NEET-2016

PHYSICS

1. A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of 15 ms^{-1}. Then, the frequency of sound that the observer hears in the echo reflected from the cliff is

(Take velocity of sound in air = 330 ms^{-1})

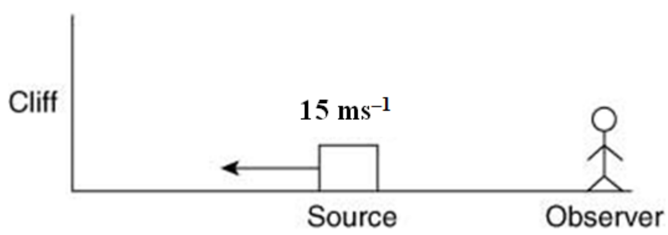
(1) 838 Hz (2) 885 Hz

(3) 765 Hz (4) 800 Hz

Solution

The frequency of the sound heard by the observer from the siren normally is

$$f' = f\left(\frac{v}{v - v_s}\right) = f\left(\frac{330}{330 - 15}\right) = 800 \times \frac{330}{315} = 838 \text{ Hz}$$



However, after the echo is reflected from the cliff, the observer hears a sound whose frequency is obtained as follows:

$$f_o = 838\left(\frac{330}{330 - 0}\right) = 838 \text{ Hz}$$

Hence, the correct option is (1).

2. Out of the following options, which one can be used to produce a propagating electromagnetic wave?

(1) A chargeless particle.

(2) A accelerating charge.

(3) A charge moving at constant velocity.

(4) A stationary charge.

Solution

- From electromagnetic theory, we know that it has been established that a charged particle at rest produces only electric field in the space surrounding it.
- If a charged particle moves with constant velocity, it produces both electric and magnetic fields, but it does not radiate energy.
- If the motion of a charged particle is accelerated, it also radiates energy in the form of electromagnetic waves.

Hence, the correct option is (2).

3. An inductor 20 mH, a capacitor 50 μF and a resistor 40 Ω are connected in series across a source of emf $V = 10\sin 340t$. The power loss in the AC circuit is

(1) 0.76 W (2) 0.89 W

(3) 0.51 W (4) 0.67 W

Solution

The electrical impedance of a series *LCR* circuit is obtained as follows:

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega c} - \omega L\right)^2}$$

$$= \sqrt{40^2 + \left[\frac{1}{340(50 \times 10^{-6})}\right] - 340(20 \times 10^{-3})^2}$$

$$= \sqrt{40^2 + (58.8 - 6.8)^2}$$

$$\Rightarrow Z = \sqrt{4304} = 65.6\,\Omega \qquad (1)$$

Now, the phase constant of the *LCR* circuit is given by

$$\tan\phi = \left[\frac{L\omega - \left(\frac{1}{\omega c}\right)}{R}\right]$$

Substituting the values, we get

$$\tan\phi = \frac{6.8 - 58.8}{40} = \frac{-52}{40} = -1.3 \qquad (2)$$

$$\Rightarrow \cos\phi = \frac{1}{\sqrt{1 + \tan^2\phi}}$$

$$\cos\phi = \frac{1}{\sqrt{1 + 1.3^2}} = 0.61 \qquad (3)$$

The power in an *LCR* circuit is given by

$$P = VI\cos\phi = \frac{V^2}{Z}\cos\phi$$

From the given source of emf, we get

$$V = \frac{10}{\sqrt{2}}$$

Substituting the values in Eq. (1), we get the power loss in the circuit as follows:

$$P = \frac{(10/\sqrt{2})^2}{65.6} \times 0.61 \approx 0.47\text{ W}$$

which is the approximate power loss in the circuit and it is closest to the option (3).

Hence, the correct option is (3).

4. Match the corresponding entries of column 1 with column 2 (where *m* is the magnification produced by the mirror):

	Column 1		Column 2
(A)	$m = -2$	(a)	Convex mirror
(B)	$m = -\frac{1}{2}$	(b)	Concave mirror
(C)	$m = +2$	(c)	Real image
(D)	$m = +\frac{1}{2}$	(d)	Virtual image

(1) A → a and d; B → b and c; C → b and d; D → b and c

(2) A → c and d; B → b and d; C → b and c; D → a and d

(3) A → b and c; B → b and c; C → b and d; D → a and d

(4) A → a and c; B → a and d; C → a and b; D → c and d

Solution

For the magnification (*m*) of the mirrors, we deduce the following cases:

- **For Convex Mirror**
  1. When $|m| < 1$, we get diminished image.
  2. When *m* is positive, we get virtual image.
- **For Concave Mirror**
  1. When $|m| < 1$, we get diminished image.
  2. When $|m| > 1$, we get enlarged image.
  3. When *m* is negative, we get real image (i.e. A→b, c).
  4. When *m* is positive, we get virtual image.

That is, for real image *m* is negative and for virtual image *m* is positive.

Hence, the correct option is (3).

5. Coefficient of linear expansion of brass and steel rods are α_1 and α_2. Lengths of brass and steel rods are l_1 and l_2, respectively. If $(l_2 - l_1)$ is maintained same at all temperatures, which one of the following relations holds good?

(1) $\alpha_1^2 l_2 = \alpha_2^2 l_1$ (2) $\alpha_1 l_1 = \alpha_2 l_2$

(3) $\alpha_1 l_2 = \alpha_2 l_1$ (4) $\alpha_1 l_2^2 = \alpha_2 l_1^2$

Solution

Since the length $l_2 - l_1$ is maintained constant at all temperatures, the rate of change in the length of both rods are equal:

$\Delta l_1 = \Delta l_2$

$\Rightarrow l_1\alpha_1\Delta T = l_2\alpha_2\Delta T$

Or, $l_1\alpha_1 = l_2\alpha_2$

Hence, the correct option is (2).

6. At what height from the surface of earth the gravitation potential and the value of *g* are -5.4×10^7 kg^{-2} and 6.0 ms^{-2}, respectively? Take the radius of earth as 6400 km.

(1) 1400 km (2) 2000 km

(3) 2600 km (4) 1600 km

Solution

The gravitational potential energy is

$$V = \frac{-GM}{R+h} = -5.4\times10^7 \text{ J/kg}^2 \quad (1)$$

and the acceleration due gravity on the surface of the earth is

$$g = \frac{GM}{(R+h)^2} = 6 \text{ m/s}^2 \quad (2)$$

Now, dividing Eq. (1) by Eq. (2), we get

$$(R+h) = \frac{5.4\times10^7}{6}$$

$\Rightarrow h = 0.9 \times 10^7 - 6400 \times 10^3 = 2600 \times 10^3$ m = 2600 km

Hence, the correct option is (3).

7. A piece of ice falls from a height h so that it melts completely. Only one-quarter of the heat produced is absorbed by the ice and all energy of ice gets converted into heat during its fall. The value of h is (latent heat of ice is 3.4×10^5 J/kg and g = 10 N/kg).

