SOLVED GATE (CIVIL ENGINEERING SET-II) 2022

Engineering Mathematics and Technical Section

Q. No. 1–25 Multiple Choice Question (MCQ) Carry One Mark Each

1. The function $f(x, y)$ satisfies the Laplace equation

$$\nabla^2 f(x,y) = 0$$

on a circular domain of radius $r = 1$ with its center at point P with coordinates $x = 0$, $y = 0$. The value of this function on the circular boundary of this domain is equal to 3. The numerical value of $f(0, 0)$ is:

(A) 0 (B) 2
(C) 3 (D) 1

Solution: As the function has a constant value of 3 at all points on the circular boundary, therefore $f(0, 0) = 3$.

Ans. (C)

2. $\int\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\right)dx$ is equal to

(A) $\frac{1}{1+x}$ + Constant

(B) $\frac{1}{1+x^2}$ + Constant

(C) $-\frac{1}{1-x}$ + Constant

(D) $-\frac{1}{1-x^2}$ + Constant

Solution: Let

$$\int\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\right)dx = \int f(x)dx,$$

where $f(x)$ is the infinite series of x.

Given function $f(x)$ is Maclaurin series expansion of $\ln(x + 1)$.

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

So, the integral becomes

$$\int \ln(x+1)dx$$

$$\int \ln(x+1)dx = (x+1)[\ln(x+1) - 1] + C$$

Ans. (*)

3. For a linear elastic and isotropic material, the correct relationship among Young's modulus of elasticity (E), Poisson's ratio (ν), and shear modulus (G) is

(A) $G = \frac{E}{2(1+\nu)}$ (B) $G = \frac{E}{(1+2\nu)}$

(C) $E = \frac{G}{2(1+\nu)}$ (D) $E = \frac{G}{(1+2\nu)}$

Solution: The correct relationship between Young's modulus of elasticity, Poisson's ratio, and shear modulus is

$$E = 2G(1+\nu) \Rightarrow G = \frac{E}{2(1+\nu)}$$

Ans. (A)

4. Read the following statements relating to flexure of reinforced concrete beams:

I. In *over-reinforced* sections, the failure strain in concrete reaches earlier than the yield strain in steel.

II. In *under-reinforced* sections, steel reaches yielding at a load lower than the load at which the concrete reaches failure strain.

III. *Over-reinforced* beams are recommended in practice as compared to the under-reinforced beams.

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IV. In *balanced* sections, the concrete reaches failure strain earlier than the yield strain in tensile steel.

Each of the above statements is either True or False. Which one of the following combinations is correct?
(A) I (True), II (True), III (False), IV (False)
(B) I (True), II (True), III (False), IV (True)
(C) I (False), II (False), III (True), IV (False)
(D) I (False), II (True), III (True), IV (False)

Solution: As per the limit state method:
In over-reinforced sections, the failure strain in concrete reaches earlier than the yield strain in steel.
In under reinforced sections, steel reaches yielding at a load lower than the load at which the concrete reaches failure strain.
Under reinforced beams are recommended in practice as compared to the over-reinforced beams.
In balanced sections, the concrete and steel reach the failure strain together.

Ans. (A)

5. Match all the possible combinations between Column X (Cement compounds) and Column Y (Cement properties):

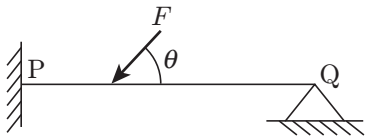
Column X	**Column Y**
(i) C_3S	(P) Early age strength
(ii) C_2S	(Q) Later age strength
(iii) C_3A	(R) Flash setting
	(S) Highest heat of hydration
	(T) Lowest heat of hydration

Which one of the following combinations is correct?
(A) (i) - (P), (ii) - (Q) and (T), (iii) - (R) and (S)
(B) (i) - (Q) and (T), (ii) - (P) and (S), (iii) - (R)
(C) (i) - (P), (ii) - (Q) and (R), (iii) - (T)
(D) (i) - (T), (ii) - (S), (iii) - (P) and (Q)

Solution:
(i) C_3S compound is used in cement for early age strength.
(ii) C_2S compound is used in cement for later age strength and has lowest heat of hydration.
(iii) C_3A compound is used in cement for flash setting and has highest heat of hydration.

Ans. (A)

6. Consider a beam PQ fixed at P, hinged at Q, and subjected to a load F as shown in figure (not drawn to scale). The static and kinematic degrees of indeterminacy, respectively, are



(A) 2 and 1
(B) 2 and 0
(C) 1 and 2
(D) 2 and 2

Solution: The static degree of indeterminacy is given by

$$D_s = \text{Number of unknown reactions} - \text{Number of equilibrium equations}$$
$$= 5 - 3 = 2$$

The kinematic degree of indeterminacy is the minimum number of movements or degree of freedom with which the kinematic configuration of the overall structure can be defined, that is, the number of unknown independent movements of the structure. Here the joint P is fixed, no movement is allowed at joint P, joint Q is hinged so only rotation is allowed at that joint. So,

$$D_k = 3j - R$$
$$= 3 \times 2 - 5 = 1$$

Ans. (A)

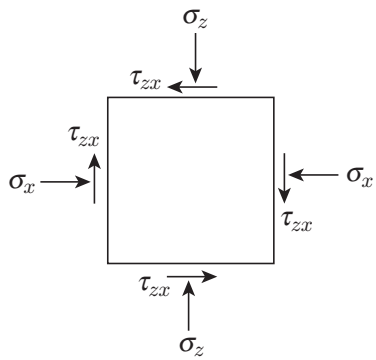
7. Read the following statements:
(P) While designing a shallow footing in sandy soil, monsoon season is considered for critical design in terms of bearing capacity.
(Q) For slope stability of an earthen dam, sudden drawdown is never a critical condition.
(R) In a sandy sea beach, quicksand condition can arise only if the critical hydraulic gradient exceeds the existing hydraulic gradient.
(S) The active earth thrust on a rigid retaining wall supporting homogeneous cohesionless backfill will reduce with the lowering of water table in the backfill.

Which one of the following combinations is correct?
(A) (P)-True, (Q)-False, (R)-False, (S)-False
(B) (P)-False, (Q)-True, (R)-True, (S)-True
(C) (P)-True, (Q)-False, (R)-True, (S)-True
(D) (P)-False, (Q)-True, (R)-False, (S)-False

Solution: While designing a shallow footing in sandy soil, monsoon season is considered for critical design in terms of bearing capacity.
For slope stability of an earthen dam, sudden drawdown is considered as critical condition as the direction of flow in reversed.
In a sandy sea beach, the quicksand condition arises when the existing hydraulic gradient exceeds the critical hydraulic gradient.
The active earth thrust on a rigid retaining wall supporting homogenous cohesionless backfill increases when the water table is lowered.

Ans. (A)

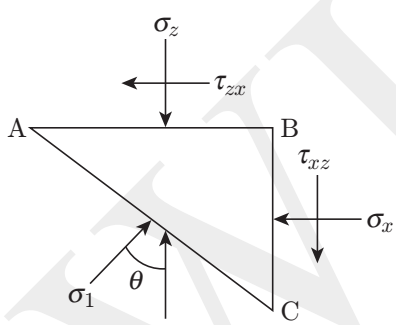
8. Stresses acting on an infinitesimal soil element are shown in the figure (with $\sigma_z > \sigma_x$). The major and minor principal stresses are σ_1 and σ_3, respectively. Considering the compressive stresses as positive, which one of the following expressions correctly represents the angle between the major principal stress plane and the horizontal plane?



