AIPMT 2015

Physics

1. In the spectrum of hydrogen, the ratio of the longest wavelength in the Lyman series to the longest wavelength in the Balmer series is:

(1) $\frac{5}{27}$ (2) $\frac{4}{9}$

(3) $\frac{9}{4}$ (4) $\frac{27}{5}$

Solution:
For Lyman Series:

$$\left(\frac{1}{\lambda_{max}}\right)_L = R(1)^2\left[\frac{1}{(1)^2}-\frac{1}{(2)^2}\right]$$

$$(\lambda_{max})_L = \frac{4}{3R}$$

For Balmer Series:

$$\left(\frac{1}{\lambda_{max}}\right)_B = R(1)^2\left[\frac{1}{(2)^2}-\frac{1}{(3)^2}\right]$$

$$(\lambda_{max})_L = \frac{36}{5R}$$

$$\frac{(\lambda_{max})_L}{(\lambda_{max})_B} = \frac{4}{3R}\times\frac{5R}{36} = \frac{5}{27}$$

Hence, the correct option is (1).

2. The energy of the em waves is of the order of 15 keV. To which part of the spectrum does it belong?

(1) γ-rays (2) X-rays

(3) Infra-red rays (4) Ultraviolet rays

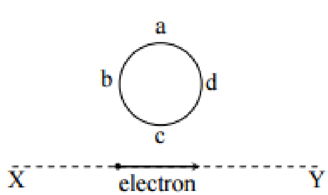
Solution:
Wavelength of the ray

$$\lambda = \frac{hc}{E} = 0.826\,\overset{0}{\mathrm{A}}$$

Since, $\lambda < 100\,\overset{0}{\mathrm{A}}$, hence is a X-ray.

Hence, the correct option is (2).

3. An electron moves on a straight line path XY as shown. The abcd is a coil adjacent to the path of electron. What will be the direction of current, if any, induced in the coil?



(1) No current induced (2) abcd

(3) adcd (4) The current will reverse its direction as the electron goes past the coil

Solution:
When e– comes towards the loop, the magnetic flux of one type starts increasing and when it goes away from the loop, the same magnetic flux starts decreasing, thus induced current will be opposite to each other.
Hence, the correct option is (4).

4. The cylindrical tube of a spray pump has radius R, one end of which has n fine holes, each of radius r. If the speed of the liquid in the tube is v, the speed of the ejection of the liquid through the holes is:

(1) $\dfrac{v^2R}{nr}$ (2) $\dfrac{vR^2}{n^2r^2}$

(3) $\dfrac{vR^2}{nr^2}$ (4) $\dfrac{vR^2}{n^3r^2}$

Solution:

$$A_1V_1 = A_2V_2$$

$$\Rightarrow V_2 = \frac{A_1V_1}{A_2} = \frac{\pi R^2 \times v}{n(\pi r^2)} = \frac{R^2 \times v}{nr^2}$$

Hence, the correct option is (3).

5. The Young's modulus of steel is twice that of brass. Two wires of same lenght and of same area of cross section, one of steel and another of brass are suspended from the same roof. If we want the lower ends of the wires to be at the same level, then the weights added to the steel and brass wires must be in the ratio of:

(1) 1 : 1 (2) 1 : 2

(3) 2 : 1 (4) 4 : 1

Solution:

$$Y = \frac{F}{A}.\frac{l}{\Delta l}$$

$$\Rightarrow \Delta l = \frac{F}{A}.\frac{l}{Y}$$

$$(\Delta l)_{\text{steel}} = (\Delta l)_{\text{brass}}$$

$$\therefore \frac{W_\text{s}}{A}.\frac{l}{Y_\text{s}} = \frac{W_\text{b}}{A}.\frac{l}{Y_\text{s}}$$

$$\Rightarrow \frac{W_\text{s}}{W_\text{b}} = \frac{Y_\text{s}}{Y_\text{b}} = 2:1$$

Hence, the correct option is (3).

6. A potentiometer wire of length L and a resistance r are connected in series with a battery of e.m.f. E_0 and a resistance r_1. An unknown e.m.f. E is balanced at a length l of the potentiometer wire. The e.m.f.
E will be given by:

(1) $\dfrac{LE_0r}{(r+r_1)l}$ (2) $\dfrac{LE_0r}{lr_2}$

(3) $\dfrac{E_0r}{(r+r_1)}.\dfrac{l}{L}$ (4) $\dfrac{E_0l}{L}$

Solution:

Current, $i = \frac{E_0}{(r_1 + r)}$

Potentiometer gradient, $x = \frac{ir}{L} = \frac{E_0}{(r_1 + r)} \frac{r}{L}$

$\therefore$ e.m.f. $E = xl = \frac{E_0 r}{(r_1 + r)} \frac{l}{L}$

Hence, the correct option is (3).