(1) 136 km (2) 68 km

(3) 34 km (4) 544 km

Solution

(1) The potential energy of the given piece of ice of mass m at height h is mgh.

Now, one-fourth of this energy is used to melt the ice. That is,

$$\frac{mgh}{4} = mL$$

where L is the latent heat of the ice. Therefore, from Eq. (1), we get the distance travelled by the ice piece till it gets completely during its fall as follows:

$$h = \frac{4L}{g} = \frac{4 \times 3.4 \times 10^5}{10} = 13.6 \times 10^4 \text{ m} = 136 \text{ km}$$

Hence, the correct option is (1).

8. In a diffraction pattern due to a single slit of width a, the first minimum is observed at an angle 30° when light of wavelength 5000 Å is incident on the slit. The first secondary maximum is observed at an angle of

(1) $\sin^{-1}\left(\frac{1}{2}\right)$ (2) $\sin^{-1}\left(\frac{3}{4}\right)$

(3) $\sin^{-1}\left(\frac{1}{4}\right)$ (4) $\sin^{-1}\left(\frac{2}{3}\right)$

Solution

For a bright fringe, we have the relation

$d\sin\theta = n\lambda$ (1)

It is given that the slit width is $d = a$.

Since it is discussed here for the first minimum, we have $n = 1$.

It is also given that wavelength of the light is λ = 5000 Å.

The first minimum is observed at an angle θ = 30°.

On substituting the values in Eq. (1), we get

$a\sin 30° = 5000 \times 10^{-10}$

$a = 2 \times 5000 \times 10^{-10}$ (2)

For nth secondary maximum, we have

$$d\sin\theta = \left(n + \frac{1}{2}\right)\lambda$$

Therefore, for the first secondary maximum, we get

$$d\sin\theta = \left(1 + \frac{1}{2}\right)\lambda = \frac{3}{2}\lambda$$

$$\sin\theta = \frac{(3/2)\lambda}{a} = \frac{(3/2)\times 5000\times 10^{-10}}{2\times 5000\times 10^{-10}} = \frac{3}{4}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{3}{4}\right)$$

Hence, the correct option is (2).

9. A potentiometer wire is 100 cm long and a constant potential difference is maintained across it. Two cells are connected in series first to support one another and then in opposite direction. The balance points are obtained at 50 cm and 10 cm from the positive end of the wire in the two cases. The ratio of emf's is

(1) 3:4 (2) 3:2

(3) 5:1 (4) 5:4

Solution

Let E_1 be the emf of the first cell and E_2 be the emf of second cell.

Since the balance points are obtained at 50 cm and 10 cm from the positive end of the wire in the two case, we have

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{50}{10}$$

$\Rightarrow E_1 + E_2 = 5E_1 - 5E_2$

$\Rightarrow 6E_2 = 4E_1$

Therefore, the ratio of the emf's, E_1 and E_2, of the cells is

$$\frac{E_1}{E_2} = \frac{3}{2}$$

Hence, the correct option is (2).

10. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion?

(1) 0.18 m/s^2 (2) 0.2 m/s^2

(3) 0.1 m/s^2 (4) 0.15 m/s^2

Solution

After starting the motion, at the end of the second revolution, the kinetic energy of the particle is

$$\frac{1}{2}mv^2 = 8\times 10^{-4} \text{ J}$$

$$\Rightarrow v^2 = \frac{2\times 8\times 10^{-4}}{10\times 10^{-3}} = \frac{1.6}{10} = \frac{16}{100}$$

$$\Rightarrow v = \frac{4}{10} = 0.4 \text{ m/s}$$

Let S be the distance covered in two revolutions. Then we have

$S = 2\pi R \times 2$

$\quad = 4\pi R = 4 \times 3.14 \times 6.4 \times 10^{-2}$ m

Let the initial speed of the particle be $u = 0$.

On substituting $u = 0$ in the equation of motion, $v^2 = u^2 + 2aS$, we get

$v^2 = 2aS$

$$\Rightarrow a = \frac{v^2}{2S} = \frac{(0.4)^2}{2(4 \times 3.14 \times 6.4 \times 10^{-2})} = 0.1 \text{ m/s}^2$$

Hence, the correct option is (3).

11. An air column, closed at one end and open at the other, resonates with a tuning fork when the smallest length of the column is 50 cm. The next larger length of the column resonating with the same tuning fork is

(1) 150 cm (2) 200 cm

(3) 66.7 cm (4) 100 cm

Solution

For n nodes of antinodes, the length of tube (l) closed at one end of the air column is given by

$$l = (2n-1)\frac{\lambda}{4} \quad (1)$$

- For n = 1: For the smallest length of the air column, we have

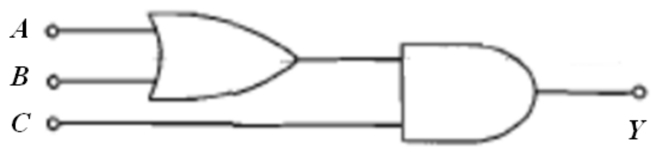
$$l = \frac{\lambda}{4} = 50 \text{ cm}$$

- For n = 2: For the next larger air column, from Eq. (1), we have

$$l = (2 \times 2 - 1)\frac{\lambda}{4} = \frac{3\lambda}{4} = 150 \text{ cm}$$

Hence, the correct option is (1).

12. To get output 1 for the following circuit, the correct choice for the input is



(1) A = 1, B = 1, C = 0 (2) A = 1, B = 0, C = 1

(3) A = 0, B = 1, C = 0 (4) A = 1, B = 0, C = 0

Solution

For the given logic gate circuit, the correct logic equation is

$(A + B) \cdot C = 1$

The truth table for this logic equation is listed as follows:

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

From this truth table, we find that when the inputs A = 1, B = 0 and C = 1, the output is 1; therefore, option (2) is the correct one.

Hence, the correct option is (2).

13. A gas is compressed isothermally to half its initial volume. The same gas is compressed separately through an adiabatic process until its volume is again reduced to half. Then

(1) Compressing the gas isothermally or adiabatically will require the same amount of work.

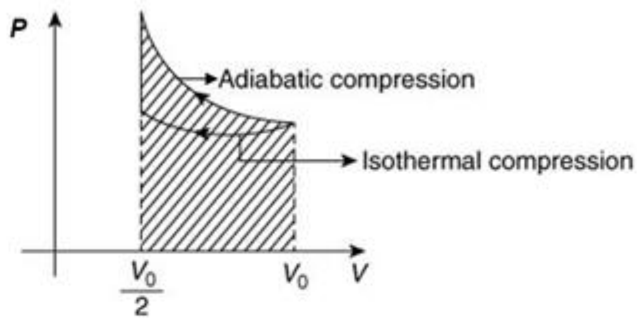
(2) Which of the case (whether compression through isothermal or through adiabatic process) requires more work will depend upon the atomicity of the gas.