(A) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_1 - \sigma_x}\right)$ (B) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_3 - \sigma_x}\right)$

(C) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_1 + \sigma_x}\right)$ (D) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_1 + \sigma_3}\right)$

Solution: Draw the transformed stress diagram for the soil particle as follows:



Equate the stresses in horizontal direction as follows:

$$\sum F_x = 0$$
$$\Rightarrow \sigma_x(\text{BC}) - \tau_{zx}(\text{AB}) + \sigma_1 \sin\theta = 0$$

Convert the lengths of the soil element in terms of AC:

$$\sigma_x(\text{AC}\sin\theta) + \tau_{zx}(\text{AC}\cos\theta) - \sigma_1(\text{AC}\sin\theta) = 0$$

Divide the whole equation by $\cos\theta$:

$$\frac{\sigma_x(\text{AC}\sin\theta)}{\cos\theta} + \frac{\tau_{zx}(\text{AC}\cos\theta)}{\cos\theta} - \frac{\sigma_1(\text{AC}\sin\theta)}{\cos\theta} = 0$$
$$\Rightarrow \sigma_x \tan\theta + \tau_{zx} = \sigma_1 \tan\theta$$
$$\Rightarrow \tan\theta(\sigma_1 - \sigma_x) = \tau_{zx}$$
$$\Rightarrow \tan\theta = \frac{\tau_{zx}}{(\sigma_x - \sigma_1)}$$

Ans. (A)

9. Match Column X with Column Y:

Column X	**Column Y**
(P) Horton equation	(I) Design of alluvial channel
(Q) Penman method	(II) Maximum flood discharge
(R) Chezy's formula	(III) Evapotranspiration
(S) Lacey's theory	(IV) Infiltration
(T) Dicken's formula	(V) Flow velocity

Which one of the following combinations is correct?
(A) (P)-(IV), (Q)-(III), (R)-(V), (S)-(I), (T)-(II)
(B) (P)-(III), (Q)-(IV), (R)-(V), (S)-(I), (T)-(II)
(C) (P)-(IV), (Q)-(III), (R)-(II), (S)-(I), (T)-(V)
(D) (P)-(III), (Q)-(IV), (R)-(I), (S)-(V), (T)-(II)

Solution: Horton equation is used to determine the infiltration as follows:

$$F_T = F_t + [F_o - F_f]e^{-kt}$$

Penman equation is used to determine the evapotranspiration by using the energy balance and mass transfer approach as follows:

$$E_{\text{mass}} = \frac{mR_n\rho_a c_p(\delta e)g_a}{\lambda_v(m+\gamma)}$$

Chezy's formula is used to calculate the flow velocity as follows:

$$V = C\sqrt{RS}$$

Lacry's theory is used to design the alluvial channel by using the concept of initial, true, and final regime. Dicken's formula is used to determine the maximum flood discharge as follows:

$$Q_p = C_D A^{3/2}$$

Ans. (A)

10. In a certain month, the reference crop evapotranspiration at a location is 6 mm/day. If the crop coefficient and soil coefficient are 1.2 and 0.8, respectively, the actual evapotranspiration in mm/day is
(A) 5.76 (B) 7.20
(C) 6.80 (D) 8.00

Solution: Actual evapotranspiration is given by

$$\text{Actual evapotranspiration} = K_s \times K_c \times \text{Evapotranspiration}$$
$$= 0.8 \times 1.2 \times 6 = 5.76 \text{ mm}$$

Ans. (A)

11. The dimension of dynamic viscosity is:

(A) $ML^{-1}T^{-1}$ (B) $ML^{-1}T^{-2}$

(C) $ML^{-2}T^{-2}$ (D) $ML^{0}T^{-1}$

Solution: The shear stress is given by

$$\tau = \mu \frac{du}{dy} \Rightarrow \mu = \tau \frac{dy}{du}$$

The dimensional formula for dynamic viscosity is

$$\mu = \frac{[MLT^{-2}]}{[L^2]} \frac{[L]}{[LT^{-1}]} = [ML^{-1}T^{-1}]$$

Ans. (A)

12. A process equipment emits 5 kg/h of volatile organic compounds (VOCs). If a hood placed over the process equipment captures 95% of the VOCs, then the fugitive emission in kg/h is

(A) 0.25 (B) 4.75

(C) 2.50 (D) 0.48

Solution: Fugitive emission is given by

$$\text{Fugitive emission} = \text{Capturing efficiency} \times \text{VOC emissions} = \frac{(100-95)}{100} \times 5 = 0.25 \text{ kg/h}$$

Ans. (A)

13. Match the following attributes of a city with the appropriate scale of measurements.

Attribute	**Scale of measurement**
(P) Average temperature (°C) of a city	(I) Interval
(Q) Name of a city	(II) Ordinal
(R) Population density of a city	(III) Nominal
(S) Ranking of a city based on ease of business	(IV) Ratio

Which one of the following combinations is correct?

(A) (P)-(I), (Q)-(III), (R)-(IV), (S)-(II)

(B) (P)-(II), (Q)-(I), (R)-(IV), (S)-(III)

(C) (P)-(II), (Q)-(III), (R)-(IV), (S)-(I)

(D) (P)-(I), (Q)-(II), (R)-(III), (S)-(IV)

Solution: The average temperature of a city is taken at different intervals of time.

Name of a city is considered as Nominal;

The population density is taken in ratio and;

The ranking of a city based on ease of business is termed as ordinal.

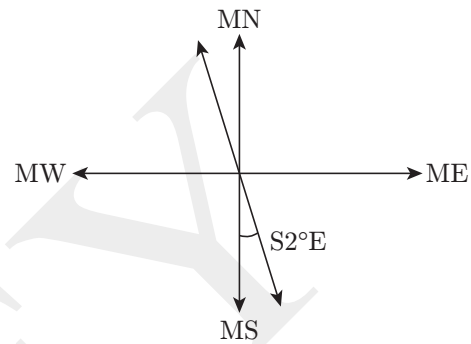
Ans. (A)

14. If the magnetic bearing of the Sun at a place at noon is S 2° E, the magnetic declination (in degrees) at that place is

(A) 2° E (B) 2° W

(C) 4° E (D) 4° W

Solution: The magnetic bearing of the Sun at a place at noon is S 2° E is shown in the following figure:



At noon, the sun should be at the true north, therefore, the magnetic declination is 2° E.

Ans. (A)

15. P and Q are two square matrices of the same order. Which of the following statement(s) is/are correct?

(A) If P and Q are invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$.

(B) If P and Q are invertible, then $[QP]^{-1} = P^{-1}Q^{-1}$.

(C) If P and Q are invertible, then $[PQ]^{-1} = P^{-1}Q^{-1}$.

(D) If P and Q are not invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$.

Solution: Properties of matrix:

If P and Q are two square matrices and are invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$,

And similarly: $[QP]^{-1} = P^{-1}Q^{-1}$.

Ans. (A, B)

16. In a solid waste handling facility, the moisture contents (MC) of food waste, paper waste, and glass waste were found to be MCf, MCp, and MCg, respectively. Similarly, the energy contents (EC) of plastic waste, food waste, and glass waste were found to be ECpp, ECf, and ECg, respectively. Which of the following statement(s) is/are correct?

(A) MCf > MCp > MCg

(B) ECpp > ECf > ECg

(C) MCf < MCp < MCg

(D) ECpp < ECf < ECg

Solution: As per the standard values of moisture and energy contents of solid waste from typical specific weight and moisture content data for

residential, commercial, industrial, and agricultural wastes, the correct orders are:

MCf > MCp > MCg; and ECpp > ECf > ECg

Ans. (A, B)

17. To design an optimum municipal solid waste collection route, which of the following is/are NOT desired:

(A) Collection vehicle should not travel twice down the same street in a day.
(B) Waste collection on congested roads should not occur during rush hours in morning or evening.
(C) Collection should occur in the uphill direction.
(D) The last collection point on a route should be as close as possible to the waste disposal facility.

Solution: The collection of the solid waste should be started at the top of the hill and occur in the downhill direction as the vehicle gets filled as it goes down.

Ans. (C)

18. For a traffic stream, v is the space mean speed, k is the density, q is the flow, v_f is the free flow speed, and k_j is the jam density. Assume that the speed decreases linearly with density. Which of the following relation(s) is/are correct?

