CAT 2020 Solved Paper

Evening Slot

SECTION: QUANTITATIVE ABILITY

1. If $f(x + y) = f(x)\,f(y)$ and $f(5) = 4$, then $f(10) - f(-10)$ is equal to

(a) 14.0625

(b) 3

(c) 15.9375

(d) 0

***Solution* (c)**

From the given condition, we can say that

$f(x) = a^x$

Since $f(5) = 4$, so $a^5 = 4$

$f(10) - f(-10) = a^{10} - a^{-10}$

$= 4^2 - 4^{-2} = 15.9375$

2. Two alcohol solutions, A and B, are mixed in the proportion 1:3 by volume. The volume of the mixture is then doubled by adding solution A such that the resulting mixture has 72% alcohol. If solution A has 60% alcohol, then the percentage of alcohol in solution B is

(a) 1.92%

(b) 2.90%

(c) 3.94%

(d) 4.89%

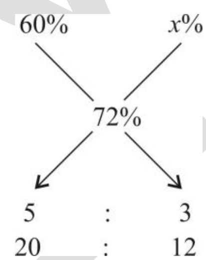
***Solution* (a)**

If mixture has 1 unit of A and 3 units of B, total volume = 4 units

After adding A, total volume = 8 units

So, 5 units of A is there in the mixture, which means ratio of volumes of A to B in the resultant mixture = 5 : 3

Using alligations, we get



Therefore, $x = 92\%$

3. A batsman played $n + 2$ innings and got out on all occasions. His average score in these $n + 2$ innings was 29 runs and he scored 38 and 15 runs in the last two innings. The batsman scored less than 38 runs in each of the first n innings. In these n innings, his average score was 30 runs and lowest score was x runs. The smallest possible value of x is

(a) 1

(b) 3

(c) 4

(d) 2

***Solution* (d)**

Total score in $n + 2$ innings = $29(n + 2)$

Average score in n innings = 30. So,

$29(n + 2) = 30n + 38 + 15 \Rightarrow n = 5$

So, total score in these n innings

$= 30n = 30 \times 5 = 150$

Smallest value of x

$= 150 - (37 + 37 + 37 + 37) = 2$

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4. Let m and n be positive integers, if $x^2 + mx + 2n = 0$ and $x^2 + 2nx + m = 0$ have real roots, then the smallest possible value of $m + n$ is

(a) 7

(b) 5

(c) 6

(d) 8

***Solution* (c)**

$m^2 - 8n \geq 0$ and $4n^2 - 4m \geq 0$

$\Rightarrow m^2 \geq 8n$ and $n^2 \geq m$

$m^2 \geq 8n \Rightarrow m^4 \geq 64n^2$ which is $\geq 64m$. So, $m^4 \geq 64m$

So, the minimum positive integral value of m is 4 and so $n^2 \geq m = 4$.

So, the minimum value of $n = 2$

So, the minimum value of $m + n = 6$

5. A man buys 35 kg of sugar and sets a marked price in order to make a 20% profit. He sells 5 kg at this price, and 15 kg at a 10% discount. Accidentally, 3 kg of sugar is wasted. He sells the remaining sugar by raising the marked price by p percent so as to make an overall profit of 15%. Then p is nearest to

(a) 25

(b) 22

(c) 31

(d) 35

***Solution* (a)**

Let cost price of 35 kg of sugar be ₹ x. Then,

$5 \times 1.2x + 15 \times (1.2x)(0.9) + (15 - 3) \times (1.2x)\left(1+\frac{p}{100}\right) = 35(1.15x)$

$\Rightarrow 6 + 16.2 + 14.4\left(1+\frac{p}{100}\right) = 40.25$

$\Rightarrow p = 25.34\% \approx 25\%$

6. Anil, Sunil, and Ravi run along a circular path of length 3 km, starting from the same point at the same time, and going in the clockwise direction. If they run at speeds of 15 km/hr, 10 km/hr, and 8 km/hr, respectively, how much distance in km will Ravi have run when Anil and Sunil meet again for the first time at the starting point?

