JEE (ADVANCED) 2018 PAPER 2

PART I–PHYSICS

One or More than One Option Correct Type

This section contains **SIX (06)** questions. Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).

Chapter: Work, Energy and Power

Topic: Kinetic Energy

1. A particle of mass *m* is initially at rest at the origin. It is subjected to a force and starts moving along the *x*-axis. Its kinetic energy *K* changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?

(A) The force applied on the particle is constant.

(B) The speed of the particle is proportional to time.

(C) The distance of the particle from the origin increases linearly with time.

(D) The force is conservative.

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Solution

(A), (B), (D) Given, a particle at rest is subjected to force. Motion of particle is along *x*-axis.

Change in kinetic energy is given as

$$K = \frac{dK}{dt} = \gamma t \qquad (1)$$

where γ is positive constant.

We know that kinetic energy is given by

$$K = \frac{1}{2}mv^2 \qquad (2)$$

Now, differentiate Eq. (2), we get

$$\frac{dK}{dt} = \frac{2}{2}mv\frac{dv}{dt} = mv\frac{dv}{dt}$$

$$\frac{dK}{dt} = mv\frac{dv}{dt} \qquad (3)$$

From Eqs. (1) and (3) we get

$$mv\frac{dv}{dt} = \gamma t$$

$$\Rightarrow mv\,dv = \gamma t\,dt \qquad (4)$$

Integrating Eq. (4) we get

$$\int mv\,dv = \int \gamma t\,dt \Rightarrow m\int v\,dv = \gamma\int t\,dt$$

$$\frac{mv^2}{2} = \frac{\gamma t^2}{2} \Rightarrow mv^2 = \gamma t^2$$

$$\Rightarrow v^2 = \frac{\gamma}{m}t^2 \Rightarrow v = \sqrt{\frac{\gamma}{m}}t$$

$$\Rightarrow v \propto t$$

Since, γ and m are constant so the speed of particle is proportional to time therefore option (B) is true.

Also
$$a = \frac{dv}{dt} \Rightarrow a = \sqrt{\frac{\gamma}{m}} = \text{constant}$$

$$\Rightarrow F = ma = \text{constant}$$

So option (A) is true.

Also
$$v = \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \sqrt{\frac{\gamma}{m}}t$$

$$\Rightarrow dr = \sqrt{\frac{\gamma}{m}}t\,dt \qquad (5)$$

Integrating Eq. (5), we get

$$\int dr = \int \sqrt{\frac{\gamma}{m}}t\,dt \Rightarrow r = \sqrt{\frac{\gamma}{m}}\frac{t^2}{2}$$

$$\Rightarrow r \propto t^2$$

So the distance of the particle from the origin increases as t^2 so option (C) is false.
Now since F is constant thus the force is conservative in nature so option (D) is true.

Chapter: Mechanical Properties of Fluids

Topic: Viscosity

2. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true?

(A) The resistive force of liquid on the plate is inversely proportional to h.

(B) The resistive force of liquid on the plate is independent of the area of the plate.

(C) The tangential (shear) stress on the floor of the tank increases with u_0.

(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid.

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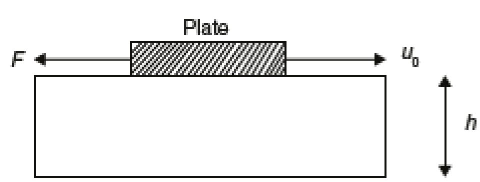
Solution

(A), (C) and (D)



We know that viscous force is given by

$$F = -\eta A \frac{dv}{dy}$$

Since height h of liquid in the tank is much less than the width of the tank. Therefore,

$$\frac{dv}{dy} = \frac{\Delta v}{\Delta y} = \frac{u_0}{h}$$

$$\Rightarrow |F| = \eta A\left(\frac{u_0}{h}\right)$$

$$\Rightarrow F \propto \frac{1}{h}$$

Hence, option (A) is correct.

$$F \propto A$$

Therefore, the resistive force of liquid on the plate is independent of the area of the plate. So option (B) is incorrect.

$$F \propto u_0$$

The tangential stress on the floor of the tank increases with u_0. So option (C) is correct.

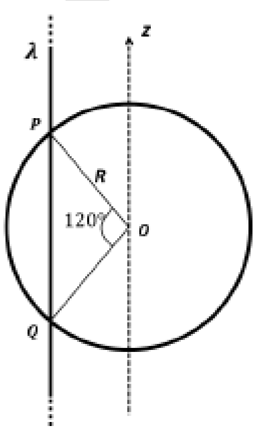
$$F \propto \eta$$

The tangential stress on the plate varies linearly with the viscosity η of the liquid. So option (D) is correct.

Chapter: Electric Charges and Fields

Topic: Gauss's Law

3. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true?



(A) The electric flux through the shell is $\sqrt{3}\, R\lambda/\varepsilon_0$.

(B) The z-component of the electric field is zero at all the points on the surface of the shell.

(C) The electric flux through the shell is $\sqrt{2}\, R\lambda/\varepsilon_0$.

(D) The electric field is normal to the surface of the shell at all points.