(A) $q = k_j k - \left(\frac{k_j}{v_f}\right)k^2$ (B) $q = v_f k - \left(\frac{v_f}{k_j}\right)k^2$

(C) $q = v_f v - \left(\frac{v_f}{k_j}\right)v^2$ (D) $q = k_j v - \left(\frac{k_j}{v_f}\right)v^2$

Solution: As per the Greenshield formula:

$$v = v_f\left[1 - \frac{k}{k_j}\right]$$

and $q = kv$

So, $$q = \left(v_f\left[1 - \frac{k}{k_j}\right]\right)k = v_f k - \frac{v_f}{k_j}k^2$$

Again substitute $k = q/v$ in the general greenshield formula:

$$q = \left(v_f\left[1 - \frac{\left(\frac{q}{v}\right)}{k_j}\right]\right)\left(\frac{q}{v}\right)$$

$$= \frac{v^2 k_j}{v_f}\left[\frac{v_f}{v} - 1\right] = k_j v - \frac{k_j}{v_f}v^2$$

Ans. (B, D)

19. The error in measuring the radius of a 5 cm circular rod was 0.2%. If the cross-sectional area of the rod was calculated using this measurement, then the resulting absolute percentage error in the computed area is _____. (round off to two decimal places)

Solution: The absolute percentage error in cross-section area is given by:

$$\text{Error\%} = \frac{dA}{A}\times 100$$

$$\frac{dA}{A}\times 100 = \frac{\frac{d(\pi r^2)}{dr}}{\pi r^2}\times 100$$

$$= \frac{2\pi r dr}{\pi r^2}\times 100 = 2\times\left(\frac{dr}{r}\right)\times 100$$

Given that, $r = 5$ cm and $dr/r = 0.2\%$. Then,

$$\text{Error\%} = 2\times\frac{0.2}{100}\times 100 = 0.40$$

Ans. (0.40)

20. The components of pure shear strain in a sheared material are given in the matrix form:

$$\varepsilon = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Here, Trace(ε) = 0. Given, P = Trace(ε^8) and Q = Trace(ε^{11}). The numerical value of $(P + Q)$ is _____. (in integer)

Solution: Express the eigen vector of the matrix as follows:

$$|A - \lambda I| = 0$$

$$\Rightarrow |\varepsilon - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (\lambda^2 - 1) - 1 = 0$$

$$\Rightarrow \lambda = \pm\sqrt{2}$$

So, the eigen values for matrix ε are $+\sqrt{2}$ and $-\sqrt{2}$.

Determine the values of P and Q with the help of trace of the matrix ε as follows:

$$P = \text{Trace}(\varepsilon^8) = (\sqrt{2})^8 + (-\sqrt{2})^8 = 32$$

$$Q = \text{Trace}(\varepsilon^{11}) = (\sqrt{2})^{11} + (-\sqrt{2})^{11} = 0$$

Therefore, the value of $P + Q$ is

$$P + Q = 32 + 0 = 32$$

Ans. (32)

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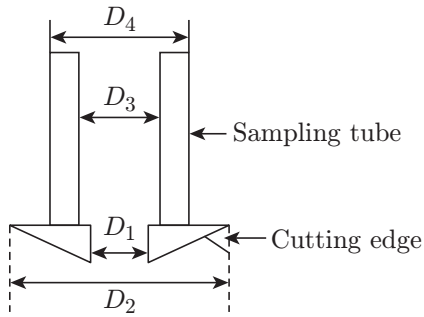
21. The inside diameter of a sampler tube is 50 mm. The inside diameter of the cutting edge is kept such that the Inside Clearance Ratio (ICR) is 1.0% to minimize the friction on the sample as the sampler tube enters into the soil. The inside diameter (in mm) of the cutting edge is _____. (round off to two decimal places)

Solution: Here, we know that:
D_1 = Inside diameter of cutting edge, D_2 = Outside diameter of cutting edge, D_3 = Inside diameter of sampling tube, and D_4 = Outside diameter of sampling tube.



So, inside clearance ratio is given by

$$\text{Inside clearance ratio} = \frac{D_3 - D_1}{D_1} \times 100$$

$$\Rightarrow 1 = \frac{50 - D_1}{D_1} \times 100$$

$$\Rightarrow D_1 = 49.5\,\text{mm}$$

Ans. (49.50)

22. A concentrically loaded isolated square footing of size 2 m × 2 m carries a concentrated vertical load of 1000 kN. Considering Boussinesq's theory of stress distribution, the maximum depth (in m) of the pressure bulb corresponding to 10% of the vertical load intensity will be _____. (round off to two decimal places)

Solution: We know that Boussinesq's equation is as follows:

$$\sigma_z = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}\frac{Q}{z^2}$$

Here, $r = 0$. So

$$\sigma_z = 0.477\left(\frac{Q}{z^2}\right) \quad (1)$$

From given data, an isobar for which

$$\sigma_z = 0.1q = 0.1\left(\frac{Q}{A}\right) = \frac{0.1Q}{4} \quad (2)$$

From Eqs. (1) and (2), we have

$$0.477\left(\frac{Q}{z^2}\right) = \frac{0.1Q}{4}$$

$$\Rightarrow z = 4.37\,\text{m}$$

Ans. (4.37)

23. In a triaxial unconsolidated undrained (UU) test on a saturated clay sample, the cell pressure was 100 kPa. If the deviatoric stress at failure was 150 kPa, then the undrained shear strength of the soil is _____ kPa. (in integer)

Solution: We know that in triaxial shear strength test:

$$\sigma_1 = \sigma_d + \sigma_3$$

where σ_1 is major principle stress, σ_3 is minor principle stress and σ_d is deviator stress. So,

$$\sigma_1 = 150 + 100 = 250\,\text{kPa}$$

The undrained shear strength of the soil is

$$\tau_f = \frac{\sigma_1 - \sigma_3}{2} = \frac{250 - 100}{2} = 75\,\text{kPa}$$

Ans. (75)

24. A flood control structure having an expected life of n years is designed by considering a flood of return period T years. When $T = n$, and $n \to \infty$, the structure's hydrologic risk of failure in percentage is _____. (round off to one decimal place)

Solution: We know that:

$$R = 1 - (1 - P)^n$$

And $$P = \frac{1}{T}$$

As per the problem, $T = n$, and $n \to \infty$. So,

$$R = 1 - \left(1 - \frac{1}{T}\right)^n$$

$$= 1 - \lim_{n\to\infty}\left(1 - \frac{1}{n}\right)^n = 1 - e^{-1} = 0.6321 \times 100$$

$$= 63.21\% \approx 63.2\%$$

Ans. (63.2)

25. The base length of the runway at the mean sea level (MSL) is 1500 m. If the runway is located at an altitude of 300 m above the MSL, the actual length (in m) of the runway to be provided is _____. (round off to the nearest integer)

Solution: It has been concluded that the elevation correction in length is increased by 7% for every 300 m rise from MSL. The actual length (in m) of the runway to be provided is

$$C_e = \frac{0.07}{300} \times 300 \times 1500 = 1605\,\text{m}$$

Ans. (1605)

Q. No. 26–55 Multiple Choice Question (MCQ) Carry Two Marks Each

26. Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ on the domain S given by $1 \le x \le 3$. The first and second derivatives are $f'(x)$ and $f''(x)$. Consider the following statements:

I. The given polynomial is zero at the boundary points $x = 1$ and $x = 3$.
II. There exists one local maxima of $f(x)$ within the domain S.
III. The second derivative $f''(x) > 0$ throughout the domain S.
IV. There exists one local minima of $f(x)$ within the domain S.

The correct option is:
(A) Only statements I, II and III are correct.
(B) Only statements I, II and IV are correct.
(C) Only statements I and IV are correct.
(D) Only statements II and IV are correct.

Solution: Given function:

$$f(x) = x^3 - 6x^2 + 11x - 6$$

At $x = 1$, we get

$$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6$$
$$= 1 - 6 + 11 - 6 = 0$$

At $x = 3$, we get

$$f(3) = (3)^3 - 6(3)^2 + 11(3) - 6$$
$$= 27 - 54 + 33 - 6 = 0$$

Therefore, statement I is correct.
Now, differentiating the equation with respect to x, we get

$$f'(x) = 3x^2 - 12x + 11$$

Put $f'(x) = 0$

$$f'(x) = 0 = 3x^2 - 12x + 11$$
$$\Rightarrow x = 2 \pm \frac{1}{\sqrt{3}}$$

Here, point of maxima is $2 - \frac{1}{\sqrt{3}}$ and point of minima is $2 + \frac{1}{\sqrt{3}}$.

