}{2}\frac{t^{-1/2}}{-1/2}\right]_0^R$$

Substituting $t = R^2 + r^2$, we get

$$\phi_{\text{flat}} = \frac{QR}{2\varepsilon_0}\left[\frac{1}{2}\frac{(R^2+r^2)^{-1/2}}{-1/2}\right]_0^R$$

$$= \frac{QR}{2\varepsilon_0}\left[\frac{-1}{\sqrt{R^2+r^2}}\right]_0^R = \frac{QR}{2\varepsilon_0}\left(\frac{-1}{\sqrt{R^2+R^2}} + \frac{1}{\sqrt{R^2}}\right)$$

$$= \frac{QR}{2\varepsilon_0}\left(\frac{-1}{\sqrt{2R^2}} + \frac{1}{\sqrt{R^2}}\right) = \frac{QR}{2\varepsilon_0}\left(\frac{-1}{\sqrt{2}R} + \frac{1}{R}\right)$$

$$= \frac{QR}{2\varepsilon_0}\frac{1}{R}\left(\frac{-1}{\sqrt{2}} + 1\right) = \frac{Q}{2\varepsilon_0}\left(\frac{-1}{\sqrt{2}} + 1\right)$$

Using Eq. (1), we get

$$\phi_{\text{curved}} = -\phi_{\text{flat}} = -\frac{Q}{2\varepsilon_0}\left(\frac{-1}{\sqrt{2}} + 1\right) = -\frac{Q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

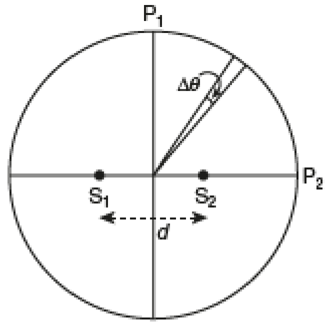
Hence, option (A) is correct.

The potential at any point on the circumference of the flat surface is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}R}$.

Thus, the circumference of flat surface is equipotential

Hence, option (D) is correct.

11. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



11. **(B), (C)** Given: λ = 600 nm; distance between sources, d = 1.8 mm.

- At point P_1, $\Delta x = 0$, where Δx is path difference and at point P_2, Δx = 1.8 mm.

  We know that $\Delta x = n\lambda$, where n is the number of fringes:

$$n = \frac{\Delta x}{\lambda} = \frac{1.8\text{ mm}}{600\text{ nm}} = \frac{1.8\times10^{-3}}{600\times10^{-9}}$$

$$n = \frac{18\times10^{-3}}{6\times10^{3}\times10^{-9}} = \frac{3\times10^{-3}}{10^{-6}} = 3\times10^{3} = 3000$$

  Hence, option (C) is correct.

- Since at point P_2, $\Delta x = 3000\lambda$, the bright fringe is formed, that is, the order of fringe is maximum.

  Hence, option (B) is correct and option (A) is incorrect.

- Now, we know that

$$\Delta x = d\cos\theta = n\lambda \Rightarrow \cos\theta\left(\frac{n\lambda}{d}\right)$$

  Differentiating this, we get

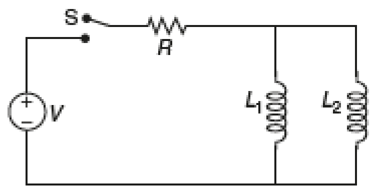
$$-\sin\theta\,\Delta\theta = (\Delta n)\frac{\lambda}{d} \Rightarrow \Delta\theta = \frac{-(\Delta n)\lambda}{d\sin\theta}$$

  Thus, as θ increases, $\Delta\theta$ decreases and with an increase in θ, Δx also increases, which results in the formation of fringes close to each other.

  Hence, the angular separation between two consecutive bright spots increases as we move from point P_1 to point P_2 along the first quadrant.

  Hence, option (D) is incorrect.

12. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?



(A) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$.

(B) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$.

(C) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$).

(D) At $t = 0$, the current through the resistance R is $\frac{V}{R}$.

