SOLVED GATE (ELECTRICAL ENGINEERING) 2022

Engineering Mathematics and Technical Section

Q. No. 1–25 Multiple Choice Question (MCQ) Carry One Mark Each

1. The transfer function of a real system, $H(s)$, is given as:

$$H(s) = \frac{As + B}{s^2 + Cs + D},$$

where A, B, C and D are positive constants. This system cannot operate as

(A) low pass filter. (B) high pass filter.
(C) band pass filter. (D) an integrator.

Solution: Given that,

$$H(s) = \frac{As + B}{s^2 + Cs + D}$$

At low frequency $s = 0$, then

$$H(0) = \frac{B}{D}$$

At high frequency $s = \infty$, then

$$H(\infty) = 0$$

For high pass filter $H(\infty)$ should be non-zero. So, the above system cannot be operated as high pass filter. High pass filter should pass high-frequency component, that is, $H(\infty)$ should be non-zero.

Ans. (B)

2. For an ideal MOSFET biased in saturation, the magnitude of the small signal current gain for a common drain amplifier is

(A) 0 (B) 1
(C) 100 (D) infinite

Solution: Ideal MOSFET in saturation region has $i_G = 0$ and voltage gain $A_V = 1$
Current gain is

$$A_I = \frac{i_S}{i_G} = \frac{i_S}{0} = \infty$$

Ans. (D)

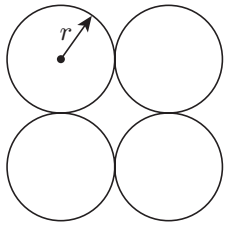
3. The most commonly used relay, for the protection of an alternator against loss of excitation, is

(A) offset Mho relay. (B) over current relay.
(C) differential relay. (D) Buchholz relay.

Solution: There are mainly two schemes available for protection against loss of field or excitation of a generator. In 1st scheme, we use an undercurrent relay connected in shunt with main field winding circuit. This relay will operate if the excitation current comes below its predetermined value.

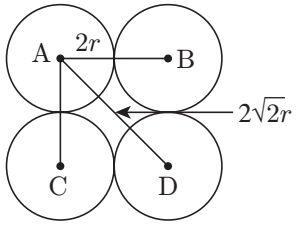
Ans. (A)

4. The geometric mean radius of a conductor, having four equal strands with each strand of radius r, as shown in the figure below, is



(A) $4r$ (B) $1.414r$
(C) $2r$ (D) $1.723r$

Solution: Let mark centres of four equal strands be A, B, C, D.
So, the distance between AB = AC = $2r$, AD = $2\sqrt{2}r$.



The geometric mean radius of a conductor is same as the geometric mean radius of one strand.

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The geometric mean radius of one strand is given by

$$\text{GMR} = [r_{AA} \times r_{AB} \times r_{AC} \times r_{AD}]^{1/4}$$
$$= [r \times 2r \times 2r \times 2\sqrt{2}r]^{1/4} = 1.723r$$

Ans. (D)

5. The valid positive, negative and zero sequence impedances (in p.u.), respectively, for a 220 kV, fully transposed three-phase transmission line, from the given choices are

(A) 1.1, 0.15 and 0.08 (B) 0.15, 0.15 and 0.35
(C) 0.2, 0.2 and 0.2 (D) 0.1, 0.3 and 0.1

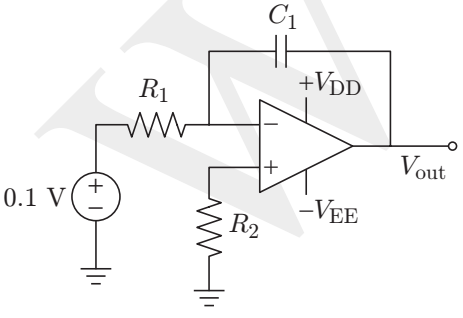
Solution: For a three-phase full transferred transmission line, we have

$$X_+ \approx X_- < X_0$$

Considering option (A): $X_1 = 1.1$, $X_2 = 0.15$ and $X_3 = 0.08$. Here, $X_+ \neq X_- > X_0$ which not satisfy the condition.
Considering option (B): $X_1 = 0.15$, $X_2 = 0.15$ and $X_3 = 0.35$. Here, $X_+ \approx X_- < X_0$ which satisfy the condition.
Considering option (C): $X_1 = 0.2$, $X_2 = 0.2$ and $X_3 = 0.2$. Here, $X_+ = X_- = X_0$ which not satisfy the condition.
Considering option (D): $X_1 = 0.1$, $X_2 = 0.3$ and $X_3 = 0.1$. Here, $X_+ \neq X_- > X_0$ which not satisfy the condition.

Ans. (B)

6. The steady state output (V_{out}), of the circuit shown below, will



(A) saturate to $+V_{DD}$
(B) saturate to $-V_{EE}$
(C) become equal to 0.1 V
(D) become equal to -0.1 V

Solution: The given circuits is opamp integrating circuit.
The output voltage of the given circuit is

$$V_{out} = \frac{-1}{R_1 C_1} \int V_{in} dt$$

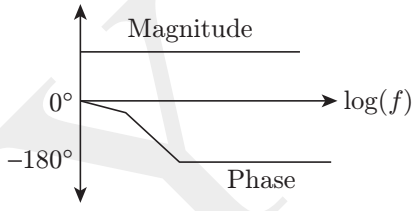
Here, $V_{in} = 0.1$ V. So,

$$V_{out} = -\frac{0.1}{R_1 C_1} \int dt = -\frac{0.1t}{R_1 C_1}$$

The steady state output will be constant, and saturate to $-V_{EE}$.

Ans. (B)

7. The Bode magnitude plot of a first order stable system is constant with frequency. The asymptotic value of the high frequency phase, for the system, is $-180°$. This system has



(A) one LHP pole and one RHP zero at the same frequency.
(B) one LHP pole and one LHP zero at the same frequency.
(C) two LHP poles and one RHP zero.
(D) two RHP poles and one LHP zero.

