SOLVED GATE
(CIVIL ENGINEERING SET-I) 2022

Engineering Mathematics and Technical Section

Q. No. 1–25 Multiple Choice Question (MCQ) Carry One Mark Each

1. Consider the following expression:

$$z = \sin(y + it) + \cos(y - it)$$

where z, y, and t are variables, and $i = \sqrt{-1}$ is a complex number. The partial differential equation derived from the above expression is

(A) $\dfrac{\partial^2 z}{\partial t^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$ (B) $\dfrac{\partial^2 z}{\partial t^2} - \dfrac{\partial^2 z}{\partial y^2} = 0$

(C) $\dfrac{\partial z}{\partial t} - i\dfrac{\partial z}{\partial y} = 0$ (D) $\dfrac{\partial z}{\partial t} + i\dfrac{\partial z}{\partial y} = 0$

Solution: We have the equation as follows:

$$z = \sin(y + it) + \cos(y - it)$$

The independent variables in the above equation are y and t and the dependent variable is z.
Therefore, to derive a differential equation, we must differentiate the equation by t and y.
Differentiate the above equation by t, we get

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial t}(\sin(y + it) + \cos(y - it))$$
$$= \cos(y + it)(i) - \sin(y - it)(-i)$$

Again, differentiate the above equation by t, we get

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2}{\partial t^2}(\cos(y + it)(i) - \sin(y - it)(-i))$$

$$\frac{\partial^2 z}{\partial t^2} = \sin(y + it) + \cos(y - it) \quad (1)$$

Differentiate the above equation by y, we get

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(\sin(y + it) + \cos(y - it))$$
$$= \cos(y + it) - \sin(y - it)$$

Again, differentiate the above equation by y, we get

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial^2}{\partial y^2}(\sin(y + it) + \cos(y - it))$$

$$\frac{\partial^2 z}{\partial y^2} = -\sin(y + it) - \cos(y - it) \quad (2)$$

Adding Eqs. (1) and (2), we get

$$\frac{\partial^2 z}{\partial t^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

Ans. (A)

2. For the equation

$$\frac{\partial^3 y}{\partial x^3} + x\left(\frac{\partial y}{\partial x}\right)^{3/2} + x^2 y = 0$$

the correct description is
(A) an ordinary differential equation of order 3 and degree 2.
(B) an ordinary differential equation of order 3 and degree 3.
(C) an ordinary differential equation of order 2 and degree 3.
(D) an ordinary differential equation of order 3 and degree 3/2.

Solution: The order of a differential equation is the highest derivative of that equation, and the degree is the power of the highest derivation in the equation.
The equation is:

$$\frac{d^3 y}{dx^3} + x\left(\frac{dy}{dx}\right)^{3/2} + x^2 y = 0$$

The highest derivative in the above equation is 3 and it's power is 1.
But to get the correct values of degree and order, the equation must have only integers powers.
As the equation is having power 3/2, it should be converted into integer value.

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Therefore, squaring this equation would give:

$$\left(\frac{d^3y}{dx^3}+x\left(\frac{dy}{dx}\right)^{3/2}+x^2y=0\right)^2$$

From the above equation, it can be concluded that the equation is an ordinary differential equation of order 3 and degree 2.

Ans. (A)

3. The hoop stress at a point on the surface of a thin cylindrical pressure vessel is computed to be 30.0 MPa. The value of maximum shear stress at this point is

(A) 7.5 MPa (B) 15.0 MPa
(C) 30.0 MPa (D) 22.5 MPa

Solution: Hoop stress and longitudinal stress are defined as follows:

$$\sigma_h=\frac{pd}{2t}$$

$$\sigma_l=\frac{pd}{4t}$$

As it is not mentioned if we have to find the maximum in plane shear stress or absolute maximum shear stress, we have to calculate both:
Absolute maximum shear stress:

$$\tau_{\max}=\frac{\sigma_h-0}{2}=\frac{\sigma_h}{2}=\frac{pd}{4t}=15\,\text{MPa}$$

Maximum in plane shear stress:

$$\tau_{\max}=\frac{\sigma_h-\sigma_t}{2}=\frac{\frac{pd}{2t}-\frac{pd}{4t}}{2}=7.5\,\text{MPa}$$

Ans. (A, B)

4. In the context of elastic theory of reinforced concrete, the modular ratio is defined as the ratio of

(A) Young's modulus of elasticity of reinforcement material to Young's modulus of elasticity of concrete.
(B) Young's modulus of elasticity of concrete to Young's modulus of elasticity of reinforcement material.
(C) Shear modulus of reinforcement material to the shear modulus of concrete.
(D) Young's modulus of elasticity of reinforcement material to the shear modulus of concrete.

Solution: The modular ratio is defined as the ratio of Young's modulus of elasticity of reinforced material and the Young's modulus of elasticity of concrete.

$$m=\frac{E_s}{E_c}$$

Ans. (A)

5. Which of the following equations is correct for the Pozzolanic reaction?

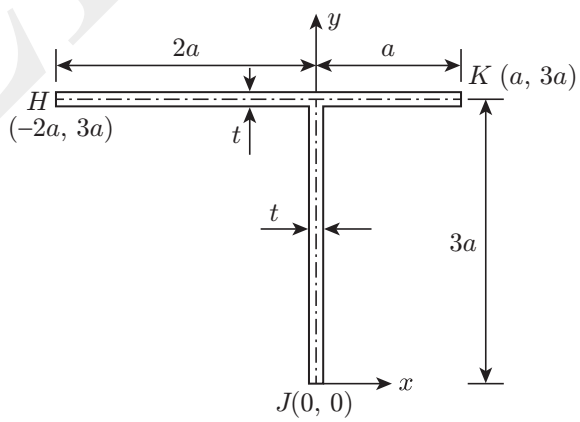
(A) $Ca(OH)_2$ + Reactive Superplasticiser + $H_2O \rightarrow$ C-S-H
(B) $Ca(OH)_2$ + Reactive Silicon dioxide + $H_2O \rightarrow$ C-S-H
(C) $Ca(OH)_2$ + Reactive Sulphates + $H_2O \rightarrow$ C-S-H
(D) $Ca(OH)_2$ + Reactive Sulphur + $H_2O \rightarrow$ C-S-H

Solution: The main constituent of pozzolana is SiO_2, and cement is $Ca(OH)_2$. Therefore, the reaction is as follows:

$Ca(OH)_2$ + Reactive Silicon dioxide + $H_2O \rightarrow$ C-S-H

Ans. (B)

6. Consider the cross-section of a beam made up of thin uniform elements having thickness $t(t \ll a)$ shown in the figure. The (x, y) coordinates of the points along the center-line of the cross-section are given in the figure.



The coordinates of the shear center of this cross-section are:

(A) $x = 0,\ y = 3a$
(B) $x = 2a,\ y = 2a$
(C) $x = -a,\ y = 2a$
(D) $x = -2a,\ y = a$

Solution: Properties of shear center:
The shear center always lies on the axis of symmetry.
It is a point through with the resultant of shearing force on the section passes.
If the section consists of two rectangular parts, the shear center lies on the intersection of their centerlines.

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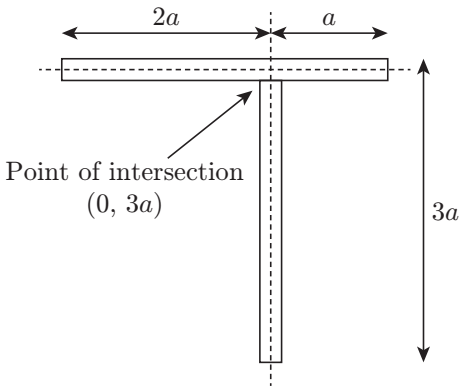


Ans. (A)

7. Four different soils are classified as CH, ML, SP, and SW, as per the Unified Soil Classification System. Which one of the following options correctly represents their arrangement in the decreasing order of hydraulic conductivity?

(A) SW, SP, ML, CH (B) CH, ML, SP, SW
(C) SP, SW, CH, ML (D) ML, SP, CH, SW

Solution: As per the Allen Hazen's formula for permeability in different soils:

$$K = CD_{10}^2$$

Here, D_{10} is the effective size of the particle of soil. Therefore, the hydraulic conductivity depends on the effective size of the particle present in the soil. In the problem,

SW is termed as Well graded sand, SP is poorly graded sand, ML is Low plastic silt, and CH is highly plastic clay.