12. **(A), (B), (C)** We discuss the correct options as follows:

- For inductors in parallel, voltage across the inductors is the same. Thus, we have

$$V_{L_1} = V_{L_2}$$

We know that $$V = \frac{LdI}{dt}$$

Therefore, $$\frac{L_1 dI_1}{dt} = \frac{L_2 dI_2}{dt}$$

where I_1 and I_2 are the current across inductor L_1 and L_2, respectively. Therefore,

$$\int_0^t d(L_1 I_1) = \int_0^t d(L_2 I_2) \Rightarrow I_1 L_1 = I_2 L_2 \Rightarrow \frac{I_1}{I_2} = \frac{L_2}{L_1}$$

Hence, option (C) is correct.

- The current along the resistor R at any time t is given by

$$I = \frac{V}{R}(1 - e^{Rt/L})$$

where $L = \frac{L_1 L_2}{L_1 + L_2}$. After a long time, the $I = \frac{V}{R}$.

We know that $I_1 L_1 = I_2 L_2$ and $I_1 + I_2 = I$.

On solving, we get

$$I_1 = \frac{I_2 L_2}{L_1} = (I - I_2) = \frac{L_2 I_2}{L_1} \quad \text{(since } I_1 + I_2 = I \text{ and } I_1 = I - I_2\text{)}$$

$$\Rightarrow I = I_2 + I_2 \frac{L_2}{L_1} \Rightarrow I = I_2\left(1 + \frac{L_2}{L_1}\right) = I_2\left(\frac{L_1 + L_2}{L_1}\right)$$

$$\Rightarrow I_2 = \frac{IL_1}{L_1 + L_2} = \frac{V}{R}\frac{L_1}{L_1 + L_2}$$

Thus, the current passing through L_2 is

$$I_2 = \frac{V}{R}\frac{L_1}{L_1 + L_2}$$

Hence, option (B) is correct.

- Similarly, the current passing through L_1 is

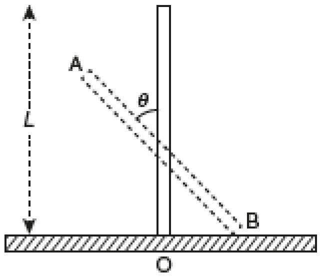
$$I_1 = \frac{V}{R}\frac{L_2}{L_1 + L_2}$$

Hence, option (A) is correct.

At $t = 0$, current $I = 0$.

Hence, option (D) is incorrect.

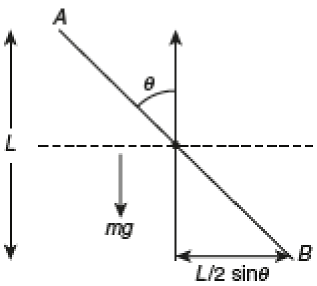
13. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?



(A) The midpoint of the bar will fall vertically downward.
(B) The trajectory of the point A is a parabola.
(C) Instantaneous torque about the point in contact with the floor is proportional to sinθ.
(D) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to (1 – cosθ).

13. **(A), (C), (D)** We discuss the options as follows:

- When the bar makes an angle θ with the floor, the height of midpoint is $\frac{L}{2}\cos\theta$. This is height of centre of mass.
  A force *mg* acts vertically downwards. Thus, the midpoint of bar falls vertically downwards.
  Hence, option (A) is correct.



- Here, we have $x = \frac{L}{2}\sin\theta$ and $y = L\cos\theta$; therefore,

$$\sin\theta = \frac{x}{L/2} \text{ and } \cos\theta = \frac{y}{L}$$

  Using the condition $\sin^2\theta + \cos^2\theta = 1$, we get

$$\frac{x^2}{(L/2)^2} + \frac{y^2}{L^2} = 1 \qquad \text{(equation of ellipse)}$$

  Thus, the trajectory of point A is not parabola.
  Hence, option (B) is incorrect.
- The torque acting about the point of contact with floor, that is, at point B is

$$\tau = \vec{F} \times \vec{r} = mg \times \frac{L}{2}\sin\theta$$

  Thus, the torque is proportional to sinθ.
  Hence, option (C) is correct.
- The displacement of midpoint is

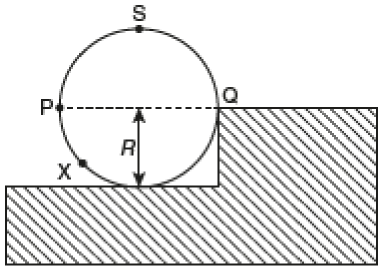
$$\frac{L}{2} - \frac{L}{2}\cos\theta = \frac{L}{2}(1 - \cos\theta)$$

  That is,

$$\text{Displacement} \propto (1 - \cos\theta)$$

  Hence, option (D) is correct.