And the point of maxima and minima exist in the domain $1 \le x \le 3$. Therefore, statement II and IV are also correct.

Now, differentiate the equation again with respect to x, we get

$$f''(x) = 6x - 12$$

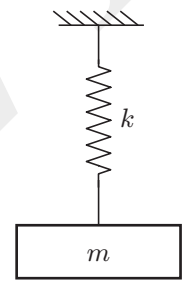
Put $f''(x) > 0$

$$6x - 12 > 0$$
$$\Rightarrow x > 2$$

For statement III to be true, x should always have a value greater than 2. Therefore, statement III is incorrect.

Ans. (B)

27. An undamped spring-mass system with mass m and spring stiffness k is shown in the figure. The natural frequency and natural period of this system are ω rad/s and T s, respectively. If the stiffness of the spring is doubled and the mass is halved, then the natural frequency and the natural period of the modified system, respectively, are



(A) 2ω rad/s and $T/2$ s (B) $\omega/2$ rad/s and $2T$ s
(C) 4ω rad/s and $T/4$ s (D) ω rad/s and T s

Solution: We know that, time period and frequency of the system is given by

$$T = \frac{2\pi}{\omega}, \omega = \sqrt{\frac{k}{m}}$$

The stiffness of the spring is doubled and the mass is halved, then the natural frequency becomes

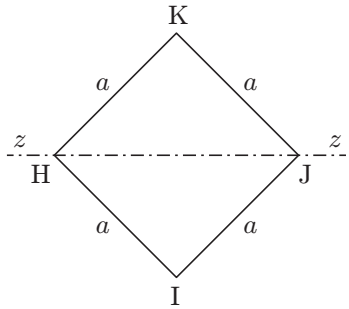
$$\omega' = \sqrt{\frac{2k}{\frac{m}{2}}} = 2\sqrt{\frac{k}{m}}$$

Substitute this value of natural frequency in time period, we get

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{2\sqrt{\frac{k}{m}}} = \frac{1}{2}\left(\frac{2\pi}{\omega}\right) = \frac{T}{2}$$

Ans. (A)

28. For the square steel beam cross-section shown in the figure, the shape factor about $z - z$ axis is S and the plastic moment capacity is M_P. Consider yield stress $f_y = 250$ MPa and $a = 100$ mm.



The values of S and M_P (rounded-off to one decimal place) are
(A) $S = 2.0$, $M_P = 58.9$ kN-m
(B) $S = 2.0$, $M_P = 100.2$ kN-m
(C) $S = 1.5$, $M_P = 58.9$ kN-m
(D) $S = 1.5$, $M_P = 100.2$ kN-m

Solution: The value of plastic moment is given by

$$M_p = f_y Z_p = f_y \times \frac{A}{2}(y_1 + y_2)$$

$$= 250 \times \frac{100 \times 100}{2}\left(\frac{100}{3\sqrt{2}} + \frac{100}{3\sqrt{2}}\right) = 58.925\,\text{kNm}$$

Shape factor for diamond shaped section is given by

$$S = \frac{M_p}{M_e} = \frac{58.925}{f_y \times \frac{a^4}{12} \times \frac{\sqrt{2}}{a}}$$

$$= \frac{58.925}{250 \times \frac{a^3}{6\sqrt{2}}} = \frac{58.925}{250 \times \frac{100^3}{6\sqrt{2}}} = 2$$

Ans. (A)

29. A post-tensioned concrete member of span 15 m and cross-section of 450 mm × 450 mm is prestressed with three steel tendons, each of cross-sectional area 200 mm^2. The tendons are tensioned one after another to a stress of 1500 MPa. All the tendons are straight and located at 125 mm from the bottom of the member. Assume the prestress to be the same in all tendons and the modular ratio to be 6. The average loss of prestress, due to elastic deformation of concrete, considering all three tendons is
(A) 14.16 MPa
(B) 7.08 MPa
(C) 28.32 MPa
(D) 42.48 MPa

Solution: The force in each tendon due to initial prestress is

$$P = 1500 \times \frac{200}{1000} = 300\,\text{kN}$$

Determine the stress in wire 2 as follows:

$$\text{Loss} = mf_c = 6\left(\frac{P}{A} + \frac{Pe^2}{I}\right)$$

$$= 6\left[\frac{300 \times 1000}{450 \times 450} + \frac{300 \times 1000 \times 100^2}{450 \times \frac{450^3}{12}}\right]$$

$$= 14.16\,\text{N/mm}^2$$

Determine the stress in wire 3 as follows:

$$\text{Loss} = mf_c = 6\left(\frac{P+P}{A} + \frac{(P+P)e^2}{I}\right)$$

$$= 6 \times 2\left[\frac{300 \times 1000}{450 \times 450} + \frac{300 \times 1000 \times 100^2}{450 \times \frac{450^3}{12}}\right]$$

$$= 28.32\,\text{N/mm}^2$$

Average loss of stress in all three wires:

$$\frac{0 + 14.16 + 28.32}{3} = 14.16\,\text{N/mm}^2$$

Ans. (A)

30. Match the following in Column X with Column Y:

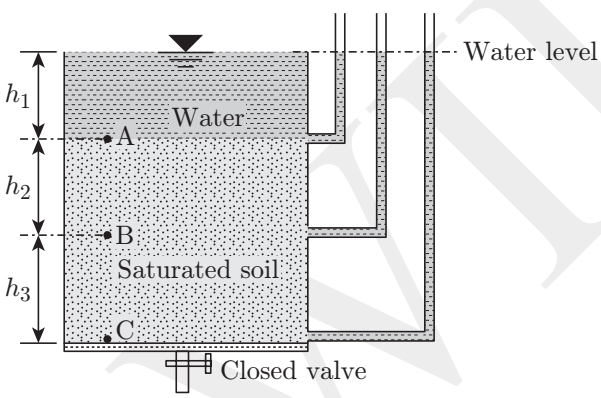
Column X	**Column Y**
(P) In a triaxial compression test, with increase of axial strain in loose sand under drained shear condition, the volumetric strain	(I) decreases.
(Q) In a triaxial compression test, with increase of axial strain in loose sand under undrained shear condition, the excess pore water pressure	(II) increases.
(R) In a triaxial compression test, the pore pressure parameter "B" for a saturated soil	(III) remains same.
(S) For shallow strip footing in pure saturated clay, Terzaghi's bearing capacity factor (N_q) due to surcharge	(IV) is always 0.0.
	(V) is always 1.0.
	(VI) is always 0.5.

Which one of the following combinations is correct?
(A) (P)-(I), (Q)-(II), (R)-(V), (S)-(V)
(B) (P)-(II), (Q)-(I), (R)-(IV), (S)-(V)
(C) (P)-(I), (Q)-(III), (R)-(VI), (S)-(IV)
(D) (P)-(I), (Q)-(II), (R)-(V), (S)-(VI)

Solution: In a triaxial compression test, with an increase of Axial strain in loose sand under drained shear conditions, the volumetric strain decreases.
In a triaxial compression test, with increase of axial strain in loose sand under undrained shear condition, the excess pore water pressure increases.
In a triaxial compression test, the pore pressure parameter "B" for a saturated soil is always 1.
For shallow strip footing in pure saturated clay, Terzaghi's bearing factor (N_q) due to surcharge is always 1.

Ans. (A)

31. A soil sample is underlying a water column of height h1, as shown in the figure. The vertical effective stresses at points A, B, and C are σ'_A, σ'_B, and σ'_C, respectively. Let γ_{sat} and γ' be the saturated and submerged unit weights of the soil sample, respectively, and γ_w be the unit weight of water. Which one of the following expressions correctly represents the sum $(\sigma'_A + \sigma'_B + \sigma'_C)$?



