JEE (ADVANCED) 2018 PAPER 1
PART–I PHYSICS

One or More than One Option Correct Type

This section contains **SIX (06)** questions. Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).

Chapter: System of Particles and Rotational Motion
Topic: Angular Momentum in Case of Rotations about a Fixed Axis

1. The potential energy of a particle of mass *m* at a distance *r* from a fixed point O is given by $V(r) = kr^2/2$, where *k* is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius *R* about the point O. If *v* is the speed of the particle and *L* is the magnitude of its angular momentum about O, which of the following statements is (are) true?

(A) $v = \sqrt{\frac{k}{2m}}R$

(B) $v = \sqrt{\frac{k}{m}}R$

(C) $L = \sqrt{mk}\,R^2$

(D) $L = \sqrt{\frac{mk}{2}}\,R^2$

(JEE Advanced 2018 Paper-1)

Solution

(B), (C) Given: $V(r) = \frac{kr^2}{2}$; where *k* is positive constant, *R* is radius of circular orbit, *V* is potential, *v* is speed of particle, *L* is angular momentum.

We know that $F = \frac{-dV}{dr} = \frac{-d}{dr}\left(\frac{kr^2}{2}\right) = -kr$

Therefore, force on particle is $F = -kr$ towards centre. Also, centripetal force on particle is $\frac{mv^2}{R}$. Thus,

$$+kr = \frac{mv^2}{R}$$

At $r = R$,

$$+kR = \frac{mv^2}{R}$$

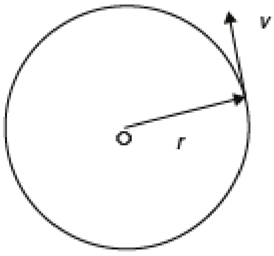
Rearranging it, we get

$$v^2 = \frac{+kR^2}{m} \Rightarrow v = \sqrt{\frac{k}{m}}R$$

Hence, option B is correct.
Also, we know $L = MvR$
Substitute the value of *v*, we get

$$L = m\sqrt{\frac{k}{m}}R \cdot R = \sqrt{mk}R^2$$

Hence, option C is correct.



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Chapter: System of Particles and Rotational Motion
Topic: Torque and Angular Momentum

2. Consider a body of mass 1.0 kg at rest at the origin at time $t = 0$. A force $\vec{F} = (\alpha t\,\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0$ N s^{-1} and $\beta = 1.0$ N. The torque acting on the body about the origin at time $t = 1.0$ s is $\vec{\tau}$. Which of the following statements is (are) true?

(A) $|\vec{\tau}| = \frac{1}{3}$ N m.

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\vec{k}$.

(C) The velocity of the body at $t = 1$ s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})$ m s^{-1}.

(D) The magnitude of displacement of the body at $t = 1$ s is $\frac{1}{6}$ m.

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Solution

(A), (C) Given, force applied on body

$$\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$$

Here $\alpha = 1.0$ N s^{-1} and $\beta = 1.0$ N. Therefore,

$$\vec{F} = t\hat{i} + \hat{j}$$

By Newton's second law of motion, $F = ma$. Given $m = 1.0$ kg. Therefore,

$$\vec{F} = \vec{a} \Rightarrow \vec{a} = (t\hat{i} + \hat{j})$$

We know

$$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \vec{v} = \int \vec{a}\,dt$$

$$\Rightarrow \vec{v} = \int (t\hat{i} + \hat{j})dt = \frac{t^2}{2}\hat{i} + t\hat{j} \quad (1)$$

Also,

$$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{r} = \int \vec{v}\,dt$$

$$\Rightarrow \vec{r} = \int \left(\frac{t^2}{2}\hat{i} + t\hat{j}\right)dt = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j} \quad (2)$$

Now,

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t^3/6 & t^2/2 & 0 \\ t & 1 & 0 \end{vmatrix}$$

$$\vec{\tau} = \hat{i}(0) - \hat{j}(0) + \hat{k}\left(\frac{t^3}{6} - \frac{t^3}{2}\right) = \frac{-1}{3}t^3\hat{k} \text{ N m}$$

At $t = 1.0$ s, $\vec{\tau} = \frac{-1}{3}\hat{k}$ N m

Hence, option A is correct.

Now, from Eq. (1), $\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$

At $t = 1.0$ s, $\vec{v} = \left(\frac{1}{2}\hat{i} + \hat{j}\right)$ m/s $= \frac{1}{2}(\hat{i} + 2\hat{j})$ m/s

Hence, option C is correct.

Chapter: Mechanical Properties of Fluids
Topic: Surface Tension

3. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ.

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Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

(A) For a given material of the capillary tube, h decreases with increase in r.

(B) For a given material of the capillary tube, h is independent of σ.

(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases.

(D) h is proportional to contact angle θ.

