CBSE 2012
Physics (Theory)

Time: 3 hours | Max. Marks: 70

General Instructions

1. *All questions are compulsory*.
2. *There are 30 questions in total. Questions 1 to 8 carry one mark each, questions 9 to 18 carry two marks each, questions 19 to 27 carry three marks each and questions 28 to 30 carry five marks each*.
3. *There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions of five marks each. You have to attempt only one of the given choices in such questions*.
4. *Use of calculators is not permitted*.
5. *You may use the following values of physical constant wherever necessary*:

$c = 3 \times 10^{8}\ \text{ms}^{-1}$
$h = 6.626 \times 10^{-34}\ \text{Js}$
$e = 1.602 \times 10^{-19}\text{C}$
$\mu_0 = 4\pi \times 10^{-7}\text{T m A}^{-1}$
$\varepsilon_0 = 8.85 \times 10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}$
$\frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\ \text{Nm}^2\text{C}^{-2}$
Mass of electron $m_e = 9.1 \times 10^{-31}$ kg

1. Two wires of equal length, one of copper and the other of manganin, have the same resistance. Which wire is thicker?

Solution

$$R_{\text{Cu}} = R_{\text{Ma}}$$

$$\frac{\rho_{\text{Cu}} l}{a_{\text{Cu}}} = \frac{\rho_{\text{Ma}} l}{a_{\text{Ma}}} \text{ or } \frac{\rho_{\text{Ma}}}{\rho_{\text{Cu}}} = \frac{a_{\text{Ma}}}{a_{\text{Cu}}}$$

As $\rho_{\text{Ma}} > \rho_{\text{Cu}}$

Therefore, $a_{\text{Ma}} > a_{\text{Cu}}$

Therefore, it is conclude that manganin wire has to be thicker.

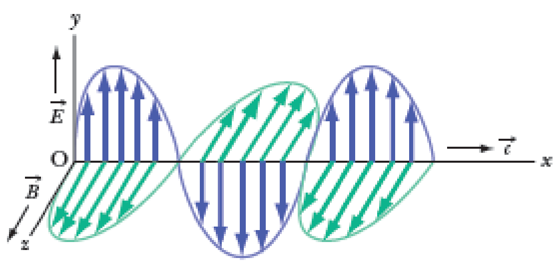
2. What are the directions of electric and magnetic field vectors relative to each other and relative to the direction of propagation of electromagnetic waves?

Solution

The direction of propagation of electromagnetic waves (as shown in the following figure) is given by

$$(\vec{E} \times \vec{B}) = (\hat{j}E \times \hat{k}B) = (\hat{j} \times \hat{k})EB = \hat{i}EB,$$

that is, along positive direction of x-axis. Here, $\vec{E} \perp \vec{B} \perp \vec{C}$.



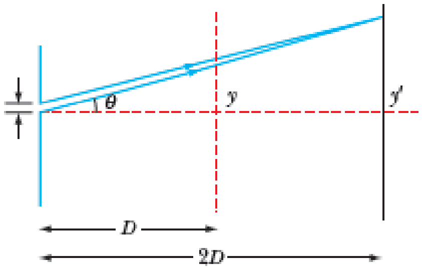
3. How does the angular separation between fringes in single-slit diffraction experiment change when the distance of separation between the slit and screen is doubled?

Solution

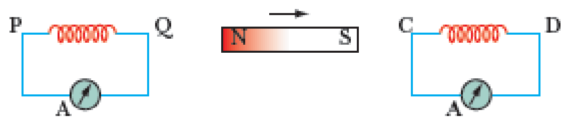
The following figure shows the situation. The angular separation is given by

$$\theta = \frac{\beta}{D} = \frac{D\lambda/d}{D} = \frac{\lambda}{d}.$$

As angular separation θ is independent of D, it would remain same when the distance of separation between the slit and screen is doubled.



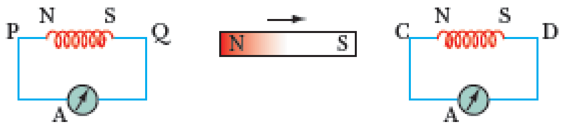
4. A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the directions of induced current in each coil.



Solution

In PQ: The movement of north pole away from the end Q induces south pole to turn toward the end Q. Due to this effect, the current in the coil nearer to the end Q is in the clockwise direction (as shown in the figure below), that is, for the end P to the end Q.

In CD: The movement of south pole toward the end C induces south pole at nearer end and due to this effect, the current passes in clockwise direction (as shown in the figure below), that is, from the end C to the end D.



5. For the same value of angle of incidence, the angles of refraction in three media A, B and C are 15°, 25° and 35°, respectively. In which medium would the velocity of light be minimum?

Solution

We have

$$v = \frac{c}{\mu} = \frac{c}{\sin i / \sin r} = \frac{c}{\sin i} \times \sin r$$

$$v = \left[\frac{c}{\sin i}\right] \cdot \sin r \text{ or } v \propto \sin r$$

Therefore, smaller the value of r, smaller would be the velocity as (c/sin i) is constant. Therefore, among the three media, 15° is the smaller value of r and hence medium A would have the minimum velocity of light.

6. A proton and an electron have same kinetic energy. Which one has greater de Broglie wavelength and why?

Solution

We know that the wavelength is expressed as

$$\lambda = \frac{h}{p}, \qquad (1)$$

where

$$\frac{p^2}{2m} = K$$

$$\Rightarrow p^2 = 2mk$$

$$\Rightarrow p = \sqrt{2mk}.$$

Substituting the value of p in Eq.(1), we get

$$\lambda = \frac{h}{\sqrt{2mk}}.$$

Since 2, h and the kinetic energy K are constant, we have

$$\lambda \propto \frac{1}{\sqrt{m}}$$

Also, since $m_p > m_e$, we have $\lambda_e > \lambda_p$.

7. Mention the two characteristic properties of the material suitable for making core of a transformer.

Solution

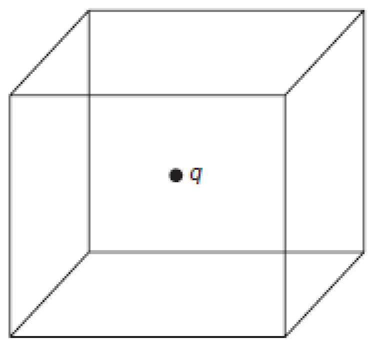
a. Iron loss: The core of a transformer on which we wind primary and secondary coils is made up of iron; therefore, loss of energy takes place due to the heating effect caused by the induction of eddy currents. Iron loss can be minimized by using laminated cores.

b. Hysteresis loss: In a transformer, repeated magnetization and demagnetization of the iron core takes place due to the oscillating input across the primary coil, due to which loss of energy takes place. Hysteresis loss in a transformer can be minimized by using a material in core which has low hysteresis loss.

Therefore, any material with low hysteresis loss and iron loss can be used to make core of a transformer.

8. A charge 'q' is placed at the centre of a cube of side l. What is the electric flux passing through each face of the cube?

Solution



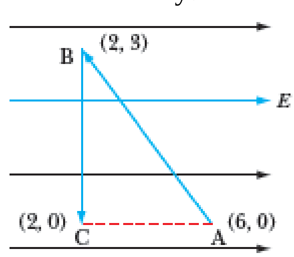
From Gauss theorem, we have the electric flux as

$$\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$

We know that a cube has six faces. Therefore, the flux through each face is given by

$$\frac{q}{6\varepsilon_0}.$$

9. A test charge 'q' is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure. (i) Calculate the potential difference between A and C. (ii) At which point (of the two) is the electric potential more and why?