We know that the diameter of particles in different soils is as follows:

$$\text{Sand}(4.75\ \text{mm} - 75\ \mu\text{m}) > \text{Silt}(75\ \mu\text{m} - 2\ \mu\text{m}) > \text{Clay}(< 2\ \mu\text{m})$$

It can also be concluded that well graded sand is more permeable than poorly graded sand and the voids can easily be filled with poor gradation. Therefore, the correct order is:

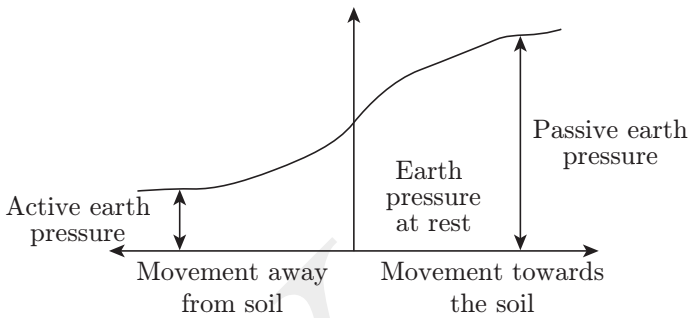
$$\text{SW} > \text{SP} > \text{ML} > \text{CH}$$

Ans. (A)

8. Let σ'_v and σ'_h denote the effective vertical stress and effective horizontal stress, respectively. Which one of the following conditions must be satisfied for a soil element to reach the failure state under Rankine's passive earth pressure condition?

(A) $\sigma'_v < \sigma'_h$ (B) $\sigma'_v > \sigma'_h$
(C) $\sigma'_v = \sigma'_h$ (D) $\sigma'_v + \sigma'_h = 0$

Solution: We know that the passive earth pressure is considered greater than the active pressure and the earth pressure at rest.



We know that:

$$K = \frac{\sigma'_h}{\sigma'_v}$$

For passive earth pressure, $K > 1$

Therefore, $\sigma'_h > \sigma'_v$

Ans. (A)

9. With respect to fluid flow, match the following in **Column X** with **Column Y**:

Column X	**Column Y**
(P) Viscosity	(I) Mach number
(Q) Gravity	(II) Reynolds number
(R) Compressibility	(III) Euler number
(S) Pressure	(IV) Froude number

(A) (P) – (II), (Q) – (IV), (R) – (I), (S) – (III)
(B) (P) – (III), (Q) – (IV), (R) – (I), (S) – (II)
(C) (P) – (IV), (Q) – (II), (R) – (I), (S) – (III)
(D) (P) – (II), (Q) – (IV), (R) – (III), (S) – (I)

Solution: As per the formulas:

$$\text{Reynold's Number} = \frac{\text{Inertial Force}}{\text{Viscous Force}}$$

$$\text{Froude's Number} = \sqrt{\frac{\text{Inertial Force}}{\text{Gravity Force}}}$$

$$\text{Mach Number} = \sqrt{\frac{\text{Inertial Force}}{\text{Elastic Force}}}$$

$$\text{Euler Number} = \sqrt{\frac{\text{Inertial Force}}{\text{Pressure Force}}}$$

Ans. (A)

10. Let ψ represent soil suction head and K represent hydraulic conductivity of the soil. If the soil moisture content θ increases, which one of the following statements is **true**?

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(A) ψ decreases and K increases.
(B) ψ increases and K decreases.
(C) Both ψ and K decrease.
(D) Both ψ and K increase.

Solution: Unsaturated hydraulic conductivity decreases as volumetric water content decreases
- Cross-sectional area of water flow decreases
- Tortuosity increases
- Drag forces increase

As the soil moisture content increases, the soil suction head decreases.

Ans. (A)

11. A rectangular channel with Gradually Varied Flow (GVF) has a changing bed slope. If the change is from a steeper slope to a steep slope, the resulting GVF profile is
(A) S_3
(B) S_1
(C) S_2
(D) either S_1 or S_2, depending on the magnitude of the slopes

Solution: S-profile, also known as steep profile, has following classifications:

Steep ($y_c > y_0$)	1	$y > y_c > y_0$	S_1
	2	$y_c > y > y_0$	S_2
	3	$y_0 > y_c > y$	S_3

S_1-profile: When the flow from steep channel is terminated by a deep pool, created by an obstruction, S_1 Profile occurs.

S_2-profile: Profile of S_2 type occurs at the entrance region of a steep channel.

S_3-profile: It occurs when a flow exists from a steeper slope to a less steep slope.

Ans. (A)

12. The total hardness in raw water is 500 milligram per liter as $CaCO_3$. The total hardness of this raw water, expressed in milligram equivalent per liter, is (Consider the atomic weights of Ca, C, and O as 40 g/mol, 12 g/mol, and 16 g/mol, respectively.)
(A) 10 (B) 100
(C) 1 (D) 5

Solution: Total hardness in raw water is given by

$$\text{Total hardness} = \frac{\left[Mg^{2+}\right]}{\text{eq. wt of } Mg^{2+}} \times \text{eq. wt of } CaCO_3 + \frac{\left[Ca^{2+}\right]}{\text{eq. wt of } Ca^{2+}} \times \text{eq. wt of } CaCO_3$$

The total hardness in milligram equivalent per liter is ratio of hardness in milligram per liter as $CaCO_3$ to equivalent weight of $CaCO_3$. The equivalent weight of $CaCO_3$ is the ratio of molecular weight to valency factor.
The molecular weight of $CaCO_3 = 40 + 12 + (3 \times 16) = 100$ gm
The equivalent weight of $CaCO_3 = 100/2 = 50$ gm
Therefore, the total hardness of this raw water is

$$500 = \frac{\left[Ca^{2+}\right]}{\text{eq. wt of } Ca^{2+}} \times 50$$

$$\Rightarrow \frac{\left[Ca^{2+}\right]}{\text{eq. wt of } Ca^{2+}} = 10\,\text{mg/L}$$

Ans. (A)

13. An aerial photograph is taken from a flight at a height of 3.5 km above mean sea level, using a camera of focal length 152 mm. If the average ground elevation is 460 m above mean sea level, then the scale of the photograph is
(A) 1 : 20000 (B) 1 : 20
(C) 1 : 100000 (D) 1 : 2800

Solution: The scale of the photograph is given by

$$\text{Scale} = \frac{f}{H - h_{\text{avg}}}$$

Here $f = 152$ mm, $H = 3.5$ km $= 3500000$ mm and $h_{\text{avg}} = 460000$ mm. So,

$$\text{Scale} = \frac{152}{3500000 - 460000} = \frac{1}{20000}$$

Ans. (A)

14. A line between stations P and Q laid on a slope of 1 in 5 was measured as 350 m using a 50 m tape. The tape is known to be short by 0.1 m. The corrected horizontal length (in m) of the line PQ will be
(A) 342.52 (B) 349.30
(C) 356.20 (D) 350.70

Solution: Determine the angle of slope as follows:

$$\tan\theta = \frac{1}{5}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{5}\right) = 11.31^\circ$$

The slope length is given as 350 m.
Apply the correct for slope to get the horizontal length as follows:

$$C_{\text{slope}} = -l(1 - \cos\theta)$$

$$= -350(1 - \cos 11.31^\circ) = -6.8 \text{ m}$$

Length after slope correction:

$$l' = l + C_{\text{slope}}$$
$$l' = 350 - 6.8$$
$$l' = 343.2 \text{ m}$$

Apply the tape correction as follows:

$$\text{Correct Length} = \text{Actual length of chain} \times \frac{\text{Length measured}}{\text{Nominal length}}$$
$$= 343.2 \times \frac{49.9}{50} = 342.51 \text{ m}$$

Ans. (A)

15. The matrix M is defined as

$$M = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

and has eigenvalues 5 and -2. The matrix Q is formed as

$$Q = M^3 - 4M^2 - 2M$$

Which of the following is/are the eigenvalue(s) of matrix Q?

(A) 15 (B) 25
(C) -20 (D) -30

Solution: Substitute the eigenvalues of the matrix M in the matrix Q equation, we get

$$Q = M^3 - 4M^2 - 2M$$

For $\lambda_1 = 5$

$$Q = 5^3 - 4 \times 5^2 - 2 \times 5 = 15$$

For $\lambda_2 = -2$

$$Q = (-2)^3 - 4 \times (-2)^2 - 2 \times (-2) = -20$$

Ans. (A, C)

16. For wastewater coming from a wood pulping industry, Chemical Oxygen Demand (COD) and 5-day Biochemical Oxygen Demand (BOD_5) were determined. For this wastewater, which of the following statement(s) is/are correct?

