JEE ADVANCED 2013
PAPER 2
MATHEMATICS

Only One Option Correct Type

This section contains **TEN** questions. Each has **FOUR** options (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. $a \in \mathbb{R}$ (the set of all real numbers), $a \neq -1$,

$$\lim_{n\to\infty} \frac{(1^a + 2^a + \ldots + n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+\ldots+(na+n)]} = \frac{1}{60},$$

Then a = ?

(A) 5 (B) 7

(C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$

Solution

We have

$$\lim_{n\to\infty} \frac{\sum_{r=1}^{n}(r)^a}{(n+1)^{a-1}\left[\sum_{r=1}^{n}(na+r)\right]} = \frac{1}{60}$$

That is,

$$\lim_{n\to\infty} \frac{n^a \sum\left(\frac{r}{n}\right)^a}{n(n+1)^{a-1}\sum\left(a+\frac{r}{n}\right)} = \frac{1}{60} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{a-1}} \frac{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^a}{\frac{1}{n}\sum_{r=1}^{n}\left(a+\frac{r}{n}\right)} = \frac{1}{60}$$

$$= \frac{\int_0^1 x^a}{\int_0^1 a+x} = \frac{1}{60}$$

$$\frac{x^{a+1}\Big|_0^1}{(a+1)\left[\left(ax+\frac{x^2}{2}\right)\Big|_0^1\right]} = \frac{1}{60}$$

$$\frac{2}{(2a+1)(a+1)} = \frac{1}{60}$$

$$2a^2 + 3a + 1 = 120$$

$$2a^2 + 3a - 119 = 0$$

Therefore,

$$a = \frac{-3 \pm \sqrt{9+8(119)}}{4}$$

$$= \frac{-3 \pm \sqrt{961}}{4}$$

$$= \frac{-3 \pm 31}{4}$$

Thus, $a = 7, -\frac{17}{2}$.

2. $a \in \mathbb{R}$ (the set of all real numbers), $a \neq -1$,

$$\lim_{n\to\infty} \frac{(1^a + 2^a + \ldots + n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+\ldots+(na+n)]} = \frac{1}{60},$$

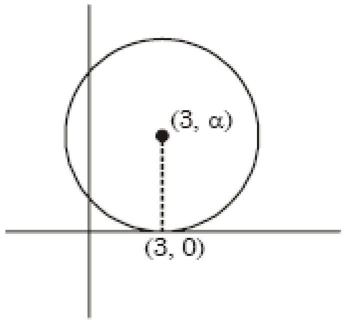
Then a = ?

(A) 5 (B) 7

(C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$

Solution

From the following figure, it is clear that the centre is (3, α) and radius is $|\alpha|$.



Therefore,

$$x^2 + y^2 - 6x - 2\alpha y + c = 0.$$

The radius is

$$9 + \alpha^2 - c = \alpha^2$$

Therefore,

$$c = 9$$

The intercept on y-axis is $2\sqrt{a^2 - c} = 2\sqrt{7}$

$$\alpha^2 - 9 = 7$$

$$\Rightarrow \alpha = \pm 4$$

Therefore, the equation is

$$x^2 + y^2 - 6x \pm 8y + 9 = 0.$$

3. Two lines $L_1 : x = 5, \frac{y}{3-a} = \frac{z}{-2}$ and $L_2 : x = a, \frac{y}{-1} = \frac{z}{2-a}$ are coplanar. Then α can take value(s)

(A) 1 (B) 2
(C) 3 (D) 4

Solution

We have

$$L_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

As the two lines are coplanar, we have

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$$(5-\alpha)[(\alpha-2)(\alpha-3)-2]=0$$
$$(5-\alpha)[\alpha^2-5\alpha+6-2]=0$$
$$(5-\alpha)[\alpha^2-5\alpha+4]=0$$

Therefore, $\alpha = 5,4,1$.