Solution

(i) As work done is independent of the path followed, we have the covered distance as

$\Delta x = (6 - 2) = 4$
Hence, in this case, the work done is given by
$W = F.\Delta x \Rightarrow W = (qE)4.$
The work is done in moving charge from lower potential difference to higher potential difference.
(ii) Since electric field is directed from C to A, the, we have $V_C > V_A$.

10. An electric dipole is held in a uniform electric field. (i) Show that the net force acting on it is zero. (ii) The dipole is aligned parallel to the field. Find the work done in rotating it through the angle 180°.

Solution

(i) Force on charge $-q$ at A is given by

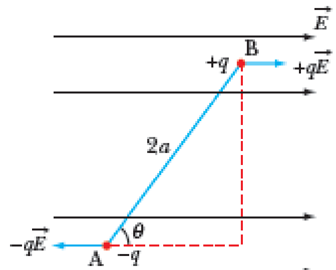
$$\vec{F}_1 = -q\vec{E}.$$

Force on charge $+q$ at B is given by

$$\vec{F}_2 = +q\vec{E}.$$

Total force is

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$
$$\vec{F} = -q\vec{E} + q\vec{E} = 0$$



(ii) We know that the work done on the dipole is given by

$$W = pE(\cos\theta_1 - \cos\theta_2). \qquad (1)$$

We have $\theta_1 = 0°$ and $\theta_2 = 180°$. Therefore, Eq. (1) becomes

$$W = pE[\cos 0° - \cos 180°] = 2pE.$$

11. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers?

Solution

A transformer works on the principle of mutual induction. For long range transmission, step-up transformer is required. With the use of step-up transformer, voltage is increased and hence the current is decreased. If R is the resistance of transmission wire, power loss is given by I^2R. Power losses are minimized. To step down the voltage at the receiver's end, step-down transformer is used to reduce the voltage.

12. A capacitor of capacitance 'C' is being charged by connecting it across a dc source along with an ammeter. Will the ammeter show a momentary deflection during the process of charging? If so, how would you explain this momentary deflection and the resulting continuity of current in the circuit? Write the expression for the current inside the capacitor.

Solution

During charging, the ammeter shows a momentary deflection due to the displacement current produced across the capacitor. We have

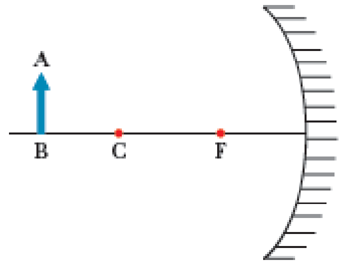
$$I_d = \frac{d}{dt}q = \frac{d}{dt}CV = \frac{d}{dt}\frac{\varepsilon_0 A}{d}.Ed \qquad \left[\text{As } C = \frac{\varepsilon_0 A}{d}; V = Ed\right]$$

$$= \varepsilon_0 \frac{d}{dt}EA.$$

Hence, the displacement current produced across the capacitor is

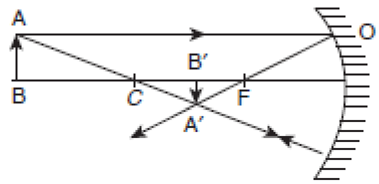
$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}.$$

13. An object AB is kept in front of a concave mirror as shown in the figure. (i) Complete the ray diagram showing the image formation of the object. (ii) How will the position and intensity of the image be affected if the lower half of the mirror's reflecting surface is painted black?



Solution

(i) When an object AB is kept in front of a concave mirror, the ray diagram showing the image formation of the object is as shown in the following figure. The ray through *C* will get reflected back. Another ray along AO parallel to the principal axis passes through the focus F. The two rays meet at A′ after reflection where the image A′B′ is formed.



(ii) If the lower half of the mirror's reflecting surface is painted black, the position of image does not change as the radius of curvature does not change. So, $f = r/2$. By using the mirror formula

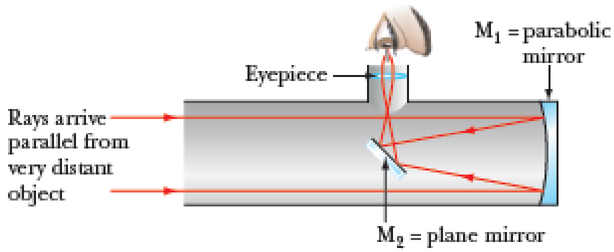
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f'}$$

we can find that *v* does not change. As rays get focused from one half, the intensity of light decreases.

14. Draw a labeled ray diagram of a reflecting telescope. Mention its two advantages over the refracting telescope.

Solution

The ray diagram of a reflecting telescope is as shown below:



(i) Size of objective (concave mirror) can be made large, as it does not suffer from chromatic aberration and spherical aberration.

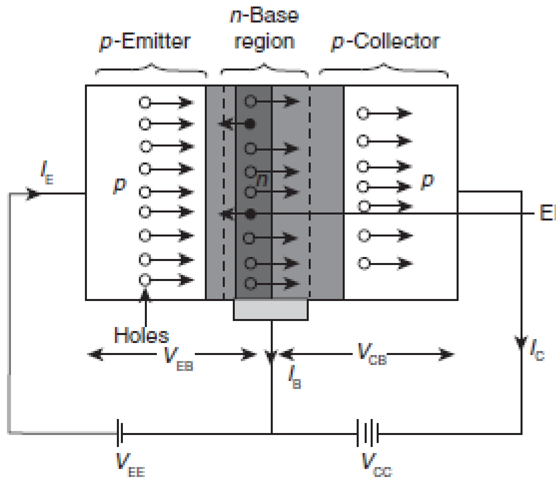
(ii) Image formed will be brighter and resolving power is more as diameter of objective is larger. Its resolving power is

$$\frac{D}{1.22\lambda}.$$

15. Describe briefly with the help of a circuit diagram, how the flow of current carriers in a *p*-*n*-*p* transistor is regulated with emitter–base junction forward biased and base–collector junction reverse biased.

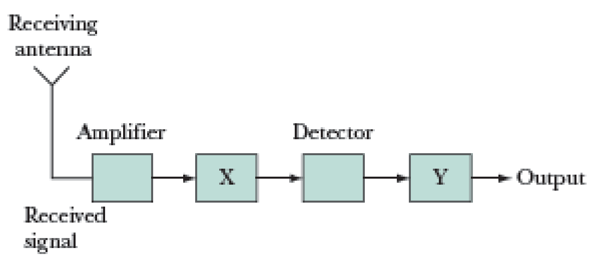
Solution

Following figure shows the working of a *p-n-p* transistor. Holes from *p* region are pushed into *n* region (base) and electrons from *n* region enter into the *p* region. The resistance of emitter–base junction is reduced and depletion layer becomes thin. In base–collector junction, being reverse biased, the holes are pulled by negative terminal of the battery. Base–collector junction becomes thick and offers more resistance.



(a)

16. In the given block diagram of a receiver, identify the boxes labeled as X and Y and write their functions.



Solution

From the given figure, it is obvious that X is demodulator. It separates the audio component from the radio frequency modulated waves. Y is AF amplifier. It amplifies the audio signal separated from the modulated wave.