Solution: The given system is non-minimum phase system. So, the possibilities for transfer function is

$$\left(\frac{s-1}{s+1}, \frac{1-s}{1+s}\right)$$

Therefore, number of poles $P = 1$ and number of zero's $Z = 1$.
Hence, one LHP pole and one RHP zero at the same frequency.

Ans. (A)

8. A balanced Wheatstone bridge ABCD has the following arm resistances:
$R_{AB} = 1\,\text{k}\Omega \pm 2.1\%$; $R_{BC} = 100\,\Omega \pm 0.5\%$; R_{CD} is an unknown resistance; $R_{DA} = 300\,\Omega \pm 0.4\%$.
The value of R_{CD} and its accuracy is

(A) $30\,\Omega \pm 3\,\Omega$ (B) $30\,\Omega \pm 0.9\,\Omega$
(C) $3000\,\Omega \pm 90\,\Omega$ (D) $3000\,\Omega \pm 3\,\Omega$

Solution: Under balanced conditions, we have

$$\frac{R_{AB}}{R_{DA}} = \frac{R_{BC}}{R_{CD}}$$
$$R_{CD} = \frac{R_{BC} \times R_{DA}}{R_{AB}}$$
$$= \frac{(100 \pm 0.5\%)(300 \pm 0.4\%)}{(1000 \pm 2.1\%)}$$

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$$= \frac{100 \times 300}{1000} \pm (0.5\% + 0.4\% + 2.1\%)$$
$$= 30 \pm 0.9\ \Omega$$

Ans. (B)

9. The open loop transfer function of a unity gain negative feedback system is given by $G(s) = \dfrac{k}{s^2 + 4s - 5}$. The range of k for which the system is stable, is

(A) $k > 3$ (B) $k < 3$
(C) $k > 5$ (D) $k < 5$

Solution: Using Routh stability criteria, the characteristics equation is

$$1 + G(s) = 0$$

Here, $G(s) = \dfrac{k}{s^2 + 4s - 5}$. So,

$$1 + \frac{k}{s^2 + 4s - 5} = 0$$

$$\Rightarrow \quad s^2 + 4s + (k - 5) = 0$$

By Routh Table analysis,

s^2	1	$k - 5$
s^1	4	0
s^0	$k - 5$	0

We have,

$$k > 5 \text{ and } k > 0$$

Hence, system is stable when

$$k > 5$$

Ans. (C)

10. Consider a 3×3 matrix A whose (i, j)-th element, $a_{i,\,j} = (i - j)^3$. Then the matrix A will be

(A) symmetric. (B) skew-symmetric.
(C) unitary. (D) null.

Solution: Given, 3×3 matrix A.

$$A = [a_{i,j}]_{3\times 3}$$

where $\quad a_{i,j} = (i - j)^3$

For $i = j$, we have

$$a_{i,j} = (i - i)^3 = 0$$

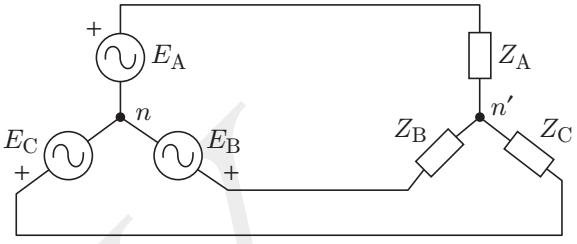
For $i \neq j$, we have

$$a_{i,j} = (i - j)^3 = -(j - i)^3 = -\,a_{i,j}$$

Hence, $A_{3\times 3}$ is a skew symmetric matrix

Ans. (B)

11. In the circuit shown below, a three-phase star-connected unbalanced load is connected to a balanced three-phase supply of $100\sqrt{3}$ V with phase sequence ABC. The star connected load has $Z_A = 10\ \Omega$ and $Z_B = 20\angle 60°\Omega$. The value of Z_C in Ω, for which the voltage difference across the nodes n and n' is zero, is



(A) $20\angle -30°$ (B) $20\angle 30°$
(C) $20\angle -60°$ (D) $20\angle 60°$

Solution: Give that, the voltage difference across the nodes n and n' is zero, that is, n and n' are at same voltage. So,

$$I_{nn'} = 0$$
$$\Rightarrow I_A + I_B + I_C = 0$$
$$\Rightarrow \frac{E_A}{Z_A} + \frac{E_B}{Z_B} + \frac{E_C}{Z_C} = 0$$
$$\Rightarrow \frac{100\angle 0°}{10} + \frac{100\angle -120°}{20\angle 60°} + \frac{100\angle 0°}{Z_C} = 0$$
$$\Rightarrow \frac{100\angle 0°}{Z_C} = -\frac{100\angle 0°}{10} - \frac{100\angle -120°}{20\angle 60°}$$
$$\Rightarrow Z_C = 20\angle 60°\ \Omega$$

Ans. (C)

12. A charger supplies 100 W at 20 V for charging the battery of a laptop. The power devices, used in the converter inside the charger, operate at a switching frequency of 200 kHz. Which power device is best suited for this purpose?

(A) IGBT (B) Thyristor
(C) MOSFET (D) BJT

Solution: Considering given options one by one, we have

Option (A): IGBT is suitable for operating at high frequency but IGBT having less stability at high frequency.

Option (B): Thyristor store charge carrier near its gate-cathode junction. For removal of these charge carrier, it takes source time. It is not suitable for fast switching.

Option (C): MOSFET is suitable for operating at high frequency and having good stability at high frequency.

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Option (D): BJT is having law conduction has but high switching loss. It is not suitable for high switching frequency.