(A) $(2h_2 + h_3)\gamma'$ (B) $(h_1 + h_2 + h_3)\gamma'$
(C) $(h_2 + h_3)(\gamma_{sat} - \gamma_w)$ (D) $(h_1 + h_2 + h_3)\gamma_{sat}$

Solution: Determine the effective stress at A:

$$\sigma'_A = 0$$

Determine the effective stress at B:

$$\sigma'_B = \gamma' h_2$$

Determine the effective stress at C:

$$\sigma'_C = \gamma'(h_2 + h_3)$$

Now, the sum of all effective stresses:

$$\sigma'_A + \sigma'_B + \sigma'_C = \gamma' h_2 + \gamma'(h_2 + h_3) = (2h_2 + h_3)\gamma'$$

Ans. (A)

32. A 100 mg of HNO_3 (strong acid) is added to water, bringing the final volume to 1.0 liter. Consider the atomic weights of H, N, and O, as 1 g/mol, 14 g/mol, and 16 g/mol, respectively. The final pH of this water is (Ignore the dissociation of water.)
(A) 2.8 (B) 6.5
(C) 3.8 (D) 8.5

Solution: The balanced reaction of HNO_3 (strong acid) as follows:

$$HNO_3 \rightarrow H^+ + NO_3^-$$

Determine the number of moles of HNO_3 present in the solution:

$$\text{Moles} = \frac{100}{1 + 14 + 16 \times 3} = 1.587 \text{ m mol}$$

According to the reaction above,

$$1 \text{ mole of } HNO^3 = 1 \text{ mole of } H^+$$

Number of $[H^+]$ moles in 1 L water

$$= 1.587 \times 10^{-3} \text{ mol/L}$$

The final pH of this water is

$$pH = -\log[H^+] = -\log[1.587 \times 10^{-3}] = 2.8$$

Ans. (A)

33. In a city, the chemical formula of biodegradable fraction of municipal solid waste (MSW) is $C_{100}H_{250}O_{80}N$. The waste has to be treated by forced-aeration composting process for which air requirement has to be estimated.
Assume oxygen in air (by weight) = 23%, and density of air = 1.3 kg/m^3. Atomic mass: C = 12, H = 1, O = 16, N = 14. C and H are oxidized completely whereas N is converted only into NH_3 during oxidation. For oxidative degradation of 1 tonne of the waste, the required theoretical volume of air (in m^3/tonne) will be (round off to the nearest integer)
(A) 4749 (B) 8025
(C) 1418 (D) 1092

Solution: The balanced reaction of $C_{100}H_{250}O_{80}N$ (MSW) is

$$C_{100}H_{250}O_{80}N + xO_2 \rightarrow 100CO_2 + 123.5H_2O + NH_3$$

By balancing the moles of oxygen, we get

$$80 + 2x = (2 \times 100) + 123.5$$
$$\Rightarrow x = 121.75$$

The number of moles of MSW is

$$\text{Moles of MSW} = \frac{1 \text{ ton}}{12 \times 100 + 1 \times 250 + 16 \times 80 + 14}$$
$$= \frac{1}{2744}$$

Therefore, the moles of oxygen required is

$$\text{Moles of } O_2 = \frac{121.75}{2744} \times 16 \times 2 = 1.4197 \text{ ton}$$

The weight of the air that contains 1.4197 ton of oxygen is

$$1.3 \text{ kg of air contains} = 0.23 \times 1.3 \text{ kg of oxygen}$$

$$0.299 \text{ kg of oxygen} = 1.3 \text{ kg of air contains}$$

$$1.419 \text{ tin of oxygen} = \frac{1.3}{0.299} \times 1.419$$

$$= 6173.15 \text{ kg of air}$$

The required volume of the air is

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\Rightarrow 1.3 = \frac{6173.15}{V}$$

$$\Rightarrow V = \frac{6173.15}{1.3} = 4748.57\,\text{m}^3 \approx 4749\,\text{m}^3$$

Ans. (A)

34. A single-lane highway has a traffic density of 40 vehicles/km. If the time-mean speed and space-mean speed are 40 kmph and 30 kmph, respectively, the average headway (in seconds) between the vehicles is

(A) 3.00 (B) 2.25
(C) 8.33×10^{-4} (D) 6.25×10^{-4}

Solution: The traffic volume is

$$q = \text{Density} \times \text{Space mean speed}$$

Also the traffic volume is

$$q = \frac{3600}{H_t}$$

So,

$$40 \times 30 = \frac{3600}{H_t}$$

$$\Rightarrow H_t = 3\,\text{s}$$

Ans. (A)

35. Let y be a non-zero vector of size 2022×1. Which of the following statement(s) is/are **true**?

(A) yy^T is a symmetric matrix.
(B) y^Ty is an eigenvalue of yy^T.
(C) yy^T has a rank of 2022.
(D) yy^T is invertible.

Solution: Let us assume a matrix as follows:

$$y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{3\times1}$$

Perform the transpose of this matrix:

$$y^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}_{1\times3}$$

Now,

$$yy^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$$

And,

$$y^Ty = \left[1^2 + 1^2 + 1^2\right]$$

$$y^Ty = [3]_{1\times1}$$

Therefore, rank of this matrix = 1,

y^Ty is an eigen value of yy^T,

yy^T is a symmetrical matrix, and $\det(yy^T) = 0$, therefore yy^T is not invertible.

Ans. (A, B)

36. Which of the following statement(s) is/are correct?

(A) If a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the deflection at any point is equal to the load applied at that point.
(B) If a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the load at any point is equal to the deflection at that point.
(C) If a structure is acted upon by two force system Pa and Pb, in equilibrium separately, the external virtual work done by a system of forces Pb during the deformations caused by another system of forces Pa is equal to the external virtual work done by the Pa system during the deformation caused by the Pb system.
(D) The shear force in a conjugate beam loaded by the M/EI diagram of the real beam is equal to the corresponding deflection of the real beam.

Solution: According to the castigliano's second theorem, if a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the deflection at any point is equal to the load applied at that point. Hence, (A) is correct.

$$\frac{\partial U}{\partial \Delta} = P$$

According to the castigliano's first theorem, if a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the load at any point is equal to the deflection at that point. Hence, (B) is correct.

$$\frac{\partial U}{\partial P} = \Delta$$

According to the Betti's Law, if a structure is acted upon by two force system Pa and Pb, in equilibrium separately, the external virtual work done by a system of forces Pb during the deformations caused by another system of forces Pa is equal to the external virtual work done by the Pa system during the deformation caused by the Pb system. Therefore, (C) is correct.

As per the conjugate beam method, the shear force in a conjugate beam loaded by the M/EI diagram of the real beam is equal to the corresponding slope of the real beam. Hence, (D) is incorrect.

Ans. (A, B, C)

37. Water is flowing in a horizontal, frictionless, rectangular channel. A smooth hump is built on the channel floor at a section and its height is gradually increased to reach choked condition in the channel. The depth of water at this section is y_2 and that at its upstream section is y_1. The correct statement(s) for the choked and unchoked conditions in the channel is/are

(A) In choked condition, y_1 decreases if the flow is supercritical and increases if the flow is subcritical.

(B) In choked condition, y_2 is equal to the critical depth if the flow is supercritical or subcritical.

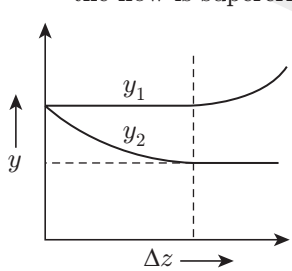
(C) In unchoked condition, y_1 remains unaffected when the flow is supercritical or subcritical.

(D) In choked condition, y_1 increases if the flow is supercritical and decreases if the flow is subcritical.

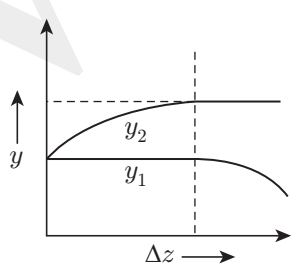
Solution: In choked condition, y_1 decreases if the flow is supercritical and increases if the flow is subcritical.