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Solution

(A), (C) We know that

$$h = \frac{2\sigma \cos\theta}{r\rho g} \quad (1)$$

where r is inner radius of capillary tube; σ is surface tension; θ is angle between water and wall of capillary tube; h is height of water in capillary tube and g is acceleration due to gravity. From Eq. (1),

$$h \propto \frac{1}{r}$$

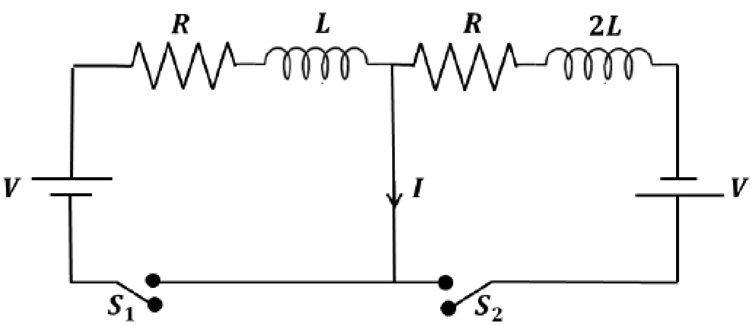
Therefore, for a given material of the capillary tube, h decreases with increase in r.
Hence, option (A) is correct.
h depends on σ therefore option (B) is incorrect.
If the experiment is performed in a lift going up with a constant acceleration (say a) then $g' = g + a$.

Therefore, $h \propto \frac{1}{g} \Rightarrow h$ decreases. So option (C) is correct.

From (1), $h \propto \cos\theta$ and not θ, therefore, option (D) is incorrect.

Chapter: Electromagnetic Induction
Topic: Inductance

4. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statements is (are) true?



(A) $I_{max} = \frac{V}{2R}$

(B) $I_{max} = \frac{V}{4R}$

(C) $\tau = \frac{L}{R} \ln 2$

(D) $\tau = \frac{2L}{R} \ln 2$

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Solution

(B), (D) Here, current $I_1 = \frac{V}{R}(1 - e^{-\tau R/L})$

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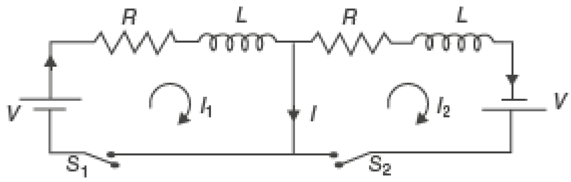
and current $I_2 = \frac{V}{R}(1 - e^{-\tau R/2L})$



$$I = I_1 - I_2 = \frac{V}{R}(1 - e^{-\tau R/L}) - \frac{V}{R}(1 - e^{-\tau R/2L})$$

$$= \frac{V}{R}[1 - e^{-\tau R/L} - 1 + e^{-\tau R/2L}]$$

$$= \frac{V}{R}[e^{-\tau R/2L} - e^{-\tau R/L}]$$

Therefore,
$$I = \frac{V}{R} e^{-\tau R/2L}(1 - e^{-\tau R/2L}) \quad (1)$$

Now, for $I_{\max} \Rightarrow \frac{dI}{d\tau} = 0$

$$\frac{dI}{d\tau} = \frac{V}{R}\left[e^{-\tau R/2L}\left(\frac{-R}{2L}\right) - e^{-\tau R/L}\left(\frac{-R}{L}\right)\right]$$

$$\frac{V}{R}\left(\frac{-R}{L}\right)\left[\frac{e^{-\tau R/2L}}{2} - e^{-\tau R/L}\right]$$

Further, $\frac{dI}{d\tau} = 0$, we get

$$\frac{V}{R}\left(\frac{-R}{L}\right)\left[\frac{e^{-\tau R/2L}}{2} - e^{\tau R/L}\right] = 0$$

$$\frac{e^{-\tau R/2L}}{2} - e^{\tau R/L} = 0 \Rightarrow \frac{e^{-\tau R/2L}}{2} = e^{-\tau R/L}$$

$$\Rightarrow e^{-\tau R/2L} = \frac{1}{2} \quad (2)$$

Put in Eq. (1), we get

$$I_{\max} = \frac{V}{R}\left(\frac{1}{2}\right)\left(1 - \frac{1}{2}\right) = \frac{1}{4}\frac{V}{R}$$

Hence, option B is correct.
Now from Eq. (2), we have

$$e^{-\tau R/2L} = \frac{1}{2}$$

$$\Rightarrow \frac{-\tau R}{2L} = \ln\left(\frac{1}{2}\right) \Rightarrow -\frac{\tau R}{2L} = -\ln 2$$

$$\Rightarrow \tau = \frac{2L}{R}\ln 2$$

Hence, option D is correct.

Chapter: Moving Charges and Magnetism
Topic: Force between Two Parallel Currents, the Ampere

5. Two infinitely long straight wires lie in the *xy*–plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the *xy*–plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the

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following statements regarding the magnetic field $\vec{B}$ is (are) true?

(A) If $I_1 = I_2$, then $\vec{B}$ **cannot** be equal to zero at the origin (0, 0, 0).

(B) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0).

(C) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0).

(D) If $I_1 = I_2$, then the z–component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$.

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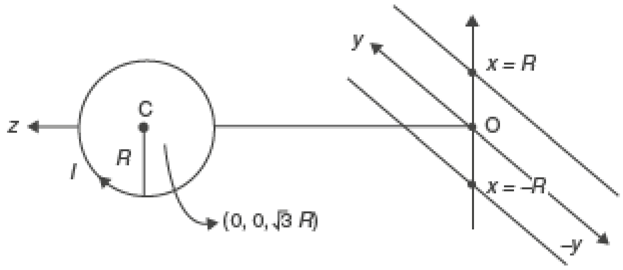
Solution

(A), (B), (D)



Statement (A) is true – If $I_1 = I_2$ then $\vec{B}$ cannot be equal to zero at the origin (0, 0, 0) because it is equal to $\vec{B}$ due to ring.