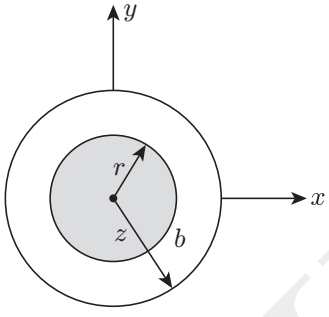
Ans. (C)

13. A long conducting cylinder having a radius b is placed along the z axis. The current density is $\vec{J} = J_a r^3 \hat{z}$ for the region $r < b$ where r is the distance in the radial direction. The magnetic field intensity $(\vec{H})$ for the region inside the conductor (i.e. for $r < b$) is

(A) $\frac{J_a}{4} r^4$ (B) $\frac{J_a}{3} r^3$

(C) $\frac{J_a}{5} r^4$ (D) $J_a r^3$

Solution: From the given data, we have the following figure:



The current density is given by

$$\vec{J} = J_a r^3 \hat{z}$$

Current enclosed in the conducting cylinder is

$$I = \int_S \vec{J} \cdot \overrightarrow{ds}$$

Where $\overrightarrow{ds} = r\,dr\,d\phi\,\hat{z}$

So,

$$I = J_a \int_{r=0}^{r} r^4 dr \int_{\phi=0}^{2\pi} d\phi = \frac{J_a (2\pi) r^5}{5}$$

From Ampere's law, we know that

$$\oint \vec{H} \cdot \overrightarrow{dl} = I_{enc} = \int_S \vec{J} \cdot \overrightarrow{ds}$$

$$\Rightarrow \quad H(2\pi r) = \frac{J_a (2\pi) r^5}{5}$$

$$\Rightarrow \quad H = \frac{J_a r^4}{5}$$

Ans. (C)

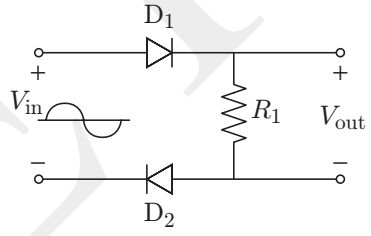
14. The type of single-phase induction motor, expected to have the maximum power factor during steady state running condition, is

(A) split phase (resistance start).
(B) shaded pole.
(C) capacitor start.
(D) capacitor start, capacitor run.

Solution: If we want to improve the power factor, we must always connect the capacitor across the power supply. We know that, in the capacitor run type single phase induction motor, the capacitor is permanently connected with the auxiliary winding. But this is not happening in any other above mentioned motors. So, the capacitor run type single phase induction motors have highest power factor at full load.

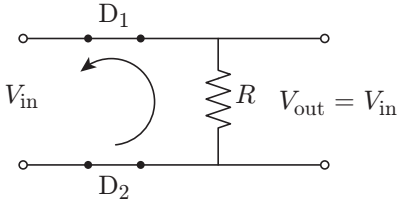
Ans. (D)

15. For the circuit shown below with ideal diodes, the output will be



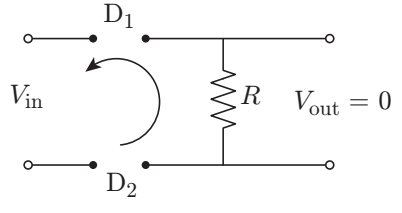
(A) $V_{out} = V_{in}$ for $V_{in} > 0$
(B) $V_{out} = V_{in}$ for $V_{in} < 0$
(C) $V_{out} = -V_{in}$ for $V_{in} > 0$
(D) $V_{out} = -V_{in}$ for $V_{in} < 0$

Solution: For positive half cycle, diodes D_1 and D_2 are forward biased. In forward biased diodes replace with short circuit or diodes are ON.



So, $V_{out} = V_{in}$

For negative half cycle, diodes D_1 and D_2 are reversed biased. In reversed biased diodes replace with open circuit or diodes are OFF.



So, $V_{out} = 0$

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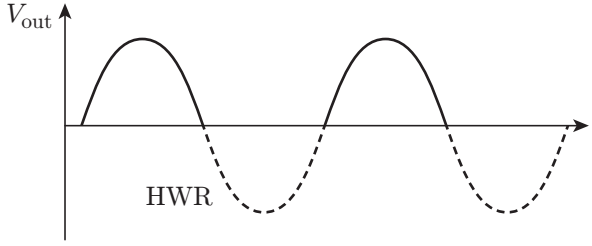
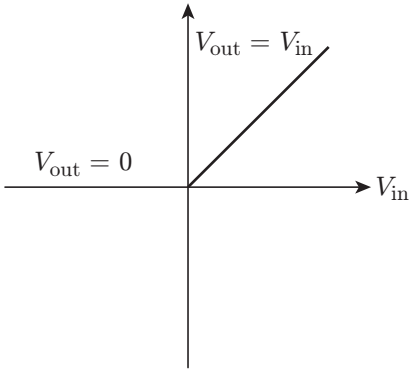
The Output waveform





Therefore, $V_{out} = V_{in}$ for $V_{in} > 0$

Ans. (A)

16. A MOD 2 and a MOD 5 up-counter when cascaded together results in a MOD _____ counter. (in integer)

Solution: Given that, MOD 2 and MOD 5 are up-counters and cascaded together.

For overall configuration $= \text{MOD}(2 \times 5) = 10$

It will be MOD 10 counter.

Ans. (10)

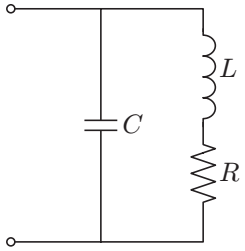
17. An inductor having a Q-factor of 60 is connected in series with a capacitor having a Q-factor of 240. The overall Q-factor of the circuit is _____. (round off to nearest integer)

Solution: Given that, $Q_L = 60$ and $Q_C = 240$. Both are connected in series. The overall Q-factor of the circuit is

$$Q = \frac{Q_L \cdot Q_C}{Q_L + Q_C} = \frac{60 \times 240}{60 + 240} = 48$$

Ans. (48)

18. The network shown below has a resonant frequency of 150 kHz and a bandwidth of 600 Hz. The Q-factor of the network is _____. (round off to nearest integer)



*Solution.*Given that, $f_o = 150$ kHz and BW $=$ 600 Hz.

