CAT 2020 Solved Paper Afternoon Slot

SECTION: QUANTITATIVE ABILITY

1. Students in a college have to choose at least two subjects from Chemistry, Mathematics and Physics. The number of students choosing all three subjects is 18, choosing Mathematics as one of their subjects is 23 and choosing Physics as one of their subjects is 25. The smallest possible number of students who could choose Chemistry as one of their subjects is

(a) 19
(b) 21
(c) 22
(d) 20

Solution **(d)**

Let number of students choosing Chemistry and Mathematics only be x, Mathematics and Physics only be y and Physics and Chemistry only be z. Also, let the union of all be T and those choosing Chemistry be C. Then,

$23 = x + y + 18$ (1)

$25 = y + z + 18$ (2)

$C = x + z + 18$ (3)

Subtracting Eq. (1) from Eq. (2), we get

$2 = z - x$

For minimum value of C, we should have minimum value of x and z. For that, $z = 2$ and $x = 0$ is one possibility. So, the minimum value of $C = 0 + 2 + 18 = 20$.

2. In a group of 10 students, the mean of the lowest 9 scores is 42 while the mean of the highest 9 scores is 47. For the entire group of 10 students, the maximum possible mean exceeds the minimum possible mean by

(a) 3
(b) 5
(c) 4
(d) 6

Solution **(c)**

If the score are $x_1, x_2, x_3, \ldots, x_{10}$, then

$x_1 + x_2 + x_3 + \ldots + x_9 = 9 \times 42 = 378$

$x_2 + x_3 + x_4 + \ldots + x_{10} = 9 \times 47 = 423$

The minimum possible value of x_{10} is 47 because of the average of highest 9 scores is 47. In that case, values of $x_2, x_3, x_4, \ldots, x_9$ are all equal to 47. So,

$x_1 = 378 - (x_2 + x_3 + \ldots + x_9)$

$= 378 - (47 \times 8) = 2$, the lowest value of x_1.

Similarly, the highest value of $x_1 = 42$ and in that case, values of $x_2, x_3, x_4, \ldots, x_9$ are all equal to 42. So,

$x_{10} = 423 - (x_2 + x_3 + \ldots + x_9)$

$= 423 - (42 \times 8) = 87$, the lowest value of x_1.

The difference of maximum and minimum mean of all 10 students

$$= \frac{423+42}{10} - \frac{378+47}{10} = 4$$

3. A and B are two points on a straight line. Ram runs from A to B while Rahim runs from B to A. After crossing each other. Ram and Rahim reach their destinations in one minute and four minutes, respectively. If they start at the same time, then the ratio of Ram's speed to Rahim's speed is

(a) $\sqrt{2}$
(b) 2

(c) $2\sqrt{2}$

(d) $\frac{1}{2}$

***Solution* (b)**

Ratio of speeds,

$$V_{\text{ram}} : V_{\text{rahim}} = \sqrt{\frac{T_{Rahim}}{T_{Ram}}} = \sqrt{\frac{4}{1}} = 2$$

4. Anil buys 12 toys and labels each with the same selling price. He sells 8 toys initially at 20% discount on the labeled price. Then he sells the remaining 4 toys at an additional 25% discount on the discounted price. Thus, he gets a total of ₹ 2112, and makes a 10% profit. With no discounts, his percentage of profit would have been

(a) 54

(b) 55

(c) 50

(d) 60

***Solution* (c)**

Since he gets a total of ₹ 2112 with 10% profit, his cost price for all the 12 toys = 2112/1.1 = 1920 and so cost price per toy = 1920 / 12 = ₹ 160.

Let label price of each toy be ₹ x. Then,

SP of 8 toys = 0.8x each

And, SP of remaining 4 toys = (0.75)(0.8x)

Total money received on the sale

= 8 × 0.8x + 4 × 0.75(0.8x) = 8.8x, which is equal to 2112.

So, x = 240

Therefore, with no discounts, percentage profit would have been

$$= \frac{240 - 160}{160} \times 100 = 50\%$$

5. In May, John bought the same amount of rice and the same amount of wheat as he had bought in April, but spent ₹ 150 more due to price increase of rice and wheat by 20% and 12%, respectively. If John had spent ₹ 450 on rice in April, then how much did he spend on wheat in May?

