CAT 2020 Solved Paper Morning Slot

SECTION: QUANTITATIVE ABILITY

1. How many distinct positive integer-valued solutions exist to the equation $(x^2 - 7x + 11)^{(x^2 - 13x + 42)} = 1$?

(a) 4

(b) 2

(c) 6

(d) 8

Solution **(c)**

The equation is valid under the following circumstances:

(i) $x^2 - 7x + 11 = 1$

(ii) $x^2 - 13x + 42 = 0$

(iii) $x^2 - 7x + 11 = -1$, and

$x^2 - 13x + 42$ = Even number

For (i) $x^2 - 7x + 11 = 1$

$\Rightarrow x^2 - 7x + 10 = 0 \Rightarrow x = 2, 5$

For (ii) $x^2 - 13x + 42 = 0$

$\Rightarrow x = 6, 7$

For (iii) $x^2 - 7x + 11 = -1$

$\Rightarrow x^2 - 7x + 12 = 0 \Rightarrow x = 3, 4$

When $x = 3$,

$x^2 - 13x + 42 = 9 - 39 + 42$ = Even number

When $x = 4$,

$x^2 - 13x + 42 = 16 - 52 + 42$ = Even number

Therefore, there are 6 distinct integer-valued solutions to the given equation.

2. A solution, of volume 40 litres, has dye and water in the proportion 2 : 3. Water is added to the solution to change this proportion to 2 : 5. If one-fourths of this diluted solution is taken out, how many litres of dye must be added to the remaining solution to bring the proportion back to 2 : 3?

Solution **(8)**

Volume of dye in the given solution = 2/5th of 40 = 16 litres and volume of water = 24 litres. To make the ratio 2 : 5, since dye is 16 litres, water must be 5/2 times 16 = 40 litres. So, 16 litres of water must be added. The volume of the solution becomes = 40 + 16 = 56 litres.

If one-fourth is taken out, the volume left is 42 litres containing dye and water in the ratio of 2 : 5. This contains dye = 2/7th of 42 = 12 litres and 30 litres of water. To make the proportion 2 : 3, since amount of water is 30 litres, the amount of dye must be 20 litres. So, amount of dye to be added = 20 – 12 = 8 litres.

3. The number of distinct real roots of the equation $\left(x+\frac{1}{x}\right)^2 - 3\left(x+\frac{1}{x}\right) + 2 = 0$ equals

Solution **(1)**

Let $x + \frac{1}{x} = t$.

The given equation becomes

$t^2 - 3t + 2 = 0 \Rightarrow t = 1, 2$

When $x + \frac{1}{x} = 1 \Rightarrow x^2 - x + 1 = 0$, which does not have real roots.

When $x + \frac{1}{x} = 2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1, 1$

So, the number of distinct real roots is 1 only.

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4. A gentleman decided to treat a few children in the following manner. He gives half of his total stock of toffees and one extra to the first child, and then the half of the remaining stock along with one extra to the second and continues giving away in this fashion. His total stock exhausts after he takes care of 5 children. How many toffees were there in his stock initially?

Solution **(62)**

To the fifth child, the gentleman gives half the remaining (after giving to fourth child) and one extra, and then he is left with none.

So, after fourth child, if he is left with x toffees, then

$$\frac{x}{2} + 1 = x \Rightarrow x = 2$$

So, after giving to the fourth child, he is left with 2 toffees.

To the fourth child, he gives half of the remaining (after giving to third child) and one extra.

So, toffees remaining after third child

= (2 + 1) × 2 = 6

Toffees remaining after second child

= (6 + 1) × 2 = 14

Toffees remaining after first child

= (14 + 1) × 2 = 30

Toffees before giving to first child

= (30 + 1) × 2 = 62

5. How many 3-digit numbers are there, for which the product of their digits is more than 2 but less than 7?

Solution **(21)**

For the given condition, three digits will be

(1, 1, 3), (1, 1, 4), (1, 1, 5), (1, 1, 6), (1, 2, 2), (1, 2, 3)

If the digits are (1, 2, 3), there will be 3! = 6 cases. For the rest, there will be 3 cases each.

So, total number of cases = 6 + 3 × 5 = 21

6. A person spent ₹ 50,000 to purchase a desktop computer and a laptop computer. He sold the desktop at 20% profit and the laptop at 10% loss. If overall he made a 2% profit then the purchase price, in rupees, of the desktop is

Solution **(₹ 20,000)**

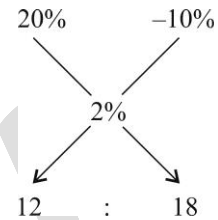
Total amount spent = ₹ 50,000

Profit on desktop = 20%

Loss on laptop = 10%

Overall profit = 2%

Using method of alligation, we have



So, ratio of cost prices of desktop to laptop = 2 : 3.