14. A wheel of radius *R* and mass *M* is placed at the bottom of a fixed step of height *R* as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following option(s) is/are correct?



(A) If the force is applied at point P tangentially, then τ decreases continuously as the wheel climbs.
(B) If the force is applied normal to the circumference at point X, then τ is constant.
(C) If the force is applied normal to the circumference at point P, then τ is zero.
(D) If the force is applied tangentially at point S, then $r \neq 0$ but the wheel never climbs the step.

14. **(C), (D)** We discuss the options as follows:

- If the force is applied normal to the circumference at point X, then the torque is
$$\tau = F \times R = FR\sin\theta$$
Therefore, torque depends on θ and it is not constant.
Hence, option (B) is incorrect.
- If the force is applied normal to the circumference at point P, then
$$\tau = F \times R = FR\sin\theta$$
At point P, the angle between F and R is 0°; therefore,
$$\sin\theta = \sin 0° = 0 \Rightarrow \tau = 0$$
Hence, option (C) is correct.
- If force is applied tangentially at point P, we have the torque as
$$\tau = 2FR\cos\theta - mgR\cos\theta$$
$$\tau \propto \cos\theta$$
Thus, the torque is proportional to θ. If θ increases, τ increases. However, as there is constant force is applied, even though the force is applied at point P tangentially, τ increases as wheel climbs the step.
Hence, option (A) is incorrect.
- If force is applied tangentially at point S, then $\tau \neq 0$. Therefore,
$$\tau = 2FR\sin\theta$$
Torque decreases with increase in θ and the wheel does not climb the step.
Hence, option (D) is correct.

Paragraph Type

This section contains 2 paragraphs. Based on each paragraph, there are 2 questions. Each question has 4 options (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 15 and 16: Consider a simple RC circuit as shown in Figure 1.

- **Process 1:** In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time $T \gg RC$). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.
- **Process 2:** In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T \gg RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T \gg RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.
These two processes are depicted in Figure 2.

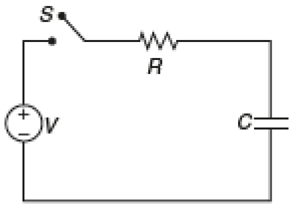


Figure 1

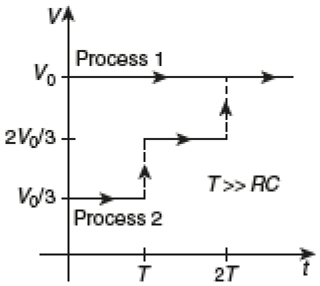


Figure 2

15. In process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by:

(A) $E_C = E_D$ (B) $E_C = E_D \ln 2$

(C) $E_C = \frac{1}{2}E_D$ (D) $E_C = 2E_D$

15. **(A)** The energy supplied to the circuit is CV_0^2.

The energy stored in capacitor is $E_C = \frac{1}{2}CV_0^2$.

Therefore, the energy dissipated is

$$E_0 = \text{Energy supplied} - \text{Energy stored}$$

$$= CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2$$

$$\Rightarrow E_C = E_D$$

Hence, option (A) is correct.