(A) $COD > BOD_5$ (B) $COD \neq BOD_5$
(C) $COD < BOD_5$ (D) $COD = BOD_5$

Solution: Chemical oxygen demand is used to measure the content of organic matter both Biodegradable and non-biodegradable.

$$\text{COD} = \text{Biodegradable Oxygen Demand} + \text{Non-biodegradable Oxygen Demand}$$

Therefore, $COD > BOD_5$ and $COD \neq BOD_5$.

Ans. (A, B)

17. Which of the following process(es) can be used for conversion of salt water into fresh water?

(A) Microfiltration (B) Electrodialysis
(C) Ultrafiltration (D) Reverse osmosis

Solution: Electrodialysis: Electro Dialysis (ED) is a membrane process, during which ions are transported through semi permeable membrane, affected by an electric potential.

Reverse Osmosis: Reverse Osmosis (RO) is a water treatment process that eliminates impurities from water by utilizing pressure to force water particles through a semipermeable membrane.

Ans. (B, D)

18. A horizontal curve is to be designed in a region with limited space. Which of the following measure(s) can be used to decrease the radius of curvature?

(A) Decrease the design speed.
(B) Increase the superelevation.
(C) Increase the design speed.
(D) Restrict vehicles with higher weight from using the facility.

Solution: We know that,

$$e + f = \frac{v^2}{gR}$$
$$R = \frac{v^2}{g(e+f)}$$

To reduce the radius of curvature, the design speed can be decreased or the superelevation can be increased.

Ans. (A, B)

19. Consider the following recursive iteration scheme for different values of variable P with the initial guess $x_1 = 1$:

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{P}{x_n}\right), \qquad n = 1, 2, 3, 4, 5$$

For $P = 2$, x_5 is obtained to be 1.414, rounded-off to three decimal places. For $P = 3$, x_5 is obtained to be 1.732, rounded-off to three decimal places. If $P = 10$, the numerical value of x_5 is _____. (round off to three decimal places)

Solution: **Short approach:**

From the results of x_5 when substituting the value of P as 2 and 3, it can be concluded that, the relation is to calculate the square root of P.

For $P = 2$, $x_5 = 1.414$ and for $P = 3$, $x_5 = 1.732$.

So, for $P = 10$, $x_5 = 3.162$

Long Approach:

Substitute, $n = \infty$ and $x_{n+1} = x_n = a$ in the given equation, we get

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{P}{x_n}\right)$$

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$$\Rightarrow a = \frac{1}{2}\left(a + \frac{P}{a}\right)$$
$$\Rightarrow a = \sqrt{P}$$

For $P = 10$, we get

$$a = \sqrt{10} = 3.162$$

Ans. (3.162)

20. The Fourier cosine series of a function is given by:

$$f(x) = \sum_{n=0}^{\infty} f_n \cos nx$$

For $f(x) = \cos^4 x$, the numerical value of $(f_4 + f_5)$ is _____. (round off to three decimal places)

Solution: For $f(x) = \cos^4 x$, we have

$$\cos^4 x = (\cos^2 x)^2 = \left(\frac{1+\cos 2x}{2}\right)^2$$
$$= \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x)$$
$$= \frac{1}{4} + \frac{\cos 2x}{2} + \frac{1+\cos 4x}{8}$$
$$= \frac{3}{8} + \frac{\cos 2x}{2} + \frac{\cos 4x}{8}$$

In exponential form, it can also be written as:

$$\cos^4 x = \frac{3}{8} + \frac{1}{4}e^{(-2ix)} + \frac{1}{4}e^{(2ix)} + \frac{1}{16}e^{(-4ix)} + \frac{1}{16}e^{(4ix)}$$

So, $f_0 = \frac{3}{8}, f_1 = 0, f_2 = \frac{1}{2}, f_3 = 0, f_4 = \frac{1}{8}$,

and $f_5 = 0$.

Therefore, $f_4 + f_5$ is

$$f_4 + f_5 = 0.125$$

Ans. (0.125)

21. An uncompacted heap of soil has a volume of 10000 m^3 and void ratio of 1. If the soil is compacted to a volume of 7500 m^3, then the corresponding void ratio of the compacted soil is _____. (round off to one decimal place)

Solution: We know that,

$$V_s = \frac{V}{1+e}$$

As the volume of solids remains constant, we can say that:

$$\frac{V_1}{1+e_1} = \frac{V_2}{1+e_2}$$
$$\Rightarrow \frac{10000}{1+1} = \frac{7500}{1+e_2}$$
$$\Rightarrow e_2 = 0.5$$

Ans. (0.5)

22. A concentrated vertical load of 3000 kN is applied on a horizontal ground surface. Points P and Q are at depths 1 m and 2 m below the ground, respectively, along the line of application of the load. Considering the ground to be a linearly elastic, isotropic, semi-infinite medium, the ratio of the increase in vertical stress at P to the increase in vertical stress at Q is _____. (in integer)

Solution: We know that, as per Boussinesq's Equation:

$$\sigma_z = \frac{3}{2\pi}\left(\frac{1}{1+(r^2/z^2)}\right)^{5/2}\frac{Q}{z^2}$$

First case: $r = 0, z = 1$

$$\sigma_{z=1} = \frac{3}{2\pi}\left(\frac{1}{1+0}\right)^{5/2}\frac{Q}{(1)^2}$$

Second case: $r = 0, z = 2$

$$\sigma_{z=2} = \frac{3}{2\pi}\left(\frac{1}{1+0}\right)^{5/2}\frac{Q}{(2)^2}$$

Therefore, the ratio of the increase in vertical stress at P to the increase in vertical stress at Q is

$$\frac{\sigma_{z=1}}{\sigma_{z=2}} = \frac{\frac{3}{2\pi}\left(\frac{1}{1+0}\right)^{5/2}\frac{Q}{(1)^2}}{\frac{3}{2\pi}\left(\frac{1}{1+0}\right)^{5/2}\frac{Q}{(2)^2}} = 4$$

Ans. (4)

23. At a site, Static Cone Penetration Test was carried out. The measured point (tip) resistance q_c was 1000 kPa at a certain depth. The friction ratio (f_r) was estimated as 1% at the same depth. The value of sleeve (side) friction (in kPa) at that depth was _____. (in integer)

Solution: We know that,

$$\text{Friction Ratio} = \frac{\text{Sleeve Friction}}{\text{Tip Resistance}}$$
$$\Rightarrow 1\% = \frac{\text{Sleeve Friction}}{1000}$$
$$\Rightarrow \text{Sleeve Friction} = \frac{1000}{100} = 10$$

Ans. (10)

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24. During a particular stage of the growth of a crop, the consumptive use of water is 2.8 mm/day. The amount of water available in the soil is 50% of the maximum depth of available water in the root zone. Consider the maximum root zone depth of the crop as 80 mm and the irrigation efficiency as 70%. The interval between irrigation (in days) will be _____. (round off to the nearest integer)

Solution: Frequency, $f = \dfrac{\text{Depth of water needed}}{\text{Consumptive use}}$

$$f = \frac{40}{2.8} = 14.28 \approx 14 \text{ days}$$

Ans. (14)

25. The bearing of a survey line is N31°17′W. Its azimuth observed from north is _____ deg. (round off to two decimal places)

Solution: Azimuth is given by

$$\text{Azimuth} = \text{WCB} = 360 - 31°17' = 328°43'$$

Convert degrees to radian:

$$\text{Azimuth} = 328°43' = 328 + \frac{43}{60} = 328.716 \approx 328.72$$

Ans. (328.72)

Q. No. 26–55 Multiple Choice Question (MCQ) Carry Two Marks Each

26. The Cartesian coordinates of a point P in a right-handed coordinate system are (1, 1, 1). The transformed coordinates of P due to a 45° clockwise rotation of the coordinate system about the positive x-axis are

(A) $(1, 0, \sqrt{2})$ (B) $(1, 0, -\sqrt{2})$

(C) $(-1, 0, \sqrt{2})$ (D) $(-1, 0, -\sqrt{2})$

Solution: The 3×3 rotation matrix is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

For the transport coordinated of the above matrix, substitute $\theta = 45°$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45° & -\sin 45° \\ 0 & \sin 45° & \cos 45° \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