So, cost price of desktop

= 2/5th of 50000 = ₹ 20,000

7. A train travelled at one-thirds of its usual speed, and hence reached the destination 30 minutes after the scheduled time. On its return journey, the train initially travelled at its usual speed for 5 minutes but then stopped for 4 minutes for an emergency. The percentage by which the train must now increase its usual speed so as to reach the destination at the scheduled time, is nearest to

(a) 50

(b) 61

(c) 67

(d) 58

Solution **(c)**

If speed becomes 1/3rd, the time taken becomes 3/1 times the usual time t. So,

$3t - t$ = 30 minutes $\Rightarrow$ t = 15 minutes, which is the usual time of travel.

On the return journey, the train travels at usual speed of 5 min. The remaining journey time is 10 minutes. Since the train stopped for 4 minutes, it has to travel such that the remaining distance is covered in 6 minutes so as to reach the destination at the scheduled time. So, the time required to be taken is 6/10th of the usual and hence speed is 10/6th of the usual, which is 4/6th more than the usual, that is, 66.66% or 67% approximately.

8. If $\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$, then y equals

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***Solution* (36)**

Given that $\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$

$$\Rightarrow \frac{\log 5}{\log 4} = \frac{\log y}{\log 4} \times \frac{1}{2} \times \frac{\log 5}{\log 6}$$

$$\Rightarrow \log 5 = \log y \left[\frac{1}{2} \times \frac{\log 5}{\log 6}\right]$$

$$\Rightarrow 1 = \log y \left[\frac{1}{2} \times \frac{1}{\log 6}\right]$$

$\Rightarrow$ 2 log 6 = (log y)

$\Rightarrow$ log 6^2 = (log y) $\Rightarrow y = 36$

9. Among 100 students, x_1 have birthdays in January, x_2 have birthdays in February, and so on.
If x_0 = max (x_1, x_2, ..., x_{12}), then the smallest possible value of x_0 is

(a) 10

(b) 12

(c) 8

(d) 9

***Solution* (*d*)**

100 students have been divided among 12 months, so average number of students in each month = 100 / 12 = 8.33. So, the maximum of x_1, x_2, ..., x_{12} cannot be 8, because if the maximum of these variables is 8, the rest of the variables values will be less than or equal to 8, in which case sum of these variables will not be 100.

If the maximum of x_1, x_2, ..., x_{12} is 9, the rest of the variables values will be less than or equal to 9, in which case sum of these variables can be 100. For example, 11 months have values 9 each, and the 12th month has value 1. So, the correct answer is 9.

10. A circle is inscribed in a rhombus with diagonals 12 cm and 16 cm. The ratio of the area of circle to the area of rhombus is

(a) $\frac{6\pi}{25}$

(b) $\frac{3\pi}{25}$

(c) $\frac{2\pi}{15}$

(d) $\frac{5\pi}{18}$

***Solution* (a)**

Area of the quadrilateral circumscribing a circle = rs, where r is inradius and s is semi-perimeter.

Now, area of quadrilateral (rhombus)

$$= \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 12 \times 16 = 96$$

Since lengths of half diagonals are 6 and 8, the length of the side of rhombus = 10 cm. So,

s = Semi-perimeter = 20 cm

Therefore, inradius,

$$r = \frac{\text{Area of quadrilateral}}{s} = \frac{96}{20} = 4.8$$

Area of circle = $\pi(4.8)^2 = 23.04\pi$

Ratio of area of circle to that of rhombus

$$= \frac{23.04\pi}{96} = \frac{5.76\pi}{24} = \frac{(2.4)^2\pi}{24}$$

$$= \frac{2.4\pi}{10} = \frac{24\pi}{100} = \frac{6\pi}{25}$$

11. An alloy is prepared by mixing three metals A, B and C in the proportion 3 : 4 : 7 by volume. Weights of the same volume of the metals A, B and C are in the ratio *5* : 2 : 6. In 130 kg of the alloy, the weight, in kg, of the metal C is

(a) 84

(b) 96

(c) 48

(d) 70

***Solution* (a)**

Weights of the same volume are in the ratio 5 : 2 : 6, which is same as the ratio of densities of the three metals. So,