Statement (B) is true – If $I_1 > 0$ and $I_2 < 0$ then $\vec{B}$ can be equal to zero at the origin (0, 0, 0) because $\vec{B}$ due to wire will be along $\hat{k}$ direction and $\vec{B}$ due to ring will be along $-\hat{k}$ direction hence net $\vec{B} = 0$ at origin.

Statement (C) is false – If $I_1 < 0$ and $I_2 > 0$ then $\vec{B}$ cannot be equal to zero at the origin (0, 0, 0) because $\vec{B}$ due to wire will be along $-\hat{k}$ direction and $\vec{B}$ due to ring will also be along $-\hat{k}$ direction hence net $\vec{B} \neq 0$ at origin.

Statement (D) is true – If $I_1 = I_2$, then z-component of the magnetic field at the centre of loop is $\left(\frac{-\mu_0 I}{2R}\right)$ since it will be only due to the ring.

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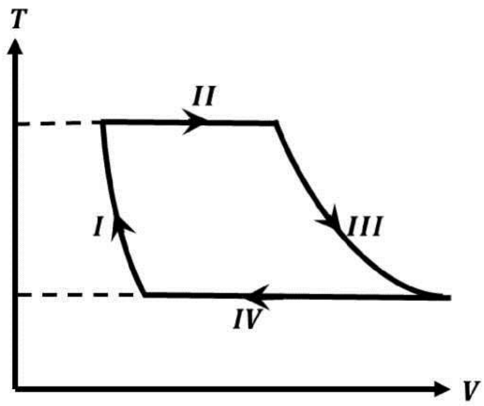
Chapter: Thermodynamics

Topic: Thermodynamic Processes

6. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the statements below is (are) true?



(A) Process I is an isochoric process.

(B) In process II, gas absorbs heat.

(C) In process IV, gas releases heat.

(D) Processes I and III are **not** isobaric.

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Solution

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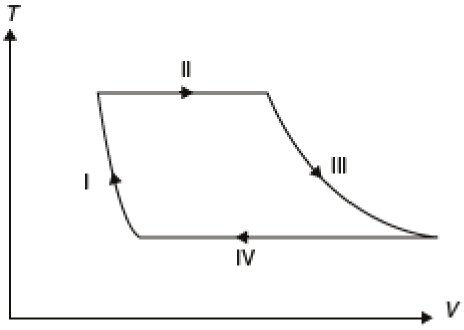
(B), (C), (D)

Isochoric process is a thermodynamic process during which volume of closed system remains constant. In process I volume is changing therefore it is not isochoric. Option (A) is incorrect.



Process II is isothermal process. An isothermal process is a thermodynamic process during which temperature of system remains constant, so by first law of thermodynamics

$$dQ = dU + dW$$

Since process is isothermal, therefore,

$$dU = 0 \Rightarrow dQ = dW$$

Now, from graph we know volume is increasing, therefore, work done is positive so dQ is positive thus the system absorbs heat in process II. Option (B) is correct.

Process IV is again isothermal since temperature is constant. Therefore, by law of thermodynamics, we have

$$dQ = dW$$

Now, from the given graph, volume is decreasing, therefore, work done is negative so dQ is negative. Hence, in process IV, gas releases heat. Option (C) is correct.

Isobaric process is a thermodynamic process in which pressure remains constant. Thus, $T \propto V$ which implies $T - V$ graph should be linear. Therefore, clearly from graph, process I and III are not isobaric. Option (D) is correct.

Integer Answer Type

This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE**.

Chapter: Motion in a Plane
Topic: Vector Addition–Analytical Method

7. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\hat{i} + \sin\omega t\hat{j})$, where a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in seconds, is_____.

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Solution

(2) Given: $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\hat{i} + \sin\omega t\hat{j})$

where a is a constant and $\omega = \pi/6$ rad s^{-1}

At $t = \tau$, $$|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}| \qquad (1)$$

Now,
$$|\vec{A}+\vec{B}| = |a\hat{i} + a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}|$$
$$= |a(1+\cos\omega t)\hat{i} + a\sin\omega t\,\hat{j}|$$
$$= \sqrt{a^2(1+\cos\omega t)^2 + a^2\sin^2\omega t} \qquad \left(\because |x\hat{i}+y\hat{j}| = \sqrt{x^2+y^2}\right)$$
$$= a\sqrt{1+\cos^2\omega t + 2\cos\omega t + \sin^2\omega t}$$
$$= a\sqrt{2+2\cos\omega t} \qquad (\because \cos^2 x + \sin^2 x = 1)$$
$$= a\sqrt{2(1+\cos\omega t)}$$

