MATHEMATICS

Q1. Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$

Sol: $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$

Operating $R_1 \to R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} x+y+z & y+z+x & z+x+y \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

Operating $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$\Delta = (x+y+z)\begin{vmatrix} 0 & 0 & 1 \\ z-x & x-y & y \\ 0 & 0 & -3 \end{vmatrix} = (x+y+z)(-3)\begin{vmatrix} 0 & 0 & 1 \\ z-x & x-y & y \\ 0 & 0 & 1 \end{vmatrix}$$

$= 0$, as R_1 and R_3 are identical.

$\therefore$ Ans: 0

Q2. Write the sum of the order and degree of the following differential $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$

Sol: Given differential equation is $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$

$$3\left(\frac{dy}{dx}\right)^2 \cdot \frac{d^2y}{dx^2} = 0$$

Order = 2, degree = 1

Order + degree = 3

$\therefore$ Ans: 3

Q3. Write the integrating factor of the following differential equation:

$$(1+y^2) + (2xy - \cot y)\frac{dy}{dx} = 0$$

Sol: Given differential equation is

$$(1+y)^2 + (2xy - \cot y)\frac{dy}{dx} = 0$$

$$\Rightarrow (1+y^2)\frac{dx}{dy} + 2xy = \cot y$$

$$\Rightarrow \frac{dx}{dy} + \left(\frac{2y}{1+y^2}\right)x = \cot y$$

It is a linear differential equation of the form $\frac{dx}{dy} + Px = Q;$ where P and Q are functions of y and its integrating factor is given by $e^{\int P dy} = e^{\int \frac{2y}{1+y^2} dy} = e^{\ln(1+y^2)} = (1+y^2)$

Ans: $(1 + y^2)$

Q4. If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $\left|2\hat{a} + \hat{b} + \hat{c}\right|$.

Sol: Given that $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors.

$\Rightarrow |\hat{a}| = |\hat{b}| = |\hat{c}| = 1$ and $\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = 0$ (1)

Now $\left|2\hat{a} + \hat{b} + \hat{c}\right|^2 = (2\hat{a} + \hat{b} + \hat{c}).(2\hat{a} + \hat{b} + \hat{c})$

$$= 4|\hat{a}|^2 + 2\hat{a}.\hat{b} + 2\hat{a}.\hat{c} + 2\hat{b}.a + |\hat{b}|^2 + \hat{b}.\hat{c} + 2\hat{c}.\hat{a} + \hat{c}.\hat{b} + |\hat{c}|^2 = 4(1)^2 + (1)^2 = 6$$

$$\Rightarrow \left|2\hat{a} + \hat{b} + \hat{c}\right| = \pm\sqrt{6},$$

but modulus of a vector is non-negative quantity.

$\Rightarrow \left|2\hat{a} + \hat{b} + \hat{c}\right| = \pm\sqrt{6}$

Ans: $\sqrt{6}$

Q5. Write a unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$.

Sol: $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j}$

A unit vector perpendicular to vectors $\vec{a}$ and $\vec{b}$ is given by $\pm\frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$ (1)

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i}(0-1) - \hat{j}(0-1) + \hat{k}(1-1)$

$= -\hat{i} + \hat{j} + 0\hat{k}$

and $|\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + (1)^2 + (0)^2} = \sqrt{2}$

$\therefore$ Required unit vector $= \pm\frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|} = \pm\frac{(-\hat{i} + \hat{j})}{\sqrt{2}} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ or $\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}$

$\therefore$ Ans: $-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ or $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$

Q6. The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

Sol: Given line is $5x - 3 = 15y + 7 = 3 - 10z$

$\Rightarrow 5\left(x - \frac{3}{5}\right) = 15\left(y + \frac{7}{15}\right) = -10\left(z - \frac{3}{10}\right)$

$\Rightarrow \frac{\left(x - \frac{3}{5}\right)}{6} = \frac{y + \frac{7}{15}}{2} = \frac{\left(z - \frac{3}{10}\right)}{-3}$

$\Rightarrow$ Direction ratios of line are $<6, 2, -3>$ or $<-6, -2, 3>$

$\Rightarrow$ Direction cosines are $\left\langle \frac{6}{k}, \frac{2}{k}, \frac{-3}{k} \right\rangle$; or $\left\langle \frac{-6}{k}, \frac{-2}{k}, \frac{3}{k} \right\rangle$; $k = \sqrt{(6)^2 + (2)^2 + (-3)^2} = 7$

$\therefore$ Direction cosines of given line are $\left\langle \frac{6}{7}, \frac{2}{7}, \frac{-3}{7} \right\rangle$ or $\left\langle \frac{-6}{7}, \frac{-2}{7}, \frac{3}{7} \right\rangle$

Ans: $\left\langle \frac{6}{7}, \frac{2}{7}, \frac{-3}{7} \right\rangle$ or $\left\langle \frac{-6}{7}, \frac{-2}{7}, \frac{3}{7} \right\rangle$

Section B

Q7. To promote the making of toilets for women, an organization tried to generate awareness through (i) house calls (ii) letters, and (iii) announcements. The cost for each mode per attempt is given below:

(i) ` 50

(ii) ` 20

(iii) ` 40

The number of attempts made in three villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organization for the three villages separately, using matrices.