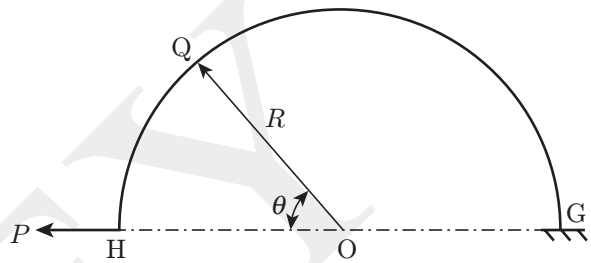
Determine the transformed coordinates as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \sqrt{2} \end{bmatrix}$$

Ans. (A)

27. A semi-circular bar of radius R m, in a vertical plane, is fixed at the end G, as shown in the figure. A horizontal load of magnitude P kN is applied at the end H. The magnitude of the axial force, shear force, and bending moment at point Q for $\theta = 45°$, respectively, are



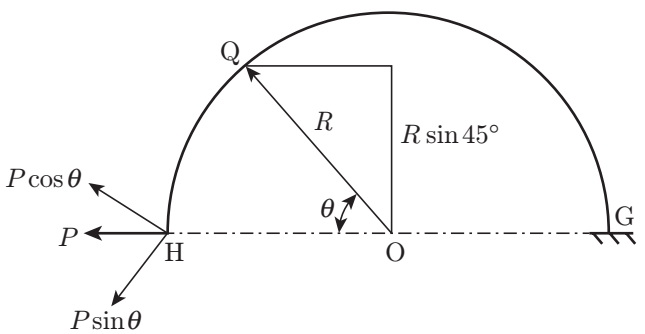
(A) $\dfrac{P}{\sqrt{2}}$ kN, $\dfrac{P}{\sqrt{2}}$ kN, and $\dfrac{PR}{\sqrt{2}}$ kNm

(B) $\dfrac{P}{\sqrt{2}}$ kN, $\dfrac{P}{\sqrt{2}}$ kN, and 0 kNm

(C) 0 kN, $\dfrac{P}{\sqrt{2}}$ kN, and $\dfrac{PR}{\sqrt{2}}$ kNm

(D) $\dfrac{P}{\sqrt{2}}$ kN, 0 kN, and $\dfrac{PR}{\sqrt{2}}$ kNm

Solution: Taking components of load, we can redraw the given figure.



Axial force at Q:

$$H_Q = P \sin 45° = \frac{P}{\sqrt{2}}$$

Shear force:

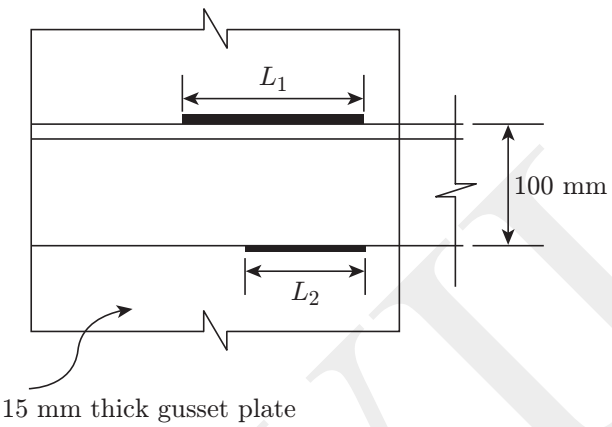
$$F_Q = P \cos 45° = \frac{P}{\sqrt{2}}$$

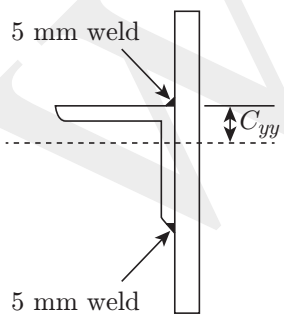
Moment at Q:

$$M_Q = P \times R \sin 45^\circ = \frac{PR}{\sqrt{2}}$$

Ans. (A)

28. A weld is used for joining an angle section ISA 100 mm × 100 mm × 10 mm to a gusset plate of thickness 15 mm to transmit a tensile load. The permissible stress in the angle is 150 MPa and the permissible shear stress on the section through the throat of the fillet weld is 108 MPa. The location of the centroid of the angle is represented by C_{yy} in the figure, where $C_{yy} = 28.4$ mm. The area of cross-section of the angle is 1903 mm^2. Assuming the effective throat thickness of the weld to be 0.7 times the given weld size, the lengths L_1 and L_2 (rounded off to the nearest integer) of the weld required to transmit a load equal to the full strength of the tension member are, respectively





(A) 541 mm and 214 mm
(B) 214 mm and 541 mm
(C) 380 mm and 151 mm
(D) 151 mm and 380 mm

Solution: As the member given in the problem is a tension member, gross area is to be considered while calculating the strength of the member.

$$T = \sigma A_g = 150 \times 1903 = 285450\,\text{N}$$

Determine the strength of both the welded connections as follows:

T = Throat Thickness × Length of Weld × Permissible Stress

So, $T_1 = (0.7s)L_1\sigma_{st} = 0.7 \times 5 \times L_1 \times 108 = 378L_1$

And $T_2 = (0.7s)L_2\sigma_{st} = 0.7 \times 5 \times L_2 \times 108 = 378L_2$

Now total weld strength would be equal to the tensile strength of the member:

$$T = T_1 + T_2 W$$
$$\Rightarrow 285450 = 378(L_1 + L_2)$$
$$L_1 + L_2 = 755.15 \quad (1)$$

Since, total bending moment around centre of gravity is zero. So,

$$\sum M_{CG} = 0$$
$$\Rightarrow T_1 \times 28.4 = T_2 \times (100 - 28.4)$$
$$\Rightarrow 378L_1 \times 28.4 = 378L_2 \times (100 - 28.4)$$
$$\Rightarrow L_1 = 2.521L_2$$

Substitute the value of L_1 in Eq. (1):

$$2.521L_2 + L_2 = 755.15$$
$$\Rightarrow L_2 = 214.47\,\text{mm}$$

And

$$L_1 = 2.521 \times L_2 = 2.521 \times 214.47 = 540.68\,\text{mm}$$

Ans. (A)

29. The project activities are given in the following table along with the duration and dependency.

Activities	Duration (days)	Depends on
P	10	—
Q	12	—
R	5	P
S	10	Q
T	10	P, Q

Which one of the following combinations is correct?

(A) Total duration of the project = 22 days, Critical path is Q → S
(B) Total duration of the project = 20 days, Critical path is Q → T
(C) Total duration of the project = 22 days, Critical path is P → T
(D) Total duration of the project = 20 days, Critical path is P → R

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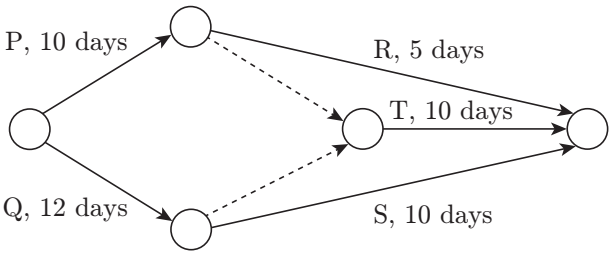
Solution: Draw the activity network diagram as follows:



Define all the path as follows:

$$P \to R = 10 + 5 = 15 \text{ days}$$
$$P \to T = 10 + 10 = 20 \text{ days}$$
$$Q \to T = 12 + 10 = 22 \text{ days}$$
$$Q \to S = 12 + 10 = 22 \text{ days}$$

The critical path has the minimum duration in which all the activities are completed, therefore it would be maximum of all the durations computed above.

Total duration of the project = 22 days, Critical path is Q → S

Ans. (A)

30. The correct match between the physical states of the soils given in Group I and the governing conditions given in Group II is

Group I	**Group II**
1. normally consolidated soil	P. sensitivity > 16
2. quick clay	Q. dilation angle = 0
3. sand in critical state	R. liquid limit > 50
4. clay of high plasticity	S. over consolidation ratio = 1

(A) 1-S, 2-P, 3-Q, 4-R (B) 1-Q, 2-S, 3-P, 4-R
(C) 1-Q, 2-P, 3-R, 4-S (D) 1-S, 2-Q, 3-P, 4-R

Solution: We know that, the over consolidation ratio is defined as the ratio of the maximum stress in the past to the stress at present. The soil having over-consolidation ratio as 1 is termed as normally consolidated soil.
Since, the sensitivity of the soil is defined as the ratio of unconfined compressive strength of an undisturbed soil to the unconfined compressive strength of the same soil after remolding. As the sensitivity increases, the soil becomes unstable. Quick clay has sensitivity greater than 16.
When the dilation angle of the sand is equals to 0, that sand is in critical state.
As the liquid limit of the soil increases, the plasticity increases. As per the Atterberg's limits, a soil having liquid limit greater than 50 is considered as clay with high plasticity.