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$$= a\sqrt{2\times 2\left(\cos\frac{\omega t}{2}\right)^2} \qquad (\because \cos 2x = 2\cos^2 x - 1)$$

$$= a\times 2\cos\frac{\omega t}{2}$$

$$|\vec{A}+\vec{B}| = 2a\cos\frac{\omega t}{2} \qquad (2)$$

Similarly,

$$|\vec{A}-\vec{B}| = |a\hat{i} - a\cos\omega t\,\hat{i} - a\sin\omega t\,\hat{j}|$$

$$= |a(1-\cos\omega t)\hat{i} - a\sin\omega t\,\hat{j}|$$

$$= \sqrt{a^2(1-\cos\omega t)^2 + a^2\sin^2\omega t}$$

$$= a\sqrt{(1-\cos\omega t)^2 + \sin^2\omega t}$$

$$= a\sqrt{1+\cos^2\omega t - 2\cos\omega t + \sin^2\omega t}$$

$$= a\sqrt{1+1-2\cos\omega t} = a\sqrt{2-2\cos\omega t}$$

$$= a\sqrt{2(1-\cos\omega t)} = a\sqrt{2\times 2\sin^2\frac{\omega t}{2}} \qquad (\because \cos 2x = 1-2\sin^2 x)$$

$$|\vec{A}-\vec{B}| = 2a\sin\frac{\omega t}{2} \qquad (3)$$

Substitute Eqs. (2) and (3) in Eq. (1), we have

$$2a\cos\frac{\omega t}{2} = \sqrt{3}\times 2a\sin\frac{\omega t}{2}$$

$$\frac{\sin\omega t/2}{\cos\omega t/2} = \frac{1}{\sqrt{3}} \Rightarrow \tan\frac{\omega t}{2} = \frac{1}{\sqrt{3}}$$

We know

$$\tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{\omega t}{2} = 30° \text{ or } \frac{\omega t}{2} = \frac{\pi}{6}$$

$$\omega t = \frac{\pi}{3} \Rightarrow t = \frac{\pi}{3\omega} = \frac{\pi}{3}\times\frac{6}{\pi} = 2$$

Therefore, $t = 2$ s or $\tau = 2$ s

Chapter: Waves
Topic: Doppler Effect

8. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 m s^{-1} and the man behind walks at a speed 2.0 m s^{-1}. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 m s^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is_____________.

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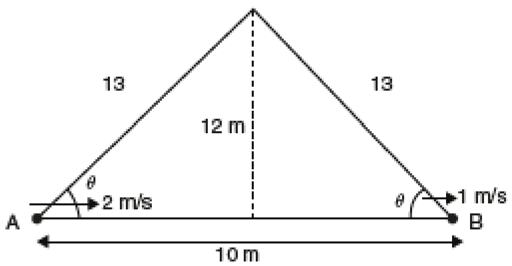
Solution

(5) From given data, we can draw the figure below.



Now, frequency of man behind is

$$f_1 = f\left(\frac{v}{v - v_S\cos\theta}\right)$$

where x is speed of sound in air, v_S is speed of man and f is frequency of whistle. Therefore,

$$f_1 = 1430\left(\frac{330}{330 - 2\times\frac{5}{13}}\right)$$

Frequency of man in front is

$$f_2 = f\left(\frac{v}{v + v_S\cos\theta}\right) = 1430\left(\frac{330}{330 + 1\times\frac{5}{13}}\right)$$

Now, frequency of beat is given as

$$\Delta f = f_1 - f_2$$

$$= 1430\left(\frac{330}{330 - \frac{10}{13}}\right) - 1430\left(\frac{330}{330 - \frac{5}{13}}\right)$$

$$= 1430\times 330\left(\left[\frac{1}{330 - \frac{10}{13}}\right] - \left[\frac{1}{330 - \frac{5}{13}}\right]\right)$$

$$= 1430\times 330\left[\frac{13}{4280} - \frac{13}{4295}\right]$$

$$= \frac{1430\times 330\times 13\times 15}{4280\times 4295} = 5.0058\text{ Hz} \approx 5.00\text{ Hz}$$

Chapter: System of Particles and Rotational Motion
Topic: Rolling Motion

9. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is____________. Take g = 10 m s^{-2}.

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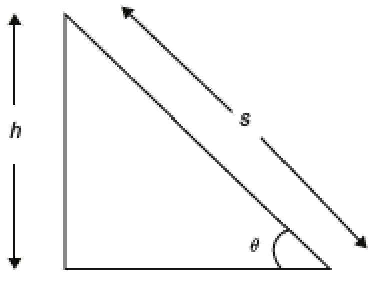
Solution

(0.75)



Given, h = height of the top of inclined plane = ?, θ = 60°, g = 10 m s^{-2} and time difference between ring and disc reaching ground $= \frac{2-\sqrt{3}}{\sqrt{10}}$ s

We know that $$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

For ring, $I = MR^2$ and for disc $I = \frac{3}{2}MR^2$.