Write one value generated by the organization in the society.

Sol: Cost of per attempt through house calls = 50 `

Cost of per attempt through letters = 20 `

Cost of per attempt through announcements = 40 `

$\therefore$ We have a column matrix $B = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$ defining per attempt cost through the given medium respectively.

Also the matrix defining the number of attempts done in *X, Y, Z.*

Villages is $A = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$

$\therefore$ Expenditures incurred by the organization is given by $E = A \times B$

$$= \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \times \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 20,000 + 6000 + 4000 \\ 15,000 + 5000 + 3000 \\ 25,000 + 8000 + 6000 \end{bmatrix} = \begin{bmatrix} 30,000 \\ 23,000 \\ 39,000 \end{bmatrix}$$

Expenditure on Village X = 30, 000 `

Expenditure on Village Y = 23,000 `

Expenditure on Village Z = 39,000 `

and total expenditure = 30,000 + 23,000 + 39,000 = 92,000

The value generated by the organization in the society is 'RESPECT AND CARE FOR WOMEN IN OUR SOCIETY'.

Q8. Solve for x:

$$\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$$

Sol: $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$

We know that, $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ for $xy<1$

$\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ for $xy>1, x>0, y>0$

$-\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ for $xy>1, x<0, y<0$

$\frac{\pi}{2}$ if $xy=1, x, y, >0$; $\frac{-\pi}{2}$ if $xy=1, x, y, <0$

Case (i) $(x+1)(x-1)<1 \Rightarrow x^2-1<1 \Rightarrow x^2<2 \Rightarrow xt(-\sqrt{2},\sqrt{2})$, then

$\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2x}{1-(x^2-1)}\right)=\tan^{-1}\left(\frac{8}{31}\right) \Rightarrow \tan^{-1}\left(\frac{2x}{2-x^2}\right)=\tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31}$ ($\because \tan^{-1}x$ is an injective function)

$\Rightarrow 62x = 16 - 8x^2 \Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow 4x^2 + 32x - x - 8 = 0 \Rightarrow 4x(x+8) - 1(x+8) = 0$

$\Rightarrow (4x-1)(x+8) = 0 \Rightarrow x = 1/4$ or $x = -8$

But $x \in (-\sqrt{2},\sqrt{2}) \Rightarrow x=\frac{1}{4}$

Case (ii) $(x+1)(x-1)>1; x+1>0, x-1>0$

$\Rightarrow x^2-1>1; x>-1; x>1$

$\Rightarrow x^2>2, x>-1, x>1 \Rightarrow x^2>2, x>1 \Rightarrow x>\sqrt{2}$

i.e., $x\in(\sqrt{2},\infty)$, then $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \pi+\tan^{-1}\left(\frac{2x}{2-x^2}\right)=\tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31} \Rightarrow x = 1/4$ or -8, both $\in(\sqrt{2},\infty)$

Case(iii) $(x+1)(x-1); x+1<0, x-1<0$

$\Rightarrow x^2 > 2; x < -1; x < 1$

$\Rightarrow \; x < -\sqrt{2} \Rightarrow x \in (-8, -\sqrt{2})$, then $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \; -\pi + \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \; \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \pi + \tan^{-1}\left(\frac{8}{31}\right)$

which is impossible as L.H.S. $\in \left(0, \frac{\pi}{2}\right)$, where as R.H.S. $\in \left(\pi, \frac{3\pi}{2}\right)$

Case(iv) $(x + 1)(x - 1) = 1; x + 1 > 0, x - 1 > 0$

$x^2 = \sqrt{2}, \; x < -1, x > 1$

$\Rightarrow \; x = \sqrt{2}$, then

$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \; \frac{\pi}{2} = \tan^{-1}\left(\frac{8}{31}\right)$ which is impossible

Case (v) $(x + 1)(x - 1) = +1; x + 1 < 0, x - 1 < 0$

$\Rightarrow \; x = -\sqrt{2}$, then

$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \; \frac{-\pi}{2} = \tan^{-1}\left(\frac{8}{31}\right)$; which is again impossible.

Ans: Thus $x = 1/4$ is the only solution.

Q8. Prove the following:

$$\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right) = 0$$

$(0 < xy, yx, zx < 1)$

Sol: $\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right)$

$= (\cot^{-1} y - \cot^{-1} x) + (\cot^{-1} z - \cot^{-1} y) + \cot^{-1} x - \cot^{-1} z$

$= 0$

Q9. Using properties of determinants, prove the following:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

Sol: $\Delta = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

$$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$ (Taking a, b and c common from C_1, C_2 and C_3 respectively)

Operating $R_1 \to R_1 + R_2 + R_3$, we get

$$\Delta = abc\begin{vmatrix} 2(a+b) & 2(c+b) & 2(a+c) \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$= 2abc\begin{vmatrix} a+b & c+b & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Operating $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = 2abc\begin{vmatrix} 2(a+b+c) & c+b & a+c \\ 2(a+b) & b & a \\ 2(b+c) & b+c & c \end{vmatrix} = 4abc\begin{vmatrix} a+b+c & c+b & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$$

