Mock Test for JEE Advanced: Mathematics

Paper – I

Maximum Marks:	**Time Allowed: 1 hr**

Instructions: The paper has four sections: Section I, Section II, Section III and Section IV.

(i) Section–I contains 7 single correct answer type objective questions. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.
(ii) Section–II contains 4 multiple (one or more) correct answer choice type questions. Each question has 4 choices (A), (B), (C) and (D), out of which One or More may be correct.
(iii) Section–III contains 5 comprehension-type questions. Each question has 4 choices (A), (B), (C) and (D), out of which One or More may be correct.
(iv) Section–IV contains 7 integer (subjective) type questions. Each question has an integer answer between 0 and 9.

SECTION - I
SINGLE CORRECT ANSWER TYPE

This part contains 7 multiple choice questions with single correct answer. Each Question has 4 choices (A), (B), (C) & (D), out of which Only One is Correct.

1. Three different numbers are selected at random from the set A = {1, 2, 3, …, 10}. The probability that the product of two of the numbers is equal to the third is

(A) 3/4 **(B)** 1/40
(C) 1/8 **(D)** None of these

2. The area bounded by $y = f(x)$, Y-axis and line $2y = \pi (x + |1)$ where

$f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1}\frac{1}{x} + \tan^{-1} x$ is

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$
(C) π **(D)** None of these b

3. Let $\vec{r} = (\vec{a}\times\vec{b})\sin x + (\vec{b}\times\vec{c})\cos y + 2(\vec{c}\times\vec{a})$ where $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors. If $\vec{r}$ is perpendicular to $\vec{a}+\vec{b}+\vec{c}$, then minimum value of $x^2 + y^2$, is

(A) π^2 **(B)** $\frac{\pi^2}{4}$
(C) $\frac{5\pi^2}{4}$ **(D)** None of these

4. If $\vec{a},\vec{b}$ are orthogonal unit vectors then for a vector $\vec{r}$ non-coplanar with $\vec{a}$ and $\vec{b}$ the vector $\vec{r}\times\vec{a}$ is equal to

(A) $[\vec{r}\,\vec{a}\,\vec{b}]\vec{b} + (\vec{r}.\vec{b})\vec{b}\times\vec{a}$ **(B)** $[\vec{r}\,\vec{a}\,\vec{b}](\vec{a}.+\vec{b})$
(C) $[\vec{r}\,\vec{a}\,\vec{b}]\vec{a} + (\vec{r}.\vec{a})\vec{a}\times\vec{b}$ **(D)** None of these

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5. Let $I_{n,m} = \int \sin^n x \cos^m dx$. Then we can relate $I_{n,m}$ with each of the following

I. $I_{n-2,m}$ **II.** $I_{n+2,m}$ **III.** $I_{n,m-2}$
IV. $I_{n,m+2}$ **V.** $I_{n-2,m+2}$ **VI.** $I_{n+2,m-2}$

The relation between $I_{4,2}$ and $I_{4,4}$ is :

(A) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x + 8I_{4,4})$ **(B)** $I_{4,2} = \frac{1}{3}(-\sin^5 x \cos^3 x + 8I_{4,4})$

(C) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x - 8I_{4,4})$ **(D)** $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x + 6I_{4,4})$

6. Let $f_n(x) = \frac{n+1}{(n-1)!}x^n$. The value of $\int_0^1 \left(\sum_{n=1}^{\infty} f_n(x) \right) dx$ is:

(A) e **(B)** 0
(C) $2e$ **(D)** None of these

7. The value of $\sum_{j=0}^{n} ({}^{4n+1}Cj + {}^{4n+1}C_{2n-j})$ is

(A) $2^{4n} + {}^{4n+1}C_n$ **(B)** 2^{4n+1}

(C) $2^{4n+1} + {}^{4n+1}C_n$ **(D)** 2^{4n}

SECTION – II
MULTIPLE CORRECT ANSWERS TYPE

This section contains 4 objective type questions with One or More correct answers. Each question has 4 choices (A), (B,) (C) & (D), out of which One or More Choices may be correct.