(a) 4.6

(b) 5.2

(c) 4.8

(d) 4.2

***Solution* (c)**

Time taken for Anil to be at starting point $= \frac{3}{15}$ hours = 12 minutes

Time taken for Sunil to be at starting point $= \frac{3}{10}$ = 18 minutes

Time taken for both of them to meet at starting point

= LCM of 12 and 18 = 36 minutes

Distance travelled by Ravi in 36 minutes

$= 8 \times \frac{36}{60} = 4.8$ km

7. If $x_1 = -1$ and $x_m = x_{m+1} + (m + 1)$ for every positive integer m, then x_{100} equals

(a) −5051

(b) −5151

(c) −5050

(d) −5150

***Solution* (c)**

$x_2 = x_1 - (1 + 1) = -1 - (2) = -3$

$x_3 = x_2 - (2 + 1) = -3 - (3) = -6$

$x_4 = x_3 - (3 + 1) = -6 - (4) = -10$, and so on.

The series becomes –1, –3, –6, –10, … which is same as –(1), –(1 + 2), –(1 + 2 + 3), –(1 + 2 + 3 + 4), …

So,

$x_{100} = -1 - 2 - 3 - \ldots$ 100 terms $= -\frac{100\times 101}{2} = -5050$

8. How many of the integers 1, 2, ..., 120, are divisible by none of 2, 5 and 7?

(a) 42

(b) 40

(c) 41

(d) 43

***Solution* (c)**

Number of integers divisible by 2 = 60

Number of integers divisible by 5 = 24

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Number of integers divisible by 7 = 17

Number of integers divisible by 2 and 5, that is, divisible by 10 = 12

Number of integers divisible by 2 and 7, or 14 = 8

Number of integers divisible by 5 and 7 = 3

Number of integers divisible by 2, 5 and 7 = 1

So, number of integers divisible by either 2, 5 or 7

= 60 + 24 + 17 – (12 + 8 + 3) + 1 = 79

Therefore, number of integers divisible by none of 2, 5 and 7 = 120 – 79 = 41

9. The vertices of a triangle are (0, 0), (4, 0) and (3, 9). The area of the circle passing through these three points is

(a) $\frac{123\pi}{7}$

(b) $\frac{205\pi}{9}$

(c) $\frac{14\pi}{3}$

(d) $\frac{12\pi}{5}$

Solution **(b)**

Let vertices of the triangle be A(0, 0), B(4, 0) and C(3, 9). Then,

Midpoint of AB = (2, 0)

Equation of perpendicular bisector of AB

$x = 2$

Midpoint of AC = (1.5, 4.5)

Slope of AC $= \frac{9}{3} = 3$

Slope of perpendicular bisector of AC $= -\frac{1}{3}$

Therefore, equation of perpendicular bisector of AC is

$y - 4.5 = -\frac{1}{3}(x - 1.5)$

$\Rightarrow 3y - 13.5 = -x + 1.5$

$\Rightarrow 3y = -x + 15$

The point of intersection of perpendicular bisectors of AB and AC = (2, 13/3)

Radius of circle, R

= Distance between circumcenter and A

$= \sqrt{2^2 + \left(\frac{13}{3}\right)^2} = \sqrt{\frac{205}{9}}$

Therefore, area of circle = $\pi R^2 = \frac{205\pi}{9}$

10. How many pairs (a, b) of positive integers are there such that $a \le b$ and $ab = 4^{2017}$?

(a) 2018

(b) 2017

(c) 2020

(d) 2019

Solution **(a)**

The number of ways in which $4^{2017} = 2^{4034}$ can be expressed as product of 2 factors

$= \frac{4035+1}{2} = 2018$, which is same as required number of pairs.

11. A person invested a certain amount of money at 10% annual interest, compounded half-yearly. After one and a half years, the interest and principal together became ₹ 18522. The amount, in rupees, that the person had invested is

Solution **(16,000)**

Let principal be ₹ x. Then,

$x(1.05)^3 = 18522 \Rightarrow x = 16,000$

12. Let k be a constant. The equations $kx + y = 3$ and $4x + ky = 4$ have a unique solution if and only if

(a) $k = 2$

(b) $|k| \ne 2$

(c) $|k| = 2$

(d) $k \ne 2$

Solution **(b)**

For unique solution, we have

$\frac{k}{4} \ne \frac{1}{k}$ $k^2 \ne 4 \Rightarrow |k| \ne 2$

13. A contractor agreed to construct a 6 km road in 200 days. He employed 140 persons for the work. After 60 days, he realized that only 1.5 km road has been completed. How many additional people would he need to employ in order to finish the work exactly on time?