(a) ₹ 590

(b) ₹ 560

(c) ₹ 570

(d) ₹ 580

***Solution* (b)**

Let the expenditure on wheat in April be x. Then,

450(1.2) + x(1.12) – (450 + x) = 150

$\Rightarrow$ x = 500

Therefore, amount spent on wheat in May

= 1.12(500) = 560

6. If x and y are positive real numbers satisfying $x + y = 102$, then the minimum possible value of $2601\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)$ is

***Solution* (2704)**

Given expression is

$$2601\left[1+\frac{1}{x}+\frac{1}{y}+\frac{1}{xy}\right] = 2601\left[1+\frac{x+y}{xy}+\frac{1}{xy}\right]$$

$$= 2601\left[1+\frac{102}{xy}+\frac{1}{xy}\right]$$

$$= 2601\left[1+\frac{103}{xy}\right]$$

For minimum value of this, xy must be maximized, which occurs when $x = y = 51$. So, the given expression

$$= 2601\left[1+\frac{103}{(51)(51)}\right] = 2704$$

7. From an interior point of an equilateral triangle, perpendiculars are drawn on all three sides. The sum of the lengths of the three perpendiculars is s. Then the area of the triangle is

(a) $\frac{\sqrt{3}s^2}{2}$

(b) $\frac{s^2}{\sqrt{3}}$

(c) $\frac{s^2}{2\sqrt{3}}$

(d) $\frac{2s^2}{\sqrt{3}}$

Solution **(b)**

Let the length of each side of the triangle be a. Also, let the length of the three perpendiculars be x, y and z, so that $x + y + z = s$. Join that interior point with the 3 vertices.

Area of triangle = Sum of the areas of the 3 triangles formed inside

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = \frac{1}{2}\times ax + \frac{1}{2}\times ay + \frac{1}{2}\times az$$

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = \frac{1}{2}\times as$$

$$\Rightarrow a = \frac{2}{\sqrt{3}}S$$

Area of the triangle

$$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\left(\frac{2}{\sqrt{3}}S\right)^2 = \frac{s^2}{\sqrt{3}}$$

8. John takes twice as much time as Jack to finish a job. Jack and Jim together take one-thirds of the time to finish the job than John takes working alone. Moreover, in order to finish the job, John takes three days more than that taken by three of them working together. In how many days will Jim finish the job working alone?

Solution **(4)**

Let time taken by Jack be x. Then, the time taken by John will be $2x$. Let Jim takes y days to finish the job.

Now, time taken by Jack and Jim together

$= \frac{xy}{x+y}$ which is 1/3rd of time taken by John, which means

$$\frac{xy}{x+y} = \frac{1}{3}\times 2x \Rightarrow 3y = 2x + 2y \Rightarrow y = 2x$$

Time taken by three of them to complete the work

$$= \frac{1}{\frac{1}{x}+\frac{1}{2x}+\frac{1}{2x}} = \frac{2x}{4} = \frac{x}{2}$$

Since John takes 3 days more than this, we get

$$2x - \frac{x}{2} = 3 \Rightarrow x = 2$$

So, time taken by Jim to finish working alone $= 2x = 4$ days

9. Let C_1 and C_2 be concentric circles such that the diameter of C_1 is 2 cm longer than that of C_2. If a chord of C_1 has length 6 cm and is a tangent to C_2. then the diameter, in cm of C_1 is

Solution **(10)**

Let radius of C_2 be r. Then,

Radius of $C_1 = r + 1$

The contact point of the chord with C_1 bisects the chord. So,

$(r + 1)^2 = r^2 + 3^2 \Rightarrow r = 4$

So, diameter of $C_1 = 2(r + 1) = 2(4 + 1) = 10$

10. The number of pairs of integers (x, y) satisfying $x \geq y \geq -20$ and $2x + 5y = 99$ is

Solution **(17)**

Since $2x$ is even, $5y$ has to be odd so that sum is odd, that is, 99. For this, y has to be odd.

Suppose $y = -19$, then $x = 97$

Similarly,

$(x, y) = (97, -19), (92, -17), (87, -15), (82, -13), (77, -11), \ldots, (17, 13)$, which are 17 in counting.

