JEE MAIN | 2013

Mathematics

1. The circle passing through (1, –2) and touching the axis of *x* at (3, 0) also passes through the point

(1) (2, –5) (2) (5, –2)

(3) (–2, 5) (4) (–5, 2)

Solution

The equation of the circle due to point (3, 0) touching the axis of *x* is given by

$$(x-3)^2+(y-0)^2+\lambda y=0$$

It is given that the circle passes through point (1, –2). Therefore,

$$(1-3)^2+(-2)^2+\lambda(-2) \Rightarrow 4+4-2\lambda=0 \Rightarrow \lambda=4$$

Therefore, the equation of the circle is: $(x-3)^2+y^2+4y=0$

from which it is clear that (5, –2) satisfies.

Hence, the correct option is (2).

2. *ABCD* is a trapezium such that *AB* and *CD* are parallel and $BC \perp CD$. If $\angle ADB=\theta, BC=p$ and $CD=q,$ then *AB* is equal to

(1) $\dfrac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

(2) $\dfrac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$

(3) $\dfrac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

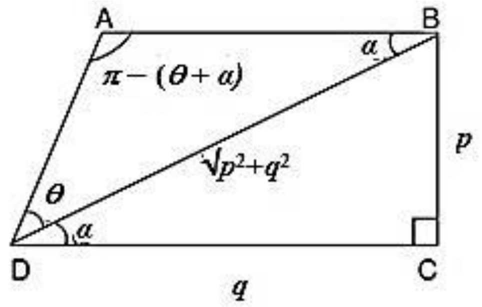
(4) $\dfrac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$

Solution

Using sine rule in the triangle ABD, as shown in the following figure, we get

$$\frac{\text{AB}}{\sin\theta}=\frac{\text{BD}}{\sin(\theta+\alpha)}=\frac{\sqrt{p^2+q^2}}{\sin(\theta+\alpha)} \Rightarrow \text{AB}=\frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha}$$

$$=\frac{\sqrt{p^2+q^2}\sin\theta}{[(\sin\theta\cdot g)/(\sqrt{p^2+q^2})]+[(\cos\theta\cdot p)/\sqrt{p^2+q^2}]}=\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)}.$$



Hence, the correct option is (4).

3. Given: A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.

Statement-I: An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement–II: If the line, $y = mx + \dfrac{\sqrt{5}}{m} (m \neq 0)$, is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(1) Statement – I is True; Statement – II is true; Statement – II is **not** a correct explanation for Statement – I.
(2) Statement – I is True; Statement – II is False.
(3) Statement – I is False; Statement– II is True.
(4) Statement – I is True; Statement – II is True; Statement – II is a **correct** explanation for Statement – I.

Solution

Let us consider that the common tangent to the parabola be

$$y + mx + \frac{\sqrt{5}}{m} \qquad (m \neq 0).$$

Its distance from the centre of the circle, (0, 0) must be equal to the radius of the circle, $\sqrt{\dfrac{5}{2}}$. Therefore, $\left|\dfrac{\sqrt{5}}{m}\right| = \dfrac{\sqrt{5}}{\sqrt{2}}\sqrt{1+m^2} \Rightarrow (1+m^2)m^2 = 2 \Rightarrow m^4 + m^2 - 2 = 0$

Hence, $(m^2 - 1)(m^2 + 2) = 0 \Rightarrow m = \pm 1$.

Therefore, the common tangents are obtained as $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$. Both statements are correct as the condition $m = \pm 1$ satisfies Statement-II.

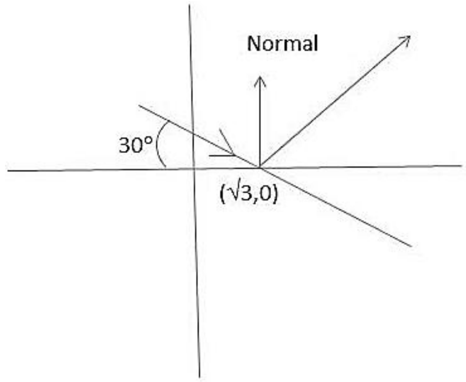
Hence, the correct option is (1).