8. Let $\vec{c} = \lambda\vec{a} + \mu\vec{b} + \gamma\vec{a} \times \vec{b}$ where $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a} \cdot \vec{b} = 0$. Then

(A) $\mu = \vec{b} \cdot \vec{c}$ **(B)** $\lambda = |\vec{a} \times \vec{c}|$

(C) $\gamma = |\vec{a}\ \vec{b}\ \vec{c}|$ **(D)** $\lambda + \mu + \gamma = (\vec{a} + \vec{b} + \vec{a} \times \vec{b}) \cdot \vec{c}$

9. Let P, A, B and C be four collinear points in order, the distances of A, B, C from P being a, b and c respectively. If the equation $(b-a)x^2 + (a-c)x + c - b = 0$ has one root doubles the other, then

(A) B divides AC in the ratio 2 : 1 internally
(B) C divides AB in the ratio 2 : 1 externally
(C) B divides AC in the ratio 1 : 2 internally
(D) None of these

10. If a, b and c are three consecutive integers then the determinant $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$ is divisible by

(A) 36 **(B)** 144
(C) 48 **(D)** None of these

11. Two real numbers, x and y are selected at random. Given that $0 \le x \le 1$, $0 \le y \le 1$. Let A be the event that $y^2 \le x$ and B be the event that $x^2 \le y$, then

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(A) $P(A \cap B) = \frac{1}{2}$

(B) A and B are exhaustive events

(C) A and B are mutually exclusive

(D) A and B are independent events

SECTION - III
COMPREHENSION TYPE

This section contains 2 groups of questions. Group one has 2 multiple choice question based on a paragraph and group two has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A) , (B) , (C) and (D) for its answer, out of which ONLY ONE is correct.

Paragraph for Questions 12–13: The general solution of a differential equation of the form: $\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \ldots + a_n y = 0$ ($a_i s$ being constants) is given by the following rules:

I. If the roots of the corresponding auxiliary equation $D^n + a_1 D^{n-1} + a_2 D^{n-2} + \ldots + a_n = 0$ in D has unequal real roots α_1, α_2, $\alpha_3, \ldots, \alpha_n$ then the general solution will be $y = c_1 e^{\alpha_1 x} + c_2 e^{\alpha_2 x} + \cdots + c_n e^{\alpha_n x}$, where $c_i\, s$ are arbitrary constants.

II. If the roots of the corresponding auxiliary equation has two equal roots (say, $\alpha_1 = \alpha_2$) and the rest are unequal real roots then the general solution will be $y = (c_1 x + c_2) e^{\alpha_1 x} + c_3^{e^{\alpha_2 x}} + \cdots + c_n e^{\alpha_n x}$.

III. If two roots of the corresponding auxiliary equation are $\alpha_1 = \alpha + i\beta$, $\alpha_2 = \alpha - i\beta$ and the rest are real and unequal then the general solution will be $y = (A\cos \beta x + B\sin \beta x) e^{\alpha x} + c_3 e^{\alpha_3 x} + \cdots + c_n e^{\alpha_n x}$.

12. The general solution of the equation $\frac{d^3 y}{dx^3} - 7\frac{d^2 y}{dx^2} + 16\frac{dy}{dx} - 12y = 0$ is:

(A) $c_1 e^{2x} + c_2 e^{-2x} + c_3 e^{-3x}$

(B) $(c_1 x + c_2) e^{2x} + c_3 e^{3x}$

(C) $(c_1 x + c_2) e^{-3x} + c_3 e^{2x}$

(D) $(A\cos x + B\sin x) e^{2x} + c_3 e^{3x}$

13. $y = (c_1 \cos x + c_2 \sin x) e^{-x} + c_3 e^{x}$ is the general solution of the equation :

(A) $\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} + 2y = 0$

(B) $\frac{d^3 y}{dx^3} - y = 0$

(C) $\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} - 2y = 0$

(D) $\frac{d^2 y}{dx^2} - y = 0$

Paragraph for Questions 14–16: Perpendiculars are drawn from the focus S of the parabola $y = ax^2 + bx + c$ upon the tangents to the parabola at the points $A(-1, 0)$ and $B(1, 2)$ meeting them at the point $C\left(-\frac{1}{4}, \frac{9}{4}\right)$ and $D\left(\frac{3}{4}, \frac{9}{4}\right)$, respectively.