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So, $a_{\text{ring}} = \dfrac{g\sin\theta}{2}$ and $a_{\text{disk}} = \dfrac{2g\sin\theta}{3}$

Now using $s = \dfrac{1}{2}at^2$

$$s_{\text{ring}} = \frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_1^2 \Rightarrow \frac{h}{\sin\theta} = \frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_1^2 \quad (1)$$

$$s_{\text{disk}} = \frac{1}{2}\left(\frac{2g\sin\theta}{3}\right)t_2^2 \Rightarrow \frac{h}{\sin\theta} = \frac{1}{2}\left(\frac{2g\sin\theta}{3}\right)t_2^2 \quad (2)$$

Given $t_1 - t_2 = \dfrac{2-\sqrt{3}}{\sqrt{10}}$

From Eq. (1), we have $t_1 = \sqrt{\dfrac{4h}{g\sin^2\theta}}$

and from Eq. (2), we have $t_2 = \sqrt{\dfrac{3h}{g\sin^2\theta}}$

So,
$$\sqrt{\frac{4h}{g\sin^2\theta}} - \sqrt{\frac{3h}{g\sin^2\theta}} = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\Rightarrow \sqrt{\frac{4h}{10\sin^2 60^\circ}} - \sqrt{\frac{3h}{10\sin^2 60^\circ}} = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\Rightarrow \sqrt{\frac{16h}{3\times 10}} - \sqrt{\frac{4h}{10}} = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\Rightarrow \frac{\sqrt{16h}}{\sqrt{3}} - \sqrt{4h} = 2-\sqrt{3}$$

$$\Rightarrow \frac{4\sqrt{h}}{\sqrt{3}} - 2\sqrt{h} = 2-\sqrt{3}$$

$$\Rightarrow \sqrt{h}\left(\frac{4}{\sqrt{3}} - 2\right) = (2-\sqrt{3})$$

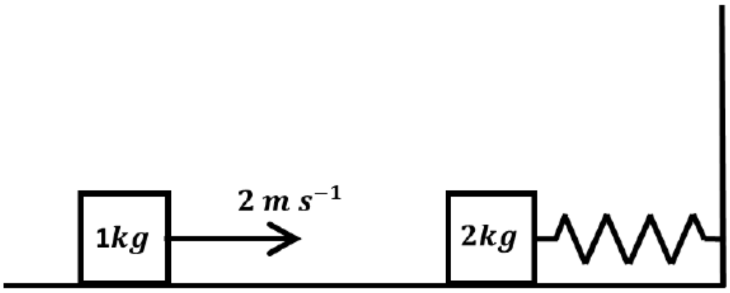
$$\Rightarrow \sqrt{h}\,\frac{(4-2\sqrt{3})}{\sqrt{3}} = (2-\sqrt{3})$$

$$\Rightarrow \sqrt{h}\times\frac{2}{\sqrt{3}}(2-\sqrt{3}) = (2-\sqrt{3}) \qquad \Rightarrow \sqrt{h} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow h = \frac{3}{4} = 0.75\text{ m}$$

Chapter: Work, Energy and Power
Topic: Collisions

10. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 N m^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m s^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is____________.



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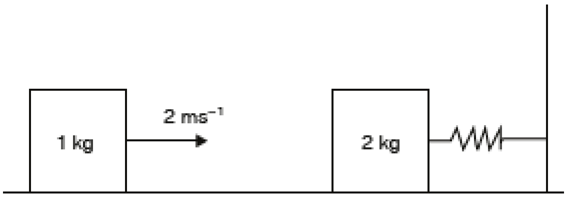
Solution

(2)



After collision the 2.0 kg block will perform simple harmonic oscillation with time period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Given m = 2.0 kg and k = 2.0 N m^{-1}, we have

$$T = 2\pi$$

Thus, the block returns to its original position in time $t = \frac{T}{2} = \pi$ s

That is, $t = 3.14$ s

Now, if v_1 and v_2 are velocities of 1.0 kg block and 2.0 kg block, respectively, before collision; v_1' and v_2' are velocities of 1.0 kg block and 2.0 kg block, respectively, after collision. So, by conservation of momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Here, m_1 = 1.0 kg, v_1 is initial speed of 1.0 kg block = 2.0 m s^{-1}, m_2 = 2.0 kg, v_2 is initial speed of 2.0 kg block = 0.0 m s^{-1}, v_1' is final speed of 1.0 kg block after collision and v_2' is final speed of 2.0 kg block after collision.

Then,

$$1.0 \text{ kg} \times 2.0 \text{ m/s} + 2.0 \text{ kg} \times 0 \text{ m/s} = 1.0v_1' + 2.0v_2'$$

$$v_1' + 2v_2' = 2 \qquad (1)$$

Also, using definition of coefficient of restitution

$$v_2' - v_1' = \varepsilon(v_1 - v_2)$$

Since collision is elastic, So $\varepsilon = 1$

$$\Rightarrow v_2' - v_1' = v_1 - v_2$$

$$\Rightarrow v_2' - v_1' = 2 - 0$$

$$\Rightarrow v_2' - v_1' = 2 \qquad (2)$$

From Eqs. (1) and (2), we get

$$v_2' = \frac{4}{3} \text{ m s}^{-1} \text{ and } v_1' = \frac{-2}{3} \text{ m s}^{-1}$$

Therefore, distance between the blocks is given as

$$s = v_1' \times t = \frac{-2}{3} \times 3.14 = 2.09 \text{ m}$$

Chapter: Electrostatic Potential and Capacitance

Topic: Combination of Capacitors

11. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) V_0 = 8 V. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 μC. The value of ε_r =____________.