7. A particle is executing a simple harmonic motion. Its maximum acceleration is a and maximum velocity is *b*. Then, its time period of vibration will be:

(1) $\frac{2\pi\beta}{\alpha}$

(2) $\frac{\beta^2}{\alpha^2}$

(3) $\frac{\alpha}{\beta}$

(4) $\frac{\beta^2}{\alpha}$

Solution:

For Simple Harmonic Motion:

Maximum acceleration = $\omega^2 A = \alpha$

Maximum velocity = $\omega A = \beta$

$\Rightarrow \omega = \frac{\alpha}{\beta} \Rightarrow \frac{2\pi}{T} = \frac{\alpha}{\beta}$

$\therefore T = \frac{2\pi\beta}{\alpha}$

Hence, the correct option is (1).

8. If vectors $\vec{A} = \cos\omega t\ \hat{i} + \sin\omega t\ \hat{j}$ and $\vec{B} = \cos\frac{\omega t}{2}\ \hat{i} + \sin\frac{\omega t}{2}\ \hat{j}$ are functions of time, then the value of *t* at which they are orthogonal to each other is:

(1) $t = 0$

(2) $t = \frac{\pi}{4\omega}$

(3) $t = \frac{\pi}{2\omega}$

(4) $t = \frac{\pi}{\omega}$

Solution:

For perpendicular vectors,

$\vec{A}.\vec{B} = 0$

$$\left[\cos\omega t\ \hat{i}\ +\ \sin\omega t\ \hat{j}\right].\left[\cos\frac{\omega t}{2}\ \hat{i}\ +\ \sin\frac{\omega t}{2}\ \hat{j}\right]=0$$

$$\cos\omega t\cos\frac{\omega t}{2}+\sin\omega t\sin\frac{\omega t}{2}\ =0$$

$$\Rightarrow\cos\left(\omega t-\frac{\omega t}{2}\right)=0$$

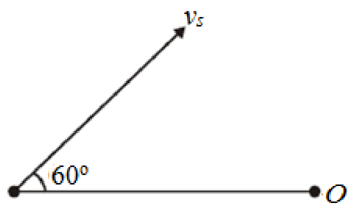
$$\Rightarrow\cos\left(\frac{\omega t}{2}\right)=0$$

$$\Rightarrow\frac{\omega t}{2}=\frac{\pi}{2}$$

$$\Rightarrow t=\frac{\pi}{\omega}$$

Hence, the correct option is (4).

9. A source of sound *S* emitting waves of frequency 100 Hz and an observer *O* are located at some distance from each other. The source is moving with a speed of 19.4 ms^{-1} at an angle of 60° with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer (velocity of sound in air 330 ms^{-1}) is:



(1) 97 Hz
(2) 100 Hz
(3) 103 Hz
(4) 106 Hz

Solution:

$$f\,'=f\left(\frac{v}{v-v_s\cos60^\circ}\right)$$

$$f\,'=100\left(\frac{330}{330-19.4\times\frac{1}{2}}\right)=100\left(\frac{330}{330-9.7}\right)=100\left(\frac{330}{320.3}\right)=103.02\text{Hz}$$

Hence, the correct option is (3).

10. An automobile moves on a road with a speed of 54 kmh^{-1}. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is 3 kgm^2. If the vehicle is brought to rest in 15s, the magnitude of average torque transmitted by its brakes to wheel is:

(1) 2.86 kg m^2s^{-2}
(2) 6.66 kg m^2s^{-2}
(3) 8.58 kg m^2s^{-2}
(4) 10.86 kg m^2s^{-2}

Solution:

Velocity of automobile, $v=54\times\frac{5}{18}=15\text{ms}^{-1}$

$$\omega_0=\frac{v}{R}=\frac{15}{0.45}=\frac{100}{3}\text{rad}\,\text{s}^{-1}\ ,\ \omega=0$$

As,

$\omega = \omega_0 + \alpha t$

$0 = \frac{100}{3} + \alpha(15)$

Thus, angular acceleration

$\alpha = \frac{100}{45}\,\text{rad}\,\text{s}^{-2}$

Torque, $\tau = I\alpha = 3 \times \frac{100}{45} = 6.66\,\text{kg}\,\text{m}^2\text{s}^{-2}$

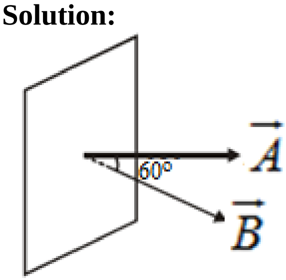
Hence, the correct option is (2).

11. A rectangular coil of length 0.12m and width 0.1m having 50 turns of wire is suspended vertically in a uniform magnetic field of strength 0.2 Weber/m^2. The coil carries a current of 2 A. If the plane of the coil is inclined at an angle of 30° with the direction of the field, the torque required to keep the coil in stable equilibrium will be:

(1) 0.12 Nm (2) 0.15 Nm

(3) 0.20 Nm (4) 0.24 Nm

Solution:



$\vec{\tau} = \vec{M} \times \vec{B}$

$|\vec{\tau}| = MB\sin\theta = nIAB\sin\theta = 50 \times 0.012 \times 2 \times 0.2 \times \sin 60^{o} = 0.20\,\text{Nm}$

Hence, the correct option is (3).