The Q factor of the given network is

$$Q = \frac{f_o}{\text{BW}} = \frac{150 \times 1000}{600} = 250$$

Ans. (250)

19. The maximum clock frequency in MHz of a 4-stage ripple counter, utilizing flipflops, with each flip-flop having a propagation delay of 20 ns, is _____. (round off to one decimal place)

Solution: Given that, one flip-flop has propagation delay $t_p = 20\,\text{ns}$.

For 4-stage ripple counter 4 flip-flops are used. So, total propagation delay in 4-stage ripple counter is

$$T = nt_p$$
$$= 4 \times 20\,\text{ns} = 80\ \text{ns}$$

Therefore, clock frequency is

$$f_{\text{CLK}} = \frac{1}{T} = \frac{1}{4 \times 20 \times 10^{-9}} = 12.5\,\text{MHz}$$

Ans. (12.5)

20. If only 5% of the supplied power to a cable reaches the output terminal, the power loss in the cable, in decibels, is _____. (round off to nearest integer)

Solution: Given that,

$$P_S = 5\%$$

So, percentage of power loss

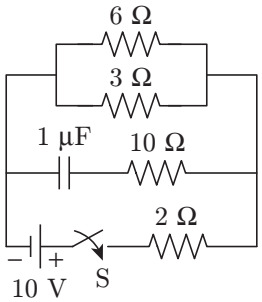
$$P_L = 100 - 5 = 95\%$$

Power loss in decibles is

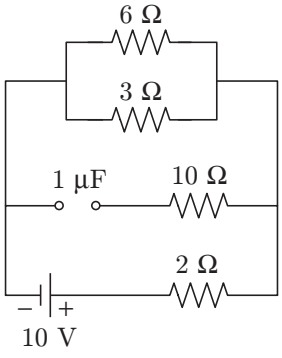
$$10 \log\left(\frac{P_L}{P_S}\right) = 10 \log\left(\frac{95}{5}\right) = 12.78\,\text{dB} \approx 13\,\text{dB}$$

Ans. (13)

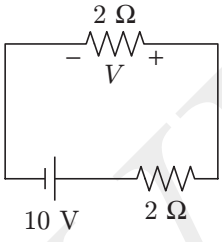
21. In the circuit shown below, the switch S is closed at $t = 0$. The magnitude of the steady state voltage, in volts, across the 6 Ω resistor is _____. (round off to two decimal places).



Solution: At steady state condition for DC supply capacitor acts as open circuit.



The circuit can be redraw as the following figure



Using voltage division, the voltage across $6\,\Omega$ is

$$V_{\text{out}} = 10 \times \frac{2}{2+2} = 5\,\text{V}$$

Ans. (5)

22. A single-phase full-bridge diode rectifier feeds a resistive load of 50 Ω from a 200 V, 50 Hz single phase AC supply. If the diodes are ideal, then the active power, in watts, drawn by the load is _____. (round off to nearest integer).

Solution: A single-phase full-bridge diode rectifier has load $R = 50\,\Omega$ and AC supply $V_S = 200$ V. The ideal diode has power

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R}$$

For full bridge diode rectifier

$$V_S = V_{S,\text{rms}} = 200\,\text{V}$$

So,
$$P_{\text{avg}} = \frac{200^2}{50} = 800\,\text{W}$$

Ans. (800)

23. The voltage at the input of an AC-DC rectifier is given by $V(t) = 230\sqrt{2}\sin\omega t$ where $\omega = 2\pi \times 50$ rad/s. The input current drawn by the rectifier is given by

$$I(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right) + 4\sin\left(3\omega t - \frac{\pi}{6}\right) + 3\sin\left(5\omega t - \frac{\pi}{3}\right).$$

The input power factor, (rounded off to two decimal places), is, _____ lag.

Solution: The input power factor is given by

$$\text{PF} = \frac{I_{1,\text{rms}}}{I_{T,\text{rms}}}\cos\left(\frac{\pi}{3}\right)$$

where $I_{1,\text{ rms}}$ is rms current of first phase and $I_{T,\text{ rms}}$ is rms current through all phases.

Here $I_{1,\text{rms}} = 10/\sqrt{2}$,

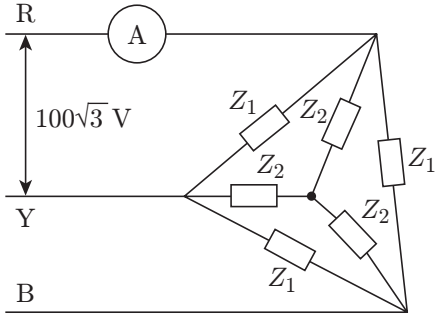
$$I_{T,\text{rms}} = \sqrt{(10/\sqrt{2})^2 + (4/\sqrt{2})^2 + (3/\sqrt{2})^2}$$

So,

$$\text{PF} = \frac{10/\sqrt{2}}{\sqrt{(10/\sqrt{2})^2 + (4/\sqrt{2})^2 + (3/\sqrt{2})^2}} \times \cos\left(\frac{\pi}{3}\right) = 0.447 \approx 0.45$$

Ans. (0.45)

24. Two balanced three-phase loads, as shown in the figure, are connected to a $100\sqrt{3}$ V, three-phase, 50 Hz main supply. Given $Z_1 = (18 + j24)\,\Omega$ and $Z_2 = (6 + j8)\,\Omega$. The ammeter reading, in amperes, is _____. (round off to nearest integer)



Solution: The given network is delta network, first we convert delta network to star network.