16. In Process 2, total energy dissipated across the resistance E_D is

(A) $E_D = \frac{1}{2}CV_0^2$ (B) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$

(C) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (D) $E_D = 3CV_0^2$

16. **(C)** We discuss the three different processes as follows:

- **Process 1:** Voltage is set to $V_0/3$.

$$\text{Charge supplied} = \frac{CV_0}{3}$$

$$\text{Energy supplied} = \frac{V_0}{3} \times \frac{CV_0}{3} = \frac{CV_0^2}{9}$$

- **Process 2:** Voltage is raised to $2V_0/3$.

$$\text{Additional charge supplied} = \frac{2V_0C}{3} - \frac{V_0C}{3} = \frac{CV_0}{3}$$

$$\text{Energy supplied} = \frac{2V_0}{3} \times \frac{CV_0}{3} = \frac{2CV_0^2}{9}$$

- **Process 3:** Voltage is raised to V_0.

$$\text{Additional charge supplied} = V_0C - \frac{2V_0C}{3} = \frac{CV_0}{3}$$

$$\text{Energy supplied} = V_0 \times \frac{CV_0}{3} = \frac{CV_0^2}{3}$$

Therefore, the total energy supplied to circuit is

$$\frac{CV_0^2}{9}+\frac{2CV_0^2}{9}+\frac{CV_0^2}{3}=\frac{6}{9}CV_0^2=\frac{2}{3}CV_0^2$$

The final energy stored in the capacitor is

$$\frac{1}{2}CV_0^2$$

Therefore, the energy dissipated is

$$E_D = \text{Energy supplied} - \text{Energy stored}$$

$$=\frac{2}{3}CV_0^2-\frac{1}{2}CV_0^2$$

$$=\frac{4CV_0^2-3CV_0^2}{6}=\frac{1}{6}CV_0^2$$

$$=\frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$$

Hence, option (C) is correct.

Paragraph for Questions 17 and 18: One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the right and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring *rolls without slipping* on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

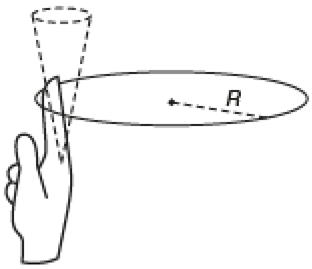


Figure 1

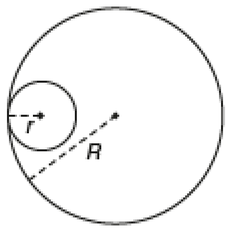


Figure 2

17. The total kinetic energy of the ring is

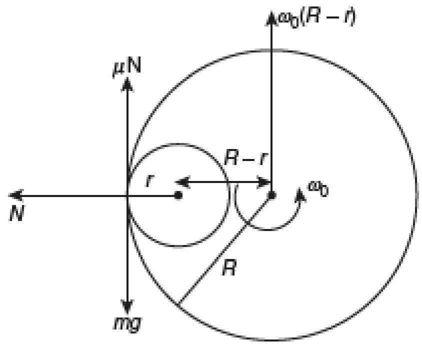
(A) $M\omega_0^2R^2$ (B) $\frac{1}{2}M\omega_0^2(R-r)^2$

(C) $M\omega_0^2(R-r)^2$ (D) $\frac{3}{2}M\omega_0^2(R-r)^2$

17. **(C)** We have

$$\omega R=\omega_0(R-r)$$

$$\omega=\frac{R-r}{R}\omega_0$$



Now, the total kinetic energy of the ring is

$$\text{K.E.} = \frac{1}{2}(2MR^2)\omega^2$$
$$= \frac{1}{2}(2MR^2)\frac{(R-r)^2}{R^2}\omega_0^2$$
$$= M\omega_0^2(R-r)^2$$

Hence, option (C) is correct.

18. The minimum value of ω_0 below which the ring will drop down is

(A) $\sqrt{\dfrac{g}{\mu(R-r)}}$ (B) $\sqrt{\dfrac{2g}{\mu(R-r)}}$

(C) $\sqrt{\dfrac{3g}{2\mu(R-r)}}$ (D) $\sqrt{\dfrac{g}{2\mu(R-r)}}$

18. **(A)** From the figure shown in the Solution of Qn. Q17, we have

$$\mu N = mg$$

We know that

$$N = m\omega_0^2(R-r)$$

Therefore,

$$\mu m\omega_0^2(R-r) = mg$$
$$\Rightarrow \omega_0^2 = \frac{g}{\mu(R-r)}$$
$$\Rightarrow \omega_0 = \sqrt{\frac{g}{\mu(R-r)}}$$

Hence, the correct option is (A).