Operating $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ gives

$$\Delta = 4abc\begin{vmatrix} a+b+c & -a & -b \\ a+b & -a & -b \\ b+c & 0 & -b \end{vmatrix} = 4a^2b^2c\begin{vmatrix} a+b+c & 1 & 1 \\ a+b & 1 & 1 \\ b+c & 0 & 1 \end{vmatrix}$$

Operating $R_2 \to R_2 \to R_1$ and $R_3 \to R_3 \to R_1$ gives

$$\Delta = 4a^2b^2c\begin{vmatrix} a+b+c & 1 & 1 \\ -c & 0 & 0 \\ -a & -1 & 0 \end{vmatrix} = 4a^2b^2c^2$$

Hence proved

Q10. Find the adjoint of the matrix $A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and hence show that A.(adj A) = $|A|I_3$.

Sol: $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$$\text{Ajoint(A)} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix};$$

Where $C_{11} = (-1)^2 (1 - 4) = -3$; $C_{12} = (-1)^3 (2 + 4) = 6$;

$C_{13} = (-1)^4 (-4-2) = -6$; $C_{21} = (-1)^3 (-2= 6$;

$C_{22} = (-1)^4 (-1 + 4) = 3$; $C_{23} = (-1)^5 (2 + 4) = -6$;

$C_{31} = (-1)^4 (4 + 2) = 6$; $C_{32} = (-1)^5 (2 + 4) = 6$; $C_{33} = (-1)^6 (-1 + 4) = 3$

$$\Rightarrow \text{Adjoint(A)} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$ Ans.

Also $|A| = (-1)C_{11} - 2C_{12} - RC_{13} = (-1)(-3) -2(-6) -2(-6)$

$= 3 + 12 + 12 = 27$

Now

$$A.(\text{Adj}.A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} = \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$

$$= 27\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 27 \quad I_3 = |A|\, I_3$$

Hence $A.(\text{Adj}.\ A) = |A|\ I_3$

Q11. Show that the function $f(x) = |x - 1| + |x + 1|$, for all $x \in R$, is not differentiable at the points $x = -1$ and $x = 1$.

Sol: $f(x) = |x - 1| + |x + 1|;\ x\ \varepsilon\ |R$

$$f(x) = \begin{cases} -2x; \text{ if } x < -1 \\ (-x+1)+(x+1) \text{ if } -1 \le x < 1 \\ (x-1)+(x+1) \text{ if } x \ge 1 \end{cases}$$

$$= \begin{cases} -2x; \text{ if } x < 1 \\ 2; \text{ if } -1 \le x < 1 \\ 2x; \text{ if } x \ge 1 \end{cases}$$

$f(x)$ is continuous at $x = -1$ and at $x = 1$

as $\lim\limits_{x\to(-1)^-} f(x) = \lim\limits_{x\to(-1)^-}(-2x) = 2;\quad \lim\limits_{x\to(1)^+} f(x) = \lim\limits_{x\to(1)^+}(2x) = 2 = f(1)$

also, $f'(x) = \begin{cases} -2; \text{ if } x < -1 \\ 0; \text{ if } -1 < x < 1 \\ 2; \text{ if } x > 1 \end{cases}$

clearly $f'(x)$ is discontinuous at $x = -1$ and $x = 1$

i.e., L.H.D. = −2, R.H.D. = 0 at $x = -1$

and L.H.D. = 0, R.H.D. = 2 at $x = 1$

$f(x)$ is non-differentiable at $x = -1$ and at $x = 1$.

Q12. If $y = e^{m\sin^{-1}x}$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$

Sol: $y = e^{m\sin^{-1}x} \Rightarrow \frac{dy}{dx} = e^{m\sin^{-1}x}.\frac{m}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = \frac{my}{\sqrt{1-x^2}} \Rightarrow (\sqrt{1-x^2})\frac{dy}{dx} = my \qquad (1)$$

$$\sqrt{1-x^2}.\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{-2x}{2\sqrt{1-x^2}} = m\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m\sqrt{1-x^2}\frac{dy}{dx} = m(my) \ \text{(using (1))}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$$

Hence proved

Q13. If $f(x) = \sqrt{x^2+1}; g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then find $f'[h'\{g'(x)\}]$.

Sol: $f(x)=\sqrt{x^2+1};\ g(x)=\dfrac{x+1}{x^2+1};\ h(x)=2x-3$

To find $f'[h'[g'(x)]]$.

Here, $f'(x)=\dfrac{x}{\sqrt{x^2+1}};\ g'(x)=\dfrac{(x^2+1)-(x+1)(2x)}{(x^2+1)^2}=\dfrac{1-x^2-2x}{(x^2+1)^2}$ which is defined $\forall\ x\in 1R$