12. A parallel plate air capacitor has capacity '*C*' distance of separation between plates is '*d*' and potential difference '*V*' is applied between the plates force of attraction between the plates of the parallel plate air capacitor is:

(1) $\frac{C^2V^2}{2d^2}$ (2) $\frac{C^2V^2}{2d}$

(3) $\frac{CV^2}{2d}$ (4) $\frac{CV^2}{d}$

Solution:

Force between plates of capacitor,

$F = qE = q\left(\frac{q}{2\varepsilon_0 A}\right)$

$\because q = CV \text{ and } C = \frac{\varepsilon_0 A}{d}$

$\therefore F = \frac{C^2V^2}{2\varepsilon_0 A} = \frac{\left(\frac{\varepsilon_0 A}{d}\right)CV^2}{2\varepsilon_0 A} = \frac{CV^2}{d}$

Hence, the correct option is (3).

13. Two vessels separately contain two ideal gases A and B at the same temperature, the pressure of A being twice that of B. Under such conditions, the density of A is found to be 1.5 times the density of B. The ratio of molecular weight of A and B is:

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{3}{4}$ (4) 2

Solution:

Ideal gas equation

$$P = \frac{\rho RT}{M} \Rightarrow M = \frac{\rho RT}{P}$$

$$\frac{M_A}{M_B} = \frac{\rho_A RT_A}{P_A} \cdot \frac{P_B}{\rho_B RT_B} = \frac{\rho_A}{\rho_B} \cdot \frac{P_B}{P_A} = (1.5)\left(\frac{1}{2}\right) = \frac{3}{4}$$

Hence, the correct option is (3).

14. A satellite *S* is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then,

(1) the acceleration of S is always directed towards the centre of the earth.

(2) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.

(3) the total mechanical energy of S varies periodically with time.

(4) the linear momentum of S remains constant in magnitude.

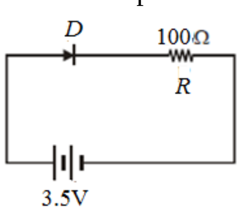
Solution:

Satellite is experiencing only the gravitational force, which will always be towards the centre of the earth.

Hence, the correct option is (1).

15. In the given figure, a diode D is connected to an external resistance R =100 W and an e.m.f of 3.5V.

If the barrier potential developed across the diode is 0.5 V, the current in the circuit will be:



(1) 35 mA (2) 30 mA

(3) 40 mA (4) 20 mA

Solution:

$$I = \frac{V_{net}}{R_{net}} = \frac{3.5 - 0.5}{100} = 0.03\,\text{A} = 30\,\text{mA}$$

Hence, the correct option is (2).

16. A remote-sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and $g = 9.8$ ms^{-2}, then the orbital speed of the satellite is:

(1) 6.67 km s^{-1} (2) 7.76 km s^{-1}

(3) 8.56 km s^{-1} (4) 9.13 km s^{-1}

Solution:
Velocity of satellite revolving around earth,

$$v_s = \sqrt{\frac{GM_e}{R_e\left(1+\frac{h}{R_e}\right)}} = \sqrt{\frac{gR_e}{1+\frac{h}{R_e}}}$$

Substituting the values,

$$v_0 = \sqrt{60\times 10^6} = 7.76\times 10^3\,\text{ms}^{-1} = 7.76\,\text{km}\,\text{s}^{-1}$$

Hence, the correct option is (2).

17. The position vector of a particle $\vec{R}$ as a function of time is given by:

$$\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$$

Where R is in meters, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x and y-directions, respectively. Which one of the following statements is wrong for the motion of particle?

(1) Path of the particle is a circle of radius 4 meter.

(2) Acceleration vector is along $-\vec{R}$.

(3) Magnitude of acceleration vector is v^2/R where v is the velocity of particle.

(4) Magnitude of the velocity of particle is 8 meter/second.

Solution:

$$\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$$

$$x = 4\sin 2\pi t$$

$$y = 4\cos 2\pi t$$

$$\vec{v} = \frac{d\vec{R}}{dt}$$

$$v_x = +4(\cos 2\pi t)(2\pi)$$

$$v_y = -4(\sin 2\pi t)(2\pi)$$

$$v = \sqrt{v_x + v_y} = \sqrt{(8\pi)^2(\cos^2 2\pi t + \sin^2 2\pi t)} = 8\pi$$

Hence, the correct option is (4).

18. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequency for this string is:

(1) 105 Hz
(2) 155 Hz
(3) 205 Hz
(4) 10.5 Hz

Solution:
For a string fixed at both ends, two consecutive resonant frequencies will be:

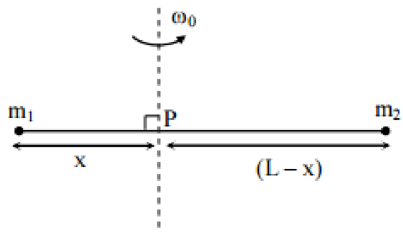
$$f_3 = \frac{nv}{2l} \text{ and } f_4 = \frac{(n+1)v}{2l}$$

$$f_4 - f_3 = \frac{(n+1)v}{2l} - \frac{nv}{2l} = 420 - 315$$

$$\therefore \frac{v}{2l} = 105\,\text{Hz}$$

Hence, the correct option is (1).

19. Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L, and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point P on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity ω_0 is minimum, is given by:



(1) $x = \dfrac{m_2 L}{m_1 + m_2}$

(2) $x = \dfrac{m_1 L}{m_1 + m_2}$

(3) $x = \dfrac{m_1}{m_2} L$

(4) $x = \dfrac{m_2}{m_1} L$

Solution:

The position of point P on rotating rod should be such that the work required to set the rod rotating with minimum angular velocity ω_0 is the centre of mass.

$$I = m_1 x^2 + m_2 (L - x)^2$$

$$I = m_1 x^2 + m_2 L^2 + m_2 x^2 - 2m_2 Lx$$

$$\frac{dI}{dx} = 2m_1 x + 0 + 2xm_2 - 2m_2 L = 0$$

$$x(2m_1 + 2m_2) = 2m_2 L$$

$$x = \frac{m_2 L}{m_1 + m_2}$$

Hence, the correct option is (1).

20. At the first minimum adjacent to the central maximum of a single-slit diffraction pattern the phase difference between the Huygen's wavelet from the edge of the slit and the wavelet from the mid-point of the slit is:

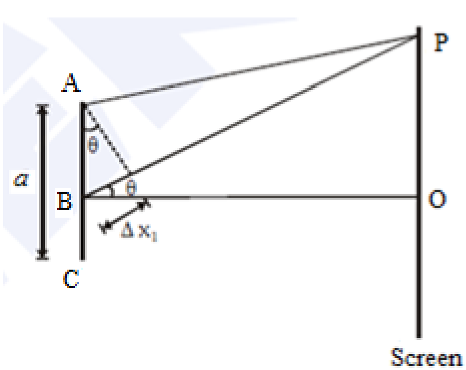
(1) $\dfrac{\pi}{8}$ radian

(2) $\dfrac{\pi}{4}$ radian

(3) $\dfrac{\pi}{2}$ radian

(4) π radian

Solution:



The first minima path difference between A and C is λ so path difference between A and B should be λ/2.

Thus, phase difference , $\Delta\phi_1 = \dfrac{\lambda}{2\lambda} \times 2\pi = \pi$ radian

Hence, the correct option is (4).

21. A force $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$ .The value of α for which angular momentum about origin is conserved is:

(1) 1 (2) -1

(3) 2 (4) zero

Solution:

For conservation of angular momentum:

$$\sum \vec{\tau}_{\text{net}} = 0$$

$$\Rightarrow \vec{r} \times \vec{F} = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -12 \\ \alpha & 3 & 6 \end{vmatrix} = 0$$

On solving it we get:

α = -1

Hence, the correct option is (2).

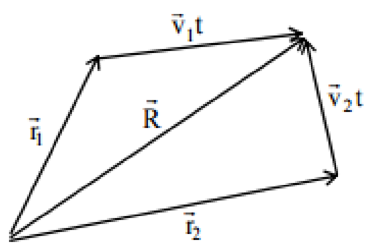
22. Two particles A and B, move with constant velocities $\vec{v}_1$ and $\vec{v}_2$. At the initial moment their position vectors are $\vec{r}_1$ and $\vec{r}_2$ respectively. The condition for particle A and B for their collision is:

(1) $\vec{r}_1 - \vec{r}_2 = \vec{v}_1 - \vec{v}_2$ (2) $\dfrac{\vec{r}_1 - \vec{r}_2}{\left|\vec{r}_1 - \vec{r}_2\right|} = \dfrac{\vec{v}_2 - \vec{v}_1}{\left|\vec{v}_2 - \vec{v}_1\right|}$

(3) $\vec{r}_1 . \vec{v}_1 = \vec{r}_2 . \vec{v}_2$ (4) $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$

Solution:

For two particles to collide with each other, the direction of the relative velocity of one with respect to other should be directed towards the relative position of the other particle.



Thus, direction of relation position of 1 with respect to 2 = $\dfrac{\vec{r}_1 - \vec{r}_2}{\left|\vec{r}_1 - \vec{r}_2\right|}$

And, direction of relation velocity of 1 with respect to 2 $= \dfrac{\vec{v}_2 - \vec{v}_1}{\left|\vec{v}_2 - \vec{v}_1\right|}$

So, for collision to take place:

$$\frac{\vec{r}_1 - \vec{r}_2}{\left|\vec{r}_1 - \vec{r}_2\right|} = \frac{\vec{v}_2 - \vec{v}_1}{\left|\vec{v}_2 - \vec{v}_1\right|}$$

Hence, the correct option is (2).