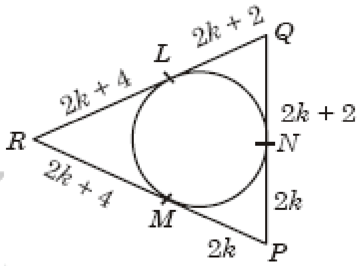
4. In a triangle *PQR*, *P* is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides *PQ*, *QR* and *RP* at *N*, *L* and *M* respectively, such that the lengths of *PN*, *QL* and *RM* are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are)

(A) 16 (B) 18

(C) 24 (D) 22

Solution

From the following figure, we see that *QR* is largest side.



Therefore,

$$PM = PN = 2k$$
$$RM = RL = 2k+4$$
$$QL = QN = 2k+2$$

Therefore,

$$\left.\begin{aligned} QR &= 4k+6 \\ RP &= 4k+6 \\ PQ &= 4k+2 \end{aligned}\right\} \qquad (1)$$

Hence,

$$\cos P = \frac{(PQ)^2 + (PR)^2 - (QR)^2}{2(PQ)(PR)}$$

$$\frac{1}{3} = \frac{(4k+2)^2 + (4k+4)^2 - (4k+6)^2}{2(4k+2)(4k+4)}$$

$$\frac{1}{3} = \frac{(2k+1)^2 + 4(k+1)^2 - (2k+3)^2}{4(2k+1)(k+1)}$$

$$\frac{1}{3} = \frac{4(k+1)^2 - (4k+4)(2)}{4(2k+1)(k+1)}$$

$$\frac{1}{3} = \frac{(k+1)^2 - 2(k+1)}{(k+1)(2k+1)}$$

$$(k+1)(2k+1) = 3(k-1)(k+1)$$

$$k = -1 \text{ or } 2k+1 = 3k-3$$

$$4 = k$$

Substituting the values in the set of equations (1), we get PQ = 18; QR = 22; and RP = 22.

5. Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, \ldots\}$. Further $H_1 = \left\{ z \in \mathbb{C} : \text{Re}\, z > \frac{1}{2} \right\}$ and $H_2 = \left\{ z \in \mathbb{C} : \text{Re}\, z < \frac{-1}{2} \right\}$, where $\mathbb{C}$ is the set of all complex numbers. If $z_1 \in P \cap H_1, z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 = ?$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

Solution

We note that $|\omega| = 1$. We also note that α_i are possible values of z_1 and β_i are possible values of z_2, whir i = 1, 2, 3. Therefore,

$$\omega = \frac{\sqrt{3}}{2} + \frac{i}{2};$$

$$\omega = e^{i\frac{\pi}{6}};$$

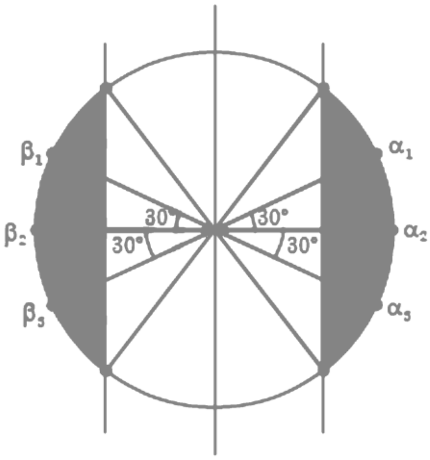
$$\omega_2 = e^{i\frac{\pi}{3}};$$

$$\omega^3 = e^{i\frac{\pi}{2}};$$

$$\omega^4 = e^{2i\frac{\pi}{3}};$$

$$\omega^5 = e^{i\frac{5\pi}{6}}.$$

Thus, $\angle z_1 o z_2$ can take the values $\frac{2\pi}{3}, \frac{5\pi}{6}$.