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***Solution* (40)**

Using the following formula, we have

$$\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$$

$$\Rightarrow \frac{140 \times 60}{1.5} = \frac{(140+x)\times(200-60)}{4.5}$$

$\Rightarrow$ 140 × 60 × 3 = (140 + x)(140)

$\Rightarrow$ x = 40 men

14. If $\log_a 30 = A$, $\log_a \frac{5}{3} = -B$ and $\log_2 a = \frac{1}{3}$, then $\log_3 a$ equals

(a) $\frac{A+B-3}{2}$

(b) $\frac{2}{A+B-3}$

(c) $\frac{A+B}{2} - 3$

(d) $\frac{2}{A+B} - 3$

***Solution* (b)**

$$\log_a 30 - \log_a \frac{5}{3} = A - (-B)$$

$$\Rightarrow \log_a 18 = A + B$$

Also, $\log_2 a = \frac{1}{3} \Rightarrow \log_a 2 = 3$

So, $A + B - 3$
$= \log_a 18 - \log_a 2 = \log_a 9 = \log_a 3^2 = 2\log_a 3$

Therefore,

$$\log_a 3 = \frac{A+B-3}{2}$$

$$\Rightarrow \log_3 a = \frac{2}{A+B-3}$$

15. If a, b, c are non-zero and $14^a = 36^b = 84^c$, then $6b\left(\frac{1}{c} - \frac{1}{a}\right)$ is equal to

***Solution* (3)**

Given that $14^a = 36^b = 84^c = x$, say. Then,

$a = \log_{14} x, b = \log_{36} x, c = \log_{84} x$. So,

$$\frac{1}{c} - \frac{1}{a} = \log_x 84 - \log_x 14$$

$$= \log_x 6$$

$$= \frac{1}{2}\log_x 36$$

$$= \frac{1}{2\log_{36} x} = \frac{1}{2b}$$

Therefore, $6b\left(\frac{1}{c} - \frac{1}{a}\right) = 3$

16. Vimla starts for office every day at 9 am and reaches exactly on time if she drives at her usual speed of 40 km/hr. She is late by 6 minutes if she drives at 35 km/hr. One day, she covers two-thirds of her distance to office in one-thirds of her usual time to reach office, and then stops for 8 minutes. The speed, in km/hr, at which she should drive the remaining distance to reach office exactly on time is

(a) 27

(b) 26

(c) 29

(d) 28

***Solution* (d)**

$40t = 35(t + 6) \Rightarrow t = 42$ minutes

Distance covered at 40 km/hr

$= 40 \times \frac{42}{60} = 28$ km

She covers 2/3rd of the distance, that is 56/3 km in 1/3rd of 42 minutes, that is 14 minutes.

After stopping for 8 min, she is left with 42 – 14 – 8 = 20 minutes to cover the remaining distance of 28/3 km.

So, required speed $= \frac{28/3}{20/60} =$ 28 km/hr

17. Let N, x and y be positive integers such that $N = x + y$, $2 < x < 10$ and $14 < y < 23$. If $N > 25$, then how many distinct values are possible for N?

***Solution* (6)**

The minimum possible value of N = 26 when x = 4 and y = 22

The maximum possible value of N

= 9 + 22 = 31

So, total 6 values of N are possible.

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18. A and B are two railway stations 90 km apart. A train leaves A at 9:00 am, heading towards B at a speed of 40 km/hr. Another train leaves B at 10:30 am, heading towards A at a speed of 20 km/hr. The trains meet each other at

(a) 11 :45 am

(b) 11 : 00 am

(c) 10 : 45 am

(d) 11 : 20 am

Solution **(b)**

By 10:30 am, the first train would have covered a distance of 40 × 1.5 = 60 km

The gap between the two trains now

= 90 – 60 = 30 km

The time taken for the two trains to meet

$= \dfrac{30}{40+20} = 0.5$ hours

So, the trains meet each other half an hour after 10:30 am, which is 11:00 am.