Also $h'(x)=2\Rightarrow h'(g'(x))=2\ \forall\ x\in 1R$

$$f'[h'(g'(x)]=f'(2)=\frac{2}{\sqrt{(2)^2+1}}=\frac{2}{\sqrt{5}}$$

$\therefore$ Ans: $2/\sqrt{5}$

Q14. Evaluate:

$$\int(3-2x).\sqrt{2+x-x^2}\,dx$$

Sol: $\int(3-2x).\sqrt{2x+x-x^2}\,dx=\int(2+(1-2x).\sqrt{2+x-x^2dx}$

$$=2\int\sqrt{2+x-x^2dx}+\int(1-2x)(2+x-x^2)^{1/2}dx$$

$$=2\int\sqrt{-(x^2-x-2)}dx+\int(2x+x-x^2)^{1/2}.(1-2x)dx$$

$$=2\int\sqrt{-\left(x^2-x+\frac{1}{4}-\frac{9}{4}\right)}+\int(2+x-x^2)^{1/2}.(1-x)dx$$

$$=2\int\sqrt{\left(\frac{3}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}+\int(2+x-x^2)^{1/2}.(1-2x)dx$$

$$=2\left[\frac{\left(x-\frac{1}{2}\right)\sqrt{\left(\frac{3}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}}{2}+\frac{\left(\frac{3}{2}\right)^2}{2}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{3}{2}}\right)\right]\frac{(2+x-x^2)^{3/2}}{3/2}+C$$

$$=\frac{(2x-1)\sqrt{2+x-x^2}}{2}+\frac{9}{4}\sin^{-1}\left(\frac{2x-1}{3}\right)+\frac{2}{3}(2+x-x^2)^{3/2}+C$$

OR

Q14. Evaluate:

$$\int\frac{x^2+x+1}{(x^2+1)(x+2)}dx$$

Sol: $\int\dfrac{x^2+x+1}{(x^2+1)(x+2)}dx=?$

Let $\dfrac{x^2+x+1}{(x^2+1)(x+2)}=\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x+2}$ (1)

$x^2+x+1=(Ax+B)(x+2)+C(x^2+1)$

For $x=-2$; $3=5C\Rightarrow$ $\boxed{C=3/5}$

For $x=0$; $1=2B+C\Rightarrow 2B=1-C=1-3/5=2/5$

$\Rightarrow$ $\boxed{B=1/5}$

Comparing the coefficients of x^2, we get

$1=A+C\Rightarrow A=1-C=1-3/5=2/5$

$\Rightarrow \boxed{A = 2/5}$

Substituting, the values of A, B and C in equation (1), we get

$$\frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{\left(\frac{2}{5}x+\frac{1}{5}\right)}{(x^2+1)} + \frac{3/5}{(x+2)}$$

$$\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx = \frac{1}{5}\int \frac{2x+1}{(x^2+1)}dx + \frac{3}{5}\int \frac{1}{(x+2)}dx$$

$$= \frac{1}{5}\left[\int \frac{2x}{x^2+1}dx + \int \frac{1}{x^2+1}dx\right] + \frac{3}{5}\int \frac{1}{(x+2)}dx$$

$$= \frac{1}{5}\ln(x^2+1) + \frac{1}{5}\tan^{-1}x + \frac{3}{5}\ln|x+2| + C$$

$$\text{or } \frac{1}{5}\left[\tan^{-1}x + \ln\left|(x^2+1)(x+2)^3\right|\right] + C$$

Ans: $\frac{1}{5}[\tan^{-1}x + \ln|(x^2+1)(x+2)^3] + C$

Q15. Find:

$$\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$$

Sol: $$\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = \int_0^{\pi/4} \frac{dx}{2(\sin x)^{1/2}(\cos x)^{7/2}}$$

$$= \int_0^{\pi/4} \frac{\sec^2 x\,dx}{2(\sin x)^{1/2}(\cos x)^{3/2}} = \int_0^{\pi/4} \frac{\sec^2 x\,dx}{\frac{2(\sin x)^{1/2}}{(\cos x)^{1/2}}.\cos^2 x}$$

$$= \int_0^{\pi/4} \frac{(1+\tan^2 x)\sec^2 x\,dx}{2\sqrt{\tan x}}$$

Put $\sqrt{\tan x} = t \Rightarrow \frac{1}{2\sqrt{\tan x}}.\sec^2 x\,dx = dt$

$I = \int_0^1 (1+t^4)dt$ as at $x = 0$, $t = 0$ and at $x = \frac{\pi}{4}$, $t = 1$

$$= \left[t + \frac{t^5}{5}\right]_0^1 = 1 + \frac{1}{5} = \frac{6}{5}$$

$\therefore$ Ans: 6/5

Q16. Find:

$$\int \frac{\log x}{(x+1)^2}dx$$

Sol: $$\int \frac{\log x}{(x+1)^2}dx = \int \underbrace{(\log x)}_{I}.\underbrace{\frac{1}{(x+1)^2}}_{II}dx$$

Integrating by parts, we get

$$= (\log x)\int \frac{1}{(x+1)^2}dx - \int\left(\frac{d}{dx}(\log x).\int \frac{1}{(x+1)^2}dx\right)dx$$

$$= (\log x)\left[\frac{-1}{(x+1)}\right] - \int \frac{1}{x}\left(\frac{-1}{x+1}\right) dx$$

$$= \frac{-\log x}{(x+1)} + \int \frac{x+1-x}{x(x+1)} dx = \frac{-\log x}{(x+1)} + \int \left(\frac{1}{x} - \frac{1}{(x+1)}\right) dx$$

$$= \frac{-\log x}{(x+1)} + \log x - \log(x+1) + C \qquad \text{Ans.}$$

Q17. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k},\ \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k},$ then find a unit vector perpendicular to both of the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.