(3) Compressing the gas isothermally will require more work to be done.

(4) Compressing the gas through adiabatic process will require more work to be done.

Solution

The work done on the gas to compress it from initial volume V_0 to $\frac{V_0}{2}$ = Area under the curve.



Therefore, compressing the gas through adiabatic process will require more work to be done:

$W_{\text{adiabatic}} > W_{\text{isothermal}}$

Hence, the correct option is (4).

14. The intensity at the maximum in a Young's double-slit experiment is I_0. Distance between two slits is $d = 5\lambda$, where λ is the wavelength of light used in the experiment. What will be the intensity in front of one of the slits on the screen placed at a distance $D = 10d$?

(1) $\frac{3}{4}I_0$ (2) $\frac{I_0}{2}$

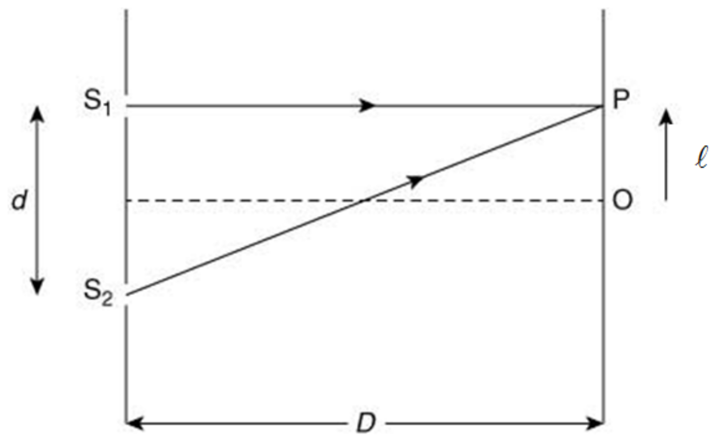
(3) I_0 (4) $\frac{I_0}{4}$

Solution

See the following figure. The path length difference at point P is

$$\Delta l = S_2P - S_1P = \frac{xd}{D}$$

$$\Delta l = = \frac{(d/2)d}{10d} = \frac{d}{20} \quad (1)$$



It is given that $d = 5\lambda$, wavelength of light used in the experiment. This implies, from Eq. (1), that

$$\Delta l = \frac{\lambda}{4}$$

Now, the phase difference is calculated as follows:

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

The intensity in front of one of the slits on the screen placed at a distance $D = 10d$ is obtained as follows:

$$I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right) = I_0 \cos^2 \frac{\pi}{4} = \frac{I_0}{2}$$

Hence, the correct option is (2).

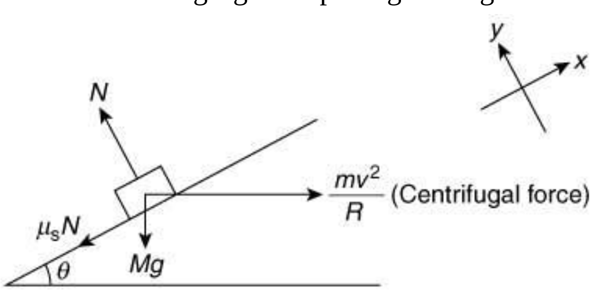
15. A car is negotiating a curved road of radius R. The road is banked at an angle θ. The coefficient of friction between the tyres of the car and the road is μ_s. The maximum safe velocity on this road is

(1) $\sqrt{\frac{g}{R}\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$ (2) $\sqrt{\frac{g}{R^2}\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$

(3) $\sqrt{gR^2\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$ (4) $\sqrt{gR\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$

Solution

See the following figure depicting the negotiation of the car on the road which is banked at an angle θ.



On applying force balance in x and y directions, we get

$$N = mg\cos\theta + \frac{mv^2}{R}\sin\theta \qquad (1)$$

For the maximum velocity, we get the maximum safe velocity of the car on this road as follows:

$$\frac{mv^2}{R} = mg\sin\theta + \mu_s N \quad (2)$$

$$= mg\sin\theta + \mu_s mg\cos\theta + \mu_s \frac{mv^2}{R}\sin\theta$$

$$\Rightarrow \frac{v^2}{R} - \mu_s \frac{v^2}{R}\sin\theta = g\sin\theta + \mu_s g\cos\theta$$

$$\Rightarrow v = \sqrt{Rg\left[\frac{\sin\theta + \mu_s\cos\theta}{1-\mu_s\sin\theta}\right]} = \sqrt{Rg\left[\frac{\tan\theta + \mu_s}{1-\mu_s\tan\theta}\right]}$$

Hence, the correct option is (4).

16. An electron of mass *m* and a photon have same energy *E*. The ratio of de Broglie wavelengths associated with them is

(1) $c(2mE)^{1/2}$ (2) $\frac{1}{c}\left(\frac{2m}{E}\right)^{1/2}$

(3) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$ (4) $\left(\frac{E}{2m}\right)^{1/2}$

(*c* being velocity of light)

Solution

The de Broglie wavelength of an electron is

$$\lambda_{\text{electron}} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

Where, $E = \frac{1}{2}mv^2$

The de Broglie wavelength of a photon is

$$E = \frac{hc}{\lambda} \Rightarrow \lambda_{\text{photon}} = \frac{hc}{E}$$

Therefore, the ratio of de Broglie wavelengths associated with electron and photon is

$$\frac{\lambda_{\text{electron}}}{\lambda_{\text{photon}}} = \frac{E}{\sqrt{2mE}\cdot c} = \frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$$

Hence, the correct option is (3).

17. A black body is at a temperature of 5760 K. The energy of radiation emitted by the body at wavelength 250 nm is U_1, at wavelength 500 nm is U_2 and that at 1000 nm is U_3. Wien's constant, $b = 2.88 \times 10^6$ nmK. Which of the following is correct?

(1) $U_1 > U_2$ (2) $U_2 > U_1$

(3) $U_1 = 0$ (4) $U_3 = 0$

Solution

Wien's wavelength–temperature displacement law is given by

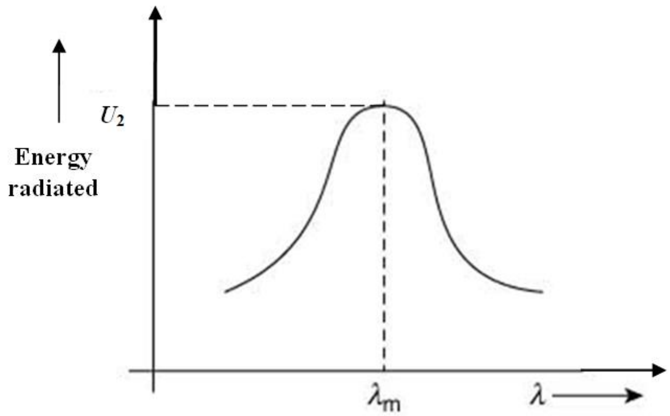
$$\frac{\lambda}{T} = \text{Constant} \quad (1)$$

Let λ_m be the wavelength at which the emissive power is the maximum. Then by Eq. (1), we get

$$\lambda_m T = b$$

where b is Wien's constant. Therefore, the wavelength corresponding to maximum energy (see the figure shown below) at a particular temperature is

$$\lambda_m = \frac{b}{T} = \frac{2.88\times 10^6 \text{ nmK}}{5760 \text{ K}} = 500 \text{ nm}$$



Since we get λ_m = 500 nm, the energy U_2 of radiation emitted by the body is associated with λ_m.