19. In the final examination, Bishnu scored 52% and Asha scored 64%. The marks obtained by Bishnu is 23 less, and that by Asha is 34 more than the marks obtained by Ramesh. The marks obtained by Geeta, who scored 84%, is

(a) 399

(b) 417

(c) 357

(d) 439

Solution **(a)**

If maximum marks of the exam is x and marks obtained by Ramesh is r, then

$0.52x + 23 = r = 0.64x - 34$

$\Rightarrow x = \dfrac{5700}{12} = 475$

Marks obtained by Geeta = 84% of x = 399

20. The area, in sq. units, enclosed by the lines $x = 2$, $y = |x - 2| + 4$, the x-axis and the y-axis is equal to

(a) 6

(b) 8

(c) 12

(d) 10

Solution **(d)**

The vertex of modulus graph is (2, 4) and it intersects y-axis at (0, 6). The enclosed shape is trapezium, whose lengths of parallel sides are 4 and 6, and distance between parallel lines is 2.

So, required area = $\dfrac{1}{2}(4+6)\times 2 = 10$

21. In a trapezium ABCD, AB is parallel to DC, BC is perpendicular to DC and $\angle BAD = 45°$. If DC = 5 cm, BC = 4 cm, then area of the trapezium in sq. cm is

Solution **(28)**

Let perpendicular from D on AB be P.

Then, DP = BC = 4 cm

Also, AP = 4, as $\angle BAD = 45°$

So, AB = AP + PB = AP + DC = 4 + 5 = 9 cm

Area of trapezium = $\dfrac{1}{2}(9+5)\times 4 = 28 \text{ cm}^2$

22. Dick is thrice as old as Tom and Harry is twice as old as Dick. If Dick's age is 1 year less than the average age of all three, then Harry's age, in years, is

Solution **(18)**

Given that

$D = 3T$, $H = 2D = 6T$, $D = \dfrac{T+D+H}{3} - 1$

$\Rightarrow 3T = \dfrac{T+3T+6T}{3} - 1$

$\Rightarrow T = 3$ and $H = 6T = 18$ years

23. How many integers in the set {100, 101, 102, …, 999} have at least one digit repeated?

Solution **(252)**

Total number of 3-digit numbers

= 9 × 10 × 10 = 900

Total number of 3-digit numbers with distinct digits = 9 × 9 × 8 = 648.

So, number of integers which have at least one digit repeated = 900 – 648 = 252

24. The points (2, 1) and (−3, −4) are opposite vertices of a parallelogram. If the other two vertices lie on the line $x + 9y + c = 0$, then c is

(a) 15

(b) 12

(c) 14

(d) 13

***Solution* (c)**

Midpoint of the given two vertices

$$=\left(\frac{2-3}{2},\frac{1-4}{2}\right)=\left(\frac{-1}{2},\frac{-3}{2}\right)$$

This point lies on the given line. So,

$x + 9y + c = 0$

$$\Rightarrow -\frac{1}{2}+9\left(-\frac{3}{2}\right)+c=0 \Rightarrow c = 14$$

25. $\dfrac{2\times4\times8\times16}{(\log_2 4)^2(\log_4 8)^3(\log_8 16)^4}$ equals

***Solution* (24)**

$\log_2 4 = 2$; $\log_4 8 = \dfrac{3}{2}$; $\log_8 16 = \dfrac{4}{3}$.

So, given expression = $\dfrac{2\times4\times8\times16}{2^2\times\left(\frac{3}{2}\right)^3\times\left(\frac{4}{3}\right)^4} = 24$

26. Let m and n be natural numbers such that n is even and $0.2<\dfrac{m}{20},\dfrac{n}{m},\dfrac{n}{11}<0.5.$ Then $m - 2n$ equals

(a) 4

(b) 2

(c) 3

(d) 1

***Solution* (d)**

$\dfrac{m}{20} > 0.2 \Rightarrow m > 4$

Also,

$\dfrac{m}{20} < 0.5 \Rightarrow m < 10$

So, values of m can be 5, 6, 7, 8 or 9.

Also,

$0.2 < \dfrac{n}{11} < 0.5 \Rightarrow 2.2 < n < 5.5$

Also, n is even. So, n can be 4 only.

Finally,

$0.2 < \dfrac{n}{m} < 0.5 \Rightarrow 0.2 < \dfrac{4}{m} < 0.5$

$\Rightarrow m < 20$ and $m > 8$

So, m can only be 9.

Therefore, $m - 2n = 9 - 2 \times 4 = 1$

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