Ratio of weights of the three metals in the alloy

= (3 × 5) : (4 × 2) : (7 × 6) = 15 : 8 : 42

So, weight of metal C in the alloy of weight 130 kg

$$= \frac{42}{65} \times 130 = 84 \text{ kg}$$

12. Let A, B and C be three positive integers such that the sum of A and the mean of B and C is 5. In addition, the sum of B and the mean of A and C is 7. Then the sum of A and B is

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(a) 7
(b) 4
(c) 5
(d) 6

Solution **(d)**

As per the question, we have

$$A+\frac{B+C}{2}=5;\ B+\frac{A+C}{2}=7$$

These expressions can be written as

$2A + B + C = 10$ (1)

$A + 2B + C = 14$ (2)

Adding Eq. (1) and (2), we get

$3(A + B) + 2C = 24$

Since $2C$ is even, $A + B$ also has to be even. So, correct answer can be either 4 or 6.

Further, subtracting the equations, we get

$B – A = 4$

But, $A + B = 4$ and $B – A = 4$ would mean $A = 0$, which is not possible.

So, the only possibility is 6.

13. The area of the region satisfying the inequalities $|x| - y \le 1$, $y \ge 0$ and $y \le 1$ is

Solution **(3)**

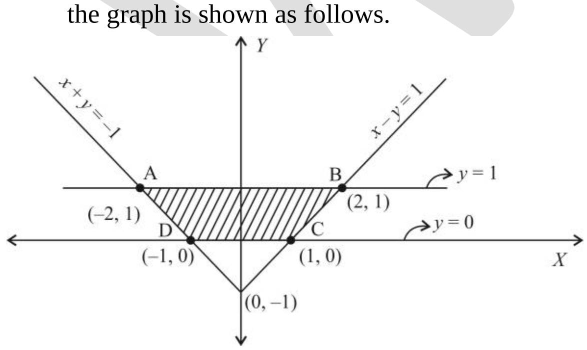
If $x > 0$, the inequality becomes

$x - y \le 1$

If $x < 0$, the inequality becomes

$-x - y \le 1$ or $x + y \ge -1$

The graph is limited between $y = 0$ and 1. So, the graph is shown as follows.



Intersection of $x – y = 1$ with $y = 1$ is (2, 1).

Intersection of $x + y = –1$ with $y = 1$ is (–2, 1).

The shape enclosed is trapezium whose lengths of parallel sides are 2 units and 4 units and height is 1 unit.

So, area $=\frac{1}{2}(2+4)\times(1)=3$

14. If y is a negative number such that $2^{y^2\log_3 5}=5^{\log_2 3}$, then y equals

(a) $\log_2(1/3)$
(b) $-\log_2(1/3)$
(c) $\log_2(1/5)$
(d) $-\log_2(1/5)$

Solution **(a)**

Given that $2^{y^2\log_3 5}=5^{\log_2 3}$

Taking logarithm on both sides, we get

$y^2\log_3 5 \log 2 = \log_2 3 \log 5$

$$\Rightarrow y^2\frac{\log 5}{\log 3}\log 2=\frac{\log 3}{\log 2}\log 5$$

$$\Rightarrow y^2\frac{\log 2}{\log 3}=\frac{\log 3}{\log 2}$$

$$\Rightarrow y^2=\left(\frac{\log 3}{\log 2}\right)^2$$

$$\Rightarrow y=-\frac{\log 3}{\log 2}$$

$$\Rightarrow y=\frac{\log(1/3)}{\log 2}=\log_2\frac{1}{3}$$

15. In a group of people, 28% of the members are young while the rest are old. If 65% of the members are literates, and 25% of the literates are young, then the percentage of old people among the illiterates is nearest to

(a) 66
(b) 59
(c) 55
(d) 62

Solution **(a)**

Let total number of persons be 100. Then, 28 are young and 72 are old people. Total 65 members are literates. Of these 65, 25% are young.

