Mock Test for JEE Advanced: Mathematics

Paper – I

Maximum Marks: **Time Allowed: 1 hr**

Instructions: The paper has three sections: Section I, Section II and Section III.

(i) Section I contains 10 objective type questions with one correct option. Each question has four choices (A), (B), (C) and (D), out of which only one is correct.

(ii) Section II contains 5 multiple correct options type questions. Each question has four choices (A), (B), (C) and (D), out of which one or more than one may be correct.

(iii) Section III contains 5 integer (subjective) type questions. Each question has an integer answer lying between 0 and 9.

SECTION - I
SINGLE CORRECT ANSWER TYPE

This part contains 10 multiple choice questions with single correct answer. Each Question has 4 choices (A), (B), (C) & (D), out of which Only One is Correct.

1. If *p*, *q*, *r* be the roots of $x^3 - ax^2 + bx - c = 0$, then the area of the triangle whose sides are *p*, *q*, *r* is

(A) $\frac{1}{4}\left[a(4ab-a)\right]^{1/2}$ **(B)** $\frac{1}{2}\left[b(4ab-a+c)\right]^{1/2}$

(C) $\frac{1}{4}\left[a(4ab-a^3-8c)\right]^{1/2}$ **(D)** None of these

2. Let $f(x) = \left[-c^2+(b-1)c-2\right]x + \int(\sin^2 x+\cos^4 x)\,dx$. If $f(x)$ be an increasing function of $x \;\forall\; x \in R$, then all possible values of b if $(c \in R)$ is

(A) $b \in R$ **(B)** $b > 0$

(C) $b < 0$ **(D)** No values of b

3. Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then

(A) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{1/2}$ **(B)** $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{1/2}$

(C) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{1/2}$ **(D)** $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{1/2}$

4. If k is an even integer and $g(x) = \sin kx \cot x$ and $f(x) = \frac{x}{k}$ then the value of $\int_0^{k\pi/2} (g \circ f)\, x\, dx$, is

(A) $2n\pi$ **(B)** $n\pi$

(C) $\frac{n\pi}{2}$ **(D)** None of these

5. The value of $I_m = \int_0^{2a} x^m \sqrt{2ax - x^2}\, dx$ is

(A) $I_m = \frac{(4m+1)a}{m+1} I_{m-1}$ **(B)** $I_m = \frac{(2m+1)a}{m+2} I_{m-1}$

(C) $I_m = \frac{(2m+1)a}{m+4} I_{m-1}$ **(D)** None of these

6. If A and B are two points in the plane such that $\frac{PA}{PB} = k$ (constant), for all P on given circle, then we have

(A) $k \in R - \{0,1\}$ **(B)** $k \in R - \{1\}$

(C) $k \in R - \{0\}$ **(D)** None of these

7. If the direction ratios of a line are $1+\lambda, 1-\lambda, 2$, and it makes an angle of 60° with the $+ve$ y-axis then λ is :

(A) $1+\sqrt{3}$ **(B)** $2+\sqrt{5}$

(C) $1-\sqrt{3}$ **(D)** None of these

8. Area bounded by the equation $[y] = [x]$ in the interval $(n, n+1)$ where $n \in I$ (where [.] denotes greatest integer function)

(A) 1 **(B)** 2

(C) $n(n+1)$ **(D)** None of these

9. If the expression $\frac{\left[\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) + i\tan(x)\right]}{\left[1 + 2i\sin\left(\frac{x}{2}\right)\right]}$ is real, then the set of all possible values of x is

(A) $2n\pi,\ n\pi - \frac{\pi}{4}$ **(B)** $n\pi,\ n\pi - \frac{\pi}{4}$

(C) $n\pi,\ 2n\pi + \frac{\pi}{3}$ **(D)** None of these

10. The maximum and minimum value of the $f(x)$ if $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x)\, f(t)\, dt$ is

(A) $\frac{\sqrt{5}}{3}, -\frac{\sqrt{5}}{3}$ **(B)** $\sqrt{5}, -\sqrt{5}$

(C) $\sqrt{3}, -\sqrt{3}$ **(D)** None of these

SECTION – II
MULTIPLE CORRECT ANSWERS TYPE

This section contains 5 objective type questions with one or more correct answers. Each question has 4 choices (A), (B,) (C) & (D), out of which One or More Choices may be correct.