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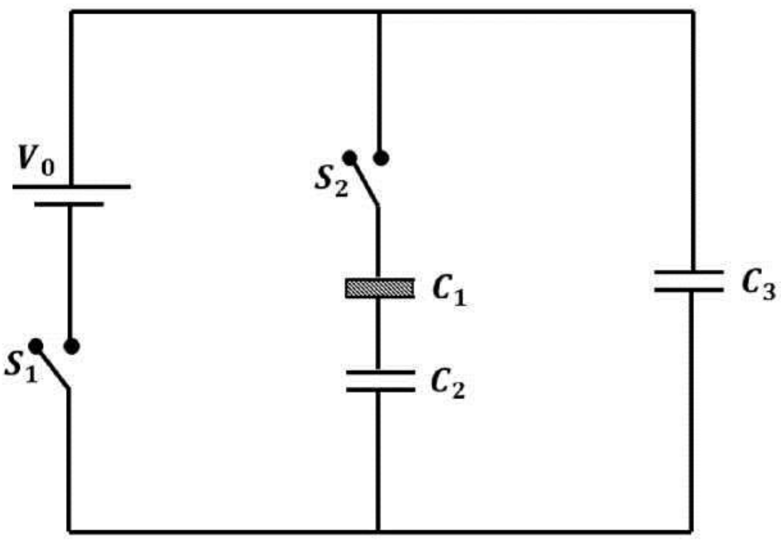


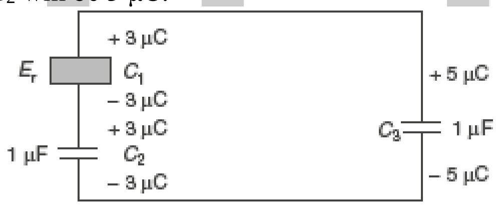
(JEE Advanced 2018 Paper-1)

Solution

(1.5) Given $C_1 = C_2 = C_3 = 1.0$ µF and $V_0 = 8$ V

When S_1 is close and S_2 is open. Charge on C_3 is 1 µC. The circuit is

When S_1 is open and S_2 is close. Charge on C_3 is 5 µC.
Therefore, charge on C_1 and C_2 will be 3 µC.



So,
$$\frac{5\,\mu\text{C}}{1\,\mu\text{F}} - \frac{3}{\varepsilon_r} - \frac{3}{1} = 0 \Rightarrow 5 - \frac{3}{\varepsilon_r} - 3 = 0$$
$$\Rightarrow 2 - \frac{3}{\varepsilon_r} = 0 \Rightarrow 2\varepsilon_r - 3 = 0 \Rightarrow \varepsilon_r = \frac{3}{2}$$
$$\varepsilon_r = 1.50$$

Chapter: Moving Charges and Magnetism
Topic: Motion in a Magnetic Field

12. In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi$ m s^{-1} at $t = 0$, as shown in the figure. Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the x–axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in m s^{-1}, along the x–axis in the time interval T is__________.

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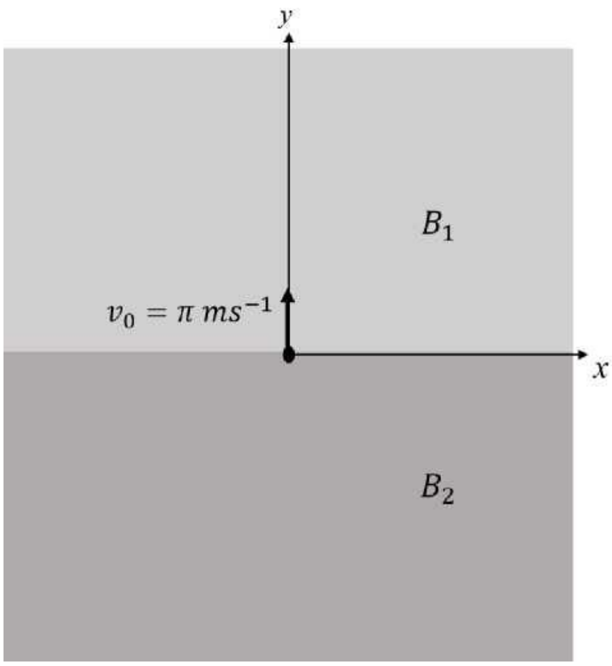
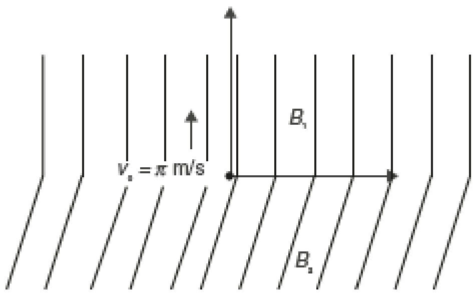
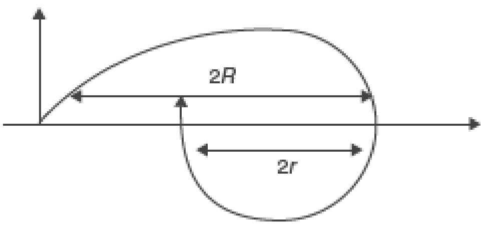


(JEE Advanced 2018 Paper-1)

Solution
(2)



Given, $B_2 = 4B_1$
The particle will follow the path shown below.