In choked condition, y_2 is equal to the critical depth if the flow is supercritical or subcritical.

In unchoked condition, y_1 remains unaffected when the flow is supercritical or subcritical.



Subcritical conditions



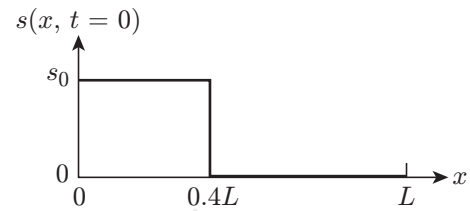
Supercritical conditions

Ans. (A, B, C)

38. The concentration $s(x, t)$ of pollutants in a one-dimensional reservoir at position x and time t satisfies the diffusion equation

$$\frac{\partial s(x, t)}{\partial t} = D\frac{\partial^2 s(x, t)}{\partial x^2}$$

on the domain $0 \le x \le L$, where D is the diffusion coefficient of the pollutants. The initial condition $s(x, 0)$ is defined by the step-function shown in the figure.



The boundary conditions of the problem are given by $\frac{\partial s(x, t)}{\partial x} = 0$ at the boundary points $x = 0$ and $x = L$ at all times. Consider $D = 0.1$ m^2/s, $s_0 = 5$ μmol/m, and $L = 10$ m. The steady-state concentration $\tilde{s}\left(\frac{L}{2}\right) = s\left(\frac{L}{2}, \infty\right)$ at the center $x = \frac{L}{2}$ of the reservoir (in μmol/m) is _____. (in integer)

Solution: From the given step function graph, the following values can be derived:

$$s_0 = s(0, t) = 5,\ s(0.4L, t) = 5,\ s(L, t) = 0$$

Calculate the concentration of the pollutants as follows:

When $0 > x > 0.4L$

$$\text{Concentration} = s_0 \times 0.4 \times L$$
$$= 5 \times 0.4 \times 10 = 20\ \mu\text{mol}$$

When $0.4L > x > L$,

$$\text{Concentration} = 0$$

So, the total concentration of the pollutants is

$$20\ \mu\text{mol} + 0\ \mu\text{mol} = 20\ \mu\text{mol}$$

Determine the concentration per meter length as follows:

$$\frac{20\ \mu\text{mol}}{10\ \text{m}} = 2\ \mu\text{mol/m}$$

Ans. (2)

39. A pair of six-faced dice is rolled thrice. The probability that the sum of the outcomes in each roll equals 4 in exactly two of the three attempts is _____. (round off to three decimal places)

Solution: To get the sum as 4 when a six faced dice is rolled twice, the possible events are as follows:

$$X = [(1, 3), (2, 2), (3, 1)]$$

The probability of the these events happening:

$$P(X) = \frac{3}{36} = \frac{1}{12}$$

The probability of these events not happening:

$$P(\bar{X}) = 1 - \frac{1}{12} = \frac{11}{12}$$

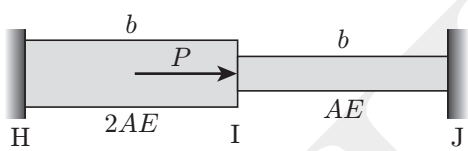
Determine the probability of the sum of the outcomes in each roll equals 4 in exactly two of the three attempts:

$$P(A) = {}^3C_2 P(X)^2 P(\bar{X})^1$$

$$= \frac{3!}{2! \times 1!} \times \left(\frac{1}{12}\right)^2 \times \left(\frac{11}{12}\right)^1 = 0.019$$

Ans. (0.019)

40. Consider two linearly elastic rods HI and IJ, each of length b, as shown in the figure. The rods are co-linear, and confined between two fixed supports at H and J. Both the rods are initially stress free. The coefficient of linear thermal expansion is α for both the rods. The temperature of the rod IJ is raised by ΔT, whereas the temperature of rod HI remains unchanged. An external horizontal force P is now applied at node I. It is given that $\alpha = 10^{-6}\ °C^{-1}$, $\Delta T = 50\,°C$, $b = 2$ m, $AE = 106$ N. The axial rigidities of the rods HI and IJ are $2AE$ and AE, respectively.



To make the axial force in rod HI equal to zero, the value of the external force P (in N) is _____. (round off to the nearest integer)

Solution: As the axial force in rod HI has to be 0, we will consider both ends of rod IJ as fixed.
Therefore, the total horizontal deflection of the rod IJ would be 0.
The horizontal deflection is given by

$$\delta_{\text{total}} = \delta_{\text{load}} + \delta_{\text{Temp}}$$

$$\Rightarrow 0 = \frac{PB}{AE} - B\alpha\Delta T$$

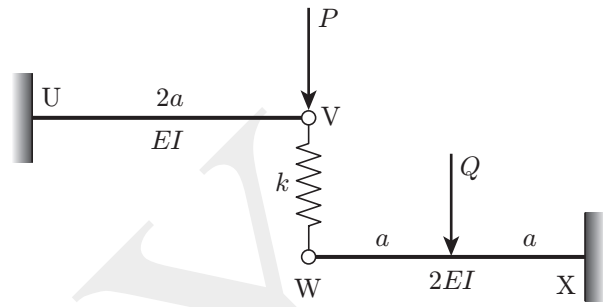
$$\Rightarrow \frac{PB}{AE} = B\alpha\Delta T$$

$$\Rightarrow P = AE\alpha\Delta T$$

$$\Rightarrow P = 10^6(10^{-6} \times 50) = 50\,\text{N}$$

Ans. (50)

41. The linearly elastic planar structure shown in the figure is acted upon by two vertical concentrated forces. The horizontal beams UV and WX are connected with the help of the vertical linear spring with spring constant $k = 20$ kN/m. The fixed supports are provided at U and X. It is given that flexural rigidity $EI = 105$ kN-m^2, $P = 100$ kN, and $a = 5$ m. Force Q is applied at the center of beam WX such that the force in the spring VW becomes zero.



The magnitude of force Q (in kN) is _____. (round off to the nearest integer)

Solution: As there should be no deflection in the spring, therefore, the deflection at V due to load P would be equal to the deflection at W due to load Q. So,

$$\delta_V = \delta_W$$

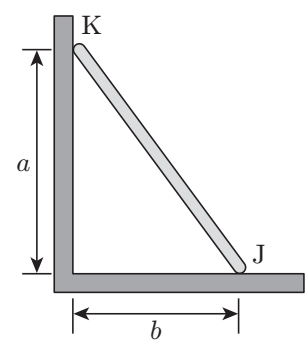
$$\Rightarrow \frac{P(2a)^3}{3EI} = \frac{Qa^3}{3(2EI)} + \frac{Qa^2}{2(2EI)} \times a$$

$$\Rightarrow \frac{100 \times (2 \times 10)^3}{3 \times 10^5} = \frac{Q \times 5^3}{3(2 \times 10^5)} + \frac{Q \times 5^2}{2(2 \times 10^5)} \times 5$$

$$\Rightarrow Q = 640 \text{ kN}$$

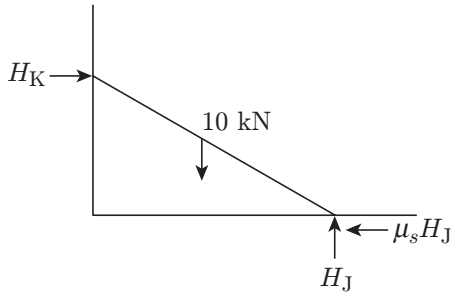
Ans. (640)

42. A uniform rod KJ of weight w shown in the figure rests against a frictionless vertical wall at the point K and a rough horizontal surface at point J. It is given that $w = 10$ kN, $a = 4$ m and $b = 3$ m.