23. A nucleus of uranium decays at rest into nuclei of thorium and helium. Then:

(1) The helium nucleus has less kinetic energy than the thorium nucleus

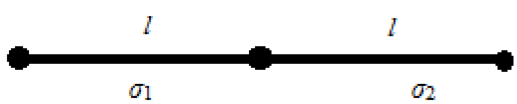
(2) The helium has more kinetic energy than the thorium nucleus.

(3) The helium nucleus has less momentum than the thorium nucleus.
(4) The helium nucleus has more momentum than the thorium nucleus.
Solution:
By law of conservation of momentum
$p_{\text{Th}} = p_{\text{He}} = p$
As, Kinetic energy = $\dfrac{p^2}{2m}$
Thus, Kinetic energy is inversely proportional to mass of elements.
$\because m_{\text{He}} < m_{\text{Th}}$
$\therefore$ Kinetic Energy of He > Kinetic Energy of Th
Hence, the correct option is (2).

24. Two metal wires of identical dimensions are connected in series. If σ_1 and σ_1 are the conductivities of the metal wires respectively, the effective conductivity of the combination is:

(1) $\dfrac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}$ (2) $\dfrac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$
(3) $\dfrac{\sigma_1+\sigma_2}{2\sigma_1\sigma_2}$ (4) $\dfrac{\sigma_1+\sigma_2}{\sigma_1\sigma_2}$

Solution:



$R_{\text{eq}} = R_1 + R_2$
$\rho_{\text{eq}}\dfrac{2L}{A} = \rho_1\dfrac{L}{A} + \rho_2\dfrac{L}{A}$
$2\rho_{\text{eq}} = \rho_1 + \rho_2$
$\dfrac{2}{\sigma_{\text{eq}}} = \dfrac{1}{\sigma_1} + \dfrac{1}{\sigma_2}$
$\dfrac{2}{\sigma_{\text{eq}}} = \dfrac{\sigma_1+\sigma_2}{\sigma_1\sigma_2}$
$\sigma_{\text{eq}} = \dfrac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$
Hence, the correct option is (2).

25. Light of wavelength 500 nm is incident on a metal with work function 2.28 eV. The de Broglie wavelength of the emitted electron is:
(1) $\leq 2.8 \times 10^{-12}$ m (2) $< 2.8 \times 10^{-10}$ m
(3) $< 2.8 \times 10^{-9}$ m (4) $\geq 2.8 \times 10^{-9}$ m
Solution:
Energy of Photon = $\dfrac{12400}{5000} = 2.48\,\text{eV}$
According to Einstein Equation:
$E = \phi_0 + (K.E.)_{\max}$
$(K.E.)_{\max} = 2.48 - 2.28 = 0.2\,\text{eV}$

For an electron, $\lambda = \frac{h}{\sqrt{(K.E.)_{\max}}} = \frac{12.27}{\sqrt{0.2}} = 2.7\times10^{-9}\,\text{m}$

Thus, $\lambda \geq 2.8\times10^{-9}\,\text{m}$

Hence, the correct option is (4).

26. 4.0 g of a gas occupies 22.4 litres at NTP. The specific heat capacity of the gas at constant volume is 5.0 JK^{-1} mol^{-1}. If the speed of sound in this gas at NTP is 952 ms^{-1}, then the heat capacity at constant pressure is: (Take gas constant R = 8.3 JK^{-1} mol^{-1})

(1) 8.5 JK^{-1} mol^{-1} (2) 8.0 JK^{-1} mol^{-1}

(3) 7.5 JK^{-1} mol^{-1} (4) 7.0 JK^{-1} mol^{-1}

Solution:

$$v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow \gamma = \frac{Mv^2}{RT} = 1.6$$

As, $C_P = \gamma C_V = 1.6\times5.0 = 8.0\,\text{JK}^{-1}\text{mol}^{-1}$

Hence, the correct option is (2).

27. A series R-C circuit is connected to an alternating voltage source. Consider two situations:

(a) When capacitor is air filled.

(b) When capacitor is mica filled.

Current through resistor is i and voltage across capacitor is V then:

(1) $V_a = V_b$ (2) $V_a < V_b$

(3) $V_a > V_b$ (4) $i_a > i_b$

Solution:

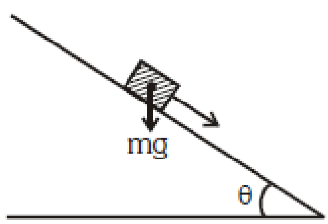
As, $X_C = \frac{1}{2\pi fC}$

Thus, when capacitor is filled with mica and its capacitance C increases, X_C decreases. Therefore, in case (b) X_C decreases and so does the voltage across capacitor.

So, $V_a > V_b$.

Hence, the correct option is (3).