4. A ray light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is

(1) $\sqrt{3}y = x - \sqrt{3}$ (2) $y = \sqrt{3}x - \sqrt{3}$
(3) $\sqrt{3}y = x - 1$ (4) $y = x + \sqrt{3}$

Solution

Let us consider a point (0, 1) on the line, that is, on the ray of light. So this point also lies on the image of this line. So the equation of the reflected ray is



$$(y-0)=(\tan 30^\circ)(x-\sqrt{3}) \Rightarrow y=\frac{1}{\sqrt{3}}\left(x-\sqrt{3}\right) \Rightarrow y\sqrt{3}=x-\sqrt{3} \Rightarrow x-\sqrt{3}y=\sqrt{3}$$

Therefore, the equation of reflected ray is $\sqrt{3}y = x-\sqrt{3}$.

Hence, the correct option is (1).

5. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?

(1) Median (2) Mode
(3) Variance (4) Mean

Solution

Before the grace marks were given, the variance of marks of the students is expressed as

$$\sigma_1^2 = \frac{\sum (x_i - \bar{x})^2}{N} \qquad (1)$$

After the grace marks were given, the variance of marks of the students is expressed as

$$\sigma_2^2 = \frac{\sum [(x_i+10)-(\bar{x}+10)]^2}{N} == \frac{\sum (x_i-\bar{x})^2}{N} \qquad (2)$$

From (1) and (2), we get $\sigma_1^2 = \sigma_2^2$.

Hence, *variance will not change* even after the grace marks were given; however, mean, median and mode will increase by 10.

Hence, the correct option is (3).

6. If x, y, z are in AP and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in AP, then

(1) $2x = 3y = 6z$ (2) $6x = 3y = 2z$
(3) $6x = 4y = 3z$ (4) $x = y = z$

Solution

If x, y, z are in AP, we have, $2y = x + z$ (1)

If $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in AP, we have, $2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

Therefore, $\tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$ (2)

Using Eq. (1) in Eq. (2), we get,

$$\tan^{-1}\left(\frac{x+z}{1-y^2}\right)=\tan^{-1}\left(\frac{x+z}{1-xz}\right)\Rightarrow y^2=xz \text{ or } x+z=0\Rightarrow x=y=z$$

Hence, the correct option is (4).

7. If $\int f(x)dx=\Psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

(1) $\frac{1}{3}x^3\Psi(x^3)-3\int x^3\Psi(x^3)dx+C$

(2) $\frac{1}{3}x^3\Psi(x^3)-\int x^2\Psi(x^3)dx+C$

(3) $\frac{1}{3}\left[x^3\Psi(x^3)-\int x^3\Psi(x^3)dx\right]+C$

(4) $\frac{1}{3}\left[x^3\Psi(x^3)-\int x^2\Psi(x^3)dx\right]+C$

Solution

We have, $\int f(x)dx=\Psi(x)$

Let $x^3 = t$ and $x^2\,dx = dt/3$. Therefore,

$$\int x^5 f(x^3)dx=\frac{1}{3}\int tf(t)dt=\frac{1}{3}\left[t\int(t)dt-\int\left[1\cdot\int f(t)dt\right]dt\right]=\frac{1}{3}x^3\Psi(x^3)-\int x^2\Psi(x^3)dx+C.$$

Hence, the correct option is (3).

8. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at (0, 3) is

(1) $x^2+y^2-6y+7=0$ (2) $x^2+y^2-6y-5=0$

(3) $x^2+y^2-6y+5=0$ (4) $x^2+y^2-6y-7=0$

Solution

Foci of the ellipse is given by $(\pm ae, 0)$. We have

Radius of the circle as, $r=\sqrt{(ae)^2+b^2}$ (1)

where $a = 4$; $b = 3$; $e=\sqrt{1-(9/16)}=\pm\sqrt{7}/4\Rightarrow ae=\pm\sqrt{7}$

Therefore, from Eq. (1), we get, $r=\sqrt{(ae)^2+b^2}=\sqrt{7+9}=4$

Therefore, equation of circle with centre $(0,3)$ and radius 4 is, $(x-0)^2+(y-3)^2=(4)^2$

That is, $x^2+y^2-6y-7=0$

Hence, the correct option is (4).