Sol: $\vec{a} = \hat{i} + 2\hat{j} + \hat{k},\ \vec{b} = 2\hat{i} + \hat{j},\ \vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k},$

To find a unit vector perpendicular to both vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$

Unit vector perpendicular to $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$

$$= \pm \frac{(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b})}{|(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b})|} \qquad (1)$$

Now $(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix} = \hat{i}(0) - \hat{j}(4) + R(4)$

$$= -4\hat{j} + 4\hat{k}$$

Unit vector = Unit Vector $= \pm \frac{(-4\hat{j} + 4\hat{k})}{\sqrt{10+16}} = \pm \frac{(-4\hat{j} + 4\hat{k})}{4\sqrt{2}} = \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ or $\frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$ Ans.

Q18. Find the equation of a line passing through the point (1, 2, −4) and perpendicular to two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

Sol: The required line is perpendicular to lines $\vec{r} = 8\hat{i} - 19\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and

$\vec{r} = (15\hat{i} + 2a\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

line is along the vector $(3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k}) = \vec{b}$ (say)

$$\therefore\ \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = \hat{i}(80 - 56) - \hat{j}(-15 - 21) + \hat{k}(24 + 48)$$

$$= 24\hat{i} + 36\hat{j} + 72\hat{k}$$

Thus the line is passing through the point (1, 2, −4) and along vector $\vec{b}$ is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(24\hat{i} + 36\hat{j} + 72\hat{k})$

In vector form $\frac{x-1}{24} = \frac{y-2}{36} = \frac{z+4}{72}$

or $\boxed{\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}}$ Ans.

OR

Q18. Find the equation of the plane passing through the points (−1, 2, 0), (2, 2, −1) and parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

Sol: Let the equation of plane passing through the points $A(-1, 2, 0)$ and $B(2, 2, -1)$ be

$a(x+1)+b(y-2)+c(z-0)=0$ (1)

It passes through $B(2, 2, -1)$, $\Rightarrow$ $3a+0b-c=0$ (2)

Also the required line is parallel to line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$

$1(a)+2(b)+(-1)(c)=0 \Rightarrow a+2b-c=0$ (3)

Solving (2) and (3), we get, $a=b$, $c=3b$

$c=3b=3a \Rightarrow$ direction ratios of normal to the plane are <1, 1, 3>

$\therefore$ The equation of required plane will be $(x+1)+(y-2)+3z=0$

or $\boxed{x+y+3z-1=0}$ Ans

Q19. Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution.

Sol: X = Number of spades drawn in three successive draws with replacement.

Clearly X can take values 0, 1, 2 and 3

Now $P(x=0)=P(\text{All three cards non spades})=\left(\frac{39}{52}\right)\left(\frac{39}{52}\right)\left(\frac{39}{52}\right)=\left(\frac{3}{4}\right)^3=\frac{27}{64}$

$P(x=1)=P(\text{exactly one card spade and other two non-spades})$

$$=\left[\left(\frac{13}{52}\right)\left(\frac{39}{52}\right)\left(\frac{39}{52}\right)\right]=3\left[\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right]=\frac{(3)^3}{(4)^3}=\frac{27}{64}$$

$P(x=2)=\text{P(exactly one non-spade and other two spades)}$

$$=3\left[\left(\frac{39}{52}\right)\left(\frac{13}{52}\right)\left(\frac{13}{52}\right)\right]=3\left[\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\right]=\frac{(3)^2}{(4)^3}=\frac{(3)^2}{64}=\frac{9}{64}$$

$P(x=3)=P(\text{All three spades})=\left(\frac{13}{52}\right)\left(\frac{13}{52}\right)\left(\frac{13}{52}\right)=\left(\frac{1}{4}\right)^3=\frac{1}{64}$

$\therefore$ Probability distribution table is as shown below,

X_i	0	1	2	3
$P(X_i)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

$$\therefore \text{ Mean}=\Sigma X_i P(x_i)=(0)\left(\frac{27}{64}\right)+1\left(\frac{27}{64}\right)+2\left(\frac{9}{64}\right)+3\left(\frac{1}{64}\right)=\frac{48}{64}=\frac{8}{4}$$

$\therefore$ $\boxed{\text{Ans}:\frac{3}{4}}$

OR

Q19 For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9P(X=4)=P(X=2)$. Find the probability of success.

Sol: Here, $n=6$, Let P = Probability of success

According to question, $qp(x=4)=p(x=2)$

$$p\left({}^6C_4\right)(p)^4(1-p)^2={}^6C_2(p)^2(1-p)^4$$

$$9p^2 = (1-p)^2 \Rightarrow \frac{1-p}{p} = \pm 3$$

but $1-p, p > 0 \Rightarrow \frac{1-p}{p} = 3$

$1-p = 3p \Rightarrow \boxed{p = \frac{1}{4}}$

SECTION C

Q20. Consider $f: R_+ \to [-9, \infty]$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54+5y}-3}{5}\right)$.