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We know that, for balanced load

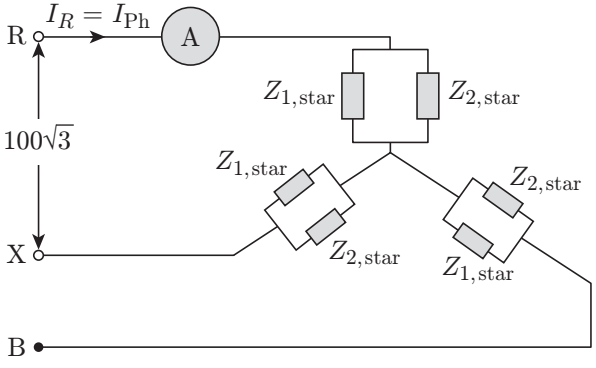
$$Z_{1,\text{star}} = \frac{Z_1 \times Z_1}{Z_1 + Z_1 + Z_1} = \frac{Z_1}{3}$$

Given that, $Z_1 = (18 + j24)\,\Omega$ and $Z_2 = (6 + j8)\,\Omega$. So,

$$Z_{1,\text{star}} = \frac{Z_1}{3} = \frac{18 + j24}{3} = (6 + j8)\,\Omega$$

And $\quad Z_{2,\text{star}} = (6 + j8)\,\Omega$

After converting network, redraw the



So, the equivalent impedance for one phase is

$$Z_{\text{eq}} = Z_{1,\text{star}} \,||\, Z_{2,\text{star}} = (6 + j8) \,||\, (6 + j8) = 3 + j4$$

The ammeter reading is

$$I_R = I_{\text{Ph}} = \frac{V_{\text{RY}}}{Z_{\text{eq}}} = \frac{100}{3 + j4} = 20\,\text{A}$$

Ans. (20)

25. The frequencies of the stator and rotor currents flowing in a three-phase 8-pole induction motor are 40 Hz and 1 Hz, respectively. The motor speed, in rpm, is _____. (round off to nearest integer)

Solution: Given that, $f_s = 40$ Hz, $f_r = 1$ Hz and $p = 8$.
We know that,

$$f_r = s f_s$$

where s is slip and its value is

$$s = \frac{f_r}{f_s} = \frac{1}{40} = 0.025$$

So, the synchronous speed is

$$N_s = \frac{120 \times 40}{8} = 600$$

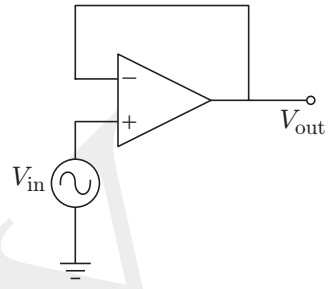
And motor speed is given by

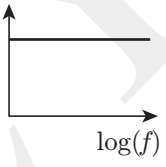
$$N = N_s(1 - s) = 600(1 - 0.025) = 585\,\text{rpm}$$

Ans. (585)

Q. No. 26–55 Multiple Choice Question (MCQ) Carry Two Marks Each

26. The output impedance of a non-ideal operational amplifier is denoted by Z_{out}. The variation in the magnitude of Z_{out} with increasing frequency, f, in the circuit shown below, is best represented by



(A)


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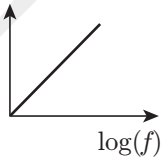
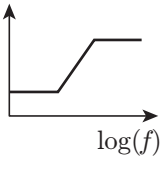
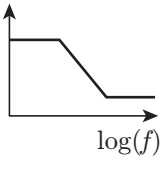


(C)


(D)


Solution: Given non-ideal operational amplifier is voltage series feedback.
Here voltage is a shunt connection so Z_{out} is the feedback and given by

$$Z_{\text{out}} = \frac{Z_o}{1 + \beta A}$$

where Z_o is impedance without feedback, β is buffer and A is open loop gain.
Here $\beta = 1$. So,

$$Z_{\text{out}} = \frac{Z_o}{1 + A}$$

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At low frequency, the open loop gain is constant. Then,

$$Z_{\text{out}} = \frac{Z_o}{1+A} = \text{constant}$$

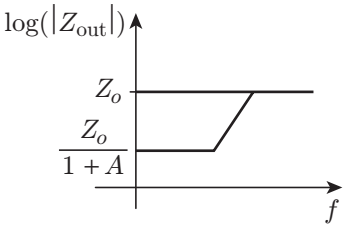
As frequency increases, the open loop gain decrease. Then,

$$Z_{\text{out}}(\uparrow) = \frac{Z_o}{1+A(\downarrow)}$$

At high frequency, the open loop gain become zero. Then,

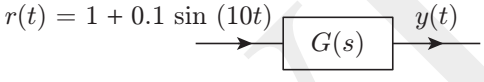
$$Z_{\text{out}} = \frac{Z_o}{1+0} = Z_o$$

Therefore, Z_{out} with frequency represented by



Ans. (C)

27. An LTI system is shown in the figure where $G(s) = \dfrac{100}{s^2 + 0.1s + 100}$. The steady state output of the system, to the input $r(t)$, is given as $y(t) = a + b\sin(10t + \theta)$. The values of a and b will be



(A) $a = 1, b = 10$ (B) $a = 10, b = 1$
(C) $a = 1, b = 100$ (D) $a = 100, b = 1$

Solution: We know that, in LTI system the steady state output is given by

$$y(t) = A\,|\,G(j\omega)\,|\,\sin(\omega t + \phi)$$

Given that,

$$\text{Input, } r(t) = 1 + 0.1\sin(10t)$$

$$\text{Output, } y(t) = a + b\sin(10t + \theta)$$

and $$G(s) = \frac{100}{s^2 + 0.1s + 100}$$

So, $$G(j\omega) = \frac{100}{-\omega^2 + j0.1\omega + 100}$$

At $\omega = 0$, we get DC component of output.