14. The coordinates of the focus are

(A) $\left(\frac{1}{2}, \frac{9}{4}\right)$

(B) $\left(\frac{1}{2}, 2\right)$

(C) $\left(0, \frac{9}{4}\right)$

(D) $(-1, 2)$

15. The normals of A and B intersect at a point P. The foot of the third normal through the point P is

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(A) $\left(\frac{1}{2}, \frac{9}{4}\right)$ **(B)** $\left(-\frac{1}{5}, \frac{5}{4}\right)$

(C) (0, 2) **(D)** $\left(\frac{3}{2}, \frac{5}{4}\right)$

16. Area of the region bounded by the parabola and the *x*-axis is

(A) $\frac{5}{4}$ **(B)** 5

(C) $\frac{5}{2}$ **(D)** $\frac{9}{2}$

SECTION - IV
INTEGER (SUBJECTIVE) TYPE

This section contains 7 integer (subjective) type questions. Each question has an integer answer between 0 and 9.

17. The latus rectum subtends a right angle at the centre of the hyperbola $\left(\frac{x^2}{a^2}\right)-\left(\frac{y^2}{b^2}\right)=1$. If the eccentricity is *e*, then the value of [*e*], where [.] represents the greatest integer function, is ________.

18. A tangent to the parabola $y = x^2$ is drawn in such way that the abscissa x_0 of the point of tangency belongs to the interval [1, 2]. The value of x_0 for which the triangle bounded by the tangent, the axis of ordinates, and the straight line $y = x_0^2$ has the greatest area is _______.

19. $\lim\limits_{x \to 0} \frac{1}{\sin x} \int_0^{\ln(1+x)} (1+\tan 2y)^{1/y}\, dy = \frac{1}{e^a}$,then *a* is equal to ________.

20. The value of $\left[\frac{k}{2}\right]$, (where [.] represents the greatest integer function) for which the system of equation $x+ky+3z=0,\ 3x+ky-2z=0;\ 2x+3y-4z=0$ possesses a non-trivial solution is _____.

21. Let $z_1 = 10+6i$ and $z_2 = 4+6i$. If *z* is any complex number such that the argument of $\frac{(z-z_1)}{(z-z_2)}$ is $\frac{\pi}{4}$, the integer value of $\frac{|z-7-9i|}{\sqrt{2}}$ is ________.

22. If $\cos^{-1}\lambda+\cos^{-1}\mu+\cos^{-1}y=3\pi$, then the value of $\lambda\mu+\mu y+\gamma\lambda$ is ________.

23. The value of $\lim\limits_{n\to\infty}\frac{1}{3^n}\sum_{p=1}^{n}\left(\sum_{m=p}^{n} {}^nC_m \times {}^mC_p\right)$ is ______________.

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Mock Test for JEE Advanced: Mathematics
Paper – II

Maximum Marks: **Time Allowed: 1 hr**

Instructions: The paper has three sections: Section I, Section II, Section III and Section IV.

(i) Section–I contains 8 objective type questions with one correct option. Each question has four choices (A), (B), (C) and (D), out of which only one is correct.

(ii) Section–II contains 4 multiple (one or more) correct answer choice type questions. Each question has 4 choices (A), (B), (C) and (D), out of which One or More may be correct.

(iii) Section–III contains 2 matrix-match type questions. Each question contains statements given in 2 columns. Statements in the first column have to be matched with statements in the second column.

(iv) Section IV contains 6 integer (subjective) type questions. Each question has an integer answer lying between 0 and 9.

SECTION – I
SINGLE CORRECT ANSWER TYPE

This part contains 8 multiple choice questions with single correct answer. Each Question has 4 choices (A), (B), (C) & (D), out of which Only One is Correct.