17. A light bulb is rated 100 W for 220 V ac supply of 50 Hz. Calculate (i) the resistance of the bulb; (ii) the rms current through the bulb.

Solution

(i) $P = 100$ W; $V = 220$ V; $f = 50$ Hz. The resistance of the bulb is calculated as follows:

$$P = VI = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$$

$$\Rightarrow \ R = \frac{V^2}{P} = \frac{220 \times 220}{100} = 484\ \Omega.$$

(ii) The rms current through the bulb is calculated as follows:

$$I_{rms} = \frac{E_{rms}}{R} = \frac{220}{484} = \frac{5}{11}\text{A} = 0.455\text{A}.$$

OR

An alternating voltage given by $V = 140 \sin 314\, t$ is connected across a pure resistor of 50 Ω. Find (i) the frequency of the source, (ii) the rms current through the resistor.

Solution

(i) The frequency of the source is calculated as follows:

$$V = 140 \sin 314\, t$$

$$V = E_0 \sin \omega t = E_0 \sin 2\pi ft$$

On comparing, we have $E_0 = 140$ V and

$$2 \times \pi ft = 314t$$
$$2 \times 3.14ft = 314t$$
$$2f = 100 \Rightarrow f = 50 \text{ Hz}.$$

(ii) The rms current through the resistor is obtained as follows:

$$I_{rms} = \frac{E_{rms}}{R} = \frac{E_0}{\sqrt{2}R} = \frac{140}{1.4142 \times 50} = 2\text{A}.$$

18. A circular coil of N turns and radius R carries a current I. It is unwound and rewound to make another coil of radius $R/2$, current I remaining the same. Calculate the ratio of the magnetic moments of the new coil and the original coil.

Solution

Magnetic dipole moment is given by

$$M = NIA.$$

For the coil before it is unwound and rewound, the magnetic dipole is given by

$$M_1 = NI\pi R^2. \quad (1)$$

For the coil after it is unwound and rewound, the magnetic dipole is given by

$$M_2 = N'I\pi\left(\frac{R}{2}\right)^2. \quad (2)$$

Now,

$$N \times 2\pi R = N' \times 2\pi \frac{R}{2}$$
$$N' = 2N$$
$$A' = \pi\left(\frac{R}{2}\right)^2$$

Therefore, Eq.(2) can be written as

$$M_2 = 2NI\frac{\pi R^2}{4}$$
$$M_2 = NI\frac{\pi R^2}{2}. \quad (3)$$

From Eqs.(1) and (3), we get

$$M_2 = \frac{M_1}{2} \text{ or } \frac{M_2}{M_1} = \frac{1}{2}.$$

19. Deduce the expression for the electrostatic energy stored in a capacitor of capacitance 'C ' and having charge 'Q'. How will the (i) energy stored and (ii) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant 'K'?

Solution

Let us consider a parallel-plate capacitor with d as separation between the plates. When charge q reaches plate A, it potential is V. By definition, potential difference is given by

$$\frac{\text{Work done}}{\text{Charge}}.$$

Work done in giving charge dq to plate A is

$$V = \frac{dW}{dq} \Rightarrow dW = V.dq.$$

Therefore,

$$dW = \frac{q}{C}.dq \quad (1)$$

The total work done in increasing charge from zero to Q can be obtained by integrating Eq. (1), that is,

$$W = \int dW$$
$$= \int_0^Q \frac{q}{C}.dq$$
$$= \frac{1}{C}\int_0^Q q dq = \frac{1}{C}\left|\frac{q^2}{2}\right|_0^Q$$
$$= \frac{1}{C}\left|\frac{Q^2}{2}\right|$$
$$= \frac{Q^2}{2C}.$$

This work done gets stored as potential energy:

$$U = \frac{Q^2}{2C}.$$

(i) On introducing dielectric, the capacity increases by K times. Charge Q is the same. Therefore, the new potential energy is

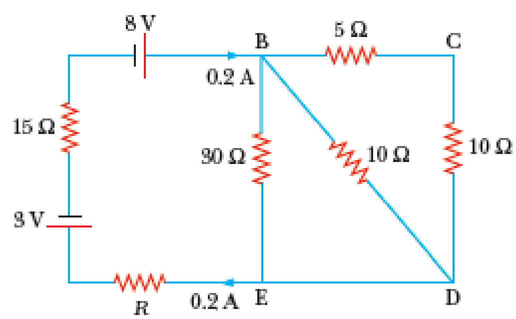
$$U' = \frac{Q^2}{2KC}.$$

(ii) When the dielectric material of dielectric constant 'K' is filled between the plates, the electric field is

$$E' = \frac{V'}{d} = \frac{Q}{KCd} = \frac{V}{Kd} = \frac{E}{K}.$$

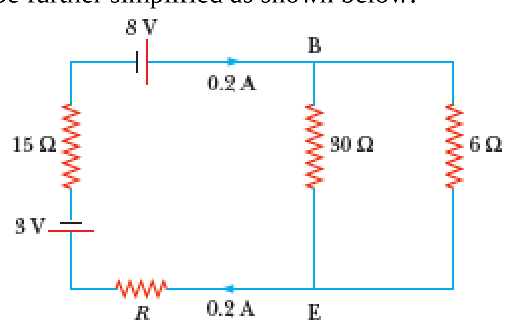
Hence, electric field decreases 'K' times.

20. Calculate the value of the resistance R in the circuit shown in the figure so that the current in the circuit is 0.2 A. What would be the potential difference between points B and E?



Solution

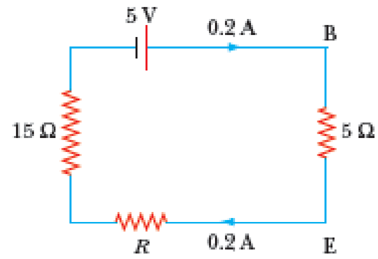
The given configuration can be further simplified as shown below:



From this simplified configuration, we calculate the resistance between B and E as follows:

$$\frac{1}{R_p} = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{1}{6}$$
$$\Rightarrow R_P = 6\Omega.$$

The above simplified configuration can further be simplified as shown below:



From this simplified configuration, we calculate the resistance between B and E as follows:

$$\frac{1}{R'_p} = \frac{1}{30} + \frac{1}{6} = \frac{1+5}{30} = \frac{6}{30} = \frac{1}{5}$$

$$\Rightarrow R'_p = 5\Omega.$$

The current is calculated as

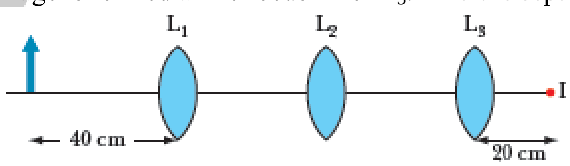
$$I = 0.2 = \frac{5}{15+5+R}$$

$$= \frac{1}{5} = \frac{5}{20+R} \Rightarrow R = 5\Omega$$

and the potential difference is

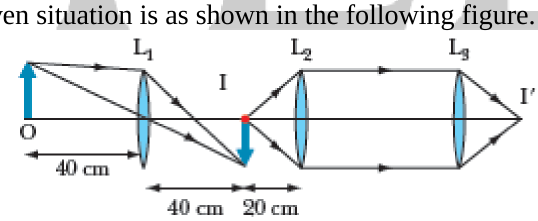
$$V_{BE} = IR = 5 \times 0.2 = 1\text{V}.$$

21. You are given three lenses L_1, L_2 and L_3 each of focal length 20 cm. An object is kept at 40 cm in front of L_1 as shown. The final real image is formed at the focus 'I' of L_3. Find the separations between L_1, L_2 and L_3.