$$G(j0) = \left|\frac{100}{-(0)^2 + j0.1(0) + 100}\right| = \left|\frac{100}{100}\right| = 1$$

At $\omega = 10$, we get AC component of output.

$$G(j10) = \left|\frac{100}{-(10)^2 + j0.1(10) + 100}\right|$$

$$= \left|\frac{100}{-100 + j1 + 100}\right| = \left|\frac{100}{j1}\right| = 100$$

So, the output is

$$y(t) = 1 + (0.1\times 100)\sin(10t + \theta) = 1 + 10\sin(10t + \theta)$$

Therefore, on comparison, we get

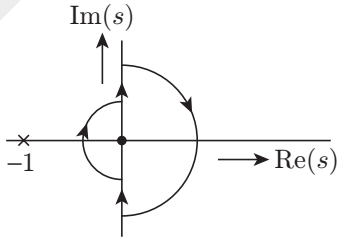
$$a = 1,\ b = 10$$

Ans. (A)

28. The open loop transfer function of a unity gain negative feedback system is given as

$$G(s) = \frac{1}{s(s+1)}.$$

The Nyquist contour in the s-plane encloses the entire right half plane and a small neighbourhood around the origin in the left half plane, as shown in the figure below. The number of encirclements of the point $(-1 + j0)$ by the Nyquist plot of $G(s)$, corresponding to the Nyquist contour, is denoted as N. Then N equals to



(A) 0 (B) 1
(C) 2 (D) 3

Solution: Since, Nyquist contour encircle one pole, that is, $s = 0$. So, $P = 1$
The number of encirclements of the point is given by

$$N = P - Z$$

$$\Rightarrow N = 1 - Z$$

Characteristic equation is given by

$$1 + G(s)H(s) = 0$$

Given that,

$$G(s) = \frac{1}{s(s+1)}$$

So,

$$1 + \frac{k}{s(s+1)} = 0$$

$$\Rightarrow \quad s^2 + s + k = 0$$

R-H criteria:

s^2	1	k
s^1	1	0
s^0	K	

From R-H criteria we can see that, there is no sign change in first column of R-H criteria. So,

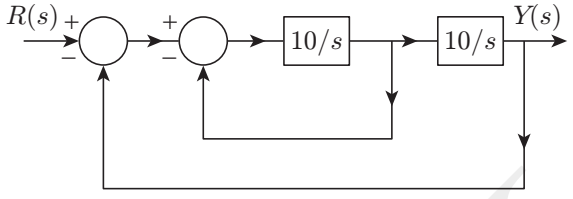
$$Z = 0$$

where, Z = closed loop pole on RHS side of s-plane.

Therefore, $N = 1 - 0 = 1$

Ans. (B)

29. The damping ratio and undamped natural frequency of a closed loop system as shown in the figure, are denoted as ξ and ω_n, respectively. The values of ξ and ω_n are



(A) $\xi = 0.5$ and $\omega_n = 10$ rad/s
(B) $\xi = 0.1$ and $\omega_n = 10$ rad/s
(C) $\xi = 0.707$ and $\omega_n = 10$ rad/s
(D) $\xi = 0.707$ and $\omega_n = 100$ rad/s

Solution: Transfer function of the given system is

$$\text{TF} = \frac{C(s)}{R(s)} = \frac{100/s(s+10)}{1+(100/s(s+10))}$$

$$= \frac{100}{s^2+10s+100}$$

Standard transfer function is

$$\text{TF} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Therefore, $\omega_n = 10\,\text{rad/s}$ and $\xi = 0.5$

Ans. (A)

30. e^A denotes the exponential of a square matrix A. Suppose λ is an eigenvalue and v is the corresponding eigen-vector of matrix A. Consider the following two statements:
Statement 1: e^λ is an eigenvalue of e^A.
Statement 2: v is an eigen-vector of e^A.
Which one of the following options is correct?
(A) Statement 1 is true and statement 2 is false.
(B) Statement 1 is false and statement 2 is true.
(C) Both the statements are correct.
(D) Both the statements are false.

Solution: Given λ is an eigen value of A. Then eigen value of e^A is given as:

$$e^A = 1 + A + \frac{A^2}{2} + \frac{A^3}{6} + ...$$

is $$e^\lambda = 1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6} + ...$$

So, statement 1 is true.
We know that eigen value of A and polynomial matrix in A is same. Eigen vector of A and e^A is same.
So, statement 2 is true.

Ans. (C)

31. Let $f(x) = \int_0^x et(t-1)(t-2)dt$. Then $f(x)$ decreases in the interval
(A) $x \in (1,2)$ (B) $x \in (2,3)$
(C) $x \in (0,1)$ (D) $x \in (0.5,1)$

Solution: The given function

$f(x) = \int_0^x e^t(t-1)(t-2)dt$ is decreasing if $f'(x) = 0$.

By Leibnitz rule

$$f'(x) = \frac{d}{dx}f(x) < 0$$

$$= e^x(x-1)(x-2)\frac{d}{dx}(x) - e^0(0-1)(0-2)\frac{d}{dx}(0)$$

$$= e^x(x-1)(x-2) - 0 < 0$$

$$= (x-1)(x-2) < 0$$

$$= x > 1 \text{ and } x < 2$$

Hence x belongs to (1, 2).