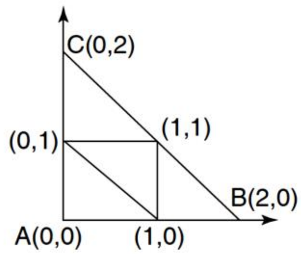
9. The x-coordinate of the incentre of the triangle that has the coordinates of midpoints of its sided as (0, 1) (1, 1) and (1, 0) is

(1) $2-\sqrt{2}$ (2) $1+\sqrt{2}$

(3) $1-\sqrt{2}$ (4) $2+\sqrt{2}$

Solution

From the following figure of the given triangle, the x-coordinate of the incentre is obtained as follows:



$$\frac{ax_1+bx_2+cx_3}{a+b+c}=\frac{2\times 2+2\sqrt{2}\times 0+2\times 0}{2+2+2\sqrt{2}}=\frac{4}{4+2\sqrt{2}}=\frac{2}{2+\sqrt{2}}=2-\sqrt{2}.$$

Hence, the correct option is (1).

10. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x |t|dt,\ x\in R,$ which are parallel to the line $y = 2x$, are equal to

(1) ±2 (2) ±3
(3) ±4 (4) ±1

Solution

Slope of the tangent to the curve will be 2.

So we can equate the slope as, $\frac{dy}{dx}=|x|=2\Rightarrow x=\pm 2$

For x = 2, we have, $y=\int_0^2 |t|dt=2$

For x = –2, we have, $y=\int_0^{-2} |t|\,dt=-2$

Therefore, one tangent passes through the point (2, 2) and has slope 2:

$$y-2=2(x-2)\Rightarrow y=2x-2$$

The other tangent passes through the point (–2, –2) and has slope 2:

$$y+2=2(x+2)\Rightarrow y=2x+2$$

Substituting y = 0, we get x-intercepts as, x = 1 and −1.

Hence, the correct option is (4).

11. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, …, is

(1) $\frac{7}{9}(99-10^{-20})$ (2) $\frac{7}{81}(179+10^{-20})$

(3) $\frac{7}{9}(99+10^{-20})$ (4) $\frac{7}{81}(179-10^{-20})$

Solution

have t_r = 0.777, …, r, which is expressed as

$$7(10^{-1}+10^{-2}+10^{-3}+\cdots+10^{-r})=7\times\frac{10^{-1}\left(1-\left(10^{-1}\right)^r\right)}{\left(1-10^{-1}\right)}=7\times\frac{10^{-1}\times 10\left(1-10^{-r}\right)}{9}=\frac{7}{9}(1-10^{-r})$$

Therefore, $S_{20} = \sum_{r=1}^{20} t_r = \frac{7}{9}\left(20 - \sum_{r=1}^{20} 10^{-r}\right) = \frac{7}{9}\left(20 - \frac{1}{9}(1-10^{-20})\right) = \frac{7}{81}(179 + 10^{-20})$.

Hence, the correct option is (2).

12. Consider

Statement-I: $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-II: $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.

(1) Statement -I is True; Statement – II is true; Statement-II is **not** a correct explanation for Statement-I
(2) Statement-I is True; Statement-II is False.
(3) Statement-I is False; Statement-II is True
(4) Statement-I is True; Statement-II is True; Statement-II is a **correct** explanation for Statement-I.

Solution

Table 1

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Table 2

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Tautology

From Table 1, it is obvious that $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is fallacy. From Table 2, it is obvious that $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is tautology. Although both statements are true, Statement-II is **not** a correct explanation for Statement-I.

Hence, the correct option is (1).

13. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is

(1) 36 (2) 18

(3) $\frac{27}{4}$ (4) 9

Solution

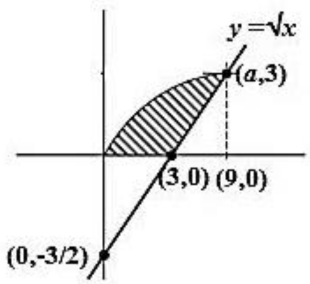
First solving the equations,

We have, $2\sqrt{x} = x - 3$ (1)

Squaring on both side of Eq. (1), we get, $4x = x^2 - 6x + 9 \Rightarrow x^2 - 10x + 9 \Rightarrow x = 9,\ x = 1$.

Since $x=1$ intersects the parabola below the x-axis, this point is extraneous.
So for $x=9$ we have, $y=3$.
Therefore, the required area under the curve (see the following figure) is

$$\int_0^3 (2y+3)-y^2 \Rightarrow \left[y^2+3y-\frac{y^3}{3}\right]_0^3 = 9+9-9=9.$$



Hence, the correct option is (4).

14. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as

(1) sec$A$ cosec$A$ + 1 (2) tan$A$ + cot$A$
(3) sec$A$ + cosec$A$ (4) sin$A$ cos$A$ + 1

Solution

We have

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=\frac{1}{\cot A(1-\cot A)}+\frac{\cot A\times\cot A}{\cot A\times(1-\tan A)}=\frac{1}{\cot A(1-\cot A)}-\frac{\cot^2 A}{(1-\cot A)}=\frac{1-\cot^3 A}{\cot A(1-\cot A)}$$

$$=\frac{\text{cosec}^2 A+\cot A}{\cot A}=1+\sec A\,\text{cosec}\,A.$$

Hence, the correct option is (1).

15. The real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]

(1) lies between 2 and 3 (2) lies between −1 and 0
(3) does not exist (4) lies between 1 and 2

Solution

When the given equation, $2x^3 + 3x + k = 0$
has two distinct real roots in [0, 1], then $f'(x)$ will change sign, but $f'(x) = 6x^2 + 3 > 0$, for all values of $x \in$ R. Therefore, no value of k exists.

Hence, the correct option is (3).

16. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(1) $\frac{1}{2}$ (2) 1

(3) 2 (4) $-\frac{1}{4}$

Solution

We have, $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}=\lim_{x\to 0}\frac{2\sin^2 x}{x^2}\frac{(3+\cos x)}{\frac{\tan 4x}{4x}\times 4x}=\frac{2(3+1)}{1\times 4}=2$

Hence, the correct option is (3).

17. Let T_n be the number of all possible triangles formed by joining vertices of an *n*-sided regular polygon. If $T_{n+1}-T_n=10$, then the value of *n* is

(1) 5 (2) 10
(3) 8 (4) 7

Solution

If $T_{n+1}-T_n=10$, then the value of *n* is obtained as follows:

$${}^{n+1}C_3-{}^{n}C_3=10\Rightarrow\frac{(n+1)(n)(n-1)}{6}-\frac{n(n-1)(n-2)}{6}=10\Rightarrow n(n-1)(\cancel{n}+1-\cancel{n}+2)=60$$

$$\Rightarrow n(n-1)=20\Rightarrow n(n-1)=5\times 4$$

$\therefore n=5$

Hence, the correct option is (1).

18. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production *P* w.r.t. additional number of workers *x* is given by $\frac{dP}{dx}=100-12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

(1) 3000 (2) 3500
(3) 4500 (4) 2500

Solution

Given that,

$$\frac{dP}{dx}=100-12\sqrt{x}\Rightarrow dP=\left(100-12\sqrt{x}\right)dx$$

Therefore, the new level of production of items is

$$\int_{2000}^{P}dP=\int_{0}^{25}(100-12\sqrt{x})dx\Rightarrow(P-2000)=25\times 100-\frac{12\times 2}{3}(25)^{3/2}\Rightarrow P=3500.$$

Hence, the correct option is (2).