Solution

The ray diagram for the given situation is as shown in the following figure.



From lens formula, we have

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

Therefore,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = -\frac{1}{-40} + \frac{1}{v} = \frac{1}{20}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40} \Rightarrow v = 40 \text{ cm}.$$

Image is formed at 40 cm from L_1. As I' is at distance 20 cm from L_3. Rays on L_3 should fall parallel. For rays to be parallel between L_2 and L_3, image I (acting as object for lens L_2) should be at principal focus of L_2, that is, 20 cm. The distance between L_1 and L_2 = 40 cm + 20 cm = 60 cm. Distance between L_2 and L_3 can be determined. We have

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow f = \frac{f_1 f_2}{f_1 + f_2} = \frac{20 \times 20}{20+20} \Rightarrow 10 \text{ cm}.$$

Consequently, we have

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\Rightarrow\ -\frac{1}{u}=\frac{1}{f}-\frac{1}{v}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$$

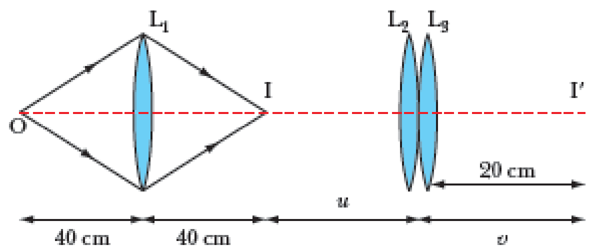
$$\Rightarrow\ u=-20\text{ cm.}$$

Therefore,

$$L_1L_2 = 40 + 20 = 60\text{ cm.}$$

$$L_2L_3 = 0$$

As per the determined distances, the ray diagram is depicted as shown in the following figure.



22. Define the terms (i) ‘cut-off voltage’ and (ii) ‘threshold frequency’ in relation to the phenomenon of photoelectric effect.
Using Einstein’s photoelectric equation, show how the cut-off voltage and threshold frequency for a given photosensitive material can be determined with the help of a suitable plot/graph.

Solution

(i) Cut-off voltage: The minimum negative potential given to the metal plate with respect to collector at which the photoelectric current becomes zero is known as cut off voltage or stopping potential.
(ii) Threshold frequency: The minimum frequency of the incident light below which photoelectrons are not ejected from the metal surface is known as threshold frequency.
The various forms of Einstein’s equation of photoelectric effect are given as follows:

$$hv = hv_0 + \frac{1}{2}mv_{\max}^2; \qquad (1)$$

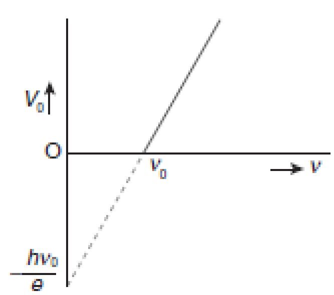
$$\frac{1}{2}mv_x^2 = eV_0; \qquad (2)$$

$$hv = hv_0 + eV_0 \Rightarrow eV_0 = hv - hv_0. \qquad (3)$$

From Eq.(3), we have the stopping potentials

$$V_0 = \left(\frac{h}{3}\right)v - \frac{hv_0}{e} \qquad (4)$$

On comparison with the equation $y = mx + c$, Eq. (4) becomes a straight line with slope $\left(\frac{h}{e}\right)$ and intercept $-\frac{hv_0}{c}$ on y-axis. If $v < v_0$, $V_0 = 0$. If $v > v_0$, V_0 increases linearly as plotted in the graph shown below:



23. A series LCR circuit is connected to an ac source. Using the phasor diagram, derive the expression for the impedance of the circuit. Plot a graph to show the variation of current with frequency of the source, explaining the nature of its variation.

Solution

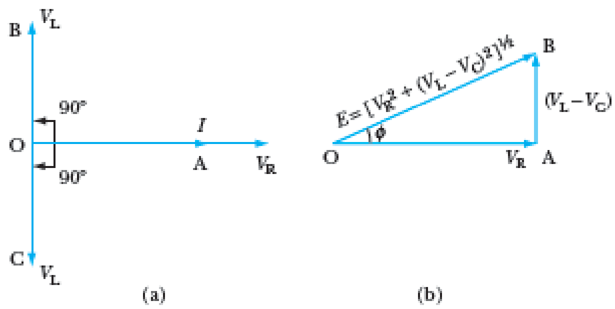
We know that

$$I = I_0 \sin \omega t;$$

$V_R = IR$, which has no phase difference with current;

$V_L = IX_L \Rightarrow (L\omega)$, which leads current by $\pi/2$;

$V_C = IX_C = I\left(\frac{1}{C\omega}\right)$, which lags behind current by $\pi/2$.



If $V_L > V_C$, then $V_L - V_C$ is along AB as in the phasor diagram shown in figure (b). The net emf is calculated as follows:

$$\varepsilon = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + [I(X_L - X_C)]^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

Substituting the values in Eq.(1), we get

$$\varepsilon = I\sqrt{\left[R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2\right]} \qquad (2)$$

We know that impedance is given by

$$Z = \frac{\varepsilon}{I}. \qquad (3)$$

Substituting Eq. (2) in eq. (3), we get the impedance as follows:

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

The following figure shows variation of current I with frequency of the ac source. I is maximum, when Z is minimum.

$$\left(L\omega - \frac{1}{C\omega}\right)^2 = 0$$

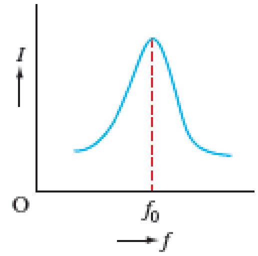
$$\Rightarrow L\omega - \frac{1}{C\omega} = 0$$

$$\Rightarrow L\omega = \frac{1}{C\omega} \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

We have frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}},$$

which is called resonant frequency.



24. Mention three different modes of propagation used in communication system. Explain with the help of a diagram how long distance communication can be achieved by ionospheric reflection of radio waves.

Solution

As represented in the following figure, electromagnetic waves can propagate in three different ways: as a ground wave, sky wave, or space wave.

Sky wave propagation is used for transmission of radio signals having frequencies between 2 MHz to 40 MHz. Ionosphere containing charged particles extends from 65 km to 400 km above the surface of the Earth. Sky waves from transformer reach the receiving antenna after getting reflected by the ionosphere.

Another way to communicate over long distances is to use the phenomenon of reflection of radio waves from the ionosphere. This mode of propagation is used by short-wave radio broadcast services. The ionosphere reflects the radio waves (or the transmitted signal) if the frequency of the radio waves lies in the frequency range 3–30 MHz. If the frequency of the transmitted signal is higher than 30 MHz, the signals will penetrate the ionosphere.