Ans. (A)

32. Consider a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars and I is the identity matrix. Then $(c + d)$ is equal to

(A) 5 (B) 17
(C) -6 (D) 11

Solution: Given matrix is

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

Now, eigen values of matrix A are

$$A - \lambda I = 0$$

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$$\Rightarrow \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 4-\lambda & -2 \\ 0 & 1 & 1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)[(4-\lambda)(1-\lambda)+2] = 0$$

$$\Rightarrow (1-\lambda)[\lambda^2 - 5\lambda + 4 + 2] = 0$$

$$\Rightarrow (1-\lambda)[\lambda^2 - 5\lambda + 6] = 0$$

$$\Rightarrow (1-\lambda)[\lambda^2 - 2\lambda - 3\lambda + 6] = 0$$

$$\Rightarrow (1-\lambda)(\lambda-2)(\lambda-3) = 0$$

$$\Rightarrow \lambda = 1, 2, 3$$

If the matrix A satisfies the given equation, so its eigen value too. So,

$$6A^{-1} = A^2 + cA + dI$$

$$\Rightarrow 6\lambda^{-1} = \lambda^2 + c\lambda + d$$

Taking $\lambda = 1$, we get

$$\frac{6}{1} = 1 + c + d$$

$$\Rightarrow c + d = 5$$

Ans. (A)

33. The fuel cost functions in rupees/hour for two 600 MW thermal power plants are given by

Plant 1: $C_1 = 350 + 6P_1 + 0.004P_1^2$

Plant 2: $C_2 = 450 + aP_2 + 0.003P_2^2$

where P_1 and P_2 are power generated by plant 1 and plant 2, respectively, in MW and a is constant. The incremental cost of power (λ) is 8 rupees per MWh. The two thermal power plants together meet a total power demand of 550 MW. The optimal generation of plant 1 and plant 2 in MW, respectively, are

(A) 200, 350 (B) 250, 300
(C) 325, 225 (D) 350, 200

Solution: Given that,

Plant 1: $C_1 = 350 + 6P_1 + 0.004P_1^2$

Plant 2: $C_2 = 450 + aP_2 + 0.003P_2^2$

The incremental cost of power is

$$\lambda = 8 \text{ rupees per MWh.}$$

Incremental equation of power plant 1 is

$$\lambda = \frac{dC_1}{dP_1} = 6 + 0.008P_1$$

and $$6 + 0.008P_1 = 8$$

$$\Rightarrow 0.008P_1 = 2$$

$$\Rightarrow P_1 = 250 \text{ MWh}$$

Total power demand is

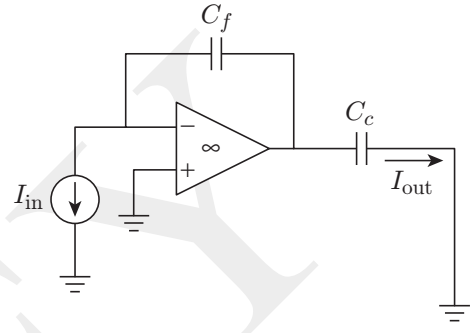
$$P_T = 550 \text{ MWh}$$

and $$P_T = P_1 + P_2 = 550 \text{ MWh}$$

$$P_2 = 550 \text{ MWh} - P_1 = 550 - 250 = 300 \text{ MWh}$$

Ans. (B)

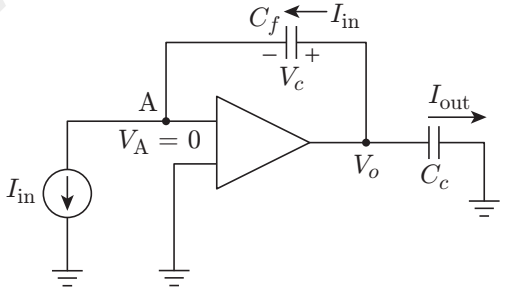
34. The current gain (I_{out}/I_{in}) in the circuit with an ideal current amplifier given below is



(A) $\frac{C_f}{C_c}$ (B) $\frac{-C_f}{C_c}$

(C) $\frac{C_c}{C_f}$ (D) $\frac{-C_c}{C_f}$

Solution: Redraw the given circuit



Voltage across feedback capacitor C_f is

$$V_{C_f} = \frac{1}{C_f}\int I_{in}dt = \frac{I_{in}}{C_f} \times t$$

$$\Rightarrow I_{in} = \frac{V_{C_f} \times C_f}{t}$$

Now, the output current through Capacitor C_c is

$$I_{out} = C_c \times \frac{dV_{C_f}}{dt} = C_c \times \frac{d\left(\frac{I_{in}}{C_f} \times t\right)}{dt} = C_c \times \frac{I_{in}}{C_f}$$

$$\Rightarrow \frac{I_{out}}{I_{in}} = \frac{C_c}{C_f}$$

Ans. (C)

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35. If the magnetic field intensity ($\vec{H}$) in a conducting region is given by the expression, $\vec{H} = x^2\hat{i} + x^2y^2\hat{j} + x^2y^2z^2\hat{k}$ A/m. The magnitude of the current density, in A/m^2, at $x = 1$ m, $y = 2$ m, and $z = 1$ m, is

(A) 8 (B) 12
(C) 16 (D) 20

Solution: Given that,

$$\vec{H} = x^2\hat{i} + x^2y^2\hat{j} + x^2y^2z^2\hat{k} \text{ A/m}$$

From Ampere's circuital law with Maxwell's addition, we have

$$\vec{J} = \nabla \times \vec{H}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2 & x^2y^2 & x^2y^2z^2 \end{vmatrix}$$

$$= \hat{i}(2xy^2z^2 - 0) - \hat{j}(2xy^2z^2 - 0) + \hat{k}(2xy^2 - 0)$$

$$= 2xy^2z^2\hat{i} - 2xy^2z^2\hat{j} + 2xy^2\hat{k}$$

At point $x = 1$, $y = 2$ and $z = 1$, we get

$$\vec{J}_{1,2,1} = 2(1)2(1)\hat{i} - 2(1)(2)^2(1)\hat{j} + 2(1)(2)^2\hat{k}$$

$$= 4\hat{i} - 8\hat{j} + 8\hat{k} = \sqrt{4^2 + 8^2 + 8^2} = 12$$

Ans. (B)

36. Let a causal LTI system be governed by the following differential equation $y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t)$, where $x(t)$ and $y(t)$ are the input and output respectively. Its impulse response is

(A) $2e^{-\frac{1}{4}t}u(t)$ (B) $2e^{-4t}u(t)$
(C) $8e^{-\frac{1}{4}t}u(t)$ (D) $8e^{-4t}u(t)$

Solution: Given that, a causal LTI system be governed by the following differential equation

$$y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t)$$

Taking Laplace transform on given differential equation, we get

$$Y(s) = \frac{1}{4}sY(s) = 2X(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{2}{1 + \frac{s}{4}} = \frac{8}{s+4}$$

We know that,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{8}{s+4}$$

Now, taking Laplace inverse transform, we get

$$h(t) = 8\,e^{-4t}\,u(t)$$

That is, impulse response of system.