That is, the energy U_2 greater than both energy U_1 and energy U_3.

Hence, the correct option is (2).

18. Given the value of Rydberg constant is 10^7 m^{-1}, the wave number of the last line of the Balmer series in hydrogen spectrum will be

(1) 0.25×10^7 m^{-1} (2) 2.5×10^7 m^{-1}

(3) 0.025×10^4 m^{-1} (4) 0.5×10^7 m^{-1}

Solution

The wavenumber of spectral lines is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For Balmer Series, $n_1 = 2$. Therefore,

$$\frac{1}{\lambda} = 10^7 \times 1^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = 0.25 \times 10^7 \text{ m}^{-1}$$

Hence, the correct option is (1).

19. An *npn*-transistor is connected in common-emitter configuration in a given amplifier. A load resistance of 800 Ω is connected in the collector circuit and the voltage drop across it is 0.8 V. If the current amplification factor is 0.96 and the input resistance of the circuits is 192 Ω, the voltage gain and the power gain of the amplifier will, respectively, be

(1) 4, 4 (2) 4, 3.69

(3) 4, 3.84 (4) 3.69, 3.84

Solution

It is given that the current amplification factor is $\beta = 0.96$; load resistance is $R_L = 800$ Ω; input resistance is $R_{input} = 192$ Ω.

- The voltage gain of the given common-emitter amplifier is

$$A_v = \beta\left(\frac{R_L}{R_{input}}\right) = 0.96 \times \frac{800}{192} = 4$$

- The power gain of the given common-emitter amplifier is

$$A_v \times \beta_{ac} = 4 \times 0.96 = 3.84$$

where A_v is the voltage gain and β_{ac} is the current gain of the configuration.

Hence, the correct option is (3).

20. Two non-mixing liquids of densities ρ and $n\rho$ ($n > 1$) are put in a container. The height of each liquid is h. A solid cylinder of length L and density d is put in this container. The cylinder floats with its axis vertical and length pL ($p < 1$) in the denser liquid. The density d is equal to

(1) $[2 + (n – 1)p]\rho$ (2) $[1 + (n – 1)p]\rho$

(3) $[1 + (n + 1)p]\rho$ (4) $[2 + (n + 1)p]\rho$

Solution

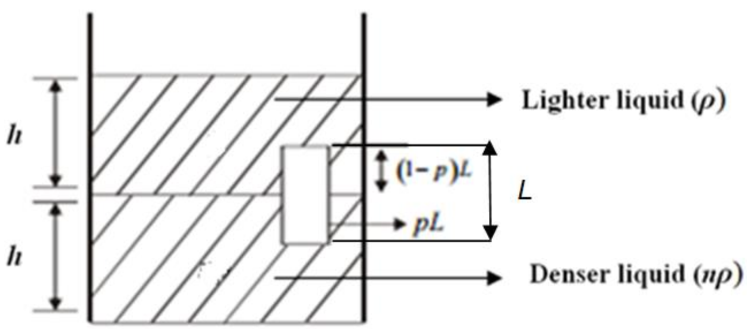
See the following figure. For balancing the forces, we relate the weight of the cylinder and the buoyant force as follows:

Weight = Buoyant force

$(LA)dg = (pL)(np)g + (1 – p)LApg$

$d = pnp + (1 – p)p$

$d = pp(n – 1) + p = p[p(n – 1 + 1]$



Hence, the correct option is (2).

21. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1 s and 2 s is

(1) $\frac{3}{2}A + \frac{7}{3}B$ (2) $\frac{A}{2} + \frac{B}{3}$

(3) $\frac{3}{2}A + 4B$ (4) $3A + 7B$

Solution

From the given velocity equation of the particle, we can deduce the distance travelled by it as follows:

$$\frac{dx}{dt} = At + Bt^2$$

$$\Rightarrow \int_0^x dx = \int_1^2 (At + Bt)^2 dt$$

$$\Rightarrow x = \left[\frac{At^2}{2} + \frac{Bt^3}{3}\right]_1^2 = \frac{A}{2}(4-1) + \frac{B}{3}(8-1) = \frac{3}{2}A + \frac{7}{3}B$$

Hence, the correct option is (1).

22. A astronomical telescope has objective and eyepiece of focal lengths 40 cm and 4 cm, respectively. To view an object 200 cm away from the objective, the lenses must be separated by a distance

(1) 50.0 cm (2) 54.0 cm

(3) 37.3 cm (4) 46.0 cm

Solution

The ray diagram for the situation is as shown in the following figure. Using lens formula for the objective, we get

$$\frac{1}{v_o} - \frac{1}{u} = \frac{1}{f_e}$$

$$\frac{1}{v_o} - \frac{1}{-200} = \frac{1}{40}$$

Here, –200 is used since due to viewing the object away from the objective. Therefore,

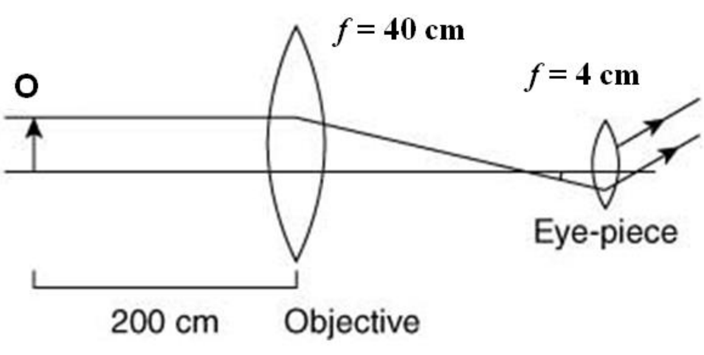
$$\frac{1}{v_o} = \frac{1}{40} - \frac{1}{200} = \frac{1}{50}$$

$\Rightarrow v_o = +50$ cm

Now, the image formed by the objective should lie at the focal length of the eye-piece so that the final image is formed at infinity.

Therefore, the distance between objective and the eye-piece is

$v_o + f_e = 50 + 4 = 54$ cm



Hence, the correct option is (2).

23. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is

(1) 1 : 4 (2) $1:\sqrt{2}$

(3) 1 : 2 (4) $1:2\sqrt{2}$

Solution

The escape velocity is given by the relation

$$v_e = \sqrt{\frac{2GM}{R}}$$

- **Escape velocity for earth:**

$$v_e = \sqrt{\frac{2G}{R}\left(\frac{4}{3}\pi R^3 p\right)}$$

$$v_e = \sqrt{\frac{8}{3}G\pi R^2 p} \qquad (1)$$

- **Escape velocity for other planet:**

$$v_p = \sqrt{\frac{2G}{(2R)}\left(\frac{4}{3}\pi (2R)^3 2p\right)}$$

The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is

$$\frac{v_e}{v_p} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

Hence, the correct option is (4).

24. A long straight wire of radius a carries a steady current I. The current is uniformly distributed over its cross-section. The ratio of the magnetic fields B and B', at a radical distances $\frac{a}{2}$ and $2a$, respectively, from the axis of the wire is

(1) 1 (2) 4

(3) $\frac{1}{4}$ (4) $\frac{1}{2}$

Solution

The magnetic field is $B = \frac{\mu_0 I}{2\pi r}$, where r is the distance from the wire due to infinite wire and I is the current enclosed. Therefore,

$$B' = B_{2a} = \frac{\mu_0 I}{2\pi(2a)} = \frac{\mu_0 I}{4\pi a} \qquad (1)$$

and $$B = B_{a/2} = \frac{\mu_0\left[I\frac{\pi(a/2)^2}{\pi a^2}\right]}{2\pi(a/2)} = \frac{\mu_0\left(\frac{I}{4}\right)}{\pi a}$$

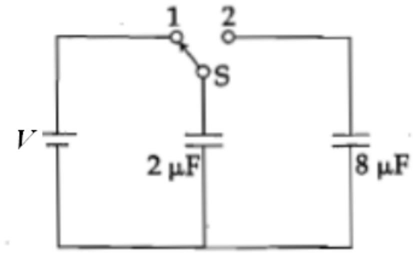
That is,

$$B_{a/2} = = \frac{\mu_0 I}{4\pi a} \qquad (2)$$

Since the value of Eq. (1) and (2) are the same, the ratio of the magnetic fields B and B', at a radical distances $\frac{a}{2}$ and $2a$, respectively, from the axis of the wire is 1.

Hence, the correct option is (1).

25. A capacitor of 2 μF is charged as shown in the diagram. When the switch S is turned to position 2, the percentage of its stored energy dissipated is



(1) 75%   (2) 80%

(3) 0%   (4) 20%

Solution

The initial energy stored in the 2 μF capacitor is

$$U_1 = \frac{1}{2}C_1V^2 = \frac{1}{2}\times 2V^2 = V^2 \quad (1)$$

When switch is changed to position 2, there will be redistribution of charges such that the total charge is conserved.

That is, let the final voltage across each capacitor be V'. Then

$$\underset{\text{Initial voltage}}{C_1V} = \underbrace{C_1V' + C_2V'}_{\text{Final voltage}}$$

$$\Rightarrow V' = \frac{C_1V}{C_1+C_2} = \frac{2V}{2+8} = \frac{V}{5} \quad (2)$$

Therefore, the final energy stored in the capacitor is

$$U_2 = \frac{1}{2}C_1V'^2 + \frac{1}{2}C_2V'^2$$

$$U_2 = \frac{V^2}{25} + \frac{4V^2}{25} = \frac{V^2}{5} \quad (3)$$

Therefore, the percentage of the stored energy dissipated is obtained as follows:

$$\frac{V^2 - (V^2/5)}{V^2}\times 100 = 80\%$$

Hence, the correct option is (2).

26. When a metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V. If the same surface is illuminated with radiation of wavelength 2λ, the stopping potential is $\frac{V}{4}$. The threshold wavelength for the metallic surface is

(1) $\frac{5}{2}\lambda$ (2) 3λ

(3) 4λ (4) 5λ

Solution

Using Einstein's photoelectric equation, we have

$$\frac{hc}{\lambda} = eV_0 + \phi_0 \quad (1)$$

where ϕ_0 is the work function

$$\frac{hc}{2\lambda} = \frac{eV_0}{4} + \phi_0 \quad (2)$$

Since we have been given the condition that the surface is illuminated with radiation of wavelength 2λ and the stopping potential is $V/4$, we have 4 × Eq. (2) – Eq. (1), which gives

$$\frac{2h}{\lambda} - \frac{hc}{\lambda} = 3\phi_0 = \frac{3hc}{\lambda_0}$$

$\Rightarrow \lambda_0 = 3\lambda$

Hence, the correct option is (2).

27. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is

(1) 45° (2) 180°

(3) 0° (4) 90°

Solution

Let the two vectors be $\vec{A}$ and $\vec{B}$.

$$\left|\vec{A} + \vec{B}\right| = \left|\vec{A} - \vec{B}\right|$$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$\Rightarrow \cos\theta = 0$

which is obvious that the angle between the two vectors is $\theta = 90°$.

Hence, the correct option is (4).

28. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (2t\,\hat{i} + 3t^2\,\hat{j})\,\text{N}$, where $\hat{i}$ and $\hat{j}$ are the unit vectors along *x*-axis and *y*-axis, respectively. What power will be developed by the force at the time *t*?

(1) $(2t^3 + 3t^4)$ W (2) $(2t^3 + 3t^5)$ W

(3) $(2t^2 + 3t^3)$ W (4) $(2t^2 + 4t^4)$ W

Solution

The force is related to mass and acceleration, which is again related as follows:

$$F = m\frac{d\vec{v}}{dt} = (2t\hat{i} + 3t^2\hat{j})\ \text{N}$$

$$\Rightarrow \int_0^{\vec{v}} d\vec{v} = \int_0^t (2t\hat{i} + 3t^2\hat{j})dt \quad (1)$$

It is given that $m = 1$ kg. Therefore, from Eq. (1), we get

$$\vec{v} = t^2\hat{i} + t^3\hat{j} \quad (2)$$

Therefore, the power developed by the force at the time *t* is

$$P = \vec{F}\cdot\vec{v} = (2t\hat{i} + 3t^2\hat{j})\cdot(t^2\hat{i} + t^3\hat{j}) = (2t^3 + 3t^5)\ \text{W}$$

Hence, the correct option is (2).

29. The angle of incidence for a ray of light at a refracting surface of a prism is 45°. The angle of prism is 60°. If the ray suffers minimum deviation through the prism, the angle of minimum deviation and refractive index of the material of the prism, respectively, are

(1) $45°; \sqrt{2}$ (2) $30°; \frac{1}{\sqrt{2}}$

(3) $45°; \frac{1}{\sqrt{2}}$ (4) $30°; \sqrt{2}$

Solution

The angle of minimum deviation of the material of the given prism is

$\delta_{\min} = 2i - A = 2 \times 45° - 60° = 30°$

From this, we can calculate the refractive index of the material of the given prism as follows:

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{60°+30°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 45°}{\sin 60°} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}$$

Hence, the correct option is (4).