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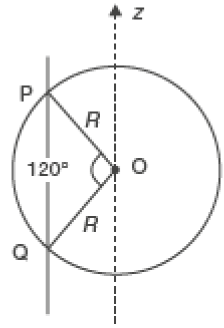
Solution

(A), (B)



From above figure, we have

$$PQ = 2\times R\sin 60° = 2R\frac{\sqrt{3}}{2} = \sqrt{3}R$$

According to Gauss's law, total flux is

$$\phi = \int \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$$

where $$Q_{enc} = \lambda(\sqrt{3}R)$$

Therefore, $$\phi = \frac{\lambda(\sqrt{3}R)}{\varepsilon_0}$$

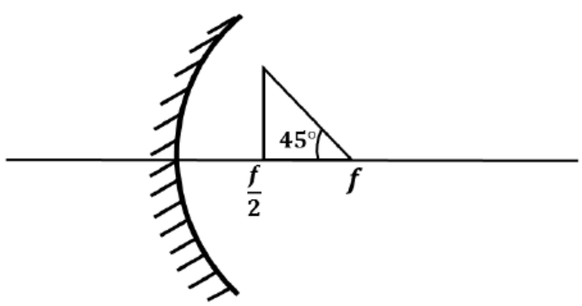
So, option (A) is correct.

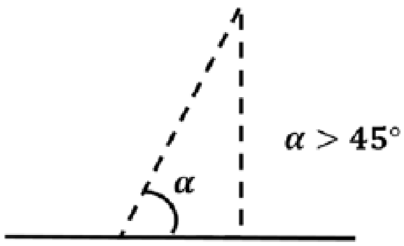
Also, electric field due to straight wire is perpendicular to the wire and radially outward, thus, the z component of the electric field is zero at all the points on the surface of the shell. So option (B) is correct.

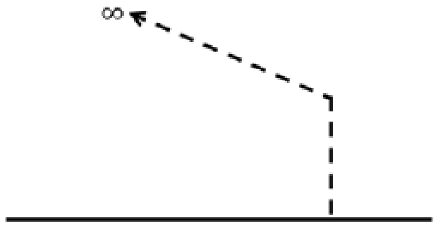
Chapter: Ray Optics and Optical Instruments

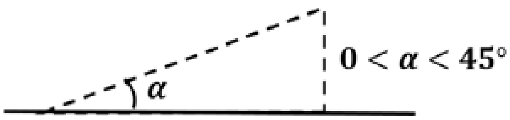
Topic: Reflection of Light by Spherical Mirrors

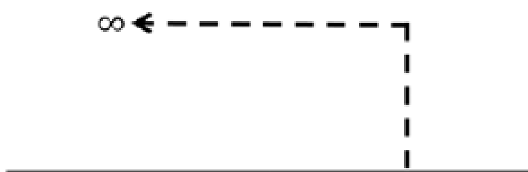
4. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)



(A) 

(B) 

(C) 

(D) 

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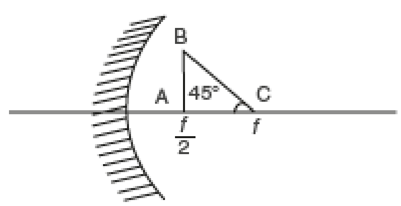
Solution

(D)



Here, distance of point A from mirror is $u = \frac{f}{2}$

Now, using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{-\frac{f}{2}} = \frac{1}{-f}$$

$$\Rightarrow \frac{1}{v} - \frac{2}{f} = \frac{-1}{f} \Rightarrow \frac{1}{v} = \frac{2}{f} - \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow v = f$$

Let height of the given right angle be h_1 and height of the image form be h_2. Then,

$$\frac{v}{u}=-\frac{h_2}{h_1}\Rightarrow\frac{f}{-f/2}=-\frac{h_2}{h_1}$$
$$\Rightarrow h_2=2f$$

The image of line AB should be perpendicular to the principle axis and image will be of some height formed at infinity.

So option (D) is correct.

Chapter: Nuclei

Topic: Radioactivity

5. In a radioactive decay chain, ${}^{232}_{90}\text{Th}$ nucleus decays to ${}^{212}_{82}\text{Pb}$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true?

(A) $N_\alpha = 5$

(B) $N_\alpha = 6$

(C) $N_\beta = 2$

(D) $N_\beta = 4$

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Solution

(A), (C)

Given

$${}^{232}_{90}\text{Th}\longrightarrow{}^{212}_{82}\text{pb}$$

where $N_\alpha =$ number of α particles, $N_\beta =$ number of β^- particles

We know that α–decay is given as

$${}^{A}_{Z}\text{X}\longrightarrow{}^{A-4}_{Z-2}\text{Y}+\alpha-\text{particle}$$

and β–decay is given as

$${}^{A}_{Z}\text{X}\longrightarrow{}^{A}_{Z+1}\text{X}+\beta^-\ \text{particle}$$

In above decay process change in mass number A is 20 and change in atomic number Z is 8.

Therefore, change in mass number $A \Rightarrow$ number of α–particles emitted are

$$N_\alpha=\frac{20}{4}=5$$

Now, emission of 5 α particles implies atomic number reduces by 10 but change in atomic number is 8, therefore, number of β^- particles emitted is 2.

Therefore, $N_\alpha = 5$ and $N_\beta = 2$

So, option (A) and option (C) be are correct.

Chapter: Waves

Topic: The Speed of a Travelling Wave

6. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true?

(A) The speed of sound determined from this experiment is 332 m s^{-1}.

(B) The end correction in this experiment is 0.9 cm.

(C) The wavelength of the sound wave is 66.4 cm.

(D) The resonance at 50.7 cm corresponds to the fundamental harmonic.