1. If the curves $\frac{x^2}{4}+y^2=1$ and $\frac{x^2}{a^2}+y^2=1$ for suitable value of 'a' cut on four concyclic points, the equation of circle passing through these four points is

(A) $x^2 + y^2 = 2$ **(B)** $x^2 + y^2 = 1$
(C) $x^2 + y^2 = 4$ **(D)** None of these

2. A man firing a distant target has 20% chance of hitting the target in one shoot. If P be the probability of hitting the target in n attempts where $20P^2 - 13P + 2 \le 0$, then maximum and minimum value of n is

(A) 3, 2 **(B)** 5, 4
(C) 4, 2 **(D)** 2, 2

3. If $f(x)=\begin{cases} e^{x-1}, & 0\le x\le 1 \\ x+1-\{x\}, & 1<x<3 \end{cases}$ and $g(x) = x^2 - ax + b$, such that $f(x).\ g(x)$ is continuous in [0, 3) then the values of a and b are

(A) 2, 3 **(B)** 3, 2
(C) $\frac{3}{2}, 1$ **(D)** None of these

4. If $f(x)=|1-x|$, then the points where $\sin^{-1}(f|x|)$ is non-differentiable, are

(A) {0, 1} **(B)** {0, –1}
(C) {0, 1, –1} **(D)** None of these

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5. Three different numbers are selected at random from the set A = {1, 2, 3, …, 10}. The probability that the product of two of the numbers is equal to the third is :

(A) 3/4 **(B)** 1/40
(C) 1/8 **(D)** None of these

6. Area bounded by the curve $y = \ln x + \tan^{-1} x$ and x-axis from $x = 1$ to $x = 2$ is

(A) $\frac{5}{2}\ln 2 - \frac{1}{2}\ln 5 + 2\tan^{-1}2 - \frac{\pi}{3} - 1$ **(B)** $\frac{5}{2}ln 2 - \frac{1}{2}\ln 5 + 2\tan^{-1}2 - \frac{\pi}{4} + 1$
(C) $\frac{5}{2}\ln 2 - \frac{1}{2}\ln 5 + 2\tan^{-1}2 + \frac{\pi}{4} - 1$ **(D)** $\frac{5}{2}\ln 2 - \frac{1}{2}\ln 5 + 2\tan^{-1}2 - \frac{\pi}{4} - 1$

7. If A and B are two matrices such that $AB = B$ and $BA = A$, then

(A) $(A^5 - B^5)^3 = A - B$ **(B)** $(A^5 - B^5)^3 = A^3 - B^3$
(C) $A - B$ is idempotent **(D)** $A - B$ is nilpotent

8. If $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ are non-coplanar vectors and $(x + y - 3)\vec{a}_1 + (2x - y + 2)\vec{a}_2 + (2x + y + \lambda)\vec{a}_3 = \vec{0}$ holds for some x and y, then λ is

(A) $\frac{7}{3}$ **(B)** 2
(C) $-\frac{10}{3}$ **(D)** $\frac{5}{3}$

SECTION –II
MULTIPLE CORRECT ANSWER TYPE

This section contains 4 objective type questions with one or more correct answers. Each question has 4 choices A, B, C & D, out of which One or More Choices may be Correct.

9. If $px^2 + qx + r = 0$ has no real roots for real values of p, q, r and $4p + 2q + r > 0$, then

(A) $r > 0$ **(B)** $p + q + r < 0$
(C) $p + q + r < 0$ **(D)** $r \geq 0$

10. A and B are two independent events such that $P(A' \cap B) = \frac{2}{15}$ and $P(A \cap B') = \frac{1}{6}$. Then $P(B)$ is equal to

(A) $\frac{4}{5}$ **(B)** $\frac{1}{6}$
(C) $\frac{1}{5}$ **(D)** $\frac{5}{6}$

11. A plane parallel to $x + y + z = 3$ and at distance $\frac{4}{\sqrt{3}}$ from it has the equation

(A) $x - 2y - z = 0$ **(B)** $x + y + z + 1 = 0$
(C) $2x - y - z = \frac{3\sqrt{3} + 4}{\sqrt{3}}$ **(D)** $x + y + z = 7$

12. In the ΔABC, $b:c = 2:1$ and $\sin(B - C) = \frac{3}{5}$. Then

(A) ΔABC is right-angled **(B)** ΔABC is obtuse-angled
(C) $a : c = 3 : 1$ **(D)** $a : c = \sqrt{5} : 1$

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SECTION – III
MATRIX-MATCH TYPE

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. Statements in Column I are labeled as (A), (B), (C) & (D) whereas statements in Column II are labeled as (p), (q), (r), (s) & (t). The answers to these questions have to be matched. More than one choice from Column II can be matched with Column I.