30. A particle moves so that its position vector is given by $\vec{r} = \cos\omega\hat{x} + \sin\omega t\hat{y},$ where ω is a constant.

Which of the following is true?

(1) Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin.

(2) Velocity is perpendicular to $\vec{r}$ and acceleration is directed from the origin.

(3) Velocity and acceleration both are perpendicular to $\vec{r}$.

(4) Velocity and acceleration both are parallel to $\vec{r}$.

Solution

It is given that

$\vec{r} = \cos\omega t\ \hat{x} + \sin\omega t\ \hat{y}$

Therefore, from this, we can calculate the velocity:

$$\vec{v} = \frac{d\vec{r}}{dt} = -\omega\sin\omega t\ \hat{x} + \omega\cos\omega t\ \hat{y}$$

Now, $\vec{r}\cdot\vec{v} = 0$. This implies that the velocity is perpendicular to $\vec{r}$:

$\vec{v} \perp \vec{r}$ (1)

Also, the acceleration is calculated as follows;

$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2\cos\omega t\ \hat{x} - \omega^2\sin\omega t\ \hat{y}$$

$\vec{a} = -\omega\vec{r}$ (2)

From Eqs. (1) and (2), it is clear that the statements in option (2) is the correct one.

Hence, the correct option is (2).

31. Consider the junction diode as ideal. The value of current flowing through AB is

(1) 10^{-1} A (2) 10^{-3} A

(3) 0 A (4) 10^{-2} A

Solution

The diode shown in the circuit is forward biased. Therefore, there is existence of current flow in the circuit. Now, the current flowing through AB is

$$I = \frac{V_{AB}}{R} = \frac{4-(-6)}{1000} = \frac{10}{1000} = \frac{1}{100} = 10^{-2}\,\text{A}$$

Hence, the correct option is (4).

32. Two identical charged spheres suspended from a common point by two massless strings of lengths *l*, are initially at a distance *d* (*d* << *l*) apart because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity *v*. The *v* varies as a function of the distance *x* between the spheres, as

(1) $v \propto x^{-1/2}$ (2) $v \propto x^{-1}$

(3) $v \propto x^{-1/2}$ (4) $v \propto x$

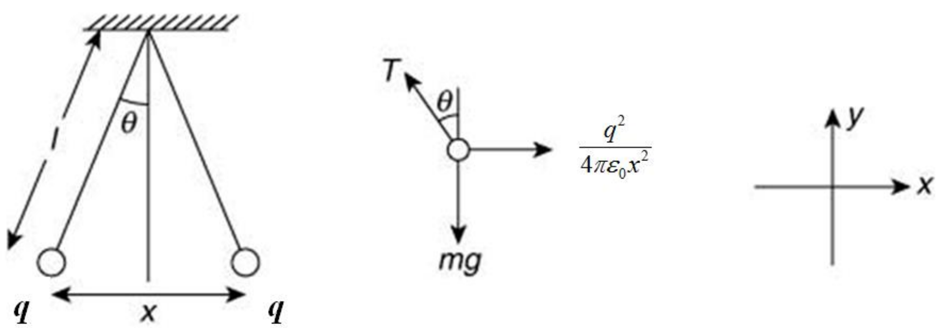
Solution

See the following figures. The balancing forces in *x* and *y* directions, respectively, are

$T\cos\theta = mg$ (1)

$T\sin\theta = \dfrac{9^2}{4\pi\varepsilon_0 x^2}$ (2)

$$\Rightarrow mg\tan\theta = \frac{q^2}{4\pi\varepsilon_0 x^2}$$



- For a small θ: $\tan\theta \approx \theta = \dfrac{x}{2l}$. Therefore,

$$mg\frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 x^2}$$

$$\Rightarrow x^3 \propto q^2$$

or $q \propto x^{3/2}$ (3)

It is given that $\dfrac{dq}{dt} = \text{Constant}$. Therefore,

$\frac{3}{2}x^{1/2}\frac{dx}{dt} = \text{Constant}$ [from Eq. (3)]

$\Rightarrow x^{1/2}v = \text{Constant}$

or $v \propto \frac{1}{x^{1/2}}$

Hence, the correct option is (1).

33. A small signal voltage $V(t) = V_0\sin\omega t$ is applied across an ideal capacitor C:

(1) Current $I(t)$ is in phase with voltage $V(t)$.

(2) Current $I(t)$ leads voltage $V(t)$ by 180°.

(3) Current $I(t)$ lags voltage $V(t)$ by 90°.

(4) Over a full cycle, the capacitor C does not consume any energy from the voltage source.

Solution

We know that in a pure capacitive circuit, the current leads the potential difference by 90°. Therefore,

$P = VI\cos\phi = 0$ $(\because \cos 90 = 0)$

Hence, it is concluded that over a full cycle, the capacitor C does not consume any energy from the voltage source.

Hence, the correct option is (4).

34. The magnetic susceptibility is negative for

(1) ferromagnetic material only.

(2) paramagnetic and ferromagnetic materials.

(3) diamagnetic material only.

(4) paramagnetic material only.

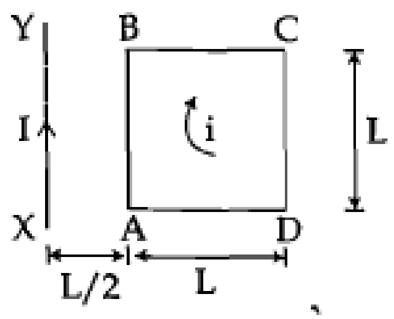
Solution

The magnetic susceptibility χ is the degree of magnetization of a material with respect to the applied magnetic field.

- For paramagnetic, ferromagnetic or anti-ferromagnetic materials, the magnetic susceptibility is positive.
- The magnetic susceptibility is negative only for diamagnetic materials.

Hence, the correct option is (3.