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Solution

(A), (B), (C)

Frequency of tuning fork = 500 Hz

Length of n_1 harmonic = 50.7 cm

Length of n_2 harmonic = 83.9 cm

We know that

$$n_2 \text{ harmonic} - n_1 \text{ harmonic} = \frac{\lambda}{2}$$

$$\Rightarrow (83.9 - 50.7) = \frac{\lambda}{2} \Rightarrow \frac{\lambda}{2} = 33.2 \text{ cm}$$

$$\Rightarrow \lambda = 66.4 \text{ cm}$$

Therefore, wavelength of sound wave is 66.4 cm, so option (C) is correct.

Now, speed of sound is given as

$$v = f\lambda = 500 \times 66.4 \times 10^{-2}$$

$$\Rightarrow v = 332 \text{ m s}^{-1}$$

Thus, speed of sound determined from this experiment is 332 m s^{-1}, so option (A) is correct.

Now $$(2n-1)\frac{\lambda}{4} = 50.7 + e$$

For $n = 2$, $$(4-1)\frac{\lambda}{4} = 50.7 + e \Rightarrow \frac{3\lambda}{4} = 50.7 + e$$

$$e = \frac{3\lambda}{4} - 50.7 \Rightarrow e = \frac{3}{4} \times 66.4 - 50.7 = 49.8 - 50.7$$

$$\Rightarrow e = -0.9 \text{ cm}$$

Thus, the end correction in this experiment is 0.9 cm. Therefore, option (B) is correct.

Integer Answer Type

This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE**.

Chapter: Laws of Motion

Topic: Solving Problems in Mechanics

7. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4 kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and τ = 4 s. The displacement of the block, in metres, at $t = \tau$ is______________. Take $e^{-1} = 0.37$.

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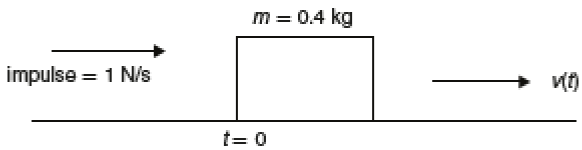
Solution

Given: $v(t) = v_0 e^{-t/\tau}$, $\tau = 4\text{s}$



Now, $$v_0 = \frac{\text{impulse}}{\text{mass}} = \frac{1}{0.4} = 2.5 \text{ m s}^{-1}$$

$$\Rightarrow v_0 = 2.5 \text{ m s}^{-1}$$

Using $v(t) = v_0 e^{-t/\tau}$ and we know that $v = \frac{dx}{dt}$, we have

$$\frac{dx}{dt} = v_0 e^{-t/\tau}$$

$$\Rightarrow dx = v_0 e^{-t/\tau} dt$$

On integrating, we get

$$\int dx = v_0 \int_0^\tau e^{-t/\tau} dt \Rightarrow x = v_0 \left(\frac{e^{-t/\tau}}{-\frac{1}{\tau}} \right) \Bigg|_0^\tau$$

$$\Rightarrow x = v_0 \left(\frac{e^{-1}}{-\frac{1}{\tau}} - \frac{e^0}{-\frac{1}{\tau}} \right) = -v_0 \tau (e^{-1} - e^0)$$

$$\Rightarrow x = -(2.5 \text{ m s}^{-1})(4 \text{ s})(0.37 - 1)$$

$$\Rightarrow x = 6.30 \text{ m}$$

Thus, displacement of block at $t = \tau$ is 6.30 m.

Chapter: Motion in a Plane

Topic: Projectile Motion

8 . A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is____________.

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Solution

(30) *For first projectile motion:* We know that height is given by

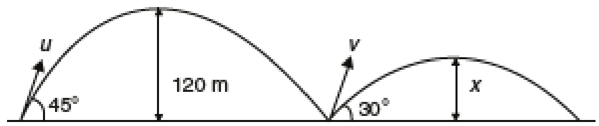
$$h = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow \frac{2h}{\sin^2 \theta} = \frac{u^2}{g} \qquad (1)$$

where h = 120 m and $\theta = 45°$. Then,

$$\Rightarrow \frac{u^2}{g} = \frac{2\times 120}{\sin^2 45°} \qquad (2)$$

For second projectile motion: We have

$$x = \frac{v^2 \sin^2 \theta}{2g}$$



Since upon hitting the ground the ball loses half of its kinetic energy. Then

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}mu^2}{\frac{1}{2}mv^2} \Rightarrow \frac{2}{1} = \frac{u^2}{v^2}$$

$$v^2 = \frac{u^2}{2}$$

and $\theta = 30°$. Therefore,

$$x = \frac{u^2}{2}\frac{\sin^2 30°}{2g} = \frac{u^2}{g}\frac{\sin^2 30°}{4}$$

Now, substitute $\frac{u^2}{g}$ from Eq. (2), we get

$$x = \frac{2\times 120}{\sin^2 45}\times\frac{\sin^2 30}{4} = \frac{2\times 120}{\frac{1}{2}}\times\frac{\frac{1}{4}}{4}$$

$$\Rightarrow x = 2\times 120\times 2\times\frac{1}{4\times 4} = 30 \text{ m}$$

Thus, maximum height ball reaches after the bounce is 30 m.

Chapter: Electric Field and Charges

Topic: Electric Field

9. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin\omega t\hat{i}$, where E_0 = 1.0 N C^{-1} and ω = 10^3 rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in m s^{-1}, attained by the particle at subsequent times is__________.