11. How many 4-digit numbers, each greater than 1000 and each having all four digits distinct, are there with 7 coming before 3?

Solution **(315)**

Consider two cases: One where there is no zero, and one with only one zero.

Case 1

In case there is no zero, then two of the digits have to be 3 and 7, and rest of the two digits can be chosen in ${}^7C_2 = 21$ ways. These 4 digits can be arranged in 4! ways, but since 7 must come before 3, the total number of ways = 21 × 4!/2! = 252.

Case 2

If one of the digits is 0, then 3 digits in the number will be 0, 3 and 7. The fourth one can be chosen in 7 ways. The digit 0 can be placed in any 3 of the locations (except the leftmost locatio*n*). In the remaining 3 locations, the 3 digits can be placed in 3! ways. Since 7 comes before 3, the total number of ways = 7 × 3 × 3!/2! = 63 ways.

Total number of ways = 252 + 63 = 315

12. If x and y are non-negative integers such that $x + 9 = z$, $y + 1 = z$ and $x + y < z + 5$, then the maximum possible value of $2x + y$ equals

***Solution* (23)**

$x = z - 9$, $y = z - 1$

So, the given inequality becomes

$(z - 9) + (z - 1) < z + 5 \Rightarrow z < 15$

So, the maximum value of $z = 14$ (as x and y are integers, so is z).

Also,

$2x + y = 2(z - 9) + (z - 1)$

$= 3z - 19 = 3 \times 14 - 19 = 23$

13. The number of integers that satisfy the equality $(x^2 - 5x + 7)^{x+1} = 1$ is

(a) 4

(b) 2

(c) 3

(d) 5

***Solution* (c)**

For the given equation to be valid

Case 1

$x^2 - 5x + 7 = 1$

$\Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$

Case 2

$x + 1 = 0 \Rightarrow x = -1$

Case 3

$x^2 - 5x + 7 = -1$ and $x + 1 = 0$, which has no common solution.

So, in all, there are only 3 different integral solutions.

14. Aron bought some pencils and sharpeners. Spending the same amount of money as Aron, Aditya bought twice as many pencils and 10 less sharpeners. If the cost of one sharpener is ₹ 2 more than the cost of a pencil, then the minimum possible number of pencils bought by Aron and Aditya together is

(a) 36

(b) 33

(c) 30

(d) 27

***Solution* (b)**

Let cost of one pencil be ₹ x. Then,

Cost of one sharpener = ₹ $(x + 2)$

If number of pencils and sharpeners bought by Aron is p and s, then

$px + s(x + 2) = 2px + (s - 10)(x + 2)$

$\Rightarrow px - 10x - 20 = 0 \Rightarrow x = \dfrac{20}{p - 10}$

Minimum value of p is 11, and so the minimum number of pencils bought by Aron and Aditya together = $p + 2p = 3p = 33$.

15. For real x, the maximum possible value of $\dfrac{x}{\sqrt{1+x^4}}$ is

(a) $\dfrac{1}{\sqrt{3}}$

(b) $\dfrac{1}{\sqrt{2}}$

(c) 1

(d) $\dfrac{1}{2}$

***Solution* (b)**

Given expression can be written as

$$\frac{x}{\sqrt{1+x^4}} = \frac{1}{\sqrt{\frac{1}{x^2} + x^2}}$$

$$= \frac{1}{\sqrt{\left(\frac{1}{x} + x\right)^2 - 2}}$$

The minimum value of $\left(x + \dfrac{1}{x}\right)^2 = 2^2 = 4$

So, maximum value of the given expression

$= \dfrac{1}{\sqrt{2^2 - 2}} = \dfrac{1}{\sqrt{2}}$

16. The distance from B to C is thrice that from A to B. Two trains travel from A to C via B. The speed of train 2 is double that of train 1 while traveling from A to B and their speeds are interchanged while traveling from B to C. The ratio of the time taken by train 1 to that taken by train 2 in travelling from A to C is

(a) 5 : 7

(b) 4 : 1

(c) 7 : 5

(d) 1 : 4

Solution **(a)**

Let distances from A to B and B to C be x and $3x$, respectively. Also, let speeds of train 1 and train 2 while going from A to B be v and $2v$, respectively.