11. If the imaginary part of the complex number $(z-1)(\cos\alpha - i\sin\alpha) + (z-1)^{-1}(\cos\alpha + i\sin\alpha)$ is zero, then

(A) $|z-1| = 1$ **(B)** $\arg(z-1) = \alpha$

(C) $\arg(z) = \alpha$ **(D)** $|z| = 1$

12. [.] represents greatest integer function,

$\int_0^x [t]\,dt = \int_0^{[x]} t\,dt$, if

(A) $x = \frac{5}{2}$ **(B)** $x = 6$

(C) $x = -\frac{7}{2}$ **(D)** $x = \frac{101}{2}$

13. If the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ is inscribed in a triangle whose two sides are axes and one side has negative slope cutting intercepts a and b on x and y axis, then

(A) $\frac{1}{a} + \frac{1}{b} - 1 = -\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$ **(B)** $\frac{1}{a} + \frac{1}{b} < 1$

(C) $\frac{1}{a} + \frac{1}{b} > 1$ **(D)** $\frac{1}{a} + \frac{1}{b} - 1 = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

14. $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to

(A) $\vec{a} \cdot \{\vec{b} \times (\vec{c} \times \vec{d})\}$ **(B)** $(\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$

(C) $\{(\vec{a} \times \vec{b}) \times \vec{c}\} \cdot \vec{d}$ **(D)** $(\vec{d} \times \vec{c}) \cdot (\vec{b} \times \vec{a})$

15. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are any four vectors then $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ is a vector

(A) perpendicular to $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$

(B) along the line of intersection of two planes, one containing vectors $\vec{a}$ and $\vec{b}$ and other containing $\vec{c}$ and $\vec{d}$

(C) equally inclined to both $\vec{a}\times\vec{b}$ and $\vec{c}\times\vec{d}$

(D) perpendicular to $\vec{b}\times\vec{c}$

SECTION - III
INTEGER (SUBJECTIVE) TYPE

This section contains 5 integer (subjective) type questions. Each question has an integer answer between 0 and 9.

16. The value of $\lim\limits_{x\to\pi/4}\dfrac{(\cos x+\sin x)^3-2\sqrt{2}}{1-\sin 2x}$ is $\dfrac{-P\sqrt{2}}{Q}$ then P + Q is __________.

17. The value of $\displaystyle\int_0^1 \frac{dx}{1+\sqrt{x}+\sqrt{1+x}}$ is $\dfrac{a}{b}-\dfrac{1}{\sqrt{b}}-\dfrac{\log(1+\sqrt{b})}{b}$ then $a-b$ is __________.

18. If $\displaystyle\int_0^n f(x)\,dx=5$, then the value of $\displaystyle\sum_{k=1}^{n}\int_0^1 f(k-1+x)\,dx=$__________.

19. If $f(x)=\tan^{-1}\left(\dfrac{1}{\sin^2 x+\sin x+1}\right)+\tan^{-1}\left(\dfrac{1}{\sin^2 x+3\sin x+3}\right)+\tan^{-1}\left(\dfrac{1}{\sin^2 x+5\sin x+7}\right)$

$+\tan^{-1}\left(\dfrac{1}{\sin^2 x+7\sin x+13}\right)+\cdots$

upto n terms, then $f'(x)$ in simplified form is $\dfrac{A}{1+(B+n)^2}-\dfrac{A}{1+B^2}$,

then A^2+B^2 is ___________.

20. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero and non-coplanar vectors and $\vec{p},\vec{q}$ and $\vec{r}$ be three vectors given by $\vec{p}=\vec{a}+3\vec{b}-2\vec{c}$, $\vec{q}=3\vec{a}-2\vec{b}+\vec{c}$ and $\vec{r}=\vec{a}-4\vec{b}+2\vec{c}$. If the volume of the parallelepiped determined by $\vec{a},\vec{b}$ and $\vec{c}$ is V_1 and that of the parallelepiped determined by $\vec{p},\vec{q},$ and $\vec{r}$ is V_2 then $V_2:V_1$ is equal to __________.

Mock Test for JEE Advanced: Mathematics
Paper – II

Maximum Marks: **Time Allowed: 1 hr**

Instructions: The paper has three sections: Section I, Section II and Section III.