Ans. (A)

31. As per Rankine's theory of earth pressure, the inclination of failure planes is $(45 + \phi/2)^\circ$ with respect to the direction of the minor principal stress. The above statement is correct for which one of the following options?
(A) Only the active state and not the passive state
(B) Only the passive state and not the active state
(C) Both active as well as passive states
(D) Neither active nor passive state

Solution: The inclination of the failure plane is always $(45 + \phi/2)^\circ$ with respect to the direction of the minor principal stress in both active as well as passive states.

Ans. (C)

32. Henry's law constant for transferring O_2 from air into water, at room temperature, is 1.3 mmol/(liter atm). Given that the partial pressure of O_2 in the atmosphere is 0.21 atm, the concentration of dissolved oxygen (mg/liter) in water in equilibrium with the atmosphere at room temperature is (Consider the molecular weight of O_2 as 32 g/mol)
(A) 8.7 (B) 0.8
(C) 198.1 (D) 0.2

Solution: Determine the concentration of oxygen as follows:

$$C_O = KP = 1.3 \times 0.21 = 0.273 \,\text{mmol/L}$$

Convert the concentration to mg/L as follows:

$$C_O = 0.273 \frac{\text{mmol}}{\text{L}} \times \text{Molecular wt. of } O_2$$
$$= 0.273 \frac{\text{mmol}}{\text{L}} \times 32 \frac{\text{g}}{\text{mol}} = 8.7 \text{ mg/L}$$

Ans. (A)

33. In a water sample, the concentrations of Ca^{2+}, Mg^{2+} and HCO_3^- are 100 mg/L, 36 mg/L and 122 mg/L, respectively. The atomic masses of various elements are:

Ca = 40, Mg = 24, H = 1, C = 12, O = 16.

The total hardness and the temporary hardness in the water sample (in mg/L as $CaCO_3$) will be
(A) 400 and 100, respectively.
(B) 400 and 300, respectively.
(C) 500 and 100, respectively.
(D) 800 and 200, respectively.

Solution: Total hardness of water is given by

$$\text{Total hardness} = \frac{[Mg^{2+}]}{\text{Eq. wt of } Mg^{2+}} \times \text{Eq. wt. of } CaCO_3 + \frac{[Ca^{2+}]}{\text{Eq. wt of } Ca^{2+}} \times \text{Eq. wt. of } CaCO_3$$
$$= \frac{36}{12} \times 50 + 100 \times \frac{50}{20} = 400 \text{ mg/L}$$

Total alkalinity in water is given by

$$\text{Total alkalinity} = \frac{[HCO_3^-]}{\text{Eq. wt of } HCO_3^-} \times \text{Eq. wt. of } CaCO_3$$
$$= \frac{122}{61} \times 50 = 100 \text{ mg/L}$$

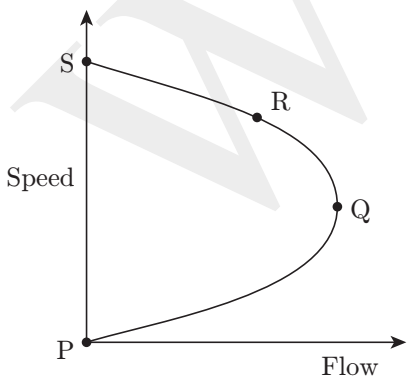
When alkalinity is present in the water, temporary hardness is given by

Temporary hardness
= min (Total hardness, Total alkalinity)

$$\text{Temporary hardness} = \min(400,100) = 100 \text{ mg/L}$$

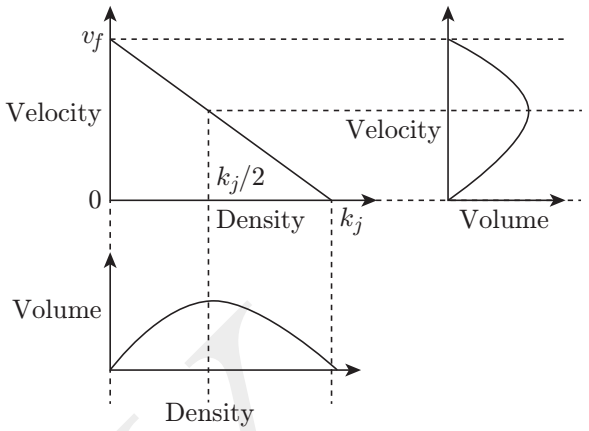
Ans. (A)

34. Consider the four points P, Q, R, and S shown in the Greenshields fundamental speed-flow diagram. Denote their corresponding traffic densities by k_P, k_Q, k_R, and k_S, respectively. The correct order of these densities is



(A) $k_P > k_Q > k_R > k_S$
(B) $k_S > k_R > k_Q > k_P$
(C) $k_Q > k_R > k_S > k_P$
(D) $k_Q > k_R > k_P > k_S$

Solution: The speed density plot is as follows:



When the speed and flow both are 0 at point P, it shows that the density is maximum which means no vehicle is moving from their position.
At point Q, the speed increases and the flow increase to a maximum point which is the most critical point.
As the speed increases more, the flow starts decreasing which shows that the density is also decreasing.
At point S, the speed is maximum, and flow is 0 which shows that there is the least density.
Therefore, $k_P > k_Q > k_R > k_S$.

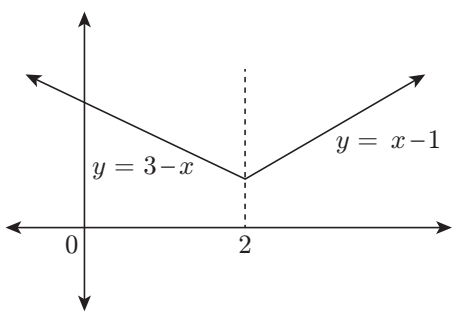
Ans. (A)

35. Let $\max\{a, b\}$ denote the maximum of two real numbers a and b. Which of the following statement(s) is/are **true** about the function $f(x) = \max\{3 - x, x - 1\}$?
(A) It is continuous on its domain.
(B) It has a local minimum at $x = 2$.
(C) It has a local maximum at $x = 2$.
(D) It is differentiable on its domain.

Solution: Determine the point where both the lines intersect with each other.

$$3 - x = x - 1$$
$$x = 2$$

Plot the line as follows:



Now, we check for the continuity of the function by substitute $x = 2$ in both the equations:

$$3 - x = x - 1$$
$$3 - 2 = 2 - 1$$
$$1 = 1$$

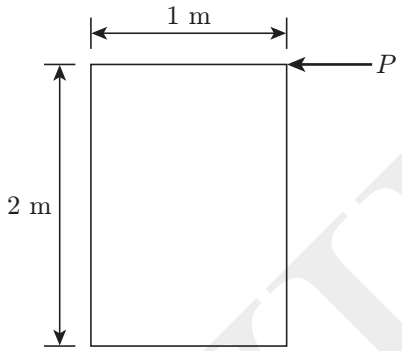
Therefore, the function is continuous.
Now, check for the differentiability of the function:

$$\frac{d}{dx}(3 - x) = \frac{d}{dx}(x - 1)$$
$$-1 \neq 1$$

Therefore, the function is not differentiable. But from the graph it has a local minimum at $x = 2$.

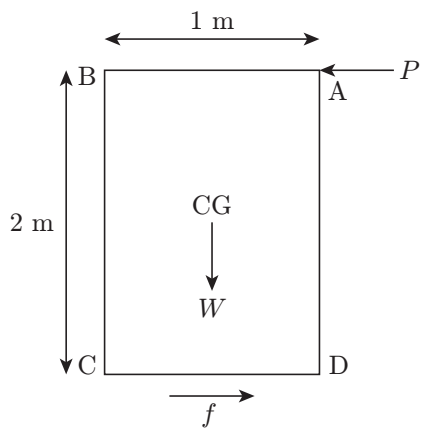
Ans. (A, B)

36. A horizontal force of P kN is applied to a homogeneous body of weight 25 kN, as shown in the figure. The coefficient of friction between the body and the floor is 0.3. Which of the following statement(s) is/are correct?