19. **Statement-I**: The value of the integral $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement-II: $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx.$

(1) Statement-I is True; Statement-II is true; Statement-II is **not** a correct explanation for Statement-I.
(2) Statement-I is True; Statement-II is False.
(3) Statement-I is False; Statement-II is True.
(4) Statement-I is True; Statement-II is True; Statement-II is a **correct** explanation for Statement-I.

Solution

We have

$$I=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}\Rightarrow\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}\Rightarrow\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}\Rightarrow\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\cot x}}$$

$$I=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\frac{1}{\tan x}}}\Rightarrow\int_{\pi/6}^{\pi/3}\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx\Rightarrow 2I=\int_{\pi/6}^{\pi/3}\frac{\left(1+\sqrt{\tan x}\right)}{\left(1+\sqrt{\tan x}\right)}dx\Rightarrow I=\frac{1}{2}\left[\frac{\pi}{3}-\frac{\pi}{6}\right]\Rightarrow I=\frac{\pi}{12}.$$

Hence, the correct option is (3).

20. If $p=\begin{bmatrix}1&\alpha&3\\1&3&3\\2&4&4\end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A|=4$, then α is equal to

(1) 11 (2) 5
(3) 0 (4) 4

Solution

We have, $p=\begin{bmatrix}1&\alpha&3\\1&3&3\\2&4&4\end{bmatrix}$

Therefore, $|Adj\ A|=|A|^2\Rightarrow|Adj\ A|=16\Rightarrow 1(12-12)-\alpha(4-6)+3(4-6)=16\Rightarrow 2\alpha-6=16$
$\Rightarrow 2\alpha=22\Rightarrow\alpha=11.$

Hence, the correct option is (1).

21. The number of values of k, for which the system of equations

$$(k+1)x+8y=4k$$
$$kx+(k+3)y=3k-1$$

has no solution, is

(1) 1 (2) 2
(3) 3 (4) infinite

Solution

For no solution the lines must be parallel for which we have,

$$\frac{k+1}{k}=\frac{8}{k+3}\neq\frac{4}{3k-1}\Rightarrow k^2+4k+3=8k\Rightarrow k^2-4k+3=0\Rightarrow k=1,3$$

$k=1$ is rejected since it gives coincident lines. Therefore number of such values of k is just one.

Hence, the correct option is (1).

22. If $y=\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at x = 1 is equal to

(1) $\frac{1}{2}$ (2) 1

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Solution

We have, $y = \sec(\tan^{-1} x)$

Therefore, $\frac{dy}{dx} = \sec(\tan^{-1} x)\tan(\tan^{-1} x).\frac{1}{1+x^2} \Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = \sqrt{2}\times 1\times \frac{1}{2} = \frac{1}{\sqrt{2}}$.

Hence, the correct option is (4).

23. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have

(1) exactly one value (2) exactly two values
(3) exactly three values (4) any value

Solution

For lines to be coplanar, scalar triple product of vectors joining the two given points of the lines and the parallel vectors to the line must be 0.

We have,

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow 1(1+2k) + 1(1+k^2) - 1(2-k) = 0 \Rightarrow k^2 + 1 + 2k + 1 - 2 + k = 0$$

$$k^2 + 3k = 0 \Rightarrow (k)(k+3) = 0$$

Therefore, there are two values of k.

Hence, the correct option is (2).

24. Let A and B be two sets containing 2 elements and 4 elements, respectively. The number of subsets of $A \times B$ having 3 or more elements is

(1) 220 (2) 219
(3) 211 (4) 256

Solution

We know that $A \times B$ will have eight elements.

Out of these 8 elements, the total number of subsets containing 3 or more elements is,

$${}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 = 256 - 1 - 8 - 28 = 219.$$

Hence, the correct option is (2).