So, number of young literates

= 25% of 65 = 16.25

So, number of old literates

= 65 – 16.25 = 48.75

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So, number of old illiterates

= 72 – 48.75 = 23.25

Total number of illiterates = 35

So, percentage of old people among the illiterates

$$= \frac{23.25}{35} \times 100 = 66.43\% \approx 66\%$$

16. Two persons are walking beside a railway track at respective speeds of 2 and 4 km per hour in the same direction. A train came from behind them and crossed them in 90 and 100 seconds, respectively. The time, in seconds, taken by the train to cross an electric post is nearest to

(a) 87

(b) 82

(c) 75

(d) 78

Solution **(b)**

Let the length and speed of train be x and v respectively. Then,

$$\frac{x}{v-2} = 90 \Rightarrow x = 90(v-2) \quad (1)$$

$$\frac{x}{v-4} = 100 \Rightarrow x = 100(v-4) \quad (2)$$

From Eqs. (1) and (2), we have

$90v – 180 = 100v – 400 \Rightarrow v = 22$

Therefore, $x = 1800$

Therefore, time taken to cross a pole

$$= \frac{x}{v} = \frac{1800}{22} = 81.81 \approx 82 \text{ seconds}$$

17. On a rectangular metal sheet of area 135 sq in, a circle is painted such that the circle touches two opposite sides. If the area of the sheet left unpainted is two-thirds of the painted area then the perimeter of the rectangle in inches is

(a) $3\sqrt{\pi}\left(5+\frac{12}{\pi}\right)$

(b) $5\sqrt{\pi}\left(3+\frac{9}{\pi}\right)$

(c) $4\sqrt{\pi}\left(3+\frac{9}{\pi}\right)$

(d) $3\sqrt{\pi}\left(\frac{5}{2}+\frac{6}{\pi}\right)$

Solution **(a)**

Let the length and breadth of the rectangle be x and y. Then,

Radius of the circle $= \frac{y}{2}$

Area of unpainted = 2/3rd of P. So,

$$P + \frac{2}{3}P = 135$$

$$\Rightarrow P = \frac{3}{5} \times 135 = 81$$

Area of circle $= \pi\left(\frac{y}{2}\right)^2 = 81$

$$\Rightarrow y = \frac{18}{\sqrt{\pi}} = \frac{18\sqrt{\pi}}{\pi}$$

$$\Rightarrow x = \frac{135\sqrt{\pi}}{18} = \frac{15\sqrt{\pi}}{2}$$

Therefore, perimeter

$$= 2(x + y) = 15\sqrt{\pi} + \frac{36\sqrt{\pi}}{\pi} = 3\sqrt{\pi}\left(5+\frac{12}{\pi}\right)$$

18. If $f(5 + x) = f(5 – x)$ for every real x, and $f(x) = 0$ has four distinct real roots, then the sum of these roots is

(a) 40

(b) 10

(c) 20

(d) 0

Solution **(c)**

The graph of $y = f(x)$ is symmetrical about $x = 5$. Since $f(x) = 0$ has 4 distinct real roots, two roots will be more than 5 and other two roots are less than 5.

Since the graph is symmetrical about $x = 5$, the four roots can be taken as $5 – a$, $5 – b$, $5 + a$ and $5 + b$. So, the sum of roots = 20.

19. A straight road connects points A and B. Car 1 travels from A to B and Car 2 travels from B to A, both leaving at the same time. After meeting each other, they take 45 minutes and 20 minutes, respectively, to complete their journeys. If Car 1 travels at the speed of 60 km/hr, then the speed of Car 2, in km/hr, is

(a) 80

(b) 100

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(c) 70

(d) 90

Solution **(d)**

Ratio of speeds, $V_A : V_B = \sqrt{T_B} : \sqrt{T_A}$

$\Rightarrow 60 : V_B = \sqrt{20} : \sqrt{45}$

$\Rightarrow \frac{60}{V_B} = \sqrt{\frac{4}{9}} = \frac{2}{3}$ $V_B = 90$ km/hr

20. Leaving home at the same time, Amal reaches office at 10:15 am if he travels at 8 km/hr, and at 9:40 am if he travels at 15 km/hr. Leaving home at 9:10 am, at what speed, in km/hr, must he travel so as to reach office exactly at 10 am?

(a) 14

(b) 11

(c) 13

(d) 12

Solution **(d)**

Speed becomes 15/8 times, so time taken becomes 8/15 times, which is 7/15 times less.

So, 7/15th of the usual travel time = Time saved in going at higher speed = 35 minutes

So, usual travel time = 75 minutes

So, when he travels at 8 km/hr, he reaches office in 75 min and so he starts from home at 9:00 am.