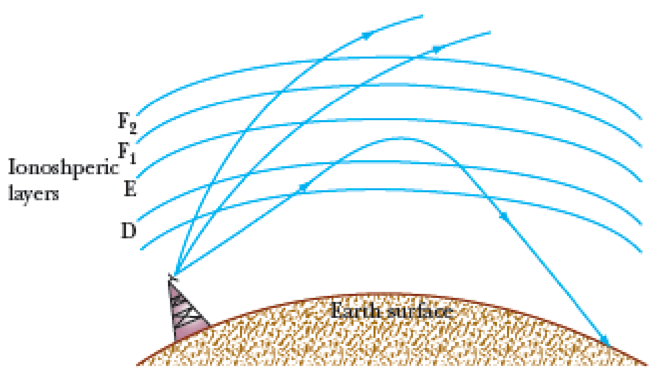
The ionosphere consists of a large number of ions, which are produced due to ionization of air molecules by ultraviolet and other high-energy radiations from the Sun. The ionosphere extends from a height of around 65 km to around 400 km above the Earth's surface and can be divided into four layers:

1. D-layer: It is at a height of 65 km above the Earth's surface. This layer exists during day time but disappears at night because it is the Sun which is responsible for ionization. The D-layer reflects low-frequency electromagnetic radiation, but partially absorbs medium- and high-frequency waves. The D-layer has an electron density of approximately $10^9\ m^{-3}$.

2. E-layer: It exists at a height of 100 km, and like the D-layer, exists only during day time. This layer helps surface-wave propagation but reflects high-frequency waves. The E-layer has an electron density of around $2\times10^{11}\ m^{-3}$.

3. F_1-layer: It is at a height of 170 km above the Earth's surface. It exists during day time and merges with the F_2-layer at night. It allows the high-frequency waves to reach the F_2-layer above it, after some absorption. This layer has an electron density of around $3\times10^1\ m^{-3}$.

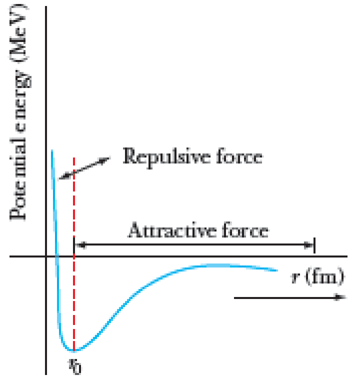
4. F_2-layer: Its height varies from 250−400 km during day time to around 300 km at night. It reflects electromagnetic waves of frequencies up to around 40 MHz. This layer has an electron density of around $8\times10^{11}\ m^{-3}$.



25. Draw a plot of potential energy of a pair of nucleons as a function of their separations. Mark the regions where the nuclear force is (i) attractive and (ii) repulsive. Write any two characteristic features of nuclear forces.

Solution

The following figure shows a rough plot of potential energy between two nucleons as a function of the distance between the nucleons.



The potential energy is minimum at a distance r_0 of about 0.8 fermi, which means that the force is attractive for distances larger than 0.8 fermi and the force is repulsive if they are separated by distances less than 0.8 fermi.
Two characteristic features of nuclear forces:
1. The nuclear force is much stronger than the coulomb's force or gravitational force.
2. It does not depend upon charge on nucleon. Force between proton-proton; proton-neutron and neutron-neutron has same value.

26. In a Geiger–Marsden experiment, calculate the distance of closest approach to the nucleus of $Z = 80$, when an a-particle of 8 MeV energy impinges on it before it comes momentarily to rest and reverses its direction. How will the distance of closest approach be affected when the kinetic energy of the a-particle is doubled?

Solution

At the distance of closest approach, potential energy of the nucleus is equal to the kinetic energy:

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(2e)(Ze)}{r}=\frac{1}{2}mv^2.$$

Therefore, the distance is

$$r=\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{(1/2)mv^2}$$

$$=\frac{9\times10^9\times2\times80\times(1.6\times10^{-19})^2}{8\times10^6\times(1.6\times10^{-19})}$$

$$= 18 \times 1.6 \times 10^{-15}\ \text{m}$$

$$= 28.8 \times 10^{-15}\ \text{m}.$$

It is obvious from the above equation that the distance of closest approach becomes half when the kinetic energy of the α-particle is doubled.

OR

The ground state energy of hydrogen atom is −13.6 eV. If an electron makes a transition from an energy level −0.85 eV to −3.4 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong?

Solution

We have

$$W_{n_2}-W_{n_1}=-0.85-(-3.4)$$

$$= 2.55\ \text{eV}.$$

Wavelength of the spectral line emitted is

$$\frac{1240}{2.55}=486.27\ \text{nm}=4862\ \text{Å}.$$

It lies in Balmer series (visible region).

27. Define relaxation time of the free electrons drifting in a conductor. How is it related to the drift velocity of free electrons? Use this relation to deduce the expression for the electrical resistivity of the material.

Solution

Electrons in a conductor move about freely with large thermal speeds colliding with each other freely. The distance between two successive collisions is called free path. The time gap between two successive collision, during which speed and direction of electrons is same, is called relaxation time.

As shown in the following figure, let us consider a section AB of conductor of length *l* and area of cross-section *A*. Volume of conductor is *Al*; number of electron per unit volume = *n*; total number of electrons = *nAl*; total charge *q* = *nAle*. Current is given by

$$I = \frac{\text{Change}}{\text{Time}} = \frac{q}{t}$$

$$= \frac{nAle}{t}$$

$$= nAv_d e. \left[\text{As } v_d = \frac{I}{t}\right]$$

Therefore, drift velocity is

$$v_d = \frac{I}{nAe} \qquad (1)$$

Between two collisions, electrical force on electron = *Ee*.

Acceleration is given by

$$a = \frac{eE}{m}.$$

Using $v = u + at$, we get

$$v_d = 0 + \frac{eE}{m}.\tau \qquad (2)$$

From Eqs.(1) and (2), we get

$$\frac{I}{nAe} = \frac{eE}{m}\tau$$

$$= e\frac{V}{ml}.\tau \qquad \left[\text{As } E = \frac{V}{\rho}\right]$$

Therefore,

$$\frac{V}{I} = \frac{ml}{nAe^2\tau}$$

$$\Rightarrow \; R\frac{A}{I} = \frac{m}{ne^2\tau} \qquad (3)$$

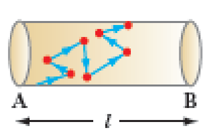
Resistance is given by

$$R = \rho\frac{I}{A}$$

$$\Rightarrow \; \rho = R\frac{A}{I} \qquad (4)$$

From Eqs.(1) and (2), we get

$$\rho = \frac{m}{ne^2\tau}.$$



28. (a) In Young's double slit experiment, derive the condition for (i) constructive interference and (ii) destructive interference at a point on the screen.

(b) A beam of light consisting of two wavelengths, 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1·4 m away. If the two slits are separated by 0·28 mm, calculate the least distance from the central bright maximum where the bright fringes of the two wavelengths coincide.

Solution

(a) Let us consider two slits S_1 and S_2 which are illuminated by a light source S. The waves which will emerge from the two slits will result in interference. For a particular point P on the screen, a maxima will occur if constructive interference takes place and a minima will occur if destructive interference takes place. We will now derive the condition for constructive and destructive interference.