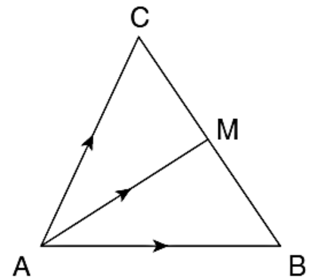
25. If the vectors $\overline{AB} = 3\hat{i} + 4\hat{k}$ and $\overline{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is

(1) $\sqrt{72}$ (2) $\sqrt{33}$
(3) $\sqrt{45}$ (4) $\sqrt{18}$

Solution

From the following figure, we see that, $\overline{AM} = \frac{\overline{AB} + \overline{AC}}{2} \Rightarrow \overline{AM} = 4\hat{i} - \hat{j} + 4\hat{k}$

Therefore, $\left|\overline{AM}\right| = \sqrt{16+16+1} = \sqrt{33}$.



Hence, the correct option is (2).

26. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(1) $\frac{13}{3^5}$ (2) $\frac{11}{3^5}$

(3) $\frac{10}{3^5}$ (4) $\frac{17}{3^5}$

Solution

We have p = Correct answer = 1/3; q = Incorrect answer = 2/3.

Therefore probability of either 4 or 5 correct answers is,

$$^5C_4\left(\frac{1}{3}\right)^{5-1}\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^{5-0} = {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5 = \frac{5\times 2}{(3)^5} + \frac{1}{(3)^5} = \frac{11}{3^5}.$$

Hence, the correct option is (2).

27. If z is a complex number of unit modulus and arguments θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals

(1) $\frac{\pi}{2}-\theta$ (2) θ

(3) $\pi-\theta$ (4) $-\theta$

Solution

We have, $|z| = 1 \Rightarrow z\bar{z} = 1$. Therefore, $\frac{1+z}{1+\bar{z}} = \frac{1+z}{1+(1/z)} = z$. Therefore $\arg z = \theta$

Hence, the correct option is (2).

28. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then $a : b : c$ is

(1) 3 : 2 : 1 (2) 1 : 3 : 2

(3) 3 : 1 : 2 (4) 1 : 2 : 3

Solution

Discriminant = 4 – 12 < 0 and $1, 2, 3 \in \mathrm{R}$

Therefore the equation has complex conjugate roots, which means both roots are common.

Hence all coefficients must be proportional, $\frac{a}{1}=\frac{b}{2}=\frac{c}{3}$.

Therefore, a:b:c = 1:2:3.

Hence, the correct option is (4).

29. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(1) $\frac{5}{2}$ (2) $\frac{7}{2}$

(3) $\frac{9}{2}$ (4) $\frac{3}{2}$

Solution

The two parallel planes can be written as, $4x+2y+4z=16; 4x+2y+4z=-5$

Let a point on the first plane is $(0, 0, 4)$

Therefore, its distance from the other plane is obtained as, $\frac{|0+0+16+5|}{\sqrt{4^2+2^2+4^2}}=\frac{21}{\sqrt{36}}=\frac{21}{6}=\frac{7}{2}$.

Hence, the correct option is (2).

30. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is

(1) 120 (2) 210

(3) 310 (4) 4

Solution

We have

$$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}=\left(\frac{(x^{1/3}+1)\cancel{(x^{2/3}-x^{1/3}+1)}}{\cancel{\left(x^{2/3}-x^{1/3}+1\right)}}-\frac{1}{\sqrt{x}}.\frac{(\sqrt{x}+1)\cancel{(\sqrt{x}-1)}}{\cancel{(\sqrt{x}-1)}}\right)^{10}$$

$$=\left(x^{1/3}+1-\frac{x^{1/2}+1}{x^{1/2}}\right)=\left(x^{1/3}+1-1-x^{-1/2}\right)=(x^{1/3}-x^{-1/2})^{10}$$

Therefore, $T_{r+1}={}^{10}C_r\left(x^{1/3}\right)^{10-r}\left(-x^{1/2}\right)^r \Rightarrow (-1)^r\,{}^{10}C_r x^{(20-5r)/6}$

For T_{r+1} to be independent of x, $20-5r=0 \Rightarrow r=4$

Therefore, $T_5=T_{4+1}=(-1)^4\,{}^{10}C_4=210$.

Hence, the correct option is (2).