28. A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4.0 m down the plank in 4.0s. The coefficients of static and kinetic friction between the box and the plank will be, respectively:



(1) 0.4 and 0.3 (2) 0.6 and 0.6

(3) 0.6 and 0.5 (4) 0.5 and 0.6

Solution:

Coefficient of static friction, $\mu_s = \tan 30^\circ = \frac{1}{\sqrt{3}} = 0.6$

$$s = v_0 t + \frac{1}{2}at^2$$

$$4 = 0 + \frac{1}{2}(g\sin 30^{\circ} - \mu_k g\cos 30^{\circ})(4)^2$$
$$\frac{4}{8} = 10 \times \frac{1}{2} - \mu_k \frac{\sqrt{3}}{2}$$
$$\mu_k = 0.5$$

Hence, the correct option is (3).

29. Two stones of masses *m* and 2 *m* are whirled in horizontal circles, the heavier one in a radius *r*/2 and the lighter one in radius *r*. The tangential speed of lighter stone is *n* times that of the value of heavier stone when they experience same centripetal forces. The value of *n* is:

(1) 1	(2) 2
(3) 3	(4) 4

Solution:
$$(F_C)_{\text{heavier}} = (F_C)_{\text{lighter}}$$
$$\frac{2mv_1^{\,2}}{(r/2)} = \frac{mv_2^{\,2}}{r}$$
$$\Rightarrow v_1^{\,2} = 4v_2^{\,2} \Rightarrow v_1 = 2v_2$$
Hence, the correct option is (2).

30. The coefficient of performance of a refrigerator is 5. If the temperature inside freezer is –20°C, the temperature of the surroundings to which it rejects heat is:

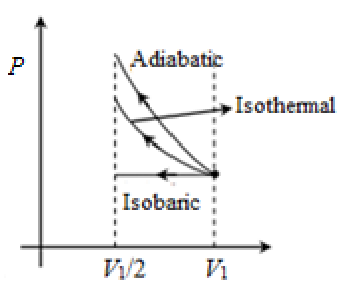
(1) 21°C	(2) 31°C
(3) 41°C	(4) 11°C

Solution:
Coefficient of performance of refrigerator, $\alpha = 5$
$$\alpha = 31\frac{T_L}{T_H - T_L} = 5$$
$$\Rightarrow T_H = \frac{6}{5}T_L = \frac{6}{5}(253) = 303.6\,\text{K} = 30.6\,^{\circ}\text{C} \simeq 31^{\circ}\text{C}$$
Hence, the correct option is (2).

31. An ideal gas is compressed to half its initial volume by means of several processes. Which of the process results in the maximum work done on the gas?

(1) Isothermal	(2) Adiabatic
(3) Isobaric	(4) Isochoric

Solution:



Work done on the gas = Area under curve
$W_{isochoric} = 0$
$W_{adiabatic} > W_{isothermal} > W_{isobaric}$
Hence, the correct option is (2).

32. A ball is thrown vertically downwards from a height of 20 m with an initial velocity v_0. It collides with the ground, loses 50 percent of its energy in collision and rebounds to the same height. The initial velocity v_0 is: (Take g = 10 ms^{-2})

(1) 10 ms^{-1}
(2) 14 ms^{-1}
(3) 20 ms^{-1}
(4) 28 ms^{-1}

Solution:
Let ball rebounds with speed v so

$$0.5\left(mgh+\frac{1}{2}mv^2\right)=mgh$$

$$0.25mv^2=0.5mgh$$

$$v=\sqrt{2gh}=\sqrt{2\times10\times20}=\sqrt{400}=20\,\text{ms}^{-1}$$

Hence, the correct option is (3).

33. On a frictionless surface, a block of mass, M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed v/3. The second block's speed after the collision is:

(1) $\frac{\sqrt{3}}{2}v$
(2) $\frac{2\sqrt{2}}{3}v$
(3) $\frac{3}{4}v$
(4) $\frac{3}{\sqrt{2}}v$

Solution:
According to law of conservation of energy, in elastic collision

$$(K.E.)_{\text{before collision}}=(K.E.)_{\text{after collision}}$$

Let speed of block after collision be v'

$$\frac{1}{2}mv^2+0=\frac{1}{2}m\left(\frac{v}{3}\right)^2+\frac{1}{2}m(v')^2$$

$$v^2=\frac{v^2}{9}+v'^2\Rightarrow v'^2=\frac{8v^2}{9}$$

$$\Rightarrow v'=\frac{2\sqrt{2}}{3}v$$

Hence, the correct option is (2).