35. A square loop ABCD carrying a current i is placed near and coplanar with a long straight conductor XY carrying a current I, the net force on the loop will be



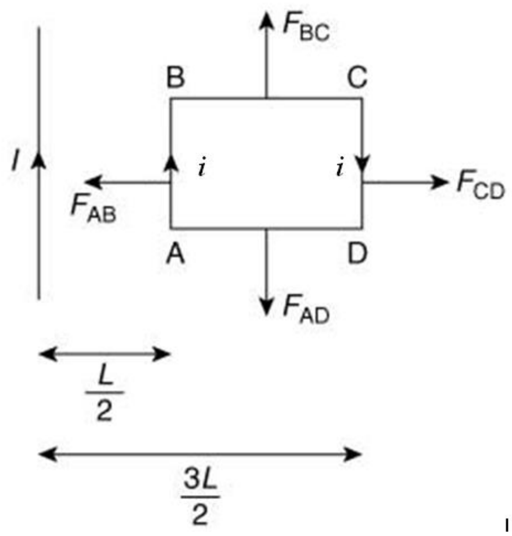
(1) $\frac{2\mu_0 IiL}{3\pi}$ (2) $\frac{\mu_0 IiL}{2\pi}$

(3) $\frac{2\mu_0 Ii}{3\pi}$ (4) $\frac{\mu_0 Ii}{2\pi}$

Solution

The force between two parallel wires (separated by distance r) carrying currents I_1 and I_2, respectively, is expressed as

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} \quad (1)$$



As seen from the figure, $F_{BC} = F_{AD}$ and they cancel each other. Therefore, using Eq. (1), the net force on the loop can be determined as follows:

$$F_{net} = \frac{\mu_0 Ii}{2\pi}\left(\frac{1}{L/2} - \frac{1}{3L/2}\right) = \frac{\mu_0 Ii}{\pi L}\left(\frac{2}{3}\right)$$

Hence, the correct option is (3).

36. A uniform rope of length L and mass m_1 hangs vertically from a rigid support. A block of mass m_2 is attached to the free end of the rope. A transverse pulse of wavelength λ_1 is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is λ_2. The ratio λ_2/λ_1 is

(1) $\sqrt{\frac{m_2}{m_1}}$ (2) $\sqrt{\frac{m_1 + m_2}{m_1}}$

(3) $\sqrt{\frac{m_1}{m_2}}$ (4) $\sqrt{\frac{m_1 + m_2}{m_2}}$

Solution

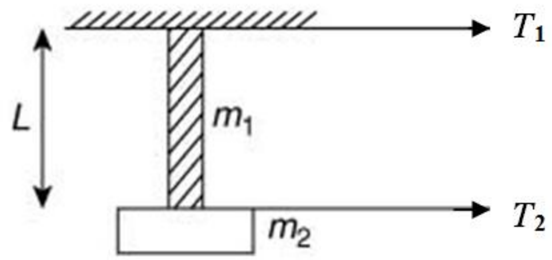
The velocity of transverse waves propagating along a string, which has tension T and mass m per unit length, is given by

$$v = \sqrt{\frac{T}{m}}$$

Since the wavelength is directly proportional to the velocity, we can express as $\lambda \propto v$.

Therefore, the wavelength is also directly proportional to the tension produced in the rope:

$$\lambda \propto \sqrt{T}$$



Now, $T_2 = (m_1 + m_2)g$ (1)

and $T_1 = mg$ (2)

Therefore, the ratio λ_2/λ_1 is obtained as follows:

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{m_1 + m_2}{m_2}}$$

Hence, the correct option is (4).

37. When an α-particle of mass m moving with velocity v bombards on a heavy nucleus of charge Ze, its distance of closest approach from the nucleus depends on m as

(1) $\frac{1}{m^2}$ (2) m

(3) $\frac{1}{m}$ (4) $\frac{1}{\sqrt{m}}$

Solution

At the distance of closest approach (d) from the nucleus (of charge Q), the initial kinetic energy of the α-particle (of charge q) gets completely converted into electrostatic potential energy:

$$\frac{1}{2}mv^2 = \frac{Qq}{4\pi\varepsilon_0 d}$$

$$\Rightarrow d \propto \frac{1}{m}$$

Hence, the correct option is (3).

38. A disc and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first?

(1) Both reach at the same time.

(2) Depends on their masses.

(3) Disc.

(4) Sphere.

Solution

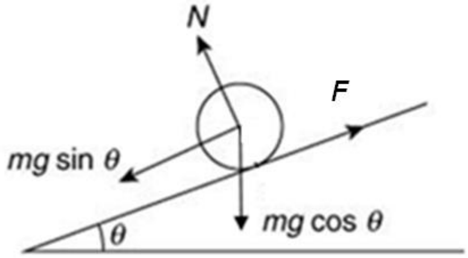
For a linear motion which should be parallel to the plane, we have

$mg\sin\theta - F = ma$ (1)

For the rotation around the axis through the centre of mass, we know that the

$\tau = I\alpha \;\Rightarrow\; FR = (mK^2)\alpha$ (2)

where K is the radius of gyration.



For no-slip condition, we know that

$V = R\omega$ and $a = R\alpha$ (3)

Solving the above equations, we can get

$$a = \frac{g\sin\theta}{1+(k^2/R^2)}$$ (4)

- **For disk:** The moment of inertia is

  $$I = \frac{1}{2}mR^2$$

  Therefore, $$\frac{k^2}{R^2} = \frac{1}{2} = 0.5$$

  Using this value in Eq. (3), we get the acceleration of the disc as follows:

  $$a = \frac{g\sin\theta}{1.5}$$

- **For sphere:** The moment of inertia is

  $$I = \frac{2}{5}mR^2 \Rightarrow \frac{k^2}{R^2} = \frac{2}{5} = 0.4$$

  Therefore, the acceleration of the sphere is

  $$a = \frac{g\sin\theta}{1.4}$$

  Therefore, the acceleration sphere is greater than the acceleration of the disc:

  $a_{\text{sphere}} > a_{\text{disc}}$

  Hence, we conclude that the sphere gets to the bottom of the plane first.

Hence, the correct option is (4).

39. From a disc of radius *R* and mass *M*, a circular hole of diameter *R*, whose rim passes through the centre, is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis passing through the centre?

(1) $11MR^2/32$ (2) $9\ MR^2/32$

(3) $15\ MR^2/32$ (4) $13\ MR^2/32$

Solution

The moment of inertia of the disc is

$$I_{\text{disc}} = \frac{1}{2}MR^2$$

The mass of the circular portion which is removed from the bigger disc is

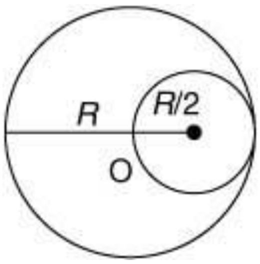
$$\frac{m\pi(R/2)^2}{\pi R^2} = \frac{m}{4}$$

The moment of inertia of the circular portion (whose rim passes through the centre of mass) which is removed from the bigger disc is

$$I_1 = \frac{1}{2}\left(\frac{m}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{32}MR^2$$

The moment of inertia of the removed disc about the centre (O) of the disc is

$$I_2 = \frac{1}{32}mR^2 + \left(\frac{m}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{3}{2}MR^2$$



Now, using parallel axis theorem, the moment of inertia of the remaining part of the disc about a perpendicular axis passing through the centre of the disc is calculated as follows:

$$I_1 = I_{\text{disc}} - I_{\text{removed disc about O}}$$
$$= \frac{1}{2}mR^2 - \frac{3}{2}mR^2$$
$$= \frac{1}{2}mR^2\left(1 - \frac{3}{16}\right) = \frac{13}{32}MR^2$$

Hence, the correct option is (4).