Sol: $f: R_+ \to [-9, \infty)$ and $f(x) = 5x^2 + 6x - 9$

$f'(s) = 10x + 6 > 0$ for $x > -3/5$

$f(x)$ is injective function for $x \in \left(-\frac{3}{5}, \infty\right) \supset [0, \infty) = R_+$

$f(x)$ is also injective on R_+, $f(x)$ being a polynomial function is continuous and also increasing

Range of $f(x) = [f(0), f(\infty)) = [-9, \infty) = \text{co}-\text{domain}$

$f(x)$ is also subjective

$f(x)$ is invertible (as bijective)

Let $y = 5x^2 + 6x - 9 \Rightarrow 5x^2 + 6x - 9 - y = 0$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(5)(-9-y)}}{2(5)}$$

$$x = \frac{-6 \pm \sqrt{36+180+20y}}{10} = \frac{-6 \pm \sqrt{20y+216}}{10}$$

$$x = \frac{-6 \pm \sqrt{5y+54}}{10} = \frac{-3 \pm \sqrt{5y+54}}{5}$$

But $x \in R_+ \Rightarrow x \geq 0 \Rightarrow x = \frac{-3+\sqrt{5y+54}}{5}$

$f^{+}(y) = \left(\frac{\sqrt{5y+54}-3}{5}\right)$; Hence proved

OR

Q20. A binary operation * is defined on the set $x = R - \{-1\}$ by $x * y = x + y + xy,\ \forall x, y \in X$.

Check whether * is commutative and associate. Find its identity element and also find the inverse of each element of X.

Sol: $*: X \times X \to X$; where $X = |R - <-1>$

and $x * y = x + y + xy,\ \forall\ x, y \in X$.

<u>Commutativity:</u> $x * y = x + y + xy = y + x + yx = y * x$

* is commutative in X.

<u>Associativity:-</u> $x, y, z \in X$, then

$x*(y * z) = x * (y + z + yz) = x + (y + z + yz) + x(y + z + yz)$

$= (x + y + xy) + z + yz + x(z + yz)$

$= [(x * y) + z] + (y + x + xy)z$

$=[(x * y) + z] + (x * y).z = (x * y) * z$

Thus * is associate in X

Clearly $0 \in X = 1R - <-1>$ and for $x \in X$, $x * (0) = x + 0 + x.(0) = x$

0 is the identity element in X.

Let $y \in X$ and be the inverse of $x \in X$

$(x * y) = 0 \ (y * x) \Rightarrow \ y = \frac{-x}{1+x} \in X = 1R - <-1>$

Thus for any $x \in X$, inverse of it is $\frac{-x}{1+x}$

Q21. Find the value of p for when the curves $x^2 = 9P(9 - y)$ and $x^2 = p(y + 1)$ cut each other at right angles.

Sol: Given curves are $x^2 = 9p(9 - y)$ and $x^2 = p(y + 1)$

At the point of intersection $9p(9 - y) = P(y + 1)$

$81 - 9y = y + 1 \Rightarrow y = 8 \Rightarrow x = \pm 3\sqrt{p}$

$\therefore$ Point of intersection is $(\pm 3\sqrt{p}, 8) = Q$ (say)

For curve, $x^2 = 9p(9 - y)$, $2x = 9p\left(-\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} = -\frac{2x}{9p}$

m_1 (at Q) $= \frac{-2(\pm 3\sqrt{p})}{9p}$

For curve, $x^2 = p(y + 1)$, $2x = p\left(\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} = \frac{2x}{p}$

m_2 (at Q) $= \frac{2(\pm 3\sqrt{p})}{p}$

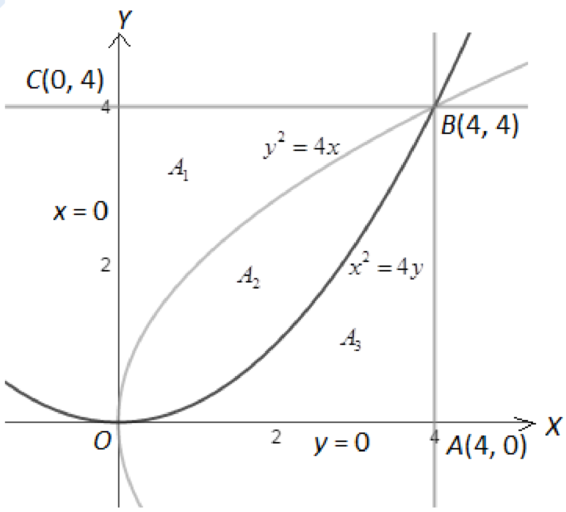
For curves to cut at light angle, $m_1 . m_2 = (-1)$

$\frac{-36}{9p^2} = -1 \Rightarrow p = 4$

$\therefore$ Ans: $\boxed{p = 4}$

Q22. Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$, and $y = 0$ into three equal parts.