(JEE Advanced 2018 Paper-2)

Solution

(2.0) Given, mass of particle = 10^{-3} kg, charge on particle = 1.0 C, electric field $\vec{E}(t) = E_0 \sin\omega t\,\hat{i}$,

$E_0 = 1.0\ \text{N}\,\text{C}^{-1}, \omega = 10^3\,\text{rad s}^{-1}$

Force on particle is given by

$$\vec{F} = q\vec{E}$$

$$\Rightarrow \vec{F} = qE_0 \sin \omega t \hat{i} = 1.0 \times 1.0 \times \sin(10^3 t)\hat{i}$$
$$\Rightarrow \vec{F} = \sin(10^3 t)\hat{i}$$

We know that
$$\vec{F} = m\vec{a} \Rightarrow \vec{a} = \frac{\vec{F}}{m}$$
$$\Rightarrow a = \frac{\sin(10^3 t)}{10^{-3}} \Rightarrow a = 10^3 \sin(10^3 t)$$

We know
$$a = \frac{dv}{dt}$$
$$\Rightarrow \frac{dv}{dt} = 10^3 \sin(10^3 t)$$
$$\Rightarrow dv = 10^3 \sin(10^3 t)dt$$

Now, integrating it, we get
$$\int_0^v dv = \int_0^t 10^3 \sin(10^3 t)dt$$
$$\Rightarrow v\Big|_0^v = 10^3 \left(\frac{-\cos(10^3 t)\Big|_0^t}{10^3} \right) \Rightarrow v\Big|_0^v = -\cos(10^3 t)\Big|_0^t$$
$$\Rightarrow v = (-\cos 10^3 t + \cos 0)$$
$$\Rightarrow v = (1 - \cos(10^3 t))$$

We know that cos θ can take values between –1 and 1. Therefore, maximum speed attained by the particle when cos θ = –1. Thus,
$$v_{\max} = 1 - (-1) = 2 \text{ m s}^{-1}$$

Chapter: Moving Charges and Magnetism

Topic: The Moving Coil Galvanometer

10. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} N m rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 – 1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is _______.

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Solution

(5.55) Given: Number of turns, N = 50, area of each turn, $A = 2 \times 10^{-4}$ m^2, magnetic field produced, B = 0.02 T, torsional constant of suspension wise $k = 10^{-4}$ N m rad^{-1}, coil rotation θ = 0.2 rad, resistance of coil, R = 50 Ω, range of ammeter = 0 – 1.0 A

Let, shunt resistance be R_S

We know that
$$\tau = BANI = k\theta \Rightarrow I = \frac{k\theta}{\text{BAN}}$$

Now, substituting the values

$$I=\frac{10^{-4}\times 0.2}{0.02\times 2\times 10^{-4}\times 50}=\frac{0.2}{2}=0.1$$

Now, $$I\times R=(1-I)R_S$$

$$\Rightarrow R_S=\frac{I\times R}{(1-I)}=\frac{0.1\times 50}{1-0.1}=\frac{0.1\times 50}{0.9}=5.555$$

$$\Rightarrow R_S=5.555\,\Omega$$

Chapter: Physical World, Units and Measurements

Topic: Accuracy, Precision of Instruments and Errors in Measurement

11. A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} N m^{-2} carries a load of mass M. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 kg, the vernier scale division which coincides with a main scale division is_________. Take g = 10 m s^{-2} and π = 3.2.

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Solution

(3) We know that $$\Delta L=\frac{W}{(YA/L)}$$

where W is weight or load = mg = 1.2 × 10 = 12 kg m s^{-2} , Y is Young's modulus $=2\times 10^{11}$ N m^{-2} , L is length of wire with load = 1.0 m, A is area of steel wire $=\pi r^2=\frac{\pi}{4}d^2=\frac{\pi}{4}\times(0.5\times 10^{-3})^2$

Therefore,

$$\Delta L=\frac{1.2\times 10}{2\times 10^{11}\times\frac{\pi}{4}(0.5\times 10^{-3})^2\times\frac{1}{1.0\text{ m}}}=\frac{1.2\times 10\times 4}{2\times 10^{11}\times\pi\times(0.5)^2\times 10^{-6}}$$

$$\Rightarrow\Delta L=0.3\times 10^{-3}\text{ m}=0.3\text{ mm}$$

Now, least count of vernier scale $=\left(1-\frac{9}{10}\right)\text{mm}=0.1\text{ mm}$

Therefore, $$\text{Vernier reading }=\frac{\Delta L}{\text{least count}}$$

$$\text{Vernier reading }=\frac{0.3\text{ mm}}{0.1\text{ mm}}=3$$

Therefore, 3rd vernier scale division coincides with the main scale division.

Chapter: Thermodynamics

Topic: Heat, Internal Energy and Work

12. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J mol^{-1}K^{-1}, the decrease in its internal energy, in Joule, is_______.