1. Section I contains 8 objective type questions with one correct option. Each question has four choices (A), (B), (C) and (D), out of which only one is correct.
2. Section II contains Section – III contains 6 comprehension-type questions. Each question has 4 choices (A), (B), (C) and (D), out of which one or more may be correct.
3. Section III contains 6 multiple correct options type questions. Each question has four choices (A), (B), (C) and (D), out of which one or more than one may be correct.

SECTION – I
SINGLE CORRECT ANSWER TYPE

This part contains 8 multiple choice questions with single correct answer. Each Question has 4 choices (A), (B), (C) & (D), out of which Only One is Correct.

1. If $f(x) = e^x$, $g(x) = x$, $x > 0$ and $F(t) = \int_0^t f(t-x) g(x)\,dx$, then

(A) $F(t) = 1 - e^{-t}(1+t)$ **(B)** $F(t) = e^t - (1+t)$

(C) $F(t) = te^t$ **(D)** $F(t) = te^{-t}$

2. Let $\vec{a}, \vec{b},$ and $\vec{c}$ be mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation $\vec{a} \times \left[(\vec{x} - \vec{b}) \times \vec{a}\right] + \vec{b} \times \left[(\vec{x} - \vec{c}) \times \vec{b}\right] + \vec{c} \times \left[(\vec{x} - \vec{a}) \times \vec{c}\right] = 0$, then $\vec{x}$ is given by

(A) $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$ **(B)** $\frac{1}{3}(2\vec{a} + \vec{b} + \vec{c})$

(C) $\frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ **(D)** $\frac{1}{2}(\vec{a} + \vec{b} - 2\vec{c})$

3. If *a*, *b*, *c* and *d* are the solutions of the equation $x^4 - bx - 3 = 0$, then an equation whose solutions are $\frac{a+b+c}{d^2}, \frac{a+b+d}{c^2}, \frac{a+c+d}{b^2}$, and $\frac{b+c+d}{a^2}$, is

(A) $3x^4 + bx + 1 = 0$ **(B)** $3x^4 - bx + 1 = 0$

(C) $3x^4 + bx^3 - 1 = 0$ **(D)** $3x^4 - bx^3 - 1 = 0$

4. The solution of differential equation $2x^3 y\,dy + (1 - y^2)(x^2y^2 + y^2 - 1)\,dx = 0$ is

(A) $x^2y^2 = (cx+1)(1-y^2)$ **(B)** $x^2y^2 = (cx+1)(1+y^2)$

(C) $x^2y^2 = (cx-1)(1-y^2)$ **(D)** None of these

5. It is given that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots$ to $\infty = \frac{\pi^4}{90}$. Then, $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots$ to ∞ is equal to

(A) $\frac{\pi^4}{96}$ **(B)** $\frac{\pi^4}{45}$

(C) $\frac{89\pi^4}{90}$ **(D)** $\frac{88\pi^4}{81}$

6. Let $f(x)=\int_0^x \cos\left(\frac{t^2+2t+1}{5}\right)dt,\ 0<x<2$. Then, $f(x)$

(A) increases monotonically **(B)** decreases monotonically

(C) has one point of local maximum **(D)** has one point of local minimum

7. The slope of the tangent to a curve $y=f(x)$ at $\{x,\ f(x)\}$ is $1-2x$ and the curve passes through the point $(2,-2)$. If the area bounded by the curve and the line $y=\alpha x$ is $\frac{32}{3}$ sq. units, then the value of α is(are)

(A) 3 **(B)** –3, 5

(C) –5 **(D)** 3, 5

8. If $w=\alpha+i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w-\bar{w}z}{1-z}\right)$ is purely real, then the set of values of z, is

(A) $|z|=1,\ z\neq 2$ **(B)** $|z|=1$ and $z\neq 1$

(C) $z=\bar{z}$ **(D)** None of these

SECTION –II
COMPREHENSION TYPE

This section contains 3 groups of questions. Each group has 2 multiple choice question based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which Only One or More is correct.

Paragraph for Questions 9 and 10: If $y=\int_{u(x)}^{v(x)} f(t)\,dt$, let us define $\frac{dy}{dx}$ in a different manner as $\frac{dy}{dx}=v'(x)f^2[v(x)]-u'(x)f^2[u(x)]$ and the equation of the tangent at (a, b) as $y-b=\left(\frac{dy}{dx}\right)_{(a,\,b)}(x-a)$.