(A) The motion of the body will occur by overturning.
(B) Sliding of the body never occurs.
(C) No motion occurs for $P \leq 6$ kN.
(D) The motion of the body will occur by sliding only.

Solution: Redraw the figure, taking free body diagram



Determine the force due to friction as follows:

$$f = \mu N = 0.3 \times 25 = 7.5\,\text{kN}$$

Determine the moment about point C:

$$\sum M_C = 0$$
$$\Rightarrow P \times 2 = W \times 0.5$$
$$\Rightarrow P = \frac{25 \times 0.5}{2} = 6.25\,\text{kN}$$

As the force to overturn the body is less, the motion of the body will occur by overturning and sliding of the body never occurs. And as the minimum force required for the moment of body is 6.25kN, there would be no motion occur for $P \leq 6$ kN.

Ans. (A, B, C)

37. In the context of cross-drainage structures, the correct statement(s) regarding the relative positions of a natural drain (stream/river) and an irrigation canal, is/are
(A) In an aqueduct, natural drain water goes under the irrigation canal, whereas in a super-passage, natural drain water goes over the irrigation canal.
(B) In a level crossing, natural drain water goes through the irrigation canal.
(C) In an aqueduct, natural drain water goes over the irrigation canal, whereas in a super-passage, natural drain water goes under the irrigation canal.
(D) In a canal syphon, natural drain water goes through the irrigation canal.

Solution: In an aqueduct, natural drain goes under the irrigation canal, whereas in a super-passage, natural water goes over the irrigation canal.
In a level crossing, the natural drain water goes through the irrigation canal.

Ans. (A, B)

38. Consider the differential equation

$$\frac{dy}{dx} = 4(x + 2) - y$$

For the initial condition $y = 3$ at $x = 1$, the value of y at $x = 1.4$ obtained using Euler's method with a step-size of 0.2 is _____. (round off to one decimal place)

Solution: Given that,

$$f(x) = \frac{dy}{dx} = 4(x + 2) - y$$

Euler equation is given by

$$y_n = y_{n-1} + hf(x_n, y_n)$$

First Iteration $x_1 = 1, y_1 = 3$:

$$y_2 = y_1 + hf(x_1, y_1)$$
$$= 3 + 0.2(4(1+2) - 3) = 4.8$$

Second Iteration $x_2 = 1.2$, $y_2 = 4.8$:

$$y_3 = y_2 + hf(x_2, y_2)$$
$$= 4.8 + 0.2(4(1.2+2) - 4.8) = 6.4$$

Ans. (6.4)

39. A set of observations of independent variable (x) and the corresponding dependent variable (y) is given below.

x	5	2	4	3
y	16	10	13	12

Based on the data, the coefficient a of the linear regression model

$$y = a + bx$$

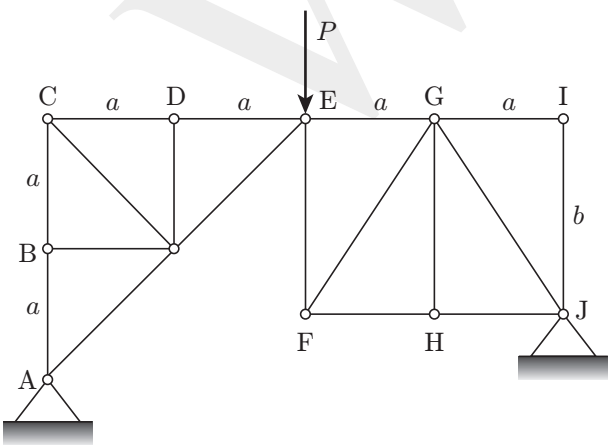
is estimated as 6.1.
The coefficient b is _____. (round off to one decimal place)

Solution: As per the linear regression method:

$$\sum y = na + b\sum x$$
$$(16 + 10 + 13 + 12) = (4 \times 6.1) + b(5 + 2 + 4 + 3)$$
$$b = 1.9$$

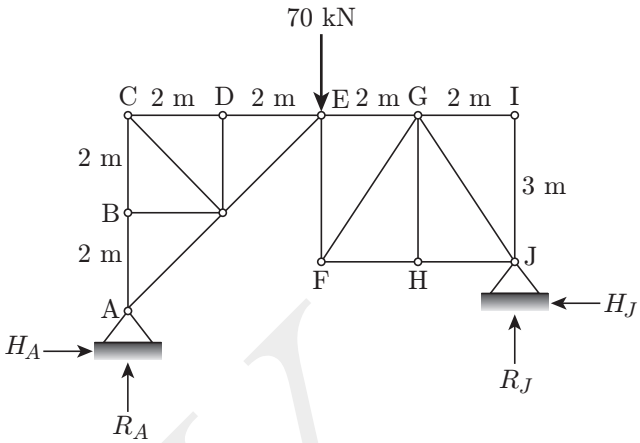
Ans. (1.9)

40. The plane truss shown in the figure is subjected to an external force P. It is given that $P = 70$ kN, $a = 2$ m, and $b = 3$ m.



The magnitude (absolute value) of force (in kN) in member EF is _____. (round off to the nearest integer)

Solution:



Determine the reactions at points E and J as follows:

$$\sum M_{\text{J}} = 0$$
$$\Rightarrow R_{\text{A}} \times 8 = (H_{\text{A}} \times 1) + (70 \times 4)$$
$$\Rightarrow R_{\text{A}} \times 8 = (R_{\text{A}} \times 1) + (70 \times 4)$$
$$\Rightarrow R_{\text{A}} = 30\,\text{kN}$$

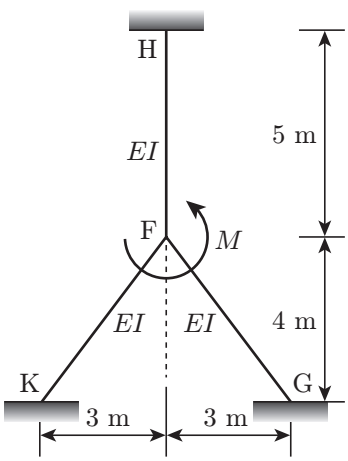
$$\sum M_E = 0$$
$$\Rightarrow R_{\text{A}} \times 4 = H_{\text{A}} \times 4$$
$$\Rightarrow R_{\text{A}} = H_{\text{A}}$$

Consider the left portion of the truss till joint E:

$$\sum M_{\text{C}} = 0$$
$$\Rightarrow (H_{\text{A}} \times 4) + (F_{\text{EF}} \times 4) - (70 \times 4) = 0$$
$$\Rightarrow F_{\text{EF}} = \frac{280 - 160}{4} = 30\,\text{kN}$$

Ans. (30)

41. Consider the linearly elastic plane frame shown in the figure. Members HF, FK and FG are welded together at joint F. Joints K, G and H are fixed supports. A counter-clockwise moment M is applied at joint F. Consider flexural rigidity $EI = 10^5$ kN-m^2 for each member and neglect axial deformations.



If the magnitude (absolute value) of the support moment at H is 10 kN-m, the magnitude (absolute value) of the applied moment M (in kN-m) to maintain static equilibrium is _____. (round off to the nearest integer)

Solution: As all the members in the frame have same end conditions, the distribution factor would be 1/3 for each member.

$$M_{HF} = 10\,\text{kNm}$$

As the far end is fixed the carry over moment is half, that is, 1/2. Therefore,

$$M_{HF} = \frac{1}{2} M_{FH}$$

$$\Rightarrow 10 = \frac{1}{2} M_{FH}$$

$$\Rightarrow M_{FH} = 20\,\text{kNm}$$

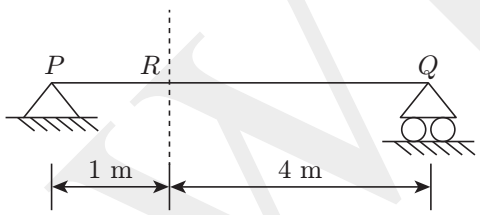
Now, the relation between applied moment and distributed moment is

$$M_{FH} = \frac{1}{3} M$$

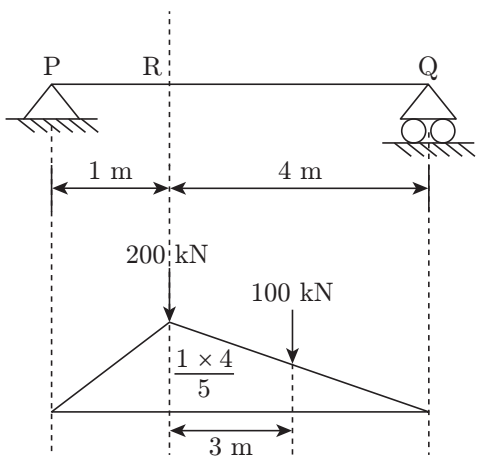
$$\Rightarrow M = 20 \times 3 = 60\,\text{kN m}$$