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Solution

(900) Given, one mole of monatomic ideal gas undergoes adiabatic expansion and $V_{\text{final}} = 8V_{\text{initial}}$

Initial temperature $T_{\text{i}} = 100\text{ K}$, $R = 8.0\text{ J mol}^{-1}\text{ K}^{-1}$

We know that for adiabatic process

$$T_{\text{i}}V_{\text{i}}^{\gamma-1} = T_{\text{f}}V_{\text{f}}^{\gamma-1}$$

For monatomic gas, $\gamma = \frac{5}{3}$

Thus, $$\gamma - 1 = \frac{5}{3} - 1 = \frac{2}{3}$$

$$\Rightarrow T_{\text{i}}V_{\text{i}}^{\frac{2}{3}} = T_{\text{f}}V_{\text{f}}^{\frac{2}{3}} \Rightarrow T_{\text{f}} = T_{\text{i}}\left(\frac{V_{\text{i}}}{V_{\text{f}}}\right)^{\frac{2}{3}}$$

Given $V_{\text{f}} = 8V_{\text{i}}$, therefore,

$$T_{\text{f}} = T_{\text{i}}\left(\frac{V_{\text{i}}}{8V_{\text{i}}}\right)^{\frac{2}{3}} = T_{\text{i}}\left(\frac{1}{8}\right)^{\frac{2}{3}}$$

$$\Rightarrow T_{\text{f}} = \frac{100}{8^{2/3}} = \frac{100}{4} = 25$$

Therefore, final temperature $T_{\text{f}} = 25\text{ K}$. Thus change in internal energy is

$$\Delta U = nC_{\text{V}}\Delta T$$

Now, $$C_{\text{P}} - C_{\text{V}} = R \Rightarrow \frac{C_{\text{P}}}{C_{\text{V}}} - 1 = \frac{R}{C_{\text{V}}}$$

But $$\frac{C_{\text{P}}}{C_{\text{V}}} = \gamma \Rightarrow \gamma - 1 = \frac{R}{C_{\text{V}}} \Rightarrow C_{\text{V}} = \frac{R}{\gamma - 1}$$

$$\Rightarrow \Delta U = n\left(\frac{R}{\gamma - 1}\right)\Delta T$$

Here, $n = 1, \quad R = 8, \quad \gamma = \frac{5}{3}, \quad \Delta T = (T_{\text{i}} - T_{\text{f}})$

$$\Rightarrow \Delta U = 1 \times \frac{8}{\left(\frac{5}{3} - 1\right)}(100 - 25) = \frac{8}{\frac{2}{3}} \times 75$$

$$\Rightarrow \Delta U = \frac{8 \times 3}{2} \times 75 = 900\text{ joule}$$

Thus, decrease in internal energy = 900 J.

Chapter: Dual Nature of Radiation and Matter

Topic: Experimental Study of Photoelectric Effect

13. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is________. Mass of the electron $m_e = 9 \times 10^{-31}$ kg and 1.0 eV = 1.6×10^{-19} J.

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Solution

(24) Given: Power of incident monochromatic light, P = 200 W, work function, $\phi = 6.25$ eV

We know that $$P = Nhf$$

where N is number of photons per second, h is Planck's constant and f is frequency of incident light.

Since, photoelectrons that are emitted have negligible kinetic energy. Therefore,

$$hf = \phi$$

$$\Rightarrow P = N\phi$$

$$\Rightarrow N = \frac{P}{\phi} = \frac{200}{6.25 \text{ eV}} = \frac{200}{6.25 \times 1.6 \times 10^{-19} \text{ J}} = 2 \times 10^{20}$$

When a potential difference is applied, photo electrons get accelerated

$$\Rightarrow KE = q\Delta V$$

$$\Rightarrow \frac{1}{2}mv^2 = q\Delta V \Rightarrow \frac{(mv)^2}{2m} = q\Delta V \Rightarrow \frac{p^2}{2m} = q\Delta V$$

$$\Rightarrow p = \sqrt{2mq\Delta V}$$

Now, it is given that, photoelectrons emission efficiency is 100%.

Therefore, number of electrons = number of photons emitted per second

Thus, force due to impact $F = \Delta p \times N$

$$\Rightarrow n \times 10^{-4} = \sqrt{2mq\Delta V} \times N$$

$$= \sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500} \times 2 \times 10^{20}$$

$$= 12 \times 10^{-24} \times 2 \times 10^{20}$$

$$= 24 \times 10^{-4}$$

$$\Rightarrow n = 24$$

Chapter: Atoms

Topic: The Line Spectra of the Hydrogen Atom

14. Consider a hydrogen-like ionised atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is________.

(JEE Advanced 2018 Paper-2)

Solution

(3) We know that

$$\Delta E_{\text{i}-\text{j}} = 13.6\left[\frac{1}{n_{\text{j}}^2} - \frac{1}{n_{\text{i}}^2}\right]Z^2$$

Therefore,

$$\Delta E_{2-1} = 13.6\left[\frac{1}{1^2} - \frac{1}{2^2}\right]Z^2 = 13.6\left[\frac{1}{1} - \frac{1}{4}\right]Z^2$$

$$\Delta E_{2-1} = 13.6 \times \frac{3}{4} \times Z^2$$

Similarly,

$$\Delta E_{3-2} = 13.6\left[\frac{1}{2^2}-\frac{1}{3^2}\right]Z^2 = 13.6\left[\frac{1}{4}-\frac{1}{9}\right]Z^2$$

$$\Delta E_{3-2} = 13.6\times\frac{5}{36}\times Z^2$$

Given, $\Delta E_{2-1} = \Delta E_{3-2} + 74.8$

$$\Rightarrow 13.6\times\frac{3}{4}\times Z^2 = 13.6\times\frac{5}{36}\times Z^2 + 74.8$$

$$\Rightarrow 13.6\times\frac{3}{4}\times Z^2 - 13.6\times\frac{5}{36}\times Z^2 = 74.8$$

$$\Rightarrow 13.6\times Z^2\left[\frac{3}{4}-\frac{5}{36}\right] = 74.8$$

$$\Rightarrow 13.6\times Z^2\left[\frac{27-5}{36}\right] = 74.8$$

$$\Rightarrow 13.6\times Z^2\times\frac{22}{36} = 74.8$$

$$\Rightarrow Z^2 = \frac{74.8\times 36}{13.6\times 22} = 9$$

$$\Rightarrow Z = 3$$

SECTION 3 (Maximum Marks: 12)