6. If $3^x = 4x^{-1}$, then x =?

(A) $\dfrac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\dfrac{2}{2 - \log_2 3}$

(C) $\dfrac{1}{1 - \log_4 3}$ (D) $\dfrac{2\log_2 3}{2\log_2 3 - 1}$

Solution

We have

$$3^x = 4^{x-1}$$
$$x\log_2 3 = (x-1)\log_2 4$$
$$x\log_2 3 = (x-1)\cdot 2$$
$$2x - x\log_2 3 = 2$$
$$x[2 - \log_2 3] = 2$$

That is,

$$x = \frac{2}{2 - \log_2 3},$$

which is option (A). Also

$$\frac{2}{2 - \dfrac{1}{\log_3 2}} = \frac{2\log_3 2}{2\log_3 2 - 1},$$

which is option (B). Also

$$\frac{1}{1 - \dfrac{1}{2}\log_2 3} = \frac{1}{1 - \log_4 3},$$

which is option (C).

7. Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n = ?

(A) 57 (B) 55

(C) 58 (D) 56

Solution

We have

$$P = [P_{ij}]_{n\times n} \qquad P^2 \neq 0$$

That is,

$$P_{ij} = \omega^{\hat{i}+\hat{j}}$$

$$P = \begin{bmatrix} \omega^2 & 1 & \omega & \omega^2 & 1 & \ldots \\ 1 & \omega & \omega^2 & 1 & \omega & \ldots \\ \omega & \omega^2 & 1 & \omega & \omega^2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix}_{n\times n}$$

$$P^2 = \begin{bmatrix} \omega^2 & 1 & \omega & \omega^2 & 1 & \ldots \\ 1 & \omega & \omega^2 & 1 & \omega & \ldots \\ \omega & \omega^2 & 1 & \omega & \omega^2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix} \begin{bmatrix} \omega^2 & 1 & \omega & \omega^2 & 1 & \ldots \\ 1 & \omega & \omega^2 & 1 & \omega & \ldots \\ \omega & \omega^2 & 1 & \omega & \omega^2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix}$$

Therefore,

$$(\omega^4 + 1 + \omega^2) + (\omega^4 + 1 + \omega^2) + \ldots = 0$$

This is possible only when n is a multiple of 3. Therefore, n can be 55, 58, 56 ($P^2 \neq 0$)

8. The function $f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$ has a local minimum or a local maximum at x =

(A) −2 (B) $\frac{-2}{3}$

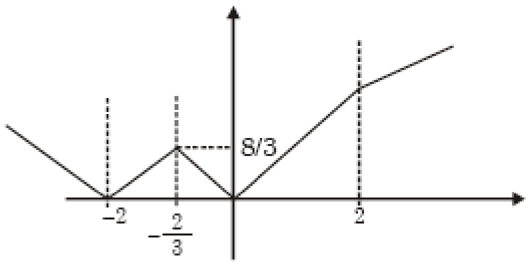
(C) 2 (D) $\frac{2}{3}$

Solution

We have

$$\begin{array}{ll} x < -2, & f(x) = -2x - 4 \\ -2 \le x < -\frac{2}{3}, & f(x) = 2x + 4 \\ -\frac{2}{3} \le x \le 0, & f(x) = -4x \\ 0 \le x < 2 & f(x) = 4x \\ x \ge 2, & f(x) = 2x + 4 \end{array}$$

It is clear from the following figure that $x = -2$ and $x = 0$ are the points of minima. Therefore, $x = -\frac{2}{3}$ is point of maxima.



Paragraph Type

This section contains **4 Paragraphs** each describing theory, experiment, data etc. **Eight questions** relate to four paragraphs with two questions on each paragraph. Each question of a paragraph has **only one correct answer** among the four choices (*A*), (*B*), (*C*) and (*D*).

Paragraph for Questions 9 and 10: Let f: [0, 1] → ℝ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \ge e^x$, $x \in [0, 1]$.

9. Which of the following is true for $0 < x < 1$?

(A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$

(C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

Solution

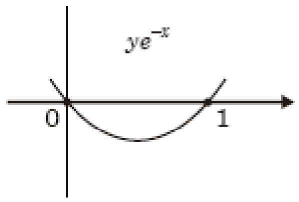
We have

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y \ge e^x$$

$$\Rightarrow e^{-x}\frac{d^2y}{dx^2} - 2e^{-x}\frac{dy}{dx} + e^{-x}y \ge 1$$

$$\Rightarrow \frac{d^2}{dx^2}(ye^{-x}) \ge 1$$

From this and the following figure, ye^{-x} is concave up. Hence, $-\infty < f(x) < 0$.