9. If $y=\int_x^{x^2} t^2\,dt$, then the equation of tangent at $x = 1$ is

(A) $y = x + 1$ **(B)** $x + y = 1$
(C) $y = x - 1$ **(D)** $y = x$

10. If $y = \int_{x^3}^{x^4} \ln t \, dt$, then $\lim_{x \to 0^+} \frac{dy}{dx}$ is

(A) 0 **(B)** 1
(C) 2 **(D)** -1

Paragraph for Questions 11 & 12: *f* is a real-valued function satisfying the functional relation

$$2f(x) + 3f\left(\frac{2x+29}{x-2}\right) = 100x + 80 \text{ for all } x \neq 2.$$

11. *f*(0) is equal to
(A) 754 **(B)** –754
(C) 854 **(D)** –854

12. $f\left(\frac{-29}{2}\right)$ is equal to
(A) 659 **(B)** –596
(C) 596 **(D)** –659

Paragraph for Questions 13 & 14: To solve equations of the form

$$(ax^2 + bx + c)(ax^2 + bx + d) = k,$$

use the substitution $ax^2 + bx = y,$ so that the given equation transforms into a quadratic equation in *y* which can be solved.

13. The number of real roots of the equation $\frac{1}{x(x+2)} - \frac{1}{(x+1)^2} = \frac{1}{12}$ is

(A) 2 **(B)** 1
(C) 0 **(D)** 4

14. The equation $\frac{24}{x^2 + 2x - 8} - \frac{15}{x^2 + 2x - 3} = 2$ has
(A) all positive solutions
(B) three positive and one negative solutions
(C) two non-negative and two negative solutions
(D) two real and two imaginary solutions

SECTION – III
MULTIPLE CORRECT ANSWER TYPE

This section contains 6 objective type questions with one or more correct answers. Each question has 4 choices (A), (B,) (C) & (D), out of which One or More Choices may be correct.

15. Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{1, 3, 5\}$. Then

(A) $n(A\times(B\cup C))=12$ **(B)** $n(A\times(B\cap C))=3$

(C) $n(A\times(B-C))=3$ **(D)** $n(B\times(A-C))=2$

16. Consider n points in a plane of which only p points are collinear. Then the number of straight lines that can be drawn by joining these points is

(A) ${}^{(n-p)}C_2+p(n-p)+1$ **(B)** ${}^{n}C_2$

(C) ${}^{n}C_2-{}^{p}C_2+1$ **(D)** ${}^{n}C_2-{}^{p}C_2$

17. Let

$$x=\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$$

Then

(A) $x=0$ if n is an odd positive integer

(B) $x=\tan^n(A-B)/2$ if n is an even positive integer

(C) $x=2\cot^n(A-B)/2$ if n is an even positive integer

(D) $x=0$ if n is an even positive integer

18. Let

$$f(x)=\begin{cases}\cos^{-1}\left(\cot(x-[x])\right) & \text{for } x<\dfrac{\pi}{2}\\ \pi[x]-1 & \text{for } x\geq\dfrac{\pi}{2}\end{cases}$$

where $[x]$ is the integer part of x. Then

(A) $\lim_{x\to\frac{\pi}{2}-0} f(x)=\frac{\pi}{2}$ **(B)** $\lim_{x\to\frac{\pi}{2}-0} f(x)=\frac{\pi}{2}-1$

(C) $\lim_{x\to\frac{\pi}{2}+0} f(x)=\frac{\pi}{2}+1$ **(D)** $\lim_{x\to\frac{\pi}{2}+0} f(x)=\frac{\pi}{2}-1$

19. If the distance of the line $8x + 15y + \lambda = 0$ from the point (2, 3) is equal to 5 units, then the value of λ is

(A) 24 **(B)** –24

(C) 146 **(D)** –146

20. PQ is a normal chord (normal at P) of the parabola $y^2=4x$ such that PQ subtends right angle at the vertex. Then the coordinates of P are

(A) $(2,2\sqrt{2})$ (B) $(2,-2\sqrt{2})$

(C) $(3,2\sqrt{3})$ (D) $(3,-2\sqrt{3})$