34. If potential (in volts) in a region is expressed as $V(x,y,z) = 6xy - y + 2yz$, the electric field (in N/C) at point (1,1,0) is:

(1) $-(6\hat{i}+9\hat{j}+\hat{k})$
(2) $-(3\hat{i}+5\hat{j}+3\hat{k})$
(3) $-(6\hat{i}+5\hat{j}+2\hat{k})$
(4) $-(2\hat{i}+3\hat{j}+\hat{k})$

Solution:

$$V=6xy-y+2yz$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

$$\vec{E} = -6y\hat{i} - (6x - 1 + 2z)\hat{j} - 2y\hat{k}$$

At (1, 1, 0)

$$E = -6\hat{i} - 5\hat{j} - 2\hat{k}$$
$$= -\left(6\hat{i} + 5\hat{j} + 2\hat{k}\right)$$

Hence, the correct option is (2).

35. Two slits in Young's experiment have widths in the ratio 1 : 25. The ratio of intensity at the maxima and minima in the interference pattern, $\frac{I_{\max}}{I_{\min}}$ is:

(1) $\frac{4}{9}$
(2) $\frac{9}{4}$
(3) $\frac{121}{49}$
(4) $\frac{49}{121}$

Solution:

$$\frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{1}{25} \Rightarrow \frac{I_2}{I_1} = \frac{25}{1}$$

$$\frac{I_{\max}}{I_{\min}} = \frac{\left(\sqrt{I_2} + \sqrt{I_1}\right)^2}{\left(\sqrt{I_2} - \sqrt{I_1}\right)^2} = \frac{\left(\sqrt{\frac{I_2}{I_1}} + 1\right)^2}{\left(\sqrt{\frac{I_2}{I_1}} - 1\right)^2}$$

$$= \left(\frac{5+1}{5-1}\right)^2 = \left(\frac{6}{4}\right)^2 = \frac{9}{4}$$

Hence, the correct option is (2).

36. The heart of a man pumps 5 litres of blood through the arteries per minute at a pressure of 150 mm of mercury. If the density of mercury be 13.6×10^3 kg/m^3 and $g = 10\text{m/s}^2$, then the power of heart in watt is:

(1) 1.50
(2) 1.70
(3) 2.35
(4) 3.0

Solution:

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{V}{t}\rho gh$$
$$= \frac{5 \times 10^{-3}}{60} \times 13.6 \times 10^3 \times 10 \times 150 \times 10^{-3}$$
$$= \frac{5 \times 13.6 \times 150}{6} \times 10^{-3} = 1700 \times 10^{-3} = 1.7\,\text{Watt}$$

Hence, the correct option is (2).

37. A proton and an alpha particle both enter a region of uniform magnetic field, *B*, moving at right angles to the field *B*. If the radius of circular orbits for both the particles is equal and the kinetic energy acquired by proton is 1 MeV, the energy acquired by the alpha particle will be:

(1) 1 MeV
(2) 4 MeV

(3) 0.5 MeV (4) 1.5 MeV

Solution:

Radius in magnetic field, $R = \dfrac{mv}{qB} = \dfrac{\sqrt{2mE}}{qB}$

$$E = \frac{q^2 B^2 R^2}{2m}$$

For proton,

$$E_1 = \frac{e^2 B^2 R^2}{2m_p}$$

For α-particles,

$$E_2 = \frac{(2e)^2 B^2 R^2}{2 \times 4m_p}$$

Therefore, $E_1 = E_2$.

Hence, the correct option is (1).

38. The input signal given to a CE amplifier having a voltage gain of 150 is Vi = 2 cos $\left(15t + \dfrac{\pi}{3}\right)$.The corresponding output signal will be:

(1) 300 cos $\left(15t + \dfrac{4\pi}{3}\right)$ (2) 300 cos $\left(15t + \dfrac{\pi}{3}\right)$

(3) 75 cos $\left(15t + \dfrac{2\pi}{3}\right)$ (4) 2 cos $\left(15t + \dfrac{5\pi}{6}\right)$

Solution:

V_i = 2 cos (15t + π/3)

$$A_v = \frac{V_0}{V_{in}} \Rightarrow V_0 = A_v \times V_{in} \Rightarrow V_0 = 150 \times 2\cos\left(15t + \frac{\pi}{3} + \pi\right)$$

$$V_0 = 300\cos\left(15t + \frac{4\pi}{3}\right)$$

Hence, the correct option is (2).

39. In dimension of critical velocity v_c, of liquid following through a tube are expressed as $(\eta^x \rho^y r^z)$, where η, ρ and r are the coefficient of viscosity of liquid, density of liquid and radius of the tube respectively, then the values of x, y and z are given by:

(1) 1, 1, 1 (2) 1, –1, –1

(3) –1, –1, 1 (4) –1, –1, –1

Solution:

$v_c = \eta^x \rho^y r^z$

$M^0 L^1 T^{-1} = (M^1 L^{-1} T^{-1})^x (M^1 L^{-3})^y (L^1)^z$

$M^0 L^1 T^{-1} = M^{x+y+z} L^{-x-3y+z} T^{-x}$

On comparing x, y and z on both the sides, we get

$x = 1$

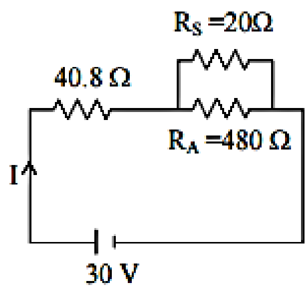
$x + y = 0 \Rightarrow y = -x = -1$

$-x - 3y + z = 1$

$-1-3(-1)+z=1$
$z=-1$
Hence, the correct option is (2).