10. If the function $e^{-x} f(x)$ assumes its minimum in the interval [0, 1] at $x = \frac{1}{4}$, which of the following is true?

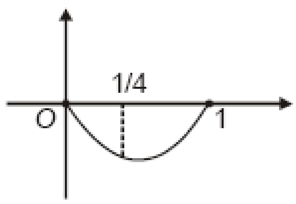
(A) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$ (B) $f'(x) > f(x), 0 < x < \frac{1}{4}$

(C) $f'(x) < f(x), 0 < x < \frac{1}{4}$ (D) $f'(x) < f(x), \frac{3}{4} < x < 1$

Solution

We know that $\frac{d}{dx}(ye^{-x})$ is an increasing function. Therefore,

$0 < x < \frac{1}{4}$	$x > \frac{1}{4}$
$\frac{d}{dx}(ye^{-x}) < 0$	$\frac{d}{dx}(ye^{-x}) > 0$
$e^{-x}\frac{dy}{dx} - e^{-x}y < 0$	$e^{-x}\frac{dy}{dx} - e^{-x}y > 0$
$\frac{dy}{dx} < y$	$\frac{dy}{dx} > y$
$f'(x) < f(x)$	$f'(x) > f(x)$



Paragraph for Questions 11 and 12: Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a$, $a > 0$.

11. Length of chord PQ is

(A) $7a$ (B) $5a$

(C) $2a$ (D) $3a$

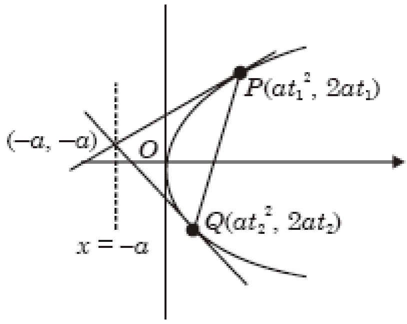
Solution

See the following figure. We have $a(t_1 + t_2) = a$. Therefore, $t_1 + t_2 = 1$. Therefore,

$$PQ = a(t_1 - t_2)^2$$
$$= a\left[(t_1 + t_2)^2 - 4t_1t_2\right]$$

From Eq. (1), we get

$$PQ = a[1+4] = 5a$$



12. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta = ?$

(A) $\frac{2}{3}\sqrt{7}$ (B) $\frac{-2}{3}\sqrt{7}$

(C) $\frac{2}{3}\sqrt{5}$ (D) $\frac{-2}{3}\sqrt{5}$

Solution

See the following figure. We have $t_1 + t_2 = 1$.

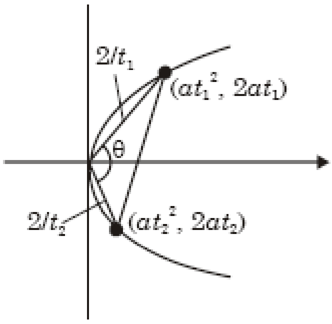
$$t_1t_2 = -1$$
$$t_1 = \frac{1 \pm \sqrt{5}}{2}, \quad t > 0$$

Therefore,

$$t_1 = \frac{1+\sqrt{5}}{2}$$

and

$$\tan\theta = \frac{\frac{2}{t_1} - \frac{2}{t_2}}{1 + \frac{4}{t_1t_2}} = \frac{2(t_2 - t_1)}{t_1t_2 + 4} = \frac{2(1-(1+\sqrt{5}))}{3} = -\frac{2\sqrt{5}}{3}$$



Paragraph for Questions 13 and 14: Let $S = S_1 \cap S_2 \cap S_3$, where

$$S_1 = \{z \in \mathbb{C} : |z| < 4\},\ S_2 = \left\{z \in \mathbb{C} : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0\right\} \text{and } S_3 = \{z \in \mathbb{C} : \text{Re } z > 0\}.$$