To reach at 10 am, starting at 9:10 am, travel time = 50 minutes

So, ratio of times taken $= \frac{50}{75} = \frac{2}{3}$

So, ratio of speeds $= \frac{3}{2}$ times usual speed

$= \frac{3}{2} \times 8 = 12$ km/hr

21. If a, b and c are positive integers such that $ab = 432$, $bc = 96$ and $c < 9$, then the smallest possible value of $a + b + c$ is

(a) 46

(b) 56

(c) 49

(d) 59

Solution **(a)**

$c < 9$ and c is a factor of 96, so it can be 1, 2, 3, 4, 6, 8.

If $c = 8$, then $b = 12$ and so $a = 36$. Therefore, $a + b + c = 56$

If $c = 6$, then $b = 16$ and so $a = 27$. Therefore, $a + b + c = 49$

If $c = 4$, then $b = 24$ and so $a = 18$. Therefore, $a + b + c = 46$

22. The number of real-valued solutions of the equation $2^x + 2^{-x} = 2 - (x - 2)^2$ is

(a) 0

(b) 2

(c) 1

(d) Infinite

Solution **(a)**

The minimum value of $2^x + 2^{-x}$ is 2 (when x = 1). The maximum value of $2 - (x - 2)^2$ is 2 at $x = 2$. And so for other values of x, the value of the expression $2 - (x - 2)^2$ is less than 2.

So, the given equation has no real-values solution.

23. A solid right circular cone of height 27 cm is cut into two pieces along *a* plane parallel to its base at a height of 18 cm from the base. If the difference in volume of the two pieces is 225 cc, the volume, in cc, of the original cone is

(a) 256

(b) 264

(c) 232

(d) 243

Solution **(d)**

The height of the smaller cone = 9 cm

Ratio of volumes of the smaller cone to original cone

$= \left(\frac{9}{27}\right)^3 = \frac{1}{27}$

If volume of smaller cone is V, the volume of the remaining piece = $26V$

Difference in volume

$= 26V - V = 225 \Rightarrow V = 9$

So, volume of original cone

$= 27V = 27 \times 9 = 243$ cc

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24. If $x = (4096)^{7+4\sqrt{3}}$, then which of the following equals 64?

(a) $\dfrac{x^7}{x^{4\sqrt{3}}}$

(b) $\dfrac{x^{\frac{7}{2}}}{x^{\frac{4}{\sqrt{3}}}}$

(c) $\dfrac{x^7}{x^{2\sqrt{3}}}$

(d) $\dfrac{x^{\frac{7}{2}}}{x^{2\sqrt{3}}}$

***Solution* (d)**

Note that $4096 = 64^2$. So,

$$x = (64)^{2(7+4\sqrt{3})} = (64)^{(14+8\sqrt{3})}$$

$$\Rightarrow 64 = x^{[1/(14+8\sqrt{3})]}$$

Now, power of x is

$$\frac{1}{14+8\sqrt{3}} = \frac{14-8\sqrt{3}}{4} = \frac{7}{2} - 2\sqrt{3}$$

So, the correct option is (d).

25. Veeru invested ₹ 10,000 at 5% simple annual interest, and exactly after two years, Joy invested ₹ 80,00 at 10% simple annual interest. How many years after Veeru's investment, will their balances, i.e., principal plus accumulated interest, be equal?

***Solution* (12)**

Veeru's balance after t years

$$= 10000\left(1 + \frac{5t}{100}\right)$$

Joy's balance after $(t - 2)$ years

$$= 8000\left[1 + \frac{10(t-2)}{100}\right]$$

As per the question, we have

$$10000\left(1 + \frac{5t}{100}\right) = 8000\left[1 + \frac{10(t-2)}{100}\right]$$

$$\Rightarrow 5\left(1 + \frac{5t}{100}\right) = 4\left[1 + \frac{10(t-2)}{100}\right]$$

$$\Rightarrow 5 + \frac{t}{4} = 4 + \frac{2(t-2)}{5}$$

$$\Rightarrow 1 = \frac{2(t-2)}{5} - \frac{t}{4}$$

$$\Rightarrow 1 = \frac{2t}{5} - \frac{t}{4} - \frac{4}{5}$$

$$\Rightarrow \frac{9}{5} = \frac{3t}{20} \Rightarrow t = 12 \text{ years}$$

26. The mean of all 4-digit even natural numbers of the form '*aabb*', where $a > 0$, is

(a) 4864

(b) 5050

(c) 4466

(d) 5544

***Solution* (d)**

The numbers of the given form are 1100, 1122, 1144, 1166, 1188, 2200, 2244, …, 9988.

The mean of all these numbers will be same as average of smallest and largest number $= \dfrac{1100 + 9988}{2} = 5544$

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