This section contains **FOUR (04)** questions. Each question has **TWO (02)** matching lists: **LIST-I** and **LIST-II**.
FOUR options are given representing matching of elements from **LIST-I** and **LIST-II. ONLY ONE** of these four options corresponds to a correct matching.

Directions for Qns. 15, 16, 17 and 18: Answer the questions by appropriately matching the information given in the three columns of the following table:

Chapter: Electric Charges and Fields

Topic: Continuous Charge Distribution

15. The electric field E is measured at a point $P(0, 0, d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

LIST-I

P. E is independent of d

Q. $E \propto \dfrac{1}{d}$

R. $E \propto \dfrac{1}{d^2}$

S. $E \propto \dfrac{1}{d^3}$

LIST-II

1. A point charge Q at the origin.

2. A small dipole with point charges Q at $(0, 0, l)$ and $-Q$ at $(0, 0, -l)$. Take $2l \ll d$

3. An infinite line charge coincident with the x-axis, with uniform linear charge density λ.

4. Two infinite wires carrying uniform linear charge density parallel to the x-axis. The one along ($y = 0$, $z = l$) has a charge density $+\lambda$ and the one along ($y = 0$, $z = -l$) has a charge density $-\lambda$. Take $2l \ll d$

5. Infinite plane charge coincident with the xy-plane with uniform surface charge density.

(A) P→5; Q→ 3, 4; R →1; S → 2

(B) P→5; Q→ 3; R→ 1, 4; S → 2

(C) P→5; Q→ 3; R→ 1, 2; S → 4

(D) P→4; Q→ 2, 3; R→ 1; S → 5

(JEE Advanced 2018 Paper-2)

Solution

(B) For a point charge Q at the origin, we have

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}$$

$$\Rightarrow E \propto \frac{1}{d^2}$$

Thus, the correct mapping is $\text{R} \to 1$.

For a small dipole with point charges Q at (0, 0, l) and $-Q$ at (0, 0, $-l$), $2l \ll d$, we have

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2Q \times 2l}{d^3}$$

$$\Rightarrow E \propto \frac{1}{d^3}$$

Thus, the correct mapping is $\text{S} \to 2$.

For an infinite line charge coincident with x-axis with uniform linear charge density λ, we have

$$E = \frac{\lambda}{2\pi\varepsilon_0 d}$$

$$\Rightarrow E \propto \frac{1}{d}$$

Thus, the correct mapping is $\text{Q} \to 3$.

For two infinite wires carrying uniform linear charge density parallel to the x-axis, we have

$$E = \frac{\lambda}{2\pi\varepsilon_0(d-l)} - \frac{\lambda}{2\pi\varepsilon_0(d+l)} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{d-l} - \frac{1}{d+l}\right)$$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{d+l-d+l}{(d-l)(d+l)}\right) = \frac{\lambda}{2\pi\varepsilon_0}\frac{(2l)}{(d^2-l^2)}$$

Since $2l \ll d$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0}\frac{2l}{d^2}$$

$$\Rightarrow E \propto \frac{1}{d^2}$$

Thus, the correct mapping is $\mathrm{R} \rightarrow 4$.

For an infinite plane charge coincident with the *x*-*y* plane with uniform surface charge density, we have

$$E = \frac{\sigma}{2\varepsilon_0}$$

Therefore, E is independent of d.

Thus, the correct mapping is $\mathrm{P} \rightarrow 5$.

Therefore, option (B) is correct.

Chapter: Gravitation

Topic: Gravitational Potential Energy

16. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2, respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

LIST-I	**LIST-II**
P. $\frac{v_1}{v_2}$	**1.** $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	**2.** 1
R. $\frac{K_1}{K_2}$	**3.** 2
S. $\frac{T_1}{T_2}$	**4.** 8

(A) $\mathrm{P} \rightarrow 4$; $\mathrm{Q} \rightarrow 2$; $\mathrm{R} \rightarrow 1$; $\mathrm{S} \rightarrow 3$

(B) $\mathrm{P} \rightarrow 3$; $\mathrm{Q} \rightarrow 2$; $\mathrm{R} \rightarrow 4$; $\mathrm{S} \rightarrow 1$

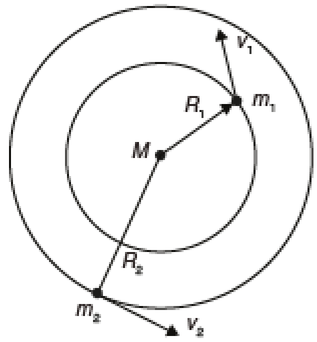
(C) $\mathrm{P} \rightarrow 2$; $\mathrm{Q} \rightarrow 3$; $\mathrm{R} \rightarrow 1$; $\mathrm{S} \rightarrow 4$

(D) $\mathrm{P} \rightarrow 2$; $\mathrm{Q} \rightarrow 3$; $\mathrm{R} \rightarrow 4$; $\mathrm{S} \rightarrow 1$