Ans. (60)

42. Consider a simply supported beam PQ as shown in the figure. A truck having 100 kN on the front axle and 200 kN on the rear axle, moves from left to right. The spacing between the axles is 3 m. The maximum bending moment at point R is _____ kNm. (in integer)



Solution: Draw the Influence Line Diagram (ILD) for the bending moment at R as follows:



Calculate the ordinate of the ILD where 100 kN load in action:

$$\frac{\frac{1 \times 4}{5}}{4} = \frac{y}{1}$$

$$\Rightarrow y = \frac{1}{5}$$

Calculate the bending moment at point R as follows:

$$BM_R = \frac{4}{5} \times 200 + \frac{1}{5} \times 100 = 180\,\text{kNm}$$

Ans. (180)

43. A reinforced concrete beam with rectangular cross section (width = 300 mm, effective depth = 580 mm) is made of M30 grade concrete. It has 1% longitudinal tension reinforcement of Fe 415 grade steel. The design shear strength for this beam is 0.66 N/mm^2. The beam has to resist a factored shear force of 440 kN. The spacing of two-legged, 10 mm diameter vertical stirrups of Fe 415 grade steel is _____mm. (round off to the nearest integer)

Solution: Calculate the nominal shear stress as follows:

$$\tau_v = \frac{V_u}{bd} = \frac{440 \times 10^3}{300 \times 580} = 2.53\,\text{N/mm}^2$$

Calculate the spacing of two-legged is

$$\frac{0.87 f_y A_{sv} d}{S_v} \geq (\tau_v - \tau_c) bd$$

$$\Rightarrow \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 10^2 \times 580}{S_v} \geq (2.53 - 0.66) \times 300 \times 580$$

$$\Rightarrow S_v \leq 101.09\ \text{mm} \Rightarrow S_v = 101\ \text{mm}$$

Ans. (101)

44. A square concrete pile of 10 m length is driven into a deep layer of uniform homogeneous clay. Average unconfined compressive strength of the clay, determined through laboratory tests on undisturbed samples extracted from the clay layer, is 100 kPa. If the ultimate compressive load capacity of the driven pile is 632 kN, the required width of the pile is _____ mm. (in integer) (Bearing capacity factor $N_c = 9$; adhesion factor $\alpha = 0.7$)

Solution: Determine the ultimate bearing capacity of the pile as follows:

$$Q_u = Q_{pu} + Q_f$$

$$= (q_{up} \times A_b) + (f_s \times A_s)$$

$$= CN_c \times A_b + \alpha C_u \times A_s$$

$$\Rightarrow 632 = \left[\frac{100}{2}\times 9\times (a^2)\right] + \left[0.7\times\frac{100}{2}\times(4a\times 10)\right]$$

$$\Rightarrow 632 = \left[\frac{100}{2}\times 9\times (a^2)\right] + \left[0.7\times\frac{100}{2}\times(4a\times 10)\right]$$

$$\Rightarrow a = 0.4\,\text{m} = 400\,\text{mm}$$

Ans. (400)

45. A raft foundation of 30 m × 25 m is proposed to be constructed at a depth of 8 m in a sand layer. A 25 m thick saturated clay layer exists 2 m below the base of the raft foundation. Below the clay layer, a dense sand layer exists at the site. A 25 mm thick undisturbed sample was collected from the mid-depth of the clay layer and tested in a laboratory oedometer under double drainage condition. It was found that the soil sample had undergone 50% consolidation settlement in 10 minutes. The time (in days) required for 25% consolidation settlement of the raft foundation will be _____. (round off to the nearest integer)

Solution: Determine the Coefficient of consolidation from the formula taking case of double drainage as follows:

$$H_1 = \frac{H}{2}$$

So,

$$T_{v1} = \frac{C_v t_1}{{H_1}^2}$$

$$\Rightarrow \frac{\pi}{4}{U_1}^2 = \frac{C_v t_1}{{H_1}^2}$$

$$\Rightarrow \frac{\pi}{4}\times 0.5^2 = \frac{C_v\times 10}{\left(\frac{25}{2}\right)^2}$$

$$\Rightarrow C_v = 3.0625\times 10^{-6}\,\text{m}^2/\text{min}$$

Determine the time required for 25% consolidation settlement as follows:

$$T_{v2} = \frac{C_v t_2}{{H_2}^2}$$

$$\Rightarrow \frac{\pi}{4}{U_2}^2 = \frac{C_v t_2}{{H_2}^2}$$

$$\Rightarrow \frac{\pi}{4}\times 0.25^2 = \frac{C_v\times t_2}{\left(\frac{25}{2}\right)^2}$$

$$\Rightarrow t_2 = 2.504\times 10^6\,\text{min} = \frac{2.504\times 10^6}{60\times 24}$$

$$= 1738.88\,\text{days} \approx 1739\,\text{days}$$

Ans. (1739)

46. A two-hour duration storm event with uniform excess rainfall of 3 cm occurred on a watershed. The ordinates of streamflow hydrograph resulting from this event are given in the table.

Time (hours)	0	1	2	3	4	5	6	7
Streamflow (m^3/s)	10	16	34	40	31	25	16	10

Considering a constant baseflow of 10 m^3/s, the peak flow ordinate (in m^3/s) of one-hour unit hydrograph for the watershed is _____. (in integer)

Solution:

Time	**Stream flow**	**Base flow**	**2hr DRH**	**2hr UH**	**SA**	**SB**	**SA − SB**	**1hr UH**
0	10	10	0	0	0		0	0
1	16	10	6	2	2	0	2	4
2	34	10	24	8	8	2	6	12
3	40	10	30	10	12	8	4	8
4	31	10	21	7	15	12	3	6
5	25	10	15	5	17	15	2	4
6	16	10	6	2	17	17	0	0
7	10	10	0	0	17	17	0	0

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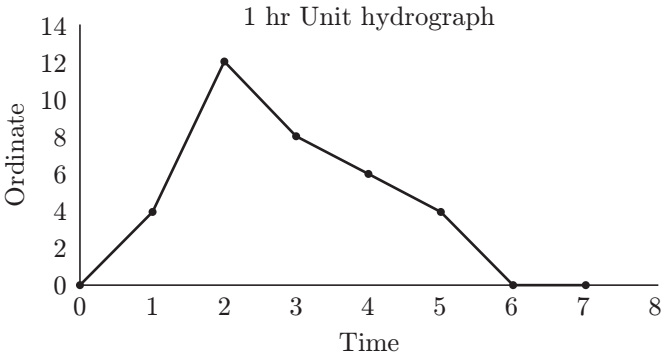
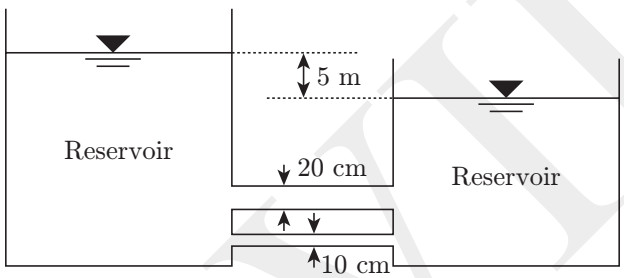
Plot the ordinates of 1hr UH vs time as follows:



Therefore, the peak flow ordinate of one-hour unit hydrograph for the watershed is 12 m^3/s.