Then, time taken by the train 1 from A to C

$$t_1 = \frac{x}{v} + \frac{3x}{2v} = \frac{5x}{2v}$$

Also, time taken by train 2 from A to C

$$t_2 = \frac{x}{2v} + \frac{3x}{v} = \frac{7x}{2v}$$

Required ratio = 5 : 7

17. In a car race, car A beats car B by 45 km, car B beats car C by 50 km, and car A beats car C by 90 km. The distance (in km) over which the race has been conducted is

(a) 475

(b) 450

(c) 500

(d) 550

Solution **(b)**

Let length of the race be x. Then

Ratio of speeds of A to B = $x : (x - 45)$

Ratio of speeds of B to C = $x : (x - 50)$

Ratio of speeds of and A to C = $x : (x - 90)$

$$A : C = \frac{A}{B} \times \frac{B}{C} = \left(\frac{x}{x-45}\right) \times \left(\frac{x}{x-50}\right)$$

So,

$$\frac{x}{x-90} = \frac{x^2}{(x-45)(x-50)}$$

$\Rightarrow x(x - 90) = (x - 45)(x - 50)$

$\Rightarrow x^2 - 90x = x^2 - 95x + 2250 \Rightarrow x = 450$ km

Alternatively,

When A reaches the finish line, B is 45 km behind, A and C is 90 km behind A. This means, C is 45 km behind B. Now, when B travels remaining 45 km, then C would travel 40 km so that gap between B and C would become 50 km. So, ratio of speeds of B to C = 45 : 40 = 9 : 8.

When B travels 9 km, then gap between B and C = 1 km. To have B beat C by 50 km, length of race = 9 × 50 = 450 km.

18. Let the m-th and n-th terms of a geometric progression be $\frac{3}{4}$ and 12, respectively, where $m < n$. If the common ratio of the progression is an integer r, then the smallest possible value of $r + n - m$ is

(a) 6

(b) −4

(c) −2

(d) 2

Solution **(c)**

We can say that,

$$t_m = ar^{m-1} = \frac{3}{4} \quad (1)$$

$$t_n = ar^{n-1} = 12 \quad (2)$$

Dividing Eq. (1) by eq. (2), we get

$$\frac{t_n}{t_m} = r^{n-m} = 16$$

So, r can be 4 or –4 with $n - m = 2$.

So, smallest possible value of r + $n - m$

= –4 + 2 = –2

19. A sum of money is split among Amal, Sunil and Mita so that the ratio of the shares of Amal and Sunil is 3:2, while the ratio of the shares of Sunil and Mita is 4:5. If the difference between the largest and the smallest of these three shares is ₹ 400, then Sunil's share, in rupees, is

Solution **(800)**

Ratio of Amal, Sunil and Mita = 6 : 4 : 5

Since difference between the largest and the smallest of these three shares = 400

$6x - 4x = 400 \Rightarrow x = 200$

So, Sunil's share = $4x$ = 800

20. Let $f(x) = x^2 + ax + b$ and $g(x) = f(x + 1) - f(x - 1)$. If $f(x) \geq 0$ for all real x, and $g(20) = 72$, then the smallest possible value of b is

(a) 4

(b) 1

(c) 0

(d) 16

***Solution* (a)**

Since $f(x) \geq 0$, $x^2 + ax + b \geq 0$ which means $a^2 - 4b \leq 0$ or $4b \geq a^2$

$g(x) = [(x + 1)^2 + a(x + 1) + b] - [(x - 1)^2 + a(x - 1) + b] = 4x + 2a$

$g(20) = 72$

$\Rightarrow 4(20) + 2a = 72 \Rightarrow a = -4$

Therefore,

$4b \geq a^2 \Rightarrow 4b \geq (-4)^2 = 16$

So, the smallest possible value of $b = \dfrac{16}{4} = 4$

21. For the same principal amount, the compound interest for two years at 5% per annum exceeds the simple interest for three years at 3% per annum by ₹ 1125. Then the principal amount in rupees is

***Solution* (90,000)**

Let x be the principal amount. Then,

$$[x(1.05)^2 - x] - \frac{x \times 3 \times 3}{100} = 1125$$

$\Rightarrow 0.1025x - 0.09x = 1125$

$\Rightarrow x =$ ₹ 90,000

22. Two circular tracks T_1 and T_2 of radii 100 m and 20 m, respectively touch at a point A. Starting from A at the same time, Ram and Rahim are walking on track T_1 and track T_2 at speeds 15 km/hr and 5 km/hr respectively. The number of full rounds that Ram will make before he meets Rahim again for the first time is

(a) 3

(b) 2

(c) 4

(d) 5

***Solution* (a)**

Ram is at A again after every multiple of (100/15) = 20/3, without caring for units; Rahim is at A after every multiple of (20/5) = 4. Both will be at A after a time which is LCM of 20/3 and 4 = 20, for which number of rounds made by Ram will be 3 rounds.