The minimum coefficient of static friction that is required at the point J to hold the rod in equilibrium is _____. (round off to three decimal places)

Solution: Draw the free body diagram of the rod as follows:



Since, the rod is static, means net external force is equal to zero. So,

$$\sum F_y = 0$$
$$\Rightarrow H_J - 10 = 0$$
$$\Rightarrow H_J = 10\,\text{kN}$$

And Total bending moment at point K is equal to zero. So,

$$\sum M_K = 0$$
$$\Rightarrow (10 \times 1.5) - (H_J \times 3) + (\mu_s H_J \times 4) = 0$$
$$\Rightarrow (10 \times 1.5) - (10 \times 3) + (\mu_s \times 10 \times 4) = 0$$
$$\Rightarrow \mu_s = 0.370$$

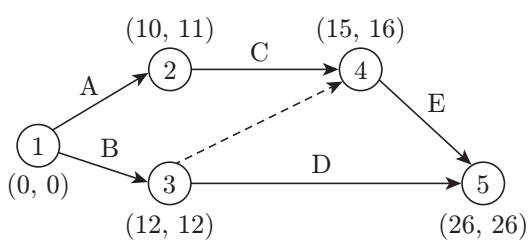
Ans. (0.370)

43. The activities of a project are given in the following table along with their durations and dependency.

Activities	**Duration (days)**	**Depends on**
A	10	–
B	12	–
C	5	A
D	14	B
E	10	B, C

The total float of the activity E (in days) is _____. (in integer)

Solution: Draw the network diagram of the activities as follows:



The total float is

$$F_T = T_L^j - T_L^i - t^{ij} = 26 - 15 - 10 = 1$$

Ans. (1)

44. A group of total 16 piles are arranged in a square grid format. The center-to-center spacing (s) between adjacent piles is 3 m. The diameter (d) and length of embedment of each pile are 1 m and 20 m, respectively. The design capacity of each pile is 1000 kN in the vertical downward direction. The pile group efficiency (η_g) is given by

$$\eta_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)m + (m-1)n}{mn}\right]$$

where m and n are number of rows and columns in the plan grid of pile arrangement, and $\theta = \tan^{-1}\left(\frac{d}{g}\right)$.

The design value of the pile group capacity (in kN) in the vertical downward direction is _____. (round off to the nearest integer)

Solution: The pile group efficiency is given by

$$n_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)m + (m-1)n}{mn}\right]$$
$$= 1 - \frac{\tan^{-1}\left(\frac{d}{s}\right)}{90}\left[\frac{(n-1)m + (m-1)n}{mn}\right]$$
$$= 1 - \frac{\tan^{-1}\left(\frac{1}{3}\right)}{90}\left[\frac{(4-1)4 + (4-1)4}{4 \times 4}\right]$$
$$= 0.693$$

Determine the design value of the pile group capacity as follows:

$$P_g = nPn_g = 16 \times 1000 \times 0.693 = 11088\,\text{kN}$$

Ans. (11088)

45. A saturated compressible clay layer of thickness h is sandwiched between two sand layers, as shown in the figure. Initially, the total vertical stress and pore water pressure at point P, which is located at the mid-depth of the clay layer, were 150 kPa and 25 kPa, respectively. Construction of a building caused an additional total vertical stress of 100 kPa at P. When the vertical effective stress at P is 175 kPa, the percentage of consolidation in the clay layer at P is _____. (in integer)

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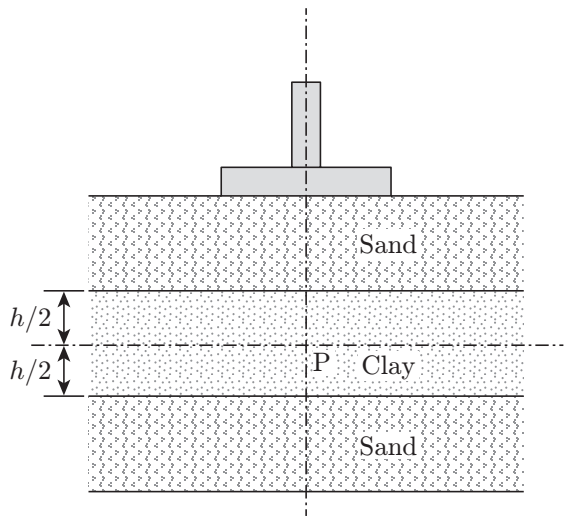


Solution: The initial effective stress before the building load is

$$\bar{\sigma}_1 = \sigma - u = 150 - 25 = 125\,\text{kPa}$$

The final effective stress after the building load is

$$\bar{\sigma}_2 = \sigma + P - u = 150 + 100 - 25 = 225\,\text{kPa}$$

The percentage of consolidation when the vertical effective stress is 175 kPa is

$$\%\ \text{consolidation} = \frac{P - \bar{\sigma}_1}{\bar{\sigma}_2 - \bar{\sigma}_1} \times 100$$

$$= \frac{175 - 125}{225 - 125} \times 100 = 50\%$$

Ans. (50)

46. A hydraulic jump takes place in a 6 m wide rectangular channel at a point where the upstream depth is 0.5 m (just before the jump). If the discharge in the channel is 30 m^3/s and the energy loss in the jump is 1.6 m, then the Froude number computed at the end of the jump is _____. (round off to two decimal places) (Consider the acceleration due to gravity as 10 m/s^2.)

Solution: The velocity of flow at the upstream as follows:

$$v_1 = \frac{Q}{By_1} = \frac{30}{6 \times 0.5} = 10\,\text{m/s}$$

The Froude's number at the upstream of the jump is

$$F_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{10}{\sqrt{10 \times 0.5}} = 4.47$$

The depth of the flow at the downstream of the jump is

$$\frac{y_2}{y_1} = \frac{-1 + \sqrt{1 + 8F_1^2}}{2}$$

$$\Rightarrow y_2 = \frac{-1 + \sqrt{1 + 8 \times 4.47^2}}{2} \times 0.5 = 2.92$$

The value of Froude's number at the end of the jump is

$$F_2^2 = \frac{Q^2 T}{gA^3} = \frac{30^2 \times 6}{10 \times (6 \times 2.92)^3}$$

$$\Rightarrow F_2 = \sqrt{\frac{30^2 \times 6}{10 \times (6 \times 2.92)^3}} = 0.316 \approx 0.32$$

Ans. (0.32)

47. A pump with an efficiency of 80% is used to draw groundwater from a well for irrigating a flat field of area 108 hectares. The base period and delta for paddy crop on this field are 120 days and 144 cm, respectively. Water application efficiency in the field is 80%. The lowest level of water in the well is 10 m below the ground. The minimum required horse power (hp) of the pump is _____. (round off to two decimal places) (Consider 1 hp = 746 W; unit weight of water = 9810 N/m^3)

Solution: The discharge required is given by

$$D = \frac{A}{Q}$$

$$\Rightarrow \frac{8.64B}{\Delta} = \frac{108 \times 10^4}{Q}$$

$$\Rightarrow \frac{8.64 \times 120}{1.44} \times 10^4 = \frac{108 \times 10^4}{Q}$$

$$\Rightarrow Q = \frac{108 \times 10^4 \times 1.44}{8.64 \times 120 \times 10^4} = 0.15\,\text{m}^3/\text{s}$$

The applied discharge is given by

$$n_a = \frac{Q}{Q_1}$$

$$\Rightarrow Q_1 = \frac{0.15}{0.8} = 0.1875\,\text{m}^3/\text{s}$$

The power required is given by

$$P = \frac{\rho Q g h}{n_a} = \frac{\gamma Q h}{n_a}$$

$$= \frac{9810 \times 0.1875 \times 10}{746 \times 0.8} = 30.82\,\text{hp}$$

Ans. (30.82)

48. Two discrete spherical particles (P and Q) of equal mass density are independently released in water. Particle P and particle Q have diameters of 0.5 mm and 1.0 mm, respectively. Assume Stokes' law is valid. The drag force on particle Q will be _____ times the drag force on particle P. (round off to the nearest integer)

Solution: The expression for the drag force on a spherical particle is

$$F_D = \frac{\pi}{4}(\rho_p - \rho_f)d^3 g$$

The ratio of drag force on a spherical particle Q and P is

$$\frac{(F_D)_Q}{(F_D)_P} = \frac{\frac{\pi}{4}(\rho_p - \rho_f)d_Q^3 g}{\frac{\pi}{4}(\rho_p - \rho_f)d_P^3 g}$$

$$\Rightarrow \frac{(F_D)_Q}{(F_D)_P} = \frac{d_Q^3}{d_P^3} = \frac{1^3}{0.5^3} = 8$$

$$\Rightarrow (F_D)_Q = 8(F_D)_P$$

Ans. (8)

49. At a municipal waste handling facility, 30 metric ton mixture of food waste, yard waste, and paper waste was available. The moisture content of this mixture was found to be 10%. The ideal moisture content for composting this mixture is 50%. The amount of water to be added to this mixture to bring its moisture content to the ideal condition is _____ metric ton. (in integer)

Solution: The weight of water initially present in the waste is given by

$$w = \frac{W_w}{W_s}$$

$$\Rightarrow 0.10 = \frac{W_w}{30}$$

$$\Rightarrow W_w = 3\,\text{MT}$$

Therefore, the weight of solids present is

$$W = W_s + W_w$$

$$\Rightarrow 30 = 3 + W_s$$

$$\Rightarrow W_s = 27\,\text{MT}$$

To make the water content as 50%, total weight of the water that should be present in the water is 27 MT.