Ans. (12)

47. Two reservoirs are connected by two parallel pipes of equal length and of diameters 20 cm and 10 cm, as shown in the figure (not drawn to scale). When the difference in the water levels of the reservoirs is 5 m, the ratio of discharge in the larger diameter pipe to the discharge in the smaller diameter pipe is _____. (round off to two decimal places)
(Consider only loss due to friction and neglect all other losses. Assume the friction factor to be the same for both the pipes)



Solution: Write the expression for head loss in terms of discharge:

$$h_f = \frac{8fLQ^2}{\pi^2 g D^5}$$

In parallel connection of pipes, the head loss remains constant and discharge changes.
Therefore:

$$\frac{8fLQ_1^2}{\pi^2 g D_1^5} = \frac{8fLQ_2^2}{\pi^2 g D_2^5}$$

$$\Rightarrow \frac{Q_1^2}{D_1^5} = \frac{Q_2^2}{D_2^5}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^{5/2} = \left(\frac{0.2}{0.1}\right)^{5/2} = 5.66$$

Ans. (5.66)

48. Depth of water flowing in a 3 m wide rectangular channel is 2 m. The channel carries a discharge of 12 m^3/s. Take $g = 9.8\ m/s^2$. The bed width (in m) at contraction, which just causes the critical flow, is _____ without changing the upstream water level. (round off to two decimal places)

Solution: Determine the initial energy of the flow in the rectangular channel as follows:

$$E_1 = y_1 + \frac{v_1^2}{2g} = 2 + \frac{\left(\frac{12}{3\times 2}\right)^2}{2\times 9.81} = 2.2\,\text{m}$$

We know that:

$$E_c = \frac{3}{2} y_c$$

$$\Rightarrow y_c = \frac{2}{3} E_c$$

Calculate the width of the channel to just cause the critical conditions:

$$y_c = \frac{2}{3}\times 2.2 = 1.46$$

We also know that:

$$y_c = \left(\frac{q_2^2}{g}\right)^{1/3}$$

So,

$$\left(\frac{q_2^2}{9.81}\right)^{1/3} = 1.46$$

$$\Rightarrow q_2 = 5.57$$

$$\Rightarrow q_2 = \frac{Q_2}{B_2} = 5.57$$

$$\Rightarrow B_2 = \frac{12}{5.57} = 2.15\,\text{m}$$

Ans. (2.15)

49. A wastewater sample contains two nitrogen species, namely ammonia and nitrate. Consider the atomic weight of N, H, and O as 14 g/mol, 1 g/mol, and 16 g/mol, respectively. In this wastewater, the concentration of ammonia is 34 mg NH_3/liter and that of nitrate is 6.2 mg NO_3^-/liter. The total nitrogen concentration in this wastewater is _____ milligram nitrogen per liter. (round off to one decimal place)

Solution: Calculate the total nitrogen concentration in the wastewater as follows:

$$\left(\frac{[NH_3]}{[N]+3[H]} + \frac{[NO_3^-]}{[N]+3[O]}\right)\times 14$$

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$$= \left(\frac{34}{14+3\times1} + \frac{6.2}{14+3\times16}\right)\times14$$
$$= 2.1\times14 \text{ m eq/L} = 29.4 \text{ m eq/L}$$

Ans. (29.4)

50. A 2 % sewage sample (in distilled water) was incubated for 3 days at 27°C temperature. After incubation, a dissolved oxygen depletion of 10 mg/L was recorded. The biochemical oxygen demand (BOD) rate constant at 27°C was found to be 0.23 day^{-1} (at base e). The ultimate BOD (in mg/L) of the sewage will be _____. (round off to the nearest integer)

Solution: Calculate the dilution factor (DF) as follows:

$$DF = \frac{\text{Volume of total sample (taken as 100 L)}}{\text{Volume of sewage}}$$
$$= \frac{100}{2} = 50$$

Calculate the BOD of 3 days as follows:

$$BOD_3 = (DO_{initial} - DO_{final})DF = 10\times50$$
$$= 500 \text{ mg/L}$$

Calculate the ultimate BOD as follows:

$$BOD_3 = BOD_u(1-e^{-kt})$$
$$\Rightarrow 500 = BOD_u(1-e^{-0.23\times3})$$
$$\Rightarrow BOD_u = 1003 \text{ mg/L}$$

Ans. (1003)

51. A water treatment plant has a sedimentation basin of depth 3 m, width 5 m, and length 40 m. The water inflow rate is 500 m^3/h. The removal fraction of particles having a settling velocity of 1.0 m/h is _____. (round off to one decimal place) (Consider the particle density as 2650 kg/m^3 and liquid density as 991 kg/m^3)

Solution: Calculate the overflow rate as follows:

$$v_O = \frac{Q}{WL} = \frac{500}{5\times40} = 2.5 \text{ m/h}$$

Calculate the removal efficiency as follows:

$$\eta = \frac{v_s}{v_o}\times100 = \frac{1}{2.5}\times100 = 40\% \text{ or } 0.40$$

Ans. (0.40)

52. A two-phase signalized intersection is designed with a cycle time of 100 s. The amber and red times for each phase are 4 s and 50 s, respectively. If the total lost time per phase due to start-up and clearance is 2 s, the effective green time of each phase is _____s. (in integer)

Solution: Total time is given by

$$\text{Total} = \text{Green} + \text{Amber} + \text{Red}$$

So, green time is

$$\text{Green} = \text{Total} - \text{Amber} - \text{Red}$$
$$= 100 - 4 - 50 = 46 \text{ s}$$

The effective green time due to the lost time in start-up phase:

$$\text{Green}_{\text{eff}} = \text{Green} + \text{Amber} - \text{Lost time}$$
$$= 46 + 4 - 2 = 48 \text{ s}$$

Ans. (48)

53. At a traffic intersection, cars and buses arrive randomly according to independent Poisson processes at an average rate of 4 vehicles per hour and 2 vehicles per hour, respectively. The probability of observing at least 2 vehicles in 30 minutes is _____. (round off to two decimal places)

Solution: Given that, the distributions of cars and buses are

$$\lambda_1 = 4 \text{ Veh/h};\ \lambda_2 = 2 \text{ Veh/h}$$

Total distribution is

$$\lambda_{\text{Veh}} = \lambda_{\text{car}} + \lambda_{\text{bus}} = 2+1 = 3 \text{ Veh/h}$$
$$= \frac{1}{20} \text{ Veh/min}$$

The probability for at least 2 vehicles as follows:

$$P(x<2) = 1 - P(x\le1) = 1 - [P(x=0) + P(x=1)]$$
$$= 1 - \left[\frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!}\right] = 1 - e^{-\lambda}(\lambda^0 + \lambda^1)$$
$$= 1 - e^{-3}(1+3^1) = 0.8$$

Ans. (0.80)

54. The vehicle count obtained in every 10 minute interval of a traffic volume survey done in peak one hour is given below.

Time Interval (in minutes)	Vehicle Count
0 — 10	10
10 — 20	11
20 — 30	12
30 — 40	15
40 — 50	13
50 — 60	11

The peak hour factor (PHF) for 10 minute sub-interval is _____. (round off to one decimal place)

Solution: The peak hour factor (PHF) is given by

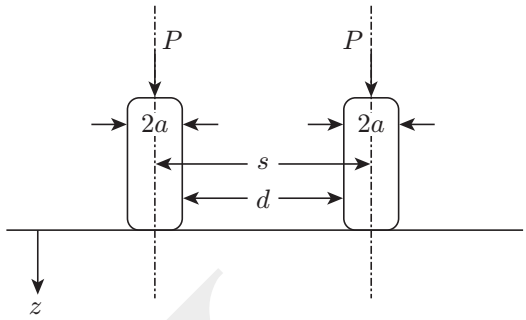
$$\text{PHF} = \frac{\text{Peak flow in 1 h}}{\dfrac{60}{\text{Time interval}} \times \text{Peak flow in 10 min}}$$

Peak flow in 1 h is given as 10 + 11 + 12 + 15 + 13 + 11 = 72
Peak flow in 10 min is given as 15
Time interval is 10
So,

$$\text{PHF} = \frac{72}{\dfrac{60}{10} \times 15} = 0.8$$

Ans. (0.8)

55. For the dual-wheel carrying assembly shown in the figure, P is the load on each wheel, a is the radius of the contact area of the wheel, s is the spacing between the wheels, and d is the clear distance between the wheels. Assuming that the ground is an elastic, homogeneous, and isotropic half space, the ratio of Equivalent Single Wheel Load (ESWL) at depth $z = d/2$ to the ESWL at depth $z = 2s$ is _____. (round off to one decimal place) (Consider the influence angle to be 45° for the linear dispersion of stress with depth)



Solution: We know that:
At $z = d/2$, there is no stress overlap. So, equivalent single wheel load at depth $d/2 = P$
At $z = s$, there is stress overlap. So, equivalent single wheel load at depth $2s = 2P$
Therefore, the ratio is

$$\frac{(\text{ESWL})_{d/2}}{(\text{ESWL})_{2S}} = \frac{P}{2P} = 0.5$$

Ans. (0.5)

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