40. A long solenoid has 1000 turns. When a current of 4 A flows through it, the magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is

(1) 2 H (2) 1 H

(3) 4 H (4) 3 H

Solution

The total flux linked with the solenoid is

$$\phi_{\text{total}} = 1000(4\times10^{-3})\text{ Wb} = 4\text{ Wb}$$

$$\phi_{\text{total}} = LI$$

Therefore, $L = \frac{4\text{ Wb}}{4\text{ A}} = 1\text{ H}$

Hence, the correct option is (2).

41. What is the minimum velocity with which a body of mass *m* must enter a vertical loop of a radius *R* so that it can complete the loop?

(1) $\sqrt{3gR}$ (2) $\sqrt{5gR}$

(3) $\sqrt{gR}$ (4) $\sqrt{2gR}$

Solution

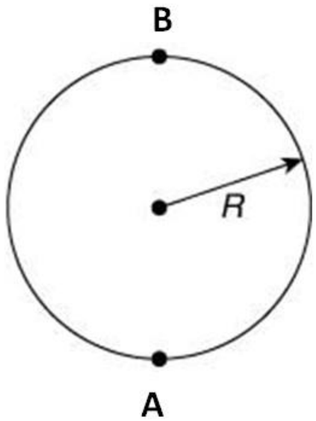
Let us consider the extreme case when the body enters the loop at the lowest point (point A) of the loop.

Now, going from point A to point B, we have the following:

Loss in K.E. = Gain in P.E.

$$\frac{1}{2}m(V_A^2 - V_B^2) = mg2R$$

$$V_A^2 = V_B^2 + 4gR \qquad (1)$$



Now, for the body to complete the loop of radius *R*, the centrifugal force on the body at point B must be greater than or equal to its weight. That is,

$$\frac{mv_B^2}{R} \geq mg$$

Or, $$V_B^2 \geq gR \qquad (2)$$

From Eqs. (1) and (2), we get

$$V_B^2 \geq gR + 4gR$$

or $$V_A \geq \sqrt{5gR}$$

Hence, the correct option is (2).

42. The molecules of a given mass of a gas have rms velocity of 200 ms^{-1} at 27°C and 1.0×10^5 Nm^{-2} pressure. When the temperature and pressure of the gas are, respectively, 127°C and 0.05×10^5 Nm^{-2}, the rms velocity of its molecules in ms^{-1} is

(1) $\frac{100\sqrt{2}}{3}$ (2) $\frac{100}{3}$

(3) $100\sqrt{2}$ (4) $\frac{400}{\sqrt{3}}$

Solution

The root mean square speed (rms) of the molecules in a gas is given by

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Therefore, $$V_{rms} \propto \sqrt{T}$$

$$\Rightarrow \frac{V_{127°C}}{V_{27°C}} = \sqrt{\frac{(127+273)}{(27+273)}} = \sqrt{\frac{4}{3}}$$

$$\Rightarrow V_{127°C} = 200 \text{ m/s} \sqrt{\frac{4}{3}} = \frac{400}{\sqrt{3}} \text{ m/s}$$

Hence, the correct option is (4).

43. The charge flowing through a resistance R varies with time t as $Q = at - bt^2$, where a and b are positive constants. The total heat produced in R is

(1) $\frac{a^3R}{2b}$ (2) $\frac{a^3R}{b}$

(3) $\frac{a^3R}{6b}$ (4) $\frac{a^3R}{3b}$

Solution

The current can be obtained as

$$I = \frac{dQ}{dt} = a - 2bt \quad (1)$$

For $I = 0$, we get $a - 2bt = 0$. Therefore,

$$t = \frac{1}{2b} \quad (2)$$

The rate of heat produced in the resistance is

$$\frac{dH}{dt} = I^2R = (a-2bt)^2R$$

Thus, the total heat produced in the resistance is

$$\int_0^H H = \int_0^{a/2b} (a-2bt)^2 Rdt$$

$$\Rightarrow H = \left[\frac{(a-2bt)^3R}{-3(2b)}\right]_0^{a/2b} = \frac{a^3R}{6b}$$

Hence, the correct option is (3).

44. A refrigerator works between 4°C and 30°C. It is required to remove 600 calories of heat every second in order to keep the temperature of the refrigerated space constant. The power required is

(Take 1 cal = 4.2 J)

(1) 236.5 W (2) 2365 W

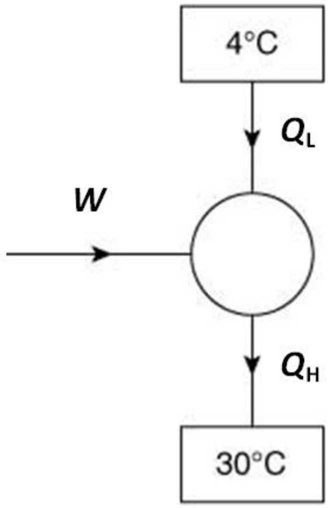
(3) 2.365 W (4) 23.65 W

Solution

For Carnot cycle, we have

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \quad (1)$$

where Q_1 and Q_2, respectively, are the heat absorbed and released isothermally [from the hot (higher temperature) and (lower temperature) reservoirs] in Carnot's engine.



Also, from conservation of energy, we can write as

$W + Q_L = Q_H$

$\Rightarrow W = Q_H - Q_L$ (2)

$= Q_L\left(\frac{T_H}{T_L} - 1\right)$

$= (600\text{ cal})\left(\frac{303}{277} - 1\right) = \frac{600 \times 26}{277}\text{ cal} = \frac{600 \times 26}{277} \times 4.2\text{ J} = 236.5\text{ J}$

It is given that t = 1 s. Therefore, the power required is

$P = \frac{W}{t} = \frac{236.5\text{ J}}{1\,s} = 236.5\text{ W}$

Hence, the correct option is (1).

45. A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2}. Its net acceleration in ms^{-2} at the end of 2.0 s is approximately:

(1) 6.0 (2) 3.0

(3) 8.0 (4) 7.0

Solution

Assuming that the particle at the end possesses both radial and tangential accelerations, we get the following cases:

- **Tangential acceleration:**

$a_T = r\alpha$

$= 0.5 \times 2.0 = 1$ m/s

- **Angular velocity:**

$\omega = \omega_0 + \alpha t$

$= 0 + (2 \times 2) = 4$ rad/s

- **Radial acceleration:**

$A_C = \omega^2 R$

$= 4^2 \times 0.5 = 16 \times 0.5 = 8\text{ m/s}^2$

Therefore, the net acceleration is obtained as follows:

$$a_{\text{net}} = \sqrt{a_{\text{T}}^2 + a_{\text{C}}^2} = \sqrt{1^2 + 8^2} = 8\ \text{m/s}^2$$

Hence, the correct option is (3).