13. Area of S = ?

(A) $\frac{10\pi}{3}$ (B) $\frac{20\pi}{3}$

(C) $\frac{16\pi}{3}$ (D) $\frac{32\pi}{3}$

Solution

As we see, S_1 represents circle with centre (0, 0) and radius 4:

$$S_1 : |z| < 4 \Rightarrow x^2 + y^2 < 16$$

Therefore,

$$S_1 : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0$$

$$\text{Im}\left[\frac{[(x-1)+(y+\sqrt{3}i)][1+\sqrt{3}i]}{2}\right] > 0$$

Also

$$S_2 \equiv y + \sqrt{3}x > 0$$

$$S_3\ \text{Re}(z) > 0, \quad (x > 0)$$

$$S = S_1 \cap S_2 \cap S_3$$

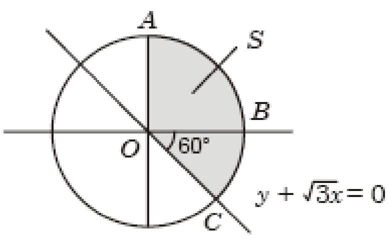
The area of the shaded region (see the following figure) is

$$OAB\ + OBC = \frac{\pi(4)^2}{4} + \frac{60}{360} \times \pi(4)^2$$

$$= 4\pi + \frac{16\pi}{6}$$

$$= 4\pi + \frac{8\pi}{3}$$

$$= \frac{20\pi}{3}$$



14. $\min_{z\in S}|1-3i-z| =$

(A) $\frac{2-\sqrt{3}}{2}$ (B) $\frac{2+\sqrt{3}}{2}$

(C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

Solution

We have $\min|1-3i-z|$. The minimum distance of z from (1, –3) from $y+\sqrt{3}x=0$ is

$$\left|\frac{-3+\sqrt{3}}{2}\right| = \frac{3-\sqrt{3}}{2}.$$

Paragraph for Questions 15 and 16: A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

15. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$

(C) $\frac{558}{648}$ (D) $\frac{566}{648}$

Solution

Following table shows the probability of draining balls from the boxes:

B_1	B_2	B_3
1 W	2 W	3 W
3 R	3 R	4 R
2 B	4 B	5 B

$$P(WWW+RRR+BBB) = \left(\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}\right)+\left(\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}\right)+\left(\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}\right)$$

$$=\frac{6+36+40}{648}=\frac{82}{648}$$

16. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$

(C) $\frac{65}{181}$ (D) $\frac{55}{181}$

Solution

$$P\left(\frac{B_2}{WR}\right)=\frac{P\left(\frac{WR}{B_2}\right)\times P(B_2)}{P\left(\frac{WR}{B_1}\right)\cdot P(B_1)+P\left(\frac{WR}{B_2}\right)\cdot P(B_2)+P\left(\frac{WR}{B_3}\right)\cdot P(B_3)}$$

$$=\frac{\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}\times\frac{1}{3}}{\left(\frac{{}^1C_1\times{}^3C_1}{{}^9C_2}\times\frac{1}{3}\right)+\left(\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}\times\frac{1}{3}\right)+\left(\frac{{}^3C_1\times{}^4C_1}{{}^{12}C_2}\times\frac{1}{3}\right)}$$

$$=\frac{\frac{2\times 3}{9\times 4}}{\frac{3\times 2}{6\times 5}+\frac{2\times 3}{9\times 4}+\frac{3\times 4\times 2}{12\times 11}}$$

$$=\frac{6\times 5\times 6\times 11}{36\times 181}$$

$$=\frac{55}{181}.$$

Matching List Type

This section contains **4 multiple choice questions. Each question has matching lists.** The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