Let the displacement of the waves from sources S_1 and S_2 at point P on the screen at any time t be given by

$$y_1 = a_1 \sin \omega t \text{ and } y_2 = a_2 \sin(\omega t + \phi),$$

where ϕ is the constant phase difference between the two waves. By the superposition principle, the resultant displacement at point P is given by

$$y = y_1 + y_2$$

$$y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

$$y = (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t \qquad (1)$$

Let

$$a_1 + a_2 \cos \phi = A \cos \theta \qquad (2)$$

$$a_2 \sin \phi = A \sin \theta \qquad (3)$$

Substituting Eqs. (2) and (3) in Eq.(1) we get

$$y = A \cos \theta \sin \omega t + A \sin \theta \cos \omega t$$

$$= A \sin(\omega t + \theta)$$

Squaring and adding both sides of the Eqs. (2) and (3), we obtain

$$A^2 + \cos^2 \theta + A^2 + \sin^2 \theta = (a_1 + a_2 \cos \phi)^2 + a_2^2 + \sin^2 \phi$$

$$A^2 = a_1^2 + a_2^2 (\cos^2 \phi + \sin^2 \phi) + 2 a_1 a_2 \cos \phi$$

$$A^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos \phi$$

The intensity of light is directly proportional to the square of amplitude of the wave. The intensity of light at point P on the screen is given by,

$$I = a_1^2 + a_2^2 + 2 a_1 a_2 \cos \phi \qquad (4)$$

For constructive interference: $\cos \phi$ is maximum; that is, $\cos \phi = +1$

$$\phi = 0, 2\pi, 4\pi, ...$$

$$\phi = 2n\pi, \text{ where } n = 0, 1, 2, \ldots \qquad (5)$$

The phase difference ϕ between the two waves will be

$$\phi = \frac{2\pi}{\lambda} x \qquad (6)$$

where x is the path difference. Using Eq. (5), the condition for constructive interference [Eq. (6)], becomes

$$\frac{2\pi}{\lambda} x = 2n\pi$$

Therefore,

$x = n\lambda$, where $n = 0, 1, 2, 3, \ldots$

which is the condition for constructive interference between two light waves in terms of path difference between them.

For destructive interference: From Eq. (4), it follows that the intensity of light at point P will be minimum if

$$\cos \phi = -1$$

$$\phi = \pi, 3\pi, 5\pi, ...$$

$$\phi = (2n+1)\pi, \text{ where } n = 0, 1, 2, \ldots \qquad (7)$$

From Eqs. (6) and (7), we have

$$\frac{2\pi}{\lambda}x = (2n+1)\pi$$

$$x = (2n+1)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, \ldots$$

which is the condition for destructive interference in terms of phase difference and path difference.

(b) Let the bright fringes coincide at distance y from centre $y_1 = y_2$.

$$\frac{n_1 D\lambda_1}{d} = \frac{n_2 D\lambda_2}{d} \Rightarrow n_1\lambda_1 = n_2\lambda_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{n_1}{n_2} \Rightarrow \frac{800}{600} = \frac{4}{3}$$

Third bright fringe of 800 nm coincides with fourth fringe of 600 nm.

$$y_1 = \frac{n_1 D\lambda_1}{d} \Rightarrow \frac{3\times 1.4\times 800\times 10^{-9}}{0.28\times 10^{-3}}$$

$$y_1 = \frac{24\times 1.4\times 10^{-4}}{0.28}\text{m}$$

$$= \frac{24}{28}\times\frac{14}{10}\times 100\times 10^{-4}\text{m}$$

$$= 12 \times 10^{-3}\text{ m}$$

$$= 0.012 \text{ m or } 1.2 \text{ cm}$$

OR

OR

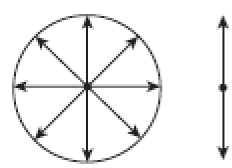
(a) How does an unpolarized light incident on a polaroid get polarized?

Describe briefly, with the help of a necessary diagram, the polarization of light by reflection from a transparent medium.

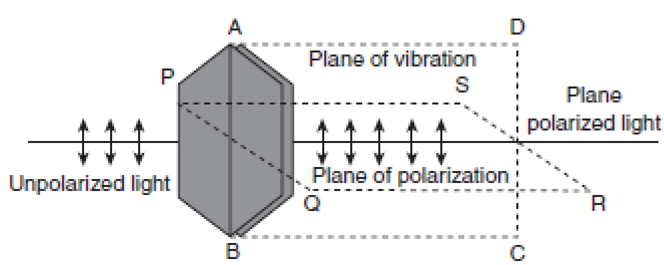
(b) Two polaroids 'A' and 'B' are kept in crossed position. How should a third polaroid 'C' be placed between them so that the intensity of polarized light transmitted by polaroid B reduces to one-eighth of the intensity of unpolarized light incident on A?

Solution

(a) Unpolarized light has vibrations in all possible directions as shown in the following figure. It can be resolved into two components. (i) Components in the plane of page (shown by arrows) and components perpendicular to plane of paper (shown by dots).



Unpolarized light can be polarized by passing it through a tourmaline or rock salt crystal, as shown in the following figure.



According to Brewster, unpolarized light can be polarized by reflection also as shown in the following figure. The amount of polarization depends upon angle of incidence. At certain angle of incidence called polarizing angle, i_p the polarization is complete for reflected beam. In this situation reflected and refracted beams are perpendicular to each other. From Snell's Law

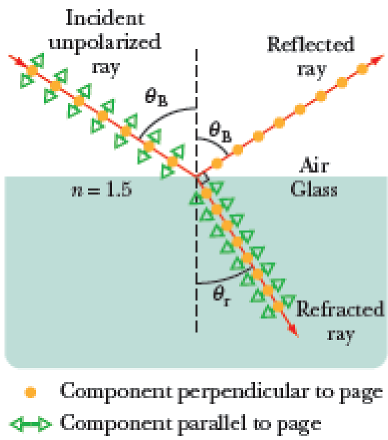
$$\mu = \frac{\sin i_P}{\sin r}$$

$$i_P + r = 90° \quad \text{or} \quad r = [90° - i_p]$$

Therefore,

$$\mu = \frac{\sin i_P}{\sin(90 - i_P)} \Rightarrow \frac{\sin i_P}{\cos i_P} \Rightarrow \tan i_P$$

tan $i_P = \mu$ is called Brewster's law.



(b) Let us consider the figure shown below. After passing through first crystal, intensity is reduced to half, that is,

$$I_1 = \frac{I}{2}$$

Let C be placed at angle θ with A.

$$I_2 = I_1 \cos^2 \theta.$$

Angle between C and B is $(90° - \theta)$.

$$I_3 = I_2 \cos^2 (90 - \theta)$$

$$I_3 = I_2 \sin^2 \theta$$

$$I_3 = I_1 \sin^2\theta \cos^2\theta.$$

Therefore,

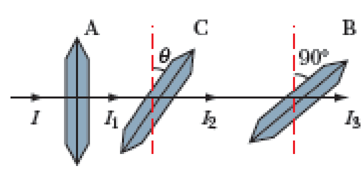
$$I_3 = \frac{I}{2}.\left[\frac{4}{4}\right]\sin^2\theta.\cos^2\theta$$

$$I_3 = \frac{I}{8}.[2\sin\theta\cos\theta]^2$$

$$= \frac{I}{8}.(\sin 2\theta)^2$$

$$= \frac{I}{8}.1 \text{ (If } \sin 2\theta = 1, 2\theta = 90° \text{ and } \theta = 45°)$$

$$I_3 = \frac{I}{8}.$$

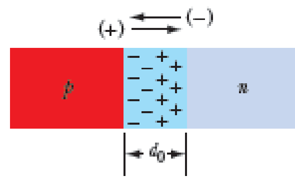


29. (a) Describe briefly, with the help of a diagram, the role of the two important processes involved in the formation of a *p-n* junction.
(b) Name the device which is used as a voltage regulator. Draw the necessary circuit diagram and explain its working.