13. Match the columns

	Column I		***Column II***
(A)	$\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ if $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$	**(p)**	26
(B)	The value of $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})+(\vec{b}\times\vec{c})\cdot(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a})\cdot(\vec{b}\times\vec{d})$	**(q)**	3
(C)	If plane $2x + 3y + 6z + k = 0$ is tangent to the sphere $x^2 + y^2 + z^2 + 2x - 2y + 2z - 6 = 0$, then a value of k is	**(r)**	0
(D)	In a ΔABC, $(a + b + c)(b + c - a) = \lambda bc$, where $\lambda \in I$, then greatest value of λ is	**(s)**	1
		(t)	$\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$

14. Match the Columns

	Column I		***Column II***
(A)	If focal distance of a point on the parabola $y = x^2 - 4$ is 25/4 and points are of the form $(\pm\sqrt{a}, b)$ then value of $a + b$ is	**(p)**	0
(B)	If locus of a point, whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola $xy = 1$ *is* $xy = c^2$, then value of c^2 is	**(q)**	4
(C)	One of the diameters of the circle circumscribing the rectangle ABCD is $4y = x + 7$. If A and B are points (–3, 4) and (5, 4) respectively, then the area of the rectangle is equal to	**(r)**	$\frac{1}{3}$
(D)	There are four boxes A_1, A_2, A_3 and A_4. Box A_i has i cards and on each card a number is printed, the numbers are from 1 to i. A box is selected randomly, the probability of selection of box i, is $\frac{i}{10}$ and then a card drawn. Let E_i represents the event that a card with number i is drawn. $P(A_3/E_2)$ is equal to	**(s)**	32
		(t)	8

SECTION – IV
INTEGER (SUBJECTIVE) TYPE

This section contains 6 integer (subjective) type questions. Each question has an integer answer between 0 and 9.

15. If $f(x)$ is an even function then $\int_{-2}^{2} \left\{x^3 f(x) + xf''(x) + 2\right\} dx$ is equal to _______.

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16. Let $f(x) = ax^4 + bx^3 + cx^2 + dx + \lambda$, where $\frac{f(1)}{1} = \frac{f(2)}{2} = \frac{f(3)}{3}$ and $\lim_{x\to\infty} \frac{f(x)}{x^4} = 1$. If $f(i) + f(\bar{i}) + 10k = 0$, where $i = \sqrt{-1}$, then k is___________.

17. The sum of the first n terms of the sequence 1, (1 + 2), (1 + 2 + 2^2), …, $(1 + 2 + 2^2 + \cdots + 2^{k-1})$ is of the form $2^{n+R} + Sn^2 + nT + U$ for all $n \in N$. Then the value of $|R + S + T + U|$ is __________.

18. If a, b, c are real numbers such that $0 < a < 1$, $0 < b < 1$, $0 < c < 1$ and $a + b + c = 2$, then the least value of $\frac{a}{1-a} \times \frac{b}{1-b} \times \frac{c}{1-c}$ is _____________.

19. Let $I = \int_0^\infty \frac{x^{2008}}{1+x^2}\,dx$ and $J = \int_0^\infty \frac{x^{2008} + x^{-2008}}{1+x^2}\,dx$, then J/I is _____________.

20. Let $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda + 1 & -2\lambda & \lambda - 4 \\ \lambda - 3 & \lambda + 4 & 3\lambda \end{vmatrix}$ be an identity in l where p, q, r, s and t are, constants. Then the value of t is ____________.

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