Therefore, the amount of water that needs to be added:

$$W_w = 27 - 3 = 24\,\text{MT}$$

Ans. (24)

50. A sewage treatment plant receives sewage at a flow rate of 5000 m^3/day. The total suspended solids (TSS) concentration in the sewage at the inlet of primary clarifier is 200 mg/L. After the primary treatment, the TSS concentration in sewage is reduced by 60%. The sludge from the primary clarifier contains 2% solids concentration. Subsequently, the sludge is subjected to gravity thickening process to achieve a solids concentration of 6%. Assume that the density of sludge, before and after thickening, is 1000 kg/m^3. The daily volume of the thickened sludge (in m^3/day) will be _____. (round off to the nearest integer)

Solution: The weight of the total suspended solids at the inlet of the primary sedimentation tank is

$$= 5000 \times 10^3 \times 200 \times 10^{-6} = 10^3\ \text{kg/d}$$

The weight of the sludge coming out of primary sedimentation tank is

$$0.6 \times 10^3 = 600\,\text{kg/d}$$

The total mass of the sludge is

$$\frac{600 \times 100}{6} = 10^4\ \text{kg/d}$$

The volume of the sludge is

$$\rho = \frac{M}{V}$$

$$\Rightarrow 1000 = \frac{10000}{V}$$

$$\Rightarrow V = 10\,\text{m}^3/\text{d}$$

Ans. (10)

51. A sample of air analyzed at 25°C and 1 atm pressure is reported to contain 0.04 ppm of SO_2. Atomic mass of S = 32, O = 16. The equivalent SO_2 concentration (in μg/m^3) will be _____. (round off to the nearest integer)

Solution: According to the Ideal Gas Law, we know that

$$PV = nRT$$

On comparing two gases, with Ideal Gas Law, we have

$$\frac{P_1V_1}{nRT_1} = \frac{P_2V_2}{nRT_2}$$

$$\Rightarrow \frac{PV_1}{nRT_1} = \frac{PV_2}{nRT_2}$$

$$\Rightarrow \frac{22.4}{273} = \frac{V_2}{(273+25)}$$

$$\Rightarrow V_2 = 24.45\,\text{L}$$

So, 10^6 m^3 of air contains 40 L of SO_2 = $\frac{40}{24.45}$ = 1.63 mol of SO_2

$$= 1.636 \times 64\,\text{g of } SO_2 = 105\ \mu\text{g/m}^3$$

Ans. (105)

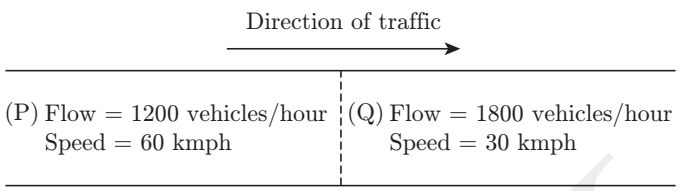
52. A parabolic vertical crest curve connects two road segments with grades +1.0% and −2.0%. If a 200 m stopping sight distance is needed for a driver at a height of 1.2 m to avoid an obstacle of height 0.15 m, then the minimum curve length should be _____ m. (round off to the nearest integer)

Solution: Considering the condition that the length of the curve is greater than the stopping sight distance. So, the curve length for the vertical curve is

$$L = \frac{NS^2}{2(\sqrt{h_1}+\sqrt{h_2})} = \frac{(1-(-2))\times 200^2}{2(\sqrt{1.2}+\sqrt{0.15})} = 273\,\text{m}$$

Ans. (273)

53. Assuming that traffic on a highway obeys the Greenshields model, the speed of a shockwave between two traffic streams (P) and (Q) as shown in the schematic is _____ kmph. (in integer)



Solution: The density of flow for the traffic streams is given by

$$k = \frac{q}{v}$$

Given that,

$$k_{\text{P}} = \frac{1200}{60} = 20\,\text{veh/min}$$

$$k_{\text{Q}} = \frac{1800}{30} = 60\,\text{veh/min}$$

Now the speed of the shock wave is determined by finding the slope of the line formed by joining the points P and Q in a graph having the vertical axis as the flow and horizontal axis as the density.

$$v_s = \frac{q_{\text{Q}} - q_{\text{P}}}{k_{\text{P}} - k_{\text{Q}}} = \frac{1800-1200}{60-20} = 15\,\text{kmph}$$

Ans. (15)

54. It is given that an aggregate mix has 260 grams of coarse aggregates and 240 grams of fine aggregates. The specific gravities of the coarse and fine aggregates are 2.6 and 2.4, respectively. The bulk specific gravity of the mix is 2.3. The percentage air voids in the mix is _____. (round off to the nearest integer)

Solution: The theoretical specific gravity of the mix is given by

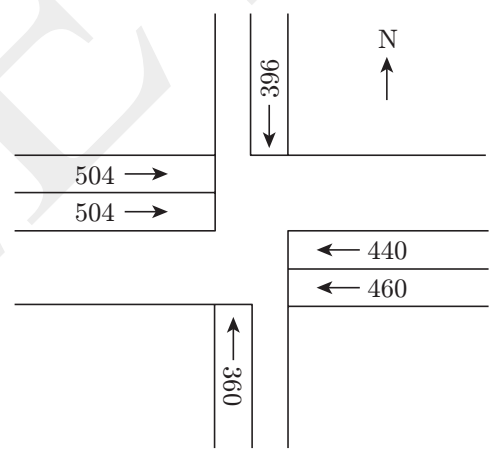
$$G_t = \frac{\sum W}{\sum \frac{W}{G}} = \frac{260+240}{\frac{260}{2.6}+\frac{240}{2.4}} = 2.5$$

The percentage air voids is

$$V_v = \frac{G_t - G_m}{G_t}\times 100 = \frac{2.5-2.3}{2.5}\times 100 = 8\%$$

Ans. (8)

55. The lane configuration with lane volumes in vehicles per hour of a four-arm signalized intersection is shown in the figure. There are only two phases: the first phase is for the East-West and the West-East through movements, and the second phase is for the North-South and the South-North through movements. There are no turning movements. Assume that the saturation flow is 1800 vehicles per hour per lane for each lane and the total lost time for the first and the second phases together is 9 seconds.



The optimum cycle length (in seconds), as per the Webster's method, is _____. (round off to the nearest integer)

Solution: The critical degree of saturation for the lanes A and B are

$$y_{\text{A}} = \frac{q_{\text{A}}}{S_{\text{A}}} = \frac{504}{1800} = 0.28$$

$$y_{\text{B}} = \frac{q_{\text{B}}}{S_{\text{B}}} = \frac{396}{1800} = 0.22$$

The optimum cycle length is given by

$$C_O = \frac{1.5L+5}{1-Y} = \frac{1.5\times 9+5}{1-(y_{\text{A}}+y_{\text{B}})}$$

$$= \frac{1.5\times 9+5}{1-(0.28+0.22)} = 37\,\text{s}$$

Ans. (37)