Here, $$R = \frac{mv}{qB_1},\ r = \frac{mv}{qB_2}$$

The particle will remain in magnetic field B_1 till

$$t_1 = \frac{\pi m}{qB_1}$$

The particle will remain in magnetic field B_2 till

$$t_2 = \frac{\pi m}{qB_2}$$

Since, total distance covered by particle along x axis is

$$2R + 2r = 2\left(\frac{mv}{qB_1} + \frac{mv}{qB_2}\right)$$

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And total time $$t_1 + t_2 = \frac{\pi m}{qB_1} + \frac{\pi m}{qB_2}$$

So, average speed is given as

$$v_{avg} = \frac{\frac{2mv}{qB_1} + \frac{2mv}{qB_2}}{\frac{\pi m}{qB_1} + \frac{\pi m}{qB_2}}$$

Using $B_2 = 4B_1$, we get

$$v_{avg} = \frac{\frac{2mv}{qB_1} + \frac{2mv}{4qB_1}}{\frac{\pi m}{qB_1} + \frac{\pi m}{4qB_1}} = \frac{\frac{2mv}{qB_1}\left(\frac{5}{4}\right)}{\frac{\pi m}{qB_1}\left(\frac{5}{4}\right)}$$

Therefore, average speed = 2.00 m s^{-1}

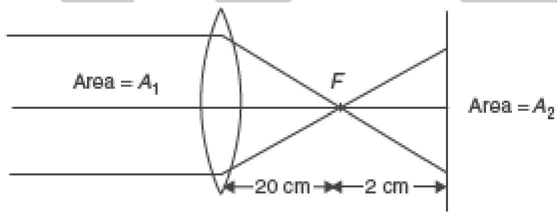
Chapter: Ray Optics and Optical Instruments
Topic: Optical Instruments

13. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is________.

(JEE Advanced 2018 Paper-1)

Solution

(130) Given, intensity of sunlight, I = 1.3 kW m^{-2} and focal length of convex lens, f = 20 cm



Now we know, $$\text{Intensity} = \frac{\text{Energy}}{\text{Area}}$$

So, intensity on screen is given as

$$I_{screen} = \frac{\text{Incident energy}}{A_2}$$

But, Incident energy = Intensity of sunlight × Area of lens
= 1.3 kWm^{-2} × A_1

So, intensity on screen is given as

$$I_{screen} = 1.3\,\text{kWm}^{-2} \times \frac{A_1}{A_2}$$

Now from figure, and using similar triangles theorem, we have

$$\frac{A_1}{A_2} = \left(\frac{20}{2}\right)^2 = 100$$

Therefore, intensity on screen = 1.3 kW m^{-2} × 100 = 130 kW m^{-2}

Chapter: Thermal Properties of Matter
Topic: Heat Transfer

14. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300 K and T_2 = 100 K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2, respectively. If the temperature at the

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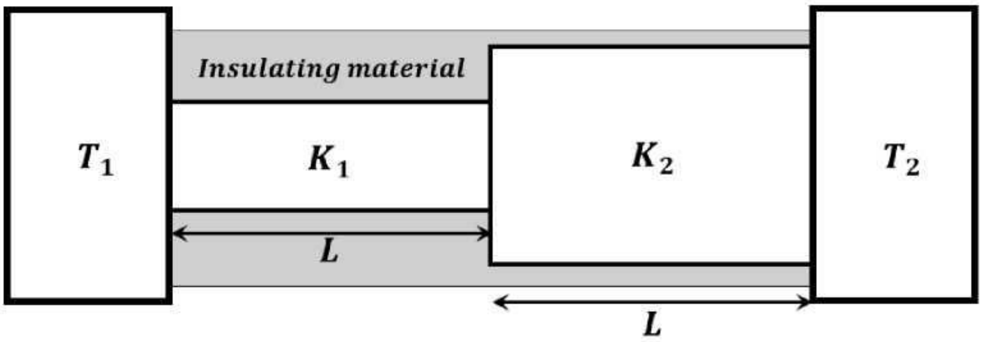
junction of the two cylinders in the steady state is 200 K, then K_1/K_2 =________.



(JEE Advanced 2018 Paper-1)

Solution

(4) Given, $T_1 = 300$ K, $T_2 = 100$ K

Temperature at junction = 200 K

At steady state, $\frac{k_1A_1(300-200)}{L} = \frac{k_2A_2(200-100)}{L}$

Given, radius of bigger cylinder is twice that of the smaller one, that is, $A_2 = 4A_1$. Therefore,

$\frac{k_1A_1(300-200)}{L} = \frac{k_2(4A_1)(200-100)}{L}$

$k_1(300 - 200) = k_2 4(200 - 100)$

$100k_1 = 4\times 100k_2 \Rightarrow \frac{k_1}{k_2} = 4.00$

Paragraph Type

This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.

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Chapter: Physical World, Units and Measurements

Topic: Dimensional Analysis and its Applications

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields, respectively, while [ε_0] and [μ_0] stand for dimensions of the permittivity and permeability of free space, respectively. [L] and [T] are dimensions of length and time, respectively. All the quantities are given in SI units.