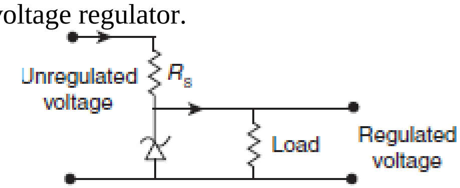
Solution

(a) When *p*-type and *n*-type semiconductors are in contact, a *p*-*n* junction diode is formed. The processes involved are (1) diffusion and (ii) drift.

Due to variation of concentration of charge carriers in two regions, diffusion takes place. A few of holes from p-region enter into n-region, while a few electrons from n-region enter into p-region. This gives rise to a sort of potential difference in form of fictitious battery of emf varying between 0.3 V and 0.7 V. This voltage is also called knee voltage. The electric field due to knee voltage controls flow of electrons and holes. Ultimately, diffusion current and drift current become equal and opposite.



(b) Zener diode is used as a voltage regulator.

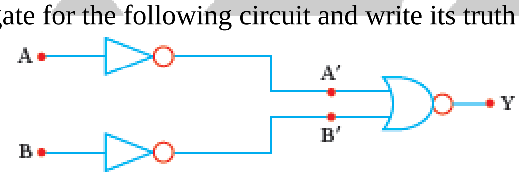


The unregulated dc voltage is connected to zener diode through a series resistance R_S such that zener diode is reverse biased. If input voltage increases, the current through R_S and zener diode also increases. This increases the voltage drop R_S, without any change in voltage drop across zener diode. This is because in breakdown region, the zener voltage remains constant, even though current through it changes. Similarly, if input voltage decreases, current through R_S and zener diode decreases. Voltage drop across R_S decreases, without any change in voltage across zener diode. Thus, any increase or decrease in input voltage does not cause any change in output voltage. So, a zener diode acts voltage regulator.

OR

(a) Explain briefly the principle on which a transistor–amplifier works as an oscillator. Draw the necessary circuit diagram and explain its working.

(b) Identify the equivalent gate for the following circuit and write its truth table.

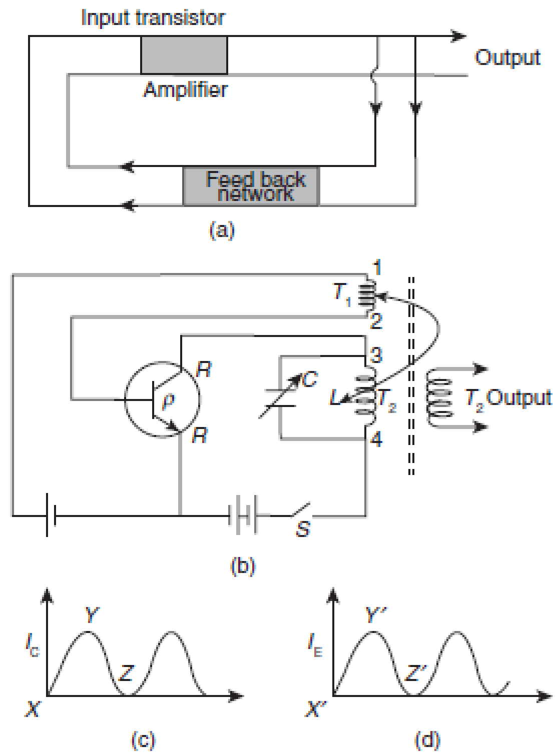


Solution

(a) Transistor amplifier as an oscillator: In an amplifier an external input is necessary to sustain ac signal in out. In oscillator no external input is required. Oscillator is self-sustained. A portion of output of amplifier is returned back to input in phase with starting power (positive feedback). It is achieved by *LC* or *RC* network coils T_1 and T_2 in figure (b) are wound over some core and have mutual inductance. When switch S_1 is closed, collector current flows in circuit. The current in coil T_2 does not reach full amplitude instantaneously but increases from X to Y as shown in figure (c). As T_1 and T_2 are coupled, current is set up in emitter circuit. As a result of this positive feedback current in coil T_1 also increases from X' to Y'. As current in T_2 reaches peak value, saturation stage is reached and there is no further increase beyond Y. without continued feedback, current in emitter begins to fall, consequently collector current decreases from Y to Z. Magnetic field around T_2 will decay causing decrease in emitter current to Z', when transistor is cut off. Both I_E and I_C cease to flow. The whole process repeats itself now. The resonance frequency of tuned circuit is

$$v = \frac{1}{2\pi\sqrt{LC}}.$$

As tank circuit [containing inductor L and capacitor C is on collector side. It is called tuned collector oscillator. If circuit is on base side, circuit is called tuned base oscillator.



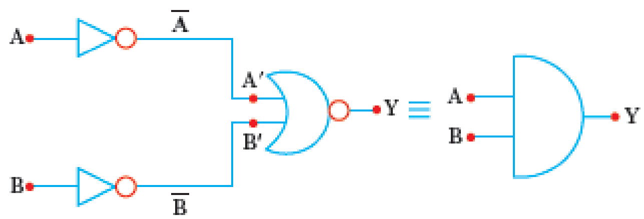
(b) For the given circuit, the output is

$$Y = \overline{A' + B'}$$

Where A' is the output from A, which is NOT gate, $\vec{A}$, and B' is the output from B< which is NOT gate, $\vec{B}$. Hence, the output is given by

$$Y = \overline{\bar{A} + \bar{B}}$$
$$= \bar{\bar{A}}.\bar{\bar{B}}$$
$$= A.B$$

Therefore, it is concluded that the equivalent gate for the given circuit is AND gate as shown in the following circuit diagram.



The truth table is as shown below:

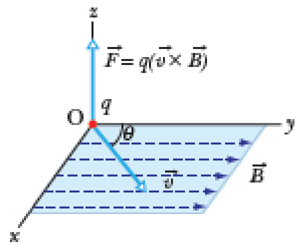
A	B	Y=A·B
0	0	0
0	1	0
1	0	0
1	1	1

30. (a) Write the expression for the force, $\vec{F}$, acting on a charged particle of charge 'q', moving with a velocity $\vec{v}$ in the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$. Obtain the condition under which the particle moves undeflected through the fields.

(b) A rectangular loop of size $l \times b$ carrying a steady current I is placed in a uniform magnetic field $\vec{B}$. Prove that the torque $\vec{\tau}$ acting on the loop is given by $\vec{\tau} = \vec{m} \times \vec{B}$ where $\vec{m}$ is the magnetic moment of the loop.

Solution

(a) Consider a charge q enters a magnetic field of induction $\vec{B}$, with velocity $\vec{v}$. The magnetic force on the charge is given by $\vec{F} = q(\vec{v} \times \vec{B})$.



It is perpendicular to $\vec{v}$ as well as $\vec{B}$. The magnitude of force is given by

$$F = qvB \sin\theta,$$

where θ is smaller of angles between velocity $\vec{v}$ and magnetic induction $\vec{B}$. Figure below shows a charge q moving downwards with velocity $\vec{v}$. Electrostatic force is

$$\vec{F}_e = q\vec{E}.$$

This force is towards right plane of paper. Magnetic force on charge q due to magnetic field of induction $\vec{B}$, directed towards reader in circular region.