Sol: Given curve are $y^2 = 4x$ and $x^2 = 4y$



We are to prove $A_1 = A_2 = A_3$.

$$A_3 = \int_0^4 \frac{x^2}{4} dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \frac{1}{12}[64-0] = \frac{16}{3} \text{sq.units}$$

$$A_2 + A_3 = \int_0^4 2\sqrt{x}\, dx = 2\left[\frac{(x)^{3/2}}{3/2}\right]_0^4 = \frac{4}{3}\left[(4)^{3/2}\right] = \frac{32}{3} \text{ sq.units}$$

$$A_2 = \frac{32}{3} - A_3 = \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{sq.units}$$

Also $A_1 + A_2 + A_3$ = Area of square *OABC* = 16 sq. units

Thus $A_1 = A_2 = A_3 = \frac{16}{3}$ sq.units

Q23. Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and also solve it.

Sol: Given differential equation is $\frac{dy}{dx} = \frac{y^2}{xy-x^2} \frac{x^2}{x^2}\left(\frac{y^2/x^2}{y/x - 1}\right)$

$$\frac{dy}{dx} = x^\circ\left[\frac{(y/x)^2}{(y/x)-1}\right] = x^\circ f(y/x)$$

Which is a homogeneous equation as it is of the form $\frac{dy}{dx} = x^n f\left(\frac{y}{x}\right)$; where *n* is a whole number.

Now put $y = vx \Rightarrow \frac{dy}{dx} = V + x\frac{dv}{dx}$,

$$v + x\frac{dv}{dx} = \left(\frac{v^2}{v-1}\right) \Rightarrow x\frac{dv}{dx} = \frac{v}{v-1} \Rightarrow \frac{dx}{x} = \frac{v-1}{v}dv$$

$$\int \frac{dx}{x} = \int \frac{v-1}{v} dv \Rightarrow \ln x = v - \ln v + c$$

$= \ln x + \ln v = v + c$

$= \ln(vx) = v + c \Rightarrow \ln y = \frac{y}{x} + c$

$= \boxed{x \ln y = y + cx}$

OR

Q23. Find the particular solution of the differential equation $(\tan^{-1}y - x)dy = (1 + y^2)dx$, given that $x = 1$ when $y = 0$.

Sol: Given differential equation is $(\tan^{-1}y - x)dy = (1 + y^2)\, dx$, and At $x = 1$, $y = 0$

$$\frac{dx}{dy} = \frac{\tan^{-1} y}{1+y^2} - \frac{x}{1+y^2} \quad \text{or} \quad \frac{dx}{dy} + \left(\frac{1}{1+y^2}\right)x = \frac{\tan^{-1} y}{1+y^2}$$

Which is a linear differential equation of the type $\frac{dx}{dy} + Px = Q$; where $P = \frac{1}{1+y^2}$, $Q = \frac{\tan^{-1} y}{1+y^2}$

Integrating factor $= e^{\int P dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

$\therefore$ Solution is given by, $x(I.F.) = \int Q(I.F)dy + C$

$$x.e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2}.e^{\tan^{-1} y} dy + c$$

$= \int t.e^t\, dt + c;$ where $t = \tan^{-1} y$

$= te^t - \int 1.e^t\, dt + c$

$= te^t - e^t + c = (\tan^{-1} y)e^{\tan^{-1} y} - e^{\tan^{-1} y} + c$

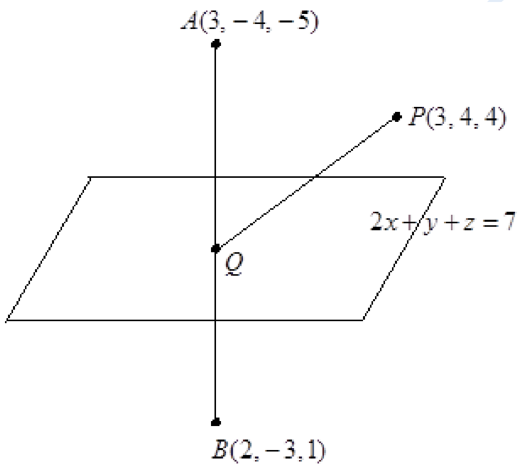
But $f(1) = 0 \Rightarrow 1 = -1 + c \Rightarrow c = 2$

$x.e^{\tan^{-1} y} = e^{\tan^{-1} y}(\tan^{-1} y - 1) + 2$ is the particular solution.

or $x = (\tan^{-1} y - 1) + 2e^{-\tan^{-1} y}$

Q24. Find the distance of the point $P(3, 4, 4)$ from the point, where the line joining the points $A(3, -4, -5)$ and $B(2, -3, 1)$ intersects the plane $2x + y + z = 7$.

Sol:



Equation of line AB is, $\dfrac{x-3}{3-2} = \dfrac{y-4}{-4+3} = \dfrac{z+5}{-5-1} = r$

Any point on line is $(3 + 4, -4 - r, -5 - 6r) = Q$ (say)

$2(3 + r) + (-4 - r) + (-5 - 6r) = 7 \Rightarrow r = -2$

$Q \equiv (1, -2, 7) \Rightarrow PQ = \sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2} = 7$

$\therefore$ Ans: 7

Q25. A company manufactures three kinds of calculators : A, B and C in the two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. the daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B, and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. the cost per day to run factory I is ` 12,000 and of factory II is ` 15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate his problem as an LPP and solve it graphically.

Sol: Let the factory *I* is operated for *x* days and factory *II* is operated for *y* days.