40. A circuit contains an ammeter, a battery of 30 V and a resistance 40.8 ohm all connected in series. If the ammeter has a coil of resistance 480 ohm and a shunt of 20 ohm, the reading in the ammeter will be:
(1) 1 A (2) 0.5 A
(3) 0.25 A (4) 2 A

Solution:



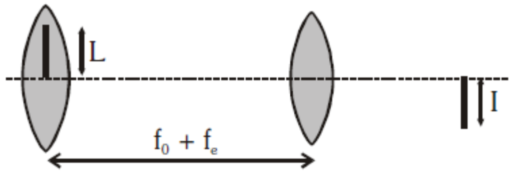
$$R_{eq}=40.8+\frac{480\times 20}{480+20}=40.8+19.2=60\ \Omega$$
$$I=\frac{V_{eq}}{R_{eq}}=\frac{30}{60}=0.5\,\text{A}$$
Hence, the correct option is (2).

41. Water rises to height '*h*' in capillary tube. If the length of capillary tube above the surface of water is made less than '*h*', then :
(1) water does not rise at all.
(2) water rises upto the tip of capillary tube and then starts overflowing like a fountain.
(3) water rises upto the top of capillary tube and stays there without overflowing.
(4) water rises upto a point a little below the top and stays there.

Solution:
Height of water column > length of tube ⇒ so liquid will be stay there
Hence, the correct option is (3).

42. In an astronomical telescope in normal adjustment a straight black line of length *L* is drawn on inside part of objective lens. The eye-piece forms a real image of this line. The length of this image is *I*. The magnification of the telescope is:
(1) $\frac{L}{I}$ (2) $\frac{L}{I}+1$
(3) $\frac{L}{I}-1$ (4) $\frac{L+I}{L-I}$

Solution:



For eye-piece lens

$$m=\frac{f}{f+u}=\frac{h_1}{h_o}\Rightarrow\frac{f_e}{f_e+[-(f_0+f_e)]}=\frac{I}{L}$$

$$\Rightarrow-\frac{f_e}{f_0}=\frac{I}{L}$$

$$\Rightarrow M=\frac{L}{I}$$

Hence, the correct option is (1).

43. The value of coefficient of volume expansion of glycerin is 5×10^{-4} K^{-1}. The fractional change in the density of glycerin for a rise of 40°C in its temperature, is:

(1) 0.010 (2) 0.015
(3) 0.020 (4) 0.025

Solution:

$$\rho=\rho_0(1+Y\Delta T)$$

$$\rho-\rho_0=\rho_0 Y\Delta T$$

Fractional change

$$\frac{\rho-\rho_0}{\rho_0}=Y\Delta T=5\times10^{-4}\times40$$

$$=200\times10^{-4}=0.020$$

Hence, the correct option is (3).

44. A photoelectric surface is illuminated successively by monochromatic light of wavelength λ and $\frac{\lambda}{2}$.

If the maximum kinetic energy of the emitted photoelectrons in the second case is 3 times that in the first case, the work function of the surface of the material is:
(h = Plank's constant, c = speed of light)

(1) $\frac{hc}{3\lambda}$ (2) $\frac{hc}{2\lambda}$
(3) $\frac{hc}{\lambda}$ (4) $\frac{2hc}{\lambda}$

Solution:

$$KE_1=\frac{hc}{\lambda}-\phi \qquad (1)$$

$$KE_2=\frac{hc}{\lambda/2}-\phi=\frac{2hc}{\lambda}-\phi \qquad (2)$$

Since, $KE_2=3KE_1$, therefore, using Eq. (1) and (2), we get

$$\frac{2hc}{\lambda} - \phi = 3\left(\frac{hc}{\lambda} - \phi\right)$$

$$\Rightarrow 2\phi = \frac{hc}{\lambda}$$

$$\Rightarrow \phi = \frac{hc}{2\lambda}$$

Hence, the correct option is (2).

45. A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive index of the material of the prism for the above red, green and blue wavelengths is 1.39, 1.44 and 1.47, respectively.

The prism will:

(1) separate the red colour part from the green and blue colours.
(2) separate the blue colour part from the red and green colours.
(3) separate all the three colours from one another.
(4) not separate the three colours at all.

Solution:

$$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45^{\circ}} = \sqrt{2} = 1.414$$

$$\because (\mu_{\text{red}} = 1.39) < \mu; \mu_{\text{violet}} > \mu \text{ and } \mu_{\text{green}} > \mu$$

Therefore, green and blue colour will not emerge out and will suffer total internal reflection.

Hence, the correct option is (1).