23. The sum of the perimeters of an equilateral triangle and a rectangle is 90 cm. The area T, of the triangle and the area, R, of the rectangle, both in sq. cm, satisfy the relationship $R = T^2$. If the sides of the rectangle are in the ratio 1:3, then the length, in cm, of the longer side of the rectangle, is

(a) 27

(b) 18

(c) 24

(d) 21

***Solution* (a)**

Let the length of side of the triangle be a, and length and breadth of rectangle be $3x$ and x. Then,

$3a + 2(3x + x) = 90 \Rightarrow 3a + 8x = 90$

$R = 3x^2$ and $T^2 = \dfrac{3a^4}{16}$

So,

$3x^2 = \dfrac{3a^4}{16} \Rightarrow x = \dfrac{a^2}{4} \Rightarrow 8x = 2a^2$

$3a + 2a^2 = 90$

$\Rightarrow 2a^2 + 3a - 90 = 0$

$\Rightarrow a = 6$ or $x = 9$

Therefore, longer side = $3x = 27$

24. The value of $\log_a\left(\dfrac{a}{b}\right) + \log_b\left(\dfrac{b}{a}\right)$, for $1 < a \leq b$ cannot be equal to

(a) 0

(b) −0.5

(c) 1

(d) −1

***Solution* (c)**

Given expression is

$$\log_a\left(\frac{a}{b}\right) + \log_b\left(\frac{b}{a}\right) = \log_a a - \log_a b + \log_b b - \log_b a$$
$$= 2 - (\log_a b + \log_b a)$$

Now, $\log_b a$ is reciprocal of $\log_a b$ and both are positive because a and b both are more than 1. So, minimum value of $(\log_a b + \log_b a)$ is 2.

Therefore, the value of the given expression is either 0 or less than 0. So, the given expression cannot be equal to 1.

25. In how many ways can a pair of integers (x, a) be chosen such that $x^2 - 2|x| + |a - 2| = 0$?

(a) 7

(b) 4

(c) 5

(d) 6

Solution **(a)**

$|a - 2| = 2|x| - x^2$

So,

$2|x| - x^2 \geq 0 \Rightarrow x^2 - 2|x| \leq 0$

If $x \geq 0$, then

$x^2 - 2x \leq 0$

$\Rightarrow x(x - 2) \leq 0 \Rightarrow 0 \leq x \leq 2$

$\Rightarrow x = 0$, 1 and 2

For $x = 0$, $a = 2$; if $x = 1$, $a = 3$ or 1; if $x = 2$, $a = 2$.

If $x < 0$, then

$x^2 + 2x \leq 0$

$\Rightarrow x(x + 2) \leq 0 \Rightarrow -2 \leq x \leq 0$

So, $x = -1$ and -2

For $x = -1$, $a = 3$ or 1; if $x = -2$, $a = 2$.

So, number of pairs of integers (x, a) is 7.

26. Let C be a circle of radius 5 meters having center at O. Let PQ be a chord of C that passes through points A and B where A is located 4 meters north of O and B is located 3 meters east of O. Then, the length of PQ, in meters, is nearest to

(a) 8.8

(b) 7.8

(c) 7.2

(d) 6.6

Solution **(a)**

Length of AB = $\sqrt{3^2 + 4^2}$ = 5 metres

Length of perpendicular from O on AB

$= \dfrac{3 \times 4}{5} = 2.4$

If the foot of perpendicular from O on AB be C, then C is the midpoint of PQ. Therefore,

CP = $\sqrt{5^2 - 2.4^2} = \sqrt{19.24} = 4.386$

So, PQ = 2 × 4.386 = 8.772 ≈ 8.8 metres