(JEE Advanced 2018 Paper-2)

Solution

(B) Given: $$\frac{m_1}{m_2} = 2 \text{ and } \frac{R_1}{R_2} = \frac{1}{4}$$



Now,

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \frac{GM}{R} = v^2 \Rightarrow v = \sqrt{\frac{GM}{R}}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{\sqrt{\frac{GM}{R_1}}}{\sqrt{\frac{GM}{R_2}}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{R_2}{R_1}}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{4}{1}} \Rightarrow \frac{v_1}{v_2} = 2$$

Thus, the correct mapping is P $\rightarrow$ 3

Now, $L = mvR$

$$\Rightarrow \frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_2 v_2 R_2} = \frac{m_1}{m_2} \times \frac{v_1}{v_2} \times \frac{R_1}{R_2}$$

$$\Rightarrow \frac{L_1}{L_2} = 2 \times 2 \times \frac{1}{4} = 1$$

Thus, the correct mapping is Q $\rightarrow$ 2 .

Now, $K = \frac{1}{2}mv^2$

$$\Rightarrow \frac{K_1}{K_2} = \frac{\frac{1}{2}m_1 v_1^2}{\frac{1}{2}m_2 v_2^2} = \frac{m_1}{m_2} \times \left(\frac{v_1}{v_2}\right)^2$$

$$\Rightarrow \frac{K_1}{K_2} = 2 \times (2)^2 = 8$$

Thus, the correct mapping is R $\rightarrow$ 4 .

Now, $T = \frac{2\pi R}{v}$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\frac{2\pi R_1}{v_1}}{\frac{2\pi R_2}{v_2}} \Rightarrow \frac{T_1}{T_2} = \frac{R_1}{R_2} \times \frac{v_2}{v_1}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{1}{4} \times \frac{1}{2} \Rightarrow \frac{T_1}{T_2} = \frac{1}{8}$$

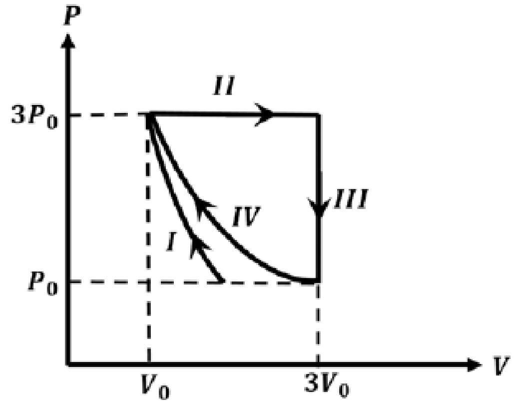
Thus, the correct mapping is $S \rightarrow 1$.

Therefore, option (B) is correct.

Chapter: Thermodynamics

Topic: Thermodynamic Processes

17. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the *PV*-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-1 with the corresponding statements in List-II.



	LIST-I	LIST-I
P.	In process I	**1.** Work done by the gas is zero
Q.	In process II	**2.** Temperature of the gas remains unchanged
R.	In process III	**3.** No heat is exchanged between the gas and its surroundings
S.	In process IV	**4.** Work done by the gas is $6P_0V_0$

(A) $P \rightarrow 4$; $Q \rightarrow 3$; $R \rightarrow 1$; $S \rightarrow 2$

(B) $P \rightarrow 1$; $Q \rightarrow 3$; $R \rightarrow 2$; $S \rightarrow 4$

(C) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 1$; $S \rightarrow 2$

(D) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 2$; $S \rightarrow 1$

(JEE Advanced 2018 Paper-2)

Solution

(C) Process I is adiabatic, therefore, $\Delta Q = 0$

$$\Rightarrow \Delta Q = \Delta U + W \Rightarrow W = -\Delta U$$

Thus, $W < 0$ since volume of gas is decreasing $\Rightarrow \Delta U > 0$

Therefore, temperature of gas increases and no heat is exchanged between gas and surroundings.

Thus, the correct mapping is $P \rightarrow 3$.

Process II is isobaric, therefore, pressure remains constant

$$\Rightarrow W = P\Delta V = 3P_0[3V_0 - V_0] = 6P_0V_0$$

Therefore, work done by gas in process II is $6P_0V_0$.

(Q $\rightarrow$ 4)

Process III is isochoric, therefore, volume remains constant

$$\Rightarrow \Delta Q = \Delta U + W \Rightarrow \Delta Q = \Delta U$$

and work done by gas is zero.

Thus, the correct mapping is R $\rightarrow$ 1

Process IV is isothermal process, therefore temperature remains constant

$$\Rightarrow \Delta Q = \Delta U + W \Rightarrow \Delta Q = W \text{ and } \Delta U = 0$$

Thus, the correct mapping is S $\rightarrow$ 2

Therefore, option (C) is correct.

Chapter: Work, Energy and Power

Topic: The Conservation of Mechanical Energy

18. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$. In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned: $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are **conserved for that path**.