(There are two questions based on PARAGRAPH "X", the question given below is one of them)

15. The relation between [E] and [B] is

(A) $[E] = [B]\,[L]\,[T]$
(B) $[E] = [B]\,[L]^{-1}\,[T]$
(C) $[E] = [B]\,[L]\,[T]^{-1}$
(D) $[E] = [B]\,[L]^{-1}\,[T]^{-1}$

(JEE Advanced 2018 Paper-1)

Solution

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(C)

We know $B = \frac{E}{c}$ or $E = Bc$

Now, $[E] = [B]$ [c]

We know that $[c] = [LT^{-1}]$

Therefore, $[E] = [B] [LT^{-1}] = [B] [L] [T]^{-1}$

Chapter: Physical World, Units and Measurements
Topic: Dimensional Analysis and its Applications

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, $[E]$ and $[B]$ stand for dimensions of electric and magnetic fields, respectively, while $[\varepsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space, respectively. $[L]$ and $[T]$ are dimensions of length and time, respectively. All the quantities are given in SI units.

(There are two questions based on PARAGRAPH "X", the question given below is one of them)

16. The relation between $[\varepsilon_0]$ and $[\mu_0]$ is

(A) $[\mu_0] = [\varepsilon_0] [L]^2 [T]^{-2}$

(B) $[\mu_0] = [\varepsilon_0] [L]^{-2} [T]^2$

(C) $[\mu_0] = [\varepsilon_0]^{-1} [L]^2 [T]^{-2}$

(D) $[\mu_0] = [\varepsilon_0]^{-1} [L]^{-2} [T]^2$

(JEE Advanced 2018 Paper-1)

Solution

(D) Since $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\Rightarrow \mu_0 \varepsilon_0 = \frac{1}{c^2} \Rightarrow \mu_0 = \frac{1}{\varepsilon_0 c^2}$$

Therefore, $[\mu_0] = [\varepsilon_0]^{-1} [c]^{-2}$

We know that $[c] = LT^{-1} \Rightarrow [c]^{-2} = [L]^{-2} [T]^2$

Thus, $[\mu_0] = [\varepsilon_0]^{-1} [L]^{-2} [T]^2$

Chapter: Physical World, Units and Measurements
Topic: Accuracy, Precision of Instruments and Errors in Measurement

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PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z = \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\Delta x/x << 1$, $\Delta y/y << 1$. Therefore, the higher powers of these quantities are neglected.

(There are two questions based on PARAGRAPH "A", the question given below is one of them)

17. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a.

If the error in the measurement of a is Δa ($\Delta a/a << 1$), then what is the error Δr in determining r?

(A) $\frac{\Delta a}{(1+a)^2}$

(B) $\frac{2\Delta a}{(1+a)^2}$

(C) $\frac{2\Delta a}{(1-a^2)}$

(D) $\frac{2a\Delta a}{(1-a^2)}$

(JEE Advanced 2018 Paper-1)

Solution

(B)

Given, ratio $r = \frac{1-a}{1+a}$ (1)

Given, error in measurement of a is Δa ($\Delta a/a << 1$)

Taking natural log of equation (1), we get

$$\ln r = \ln\left(\frac{1-a}{1+a}\right)$$

$\ln r = \ln(1-a) - \ln(1+a)$

Therefore, $\frac{\Delta r}{r} = \frac{\Delta a}{1-a} + \frac{\Delta a}{1+a} = \frac{(1+a)\Delta a + (1-a)\Delta a}{(1-a)(1+a)}$

$$\Rightarrow \frac{\Delta r}{r} = \frac{2\Delta a}{(1-a)(1+a)}$$

$$\Rightarrow \Delta r = r \cdot \frac{2\Delta a}{(1-a)(1+a)}$$

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Substituting value of r, we get

$$\Delta r = \frac{1-a}{1+a}\frac{2\Delta a}{(1-a)(1+a)} = \frac{2\Delta a}{(1+a)^2}$$

Therefore, error Δr in determining r is $\frac{2\Delta a}{(1+a)^2}$

Chapter: Physical World, Units and Measurements
Topic: Accuracy, Precision of Instruments and Errors in Measurement

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\Delta x/x << 1$, $\Delta y/y << 1$. Therefore, the higher powers of these quantities are neglected.
(There are two questions based on PARAGRAPH "A", the question given below is one of them)

18. In an experiment, the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| << 1$, $\ln(1 + x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

(A) 0.04
(B) 0.03
(C) 0.02
(D) 0.01

(JEE Advanced 2018 Paper-1)

Solution

(C)

We know, $N = N_0(1 - e^{-\lambda t})$

where N is number of nuclei decayed, N_0 is initial number of nuclei, λ is decay constant.

On rearranging Eq. (1), we get

$$N = N_0 - N_0 e^{-\lambda t}$$
$$N_0 - N = N_0 e^{-\lambda t}$$
$$\frac{N_0 - N}{N_0} = e^{-\lambda t}$$
$$\Rightarrow -\lambda t = \ln\frac{N_0 - N}{N_0}$$
$$\Rightarrow -\lambda t = \ln(N_0 - N) - \ln N_0$$
$$\Rightarrow \lambda t = \ln N_0 - \ln(N_0 - N)$$
$$\Rightarrow (\Delta\lambda)t = \frac{\Delta N}{N_0 - N} \Rightarrow \Delta\lambda = \frac{\Delta N}{(N_0 - N)t}$$

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Substituting, N_0 = 3000, N = 1000, t = 1.0s, ΔN = 40

$$\Delta\lambda = \frac{40}{(3000-1000)\times 1} = \frac{40}{2000} = 0.02\ \text{s}^{-1}$$

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