17. Match List I with List II and select the correct answer using the code given below the lists :

List I	*List II*
P. $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)^2}{\cot(\sin^{-1}y)+\tan(\sin^{-1})y}\right)+y^4\right)^{1/2}$	1. $\frac{1}{2}\sqrt{\frac{5}{3}}$
Q. If $\cos x+\cos y+\cos z=0=\sin x+\sin y+\sin z$ then possible value of $\cos\frac{x-y}{2}$ is	2. $\sqrt{2}$
R. If $\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x=\cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$ then possible value of $\sec x$ is	3. $\frac{1}{2}$
S. If $\cot(\sin^{-1}\sqrt{1-x^2})=\sin(\tan^{-1}(x\sqrt{6})), x\neq 0,$ then possible value of x is	4. 1

Codes:

	P	Q	R	S
(A)	4	3	1	2
(B)	4	3	2	1
(C)	3	4	2	1
(D)	3	4	1	2

Solution

(P) $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)^2}{\cot(\sin^{-1}y)+\tan(\sin^{-1})y}\right)+y^4\right)^{1/2}$

$$=\left\{\frac{1}{y^2}\left(\frac{\frac{1}{\sqrt{1+y^2}}+\frac{y^2}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y}+\frac{y}{\sqrt{1-y^2}}}\right)^2+y^4\right\}^{1/2}$$

$$=\left\{\frac{1}{y^2}\left(\frac{y\sqrt{1+y^2}\sqrt{1-y^2}}{1}\right)^2+y^4\right\}^{1/2}$$

$$=\{1-y^4+y^4\}^{1/2}=1$$

(Q) If $\cos x+\cos y+\cos z=\sin x+\sin y+\sin z=0$, then the possible value of $x-y$ is $\pm\frac{2\pi}{3}$. That is, $\cos\left(\frac{x-y}{2}\right)=\frac{1}{2}$.

(R) Given equation can be written is

$$\cos 2x\left\{\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{\pi}{4}+x\right)\right\}=\sin 2x(1-\tan x)$$

$$\Rightarrow \cos 2x\cdot 2\sin\frac{\pi}{4}\cdot\sin x=\sin 2x(1-\tan x)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos 2x=\cos x-\sin x$$

$$\Rightarrow \frac{1}{\sqrt{2}}(\cos x+\sin x)=1$$

$$\Rightarrow x=\frac{\pi}{4}$$

So, $\sec x=\sqrt{2}$

(S) Given equation is $\frac{x}{\sqrt{1-x^2}}=\frac{\sqrt{6}x}{\sqrt{1+6x^2}}$. Either $x = 0$ or $1+6x^2=6-6x^2$

$$\Rightarrow x=\pm\frac{1}{2}\sqrt{\frac{5}{3}}$$

18. A line L: $y = mx + 3$ meets y –axis at $E(0, 3)$ and the arc of the parabola $y^2 = 16x$, $0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum. Match List I with List II and select the correct answer using the code given below the lists:

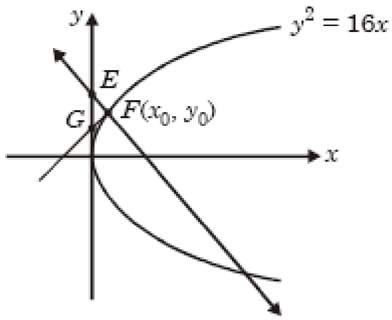
	List I	List II
P.	$m =$	1. $\frac{1}{2}$
Q.	Maximum area of ΔEFG is	2. 4
R.	$y_0 =$	3. 2
S.	$y_1 =$	4. 1

Codes:

P	Q	R	S

(A)	4	1	2	3
(B)	3	4	1	2
(C)	1	3	2	4
(D)	1	3	4	2

Solution



The parabola is $y^2 = 16x$ and line is $y = mx + 3$. Solving, we get

$$(mx+3)^2 = 16x$$

$$\Rightarrow m^2x^2 + (6m-16)x + 9 = 0 \quad (1)$$

Also, the tangent at F is

$$yy_0 = 8(x + x_0)$$

Thus,

$$y_1 = \frac{8x_0}{y_0} \quad (2)$$

Now, area of ΔEFG,

$$\Delta = \frac{1}{2}(3 - y_1)\cdot x_0$$

$$= \frac{1}{2}\left(3 - \frac{8x_0}{y_0}\right)x_0$$

$$= \frac{1}{2}\left(3x_0 - \frac{8x_0^2}{y_0}\right)$$

$$= \frac{1}{2}\left(3x_0 - \frac{8x_0^2}{4\sqrt{x_0}}\right)$$

For Δ to be maximum, we have

$$\frac{d\Delta}{dx_0} = 0$$

$$\Rightarrow 3 - 2\times\frac{3}{2}\sqrt{x_0} = 0$$

$$\Rightarrow x_0 = 1$$

Thus, $y_0 = 4$. Therefore,

$$y_1 = \frac{8x_0}{y_0} = 2$$

From $y_0 = mx_0 + 3 \Rightarrow m = 1$.

19. Match List-I with List-II and select the correct answer using the code given below the lists

List-I	*List-II*

P. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a}\times\vec{b})$, $3(\vec{b}\times\vec{c})$ and $(\vec{c}\times\vec{a})$ is	1.	100
Q. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a}+\vec{b})$, $(\vec{b}+\vec{c})$ and $2(\vec{c}+\vec{a})$ is	2.	30
R. Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is	3.	24
S. Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a}+\vec{b})$ and $\vec{a}$ is	4.	60

Codes:

	P	Q	R	S
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

Solution

(P) We have $[\vec{a}\ \vec{b}\ \vec{c}] = 2$. Therefore,

$$\begin{aligned} V &= [2(\vec{a}\times\vec{b})3(\vec{b}\times\vec{c})\ \vec{c}\times\vec{a}] \\ &= 6[\vec{a}\ \vec{b}\ \vec{c}]^2 \\ &= 24 \end{aligned}$$

(Q) We have $[\vec{a}\ \vec{b}\ \vec{c}] = 5$. Therefore,

$$\begin{aligned} V &= [3(\vec{a}+\vec{b})\vec{b}+\vec{c}\ 2(\vec{c}+\vec{a})] \\ &= 6[\vec{a}+\vec{b}\ \vec{b}+\vec{c}\ \vec{c}+\vec{a}] \\ &= 12[\vec{a}\ \vec{b}\ \vec{c}] \\ &= 60 \end{aligned}$$

(R) We have $|\vec{a}\times\vec{b}| = 40$. Therefore,

$$\begin{aligned} A &= \frac{1}{2}\left|(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})\right| \\ &= \frac{1}{2}\cdot 5\left|\vec{a}+\vec{b}\right| \\ &= \frac{5}{2}\times 40 \\ &= 100 \end{aligned}$$

(S) We have $|\vec{a}\times\vec{b}| = 30$. Therefore,

$$\begin{aligned} A &= \left|(\vec{a}+\vec{b})\times\vec{a}\right| \\ &= \left|\vec{a}+\vec{b}\right| \\ &= 30 \end{aligned}$$

20. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{-1}, L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3, P_2 = 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2. Match List I with List II and select the correct answer using the code given below the lists:

List I	*List II*
P. a =	1. 13
Q. b =	2. −3
R. c =	3. 1
S. d =	4. −2

Codes:

	P	**Q**	**R**	**S**
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

Solution

We have

$$L_1 \equiv \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1} = t_1;$$

$$L_2 \equiv \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2} = t_2.$$

For finding point of intersection, we have

$$1 + 2t_1 = 4 + t_2 \quad (1)$$

and

$$-t_1 = -3 + t_2 \quad (2)$$

Solving, we get $t_1 = 2$, $t_2 = 1$. The point of intersection is (5, −2, −1). The equation of plane P will be as follows:

$$\begin{vmatrix} x-5 & y+2 & z+1 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 0$$

$$\Rightarrow (-16)(x-5) + 48(y+2) + 32(z+1) = 0$$

$$\Rightarrow (x-5) - 3(y+2) - 2(z+1) = 0$$

$$\Rightarrow x - 3y - 2z = 13$$

Therefore, $a = 1$, $b = -3$, $c = -2$, $d = 13$.