LIST–I	**LIST–II**
P. $\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$	**1.** $\vec{p}$
Q. $\vec{r}(t) = \alpha \cos\omega t\hat{i} + \beta \sin\omega t\hat{j}$	**2.** $\vec{L}$
R. $\vec{r}(t) = \alpha(\cos\omega t\hat{i} + \sin\omega t\hat{j})$	**3.** K
S. $\vec{r}(t) = \alpha t\hat{i} + \dfrac{\beta}{2}t^2\hat{j}$	**4.** U
	5. E

(A) P $\rightarrow$ 1, 2, 3, 4, 5; Q $\rightarrow$ 2, 5; R $\rightarrow$ 2, 3, 4, 5; S $\rightarrow$ 5

(B) P $\rightarrow$ 1, 2, 3, 4, 5; Q $\rightarrow$ 3, 5 R $\rightarrow$ 2, 3, 4, 5; S $\rightarrow$ 2, 5

(C) P $\rightarrow$ 2, 3, 4; Q $\rightarrow$ 5; R $\rightarrow$ 1, 2, 4; S $\rightarrow$ 2, 5

(D) P $\rightarrow$ 1, 2, 3, 5; Q $\rightarrow$ 2, 5; R $\rightarrow$ 2, 3, 4, 5; S $\rightarrow$ 2, 5

(JEE Advanced 2018 Paper-2)

Solution

(A) For P:

$$\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \alpha\hat{i} + \beta\hat{j} = \text{constant}$$

$\Rightarrow \vec{p}$ is constant (conserved)

Now, $|\vec{v}| = \sqrt{\alpha^2 + \beta^2} = \text{constant}$

Therefore, $K = \frac{1}{2}mv^2$ is constant (conserved)

Now $$\vec{a}=\frac{d\vec{v}}{dt}=0 \Rightarrow \vec{F}=m\vec{a}=0$$

Thus, $F=\nabla U \Rightarrow U$ is constant (conserved)

Now, $E=U+K \Rightarrow E$ is constant, since U and k are constant

Now, $\vec{L}=m(\bar{r}\times\bar{v})=0 \Rightarrow \vec{L}$ is constant (conserved)

Thus, the correct mapping is $\text{P}\rightarrow 1,2,3,4,5$.

For Q: $$\vec{r}(t)=\alpha\cos\omega t\,\hat{i}+\beta\sin\omega t\,\hat{j}$$

$$\Rightarrow v=\frac{d\vec{r}}{dt}=-\alpha\omega\sin\omega t\,\hat{i}+\beta\omega\cos\omega t\,\hat{j}$$

So v is not a constant $\Rightarrow \vec{p}$ is not conserved for the path.

Now, $$|\vec{v}|=\sqrt{\alpha^2\omega^2\sin^2\omega t+\beta^2\omega^2\cos^2\omega t}$$

which is not constant. Therefore,

$K=\frac{1}{2}mv^2$ is not constant (not conserved)

Now, $$\vec{a}=\frac{d\vec{v}}{dt}=-\alpha\omega^2\cos\omega t\,\hat{i}+\beta\omega^2\sin\omega t\,\hat{j}$$

$\Rightarrow \vec{a}$ is not constant

$\Rightarrow \vec{F}$ is not constant

$\Rightarrow U$ is not constant (not conserved)

$E=U+K$ is constant (conserved)

Now, $$\vec{L}=m(\vec{r}\times\vec{v})=m\omega\alpha\beta\hat{k}$$

$\Rightarrow \vec{L}$ is constant (conserved)

Thus, the correct mapping is $\text{Q}\rightarrow 2,5$.

For R: $$r(t)=\alpha(\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j})$$

$$\vec{v}=\frac{d\vec{r}}{dt}=\alpha\omega(-\sin\omega t\,\hat{i}+\cos\omega t\,\hat{j})$$

$\Rightarrow \vec{v}$ is not a constant

$\Rightarrow \vec{p}$ is not a constant (not conserved)

Now $$|\vec{v}|=\sqrt{\alpha^2\omega^2(\sin^2\omega t+\cos^2\omega t)}=\alpha\omega$$

$\Rightarrow |\vec{v}|$ is constant

$\Rightarrow K=\frac{1}{2}mv^2$ is a constant (conserved)

Now $$\vec{a}=\frac{dv}{dt}=\alpha\omega^2(-\sin\omega t\,\hat{i}-\cos\omega t\,\hat{j})$$

$\Rightarrow \vec{a}$ is not constant

Since $E=K+U$ is constant (conserved)

$\Rightarrow U$ is constant (conserved)

Now $$\vec{L}=m(\vec{r}\times\vec{v})=m\omega\alpha^2\hat{k}$$

$\Rightarrow \vec{L}$ is constant (conserved)

Thus, the correct mapping is $R \to 2,3,4,5$.

For S: $r(t) = \alpha t\,\hat{i} + \frac{\beta}{2}t^2\,\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \alpha\,\hat{i} + \beta t\,\hat{j}$$

$$\Rightarrow \vec{v} \text{ is not conserved}$$

$$\Rightarrow \vec{p} \text{ is not conserved}$$

Now,

$$|\vec{v}| = \sqrt{\alpha^2 + \beta^2 t^2}$$

$$\Rightarrow |\vec{v}| \text{ is not conserved}$$

$$\Rightarrow K = \frac{1}{2}mv^2 \text{ is not conserved}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \beta\hat{j}$$

$$\Rightarrow E = U + K \text{ is conserved}$$

But, U is not conserved.

Now,

$$\vec{L} = m(\vec{r} \times \vec{v}) = \frac{1}{2}\alpha\beta t^2\hat{k}$$

$$\Rightarrow \vec{L} \text{ is not conserved}$$

Thus, the correct mapping is $S \to 5$.

Therefore, option (A) is correct.