$\therefore$ Production of calculators of type $A = 50x + 40y \geq 6400$ (given)

Production of calculators of type $B = 50x + 20y \geq 4000$

Production of calculators of type $C = 30x + 40y \geq 4800$

Cost = $12000x + 15000y$.

Thus the L.P.P. is

To minimize cost $c = 12000x + 15000y$;

Subject to constraints:

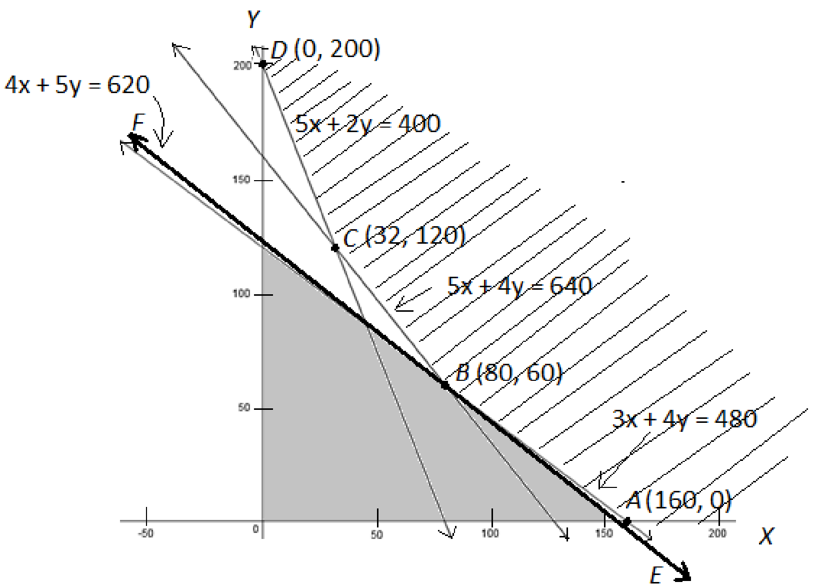
$50x + 40y \geq 6400$ or $5x + 4y \geq 640$ (1)

$50x + 20y \geq 4000$ or $5x + 2y \geq 400$ (2)

and $30x + 40y \geq 4800$ or $3x + 4y \geq 480$ (3)

$x \geq 0, y \geq 0$ (4)

Draw the graphs of $5x + 4y = 640$, $5x + 2y = 400$ and $3x + 4y = 480$ on same frame of reference and then find the feasible region satisfying the inequalities (1) to (4) as shown below.



The feasible region is unbounded.

At A(160, 0)	Cost = 19,20,000 `
At B (80, 60)	Cost = 18,60,000 `
At C (32, 120)	Cost = 21, 84,000 `
At D (0, 200)	Cost = 30,00,000 `

We observe that the minimum cost appears to be at B(80, 60). But the feasible region is unbounded so we are to analyze further. Draw the region represented by inequality $12000x + 15000y < 18,60,000$

or $12x + 15y < 180$

or $4x + 5y < 620$

The corresponding region is represented by shaded region below line EF (excluding the line itself). Thus we observe, that there is no common region between the feasible region and the region represented by inequality $4x + 5y < 620$ or $12000x + 15000y < 18,60,000$

Cost below 18,60,000 is impossible

Minimum cost = 18,60,000 ` at x = 80, y = 60 i.e., Factory *A* should work for 80 days and Factory *B* should work for 60 days.

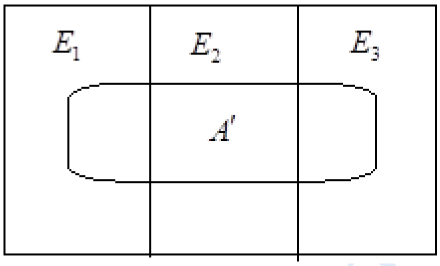
Q26. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 percent respectively are defective bolts. A bolts is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

Sol: Let E_1 = Event that the bolt is manufactured by Machine A; $P(E_1)$ = 30/100

E_2 = It is produced by machine B; $P(E_2)$ = 50/100

E_3 = It is produced by machine C; $P(E_3)$ = 20/100

A' = Event that the drawn bolt is defective



We are to find $P\left(\frac{E_2}{A'}\right)$ = Conditional Probability that the defective drawn bolt is manufactured by machine *B*.

Also $P\left(\frac{A'}{E_1}\right) = \frac{3}{100}$ = Probability of defective bolt produced by machine A.

Similarly, $P\left(\frac{A'}{E_2}\right) = \frac{4}{100}$ and $P\left(\frac{A'}{E_3}\right) = \frac{1}{100}$

By Baye's theorem,

$$P\left(\frac{E_2}{A'}\right) = \frac{P\left(\frac{A'}{E_2}\right).P(E_2)}{P\left(\frac{A'}{E_1}\right).P(E_1) + P\left(\frac{A'}{E_2}\right).P(E_2) + P\left(\frac{A'}{E_3}\right)P(E_3)}$$

$$= \frac{\frac{4}{100} \times \frac{50}{100}}{\frac{3}{100} \times \frac{3}{100} + \frac{4}{100} \times \frac{50}{100} + \frac{1}{100} \times \frac{2}{100}} = \frac{20}{90 + 200 + 20} = \frac{20}{31} \qquad \text{Ans.}$$