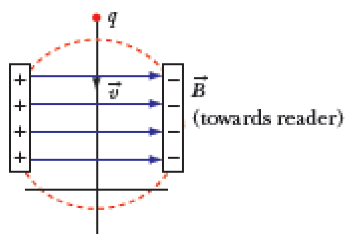
$$\vec{F}_m = q(\vec{v} \times \vec{B}).$$

This force is towards left in the plane of paper. If the two forces are equal, we have

$$|\vec{F}_e| = |\vec{F}_m|$$

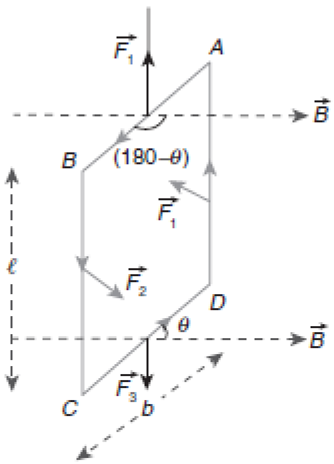
$$\cancel{q}E = \cancel{q}vB$$

$$v = \frac{E}{B}$$



Charges with this velocity will enter the region through a narrow bore. Other charges with different velocities will be blocked. The arrangement is called velocity selector in which $\vec{E}$, $\vec{B}$ and $\vec{v}$ are mutually perpendicular.

(b) Let us consider a coil of length l and breadth b be suspended inside a magnetic field of induction $\vec{B}$, at an angle θ with breadth b as shown in the following figure. N is number of turns of coil.



1. Force on side *AB*

$$\vec{F}_1 = Ni(\vec{b} \times \vec{B})$$
$$F_1 = NibB \sin(180 - \theta)$$
$$F_1 = NibB \sin\theta$$

This force acts upwards in the plane of paper.

2. Force on side *BC*

$$\vec{F}_2 = Ni(\vec{l} \times \vec{B})$$
$$F_2 = NilB \sin 90°$$
$$F_2 = NilB$$

This force is directed towards the reader.

3. Force on side *CD*

$$\vec{F}_3 = Ni(\vec{b} \times \vec{B})$$
$$F_3 = Nilb \sin\theta$$

The force acts downwards in the plane of paper.

4. Force on side *DA*

$$\vec{F}_4 = Ni[\vec{l} \times \vec{B}]$$
$$F_4 = NilB \sin 90°$$
$$F_4 = NilB$$

This force is directed away from the reader.

Forces F_1 and F_3 being equal and opposite cancel each other, while forces F_2 and F_4 being equal unlike, parallel and non collinear constitute a couple whose m current is given by

Torque = Either force × Perpendicular distance

$= NilB \times b \cos\theta$

That is,

$$\tau = Nilb\,B\cos\theta,$$

where θ is angle between breadth $\vec{b}$ and magnetic induction $\vec{B}$. If α is angle between area vector $\vec{A}$ and magnetic induction $\vec{B}$.

$$(\theta + \alpha) = 90° \Rightarrow \theta = (90° - \alpha)$$

Therefore,

$$\tau = NilbB \cos(90 - \alpha)$$
$$\tau = NiA\,B \sin\alpha \qquad \text{(As } Ib = A\text{)}$$

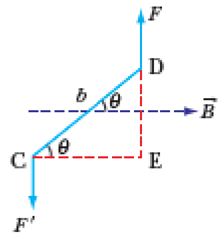
In vector form, torque is expressed as

$$\vec{\tau} = Ni(\vec{A} \times \vec{B})$$

or

$$\vec{\tau} = \vec{m} \times \vec{B}$$

where $\vec{m} = Ni\vec{A}$.



OR

(a) Explain, giving reasons, the basic difference in converting a galvanometer into (i) a voltmeter and (ii) an ammeter.

(b) Two long straight parallel conductors carrying steady currents I_1 and I_2 are separated by a distance 'd'. Explain briefly, with the help of a suitable diagram, how the magnetic field due to one conductor acts on the other. Hence, deduce the expression for the force acting between the two conductors. Mention the nature of this force.

Solution

(a) Galvanometer into ammeter: A galvanometer is a sensitive instrument and shows full scale deflection for a current of few milliampere. In order to convert galvanometer into ammeter, we connect a low resistance in parallel to the galvanometer coil as shown in the following figure. The potential difference across G is equal to the potential difference across S. Therefore,

$$I_g G = (I_m - I_g)S.$$

Hence,

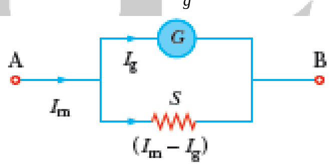
$$\frac{I_g G}{I_g} = \left(\frac{I_m - I_g}{I_g}\right).S \text{ or } G = \left(\frac{I_m}{I_g} - 1\right)S \Rightarrow G = (n-1)S$$

or

$$S = \frac{G}{n-1},$$

where

$$n = \frac{I_m}{I_g}.$$



Galvanometer into voltmeter: To measure high potential difference, resistance has to be increased by connecting resistance R with galvanometer coil of resistance G.

$$E = [R + G].I_g$$

$$\Rightarrow \frac{E}{I_g} = [R + G]$$

$$\Rightarrow R = \left[\frac{E}{I_g} - G\right]$$

is the resistance to be connected in series with galvanometer.

(b) Figure below shows two wires carrying currents I_1 and I_2 separated by a distance d. The magnetic induction at Q to the current element $I_1(d\vec{l}_1)$.

$$d\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{I_1(d\vec{l}_1 \times \vec{d}_{12})}{d_{12}^3}. \qquad (1)$$

The direction of magnetic induction at Q is into pages magnetic force on element Q of length $d\vec{l}_2$ carrying current I_2 is given by.

$$d\vec{F}_2 = I_1[d\vec{l}_2 \times d\vec{B}_1]. \qquad (2)$$

From Eqs. (1) and (2), we get

$$d\vec{F}_2 = I_1\left[d\vec{l}_2 \times \frac{\mu_0}{4\pi} I_1 \frac{(d\vec{l}_1 \times \vec{d}_{12})}{d_{12}^3}\right]$$

$$d\vec{F}_2 = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{d_{12}^3}[d\vec{l}_2 \times d\vec{l}_1 \times \vec{d}_{12}]$$

Using $\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{c}$, we get

$$d\vec{F}_2 = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{d_{12}^3}[(d\vec{l}_2.\vec{d}_{12})d\vec{l}_1 - (d\vec{l}_2.d\vec{l}_1)\vec{d}_{12}]$$

We have $(d\vec{l}_2.\vec{d}_{12}) = 0$ (being perpendicular) and $d\vec{l}_2.d\vec{l}_1 = dl_1 dl_2$ (being parallel). Therefore,

$$d\vec{F}_2 = -\frac{\mu_0}{4\pi}.\frac{I_1 I_2}{d_{12}^3}(dl_1 dl_2 \vec{d}_{12}) \qquad (3)$$

The negative sign indicates that $d\vec{F}_2$ has direction opposite to that of $\vec{d}_{12}$. Similarly, it can be proved that

$$d\vec{F}_1 = -\frac{\mu_0}{4\pi}.\frac{I_1 I_2}{d_{21}^3}(dl_1.dl_2.\vec{d}_{21})$$

The two wires will attract each other.