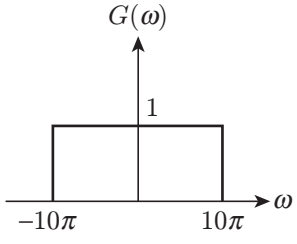
Ans. (D)

37. Let an input $x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t)$ is passed through an LTI system having an impulse response, $h(t) = 2\left(\frac{\sin(10\pi t)}{\pi t}\right)\cos(40\pi t)$. The output of the system is

(A) $2\sin(10\pi t) + 5\cos(15\pi t)$
(B) $5\cos(15\pi t) + 7\sin(42\pi t)$
(C) $7\sin(42\pi t) + 4\cos(45\pi t)$
(D) $2\sin(10\pi t) + 4\cos(45\pi t)$

Solution: Given that, input $x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t)$ and impulse response $h(t) = 2\left[\frac{\sin(10\pi t)}{\pi t}\right]\cos(40\pi t)$

Fourier transformation of $\frac{\sin(10\pi t)}{\pi t}$ is given by

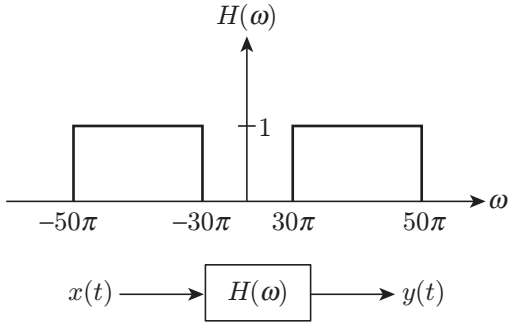


$$H(\omega) = 2g(t)\cos(40\pi t)$$

Applying Fourier transform, again

$$H(\omega) = 2\frac{[G(\omega - 40\pi) + G(\omega + 40\pi)]}{2}$$

$$= G(\omega - 40\pi) + G(\omega + 40\pi)$$



Applying inverse Fourier transform, again

$$y(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t)$$

From the output, we get

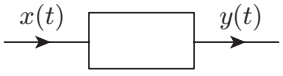
$$\omega_1 = 10\pi, \omega_2 = 15\pi, \omega_3 = 42\pi, \omega_4 = 45\pi$$

The output of LTI system is

$$y(t) = 7\sin(42\pi t) + 4\cos(45\pi t)$$

Ans. (C)

38. Consider the system as shown below



where $y(t) = x(e^t)$. The system is

(A) linear and causal.
(B) linear and non-causal.
(C) non-linear and causal.
(D) non-linear and non-causal.

Solution: We know, a linear system follows the law of superposition.
It is a combination of two laws:
(i) Law of additivity:

$$y_1(t) = x_1(e^t), y_2(t) = x_2(e^t)$$

$$y_1(t) + y_2(t) = x_1(e^t) + x_2(e^t)$$

And $$(y_1 + y_2)(t) = (x_1 + x_2)(e^t) = x_1(e^t) + x_2(e^t)$$

Both results are same, hence, it follows law of additivity.
(ii) Law of Homogeneity:

$$ay(t) = ax(e^t)$$

And $$y(at) = x(ae^t) = ax(e^t)$$

Here also both results are same, hence it follows law of Homogeneity.
Therefore, System is linear.
We know, a causal system is independent of future values of input at each and every instant of time them system will be causal.
Now put $t = 0$, we get

$$y(0) = x(e^0) = x(1)$$

Since, its depends on future value. Therefore, system is non-causal.

Ans. (B)

39. The discrete time Fourier series representation of a signal $x[n]$ with period N is written as $x[n] = \sum_{k=0}^{N-1} a_k e^{j(2kn\pi/N)}$. A discrete time periodic signal with period $N = 3$, has the non-zero Fourier series coefficients: $a_{-3} = 2$ and $a_4 = 1$. The signal is

(A) $2 + 2e^{-\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

(B) $1 + 2e^{\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

(C) $1 + 2e^{\left(j\frac{2\pi}{3}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

(D) $2 + 2e^{\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

Solution: Given that,

$$x[n] = \sum_{k=0}^{N-1} a_k e^{j(2kn\pi/N)}$$

So, $$x(n) = a_0 + a_1 e^{\frac{j2\pi n}{3}} + 0 + \ldots \quad (1)$$

We have,

$$a_k = a_{k+N}$$

$$a_0 = a_3, a_1 = a_4 = 1, a_{-3} = a_0 = 2$$

Put $n = 0$ in Eq. (1), we get

$$x(0) = a_0 + a_1 = 2 + 1 = 3$$

Put $n = 1$ in Eq. (1), we get

$$x(1) = a_0 + a_1 e^{\frac{j2\pi}{3}} = 2 + 1\left(\cos\left(\frac{2\pi}{3}\right) + j\sin\left(\frac{2\pi}{3}\right)\right)$$

$$= 2 + (-0.5 + j0.866) = 1.5 + j0.866$$

These conditions satisfied by the option (B).

Ans. (B)

40. Let, $f(x, y, z) = 4x^2 + 7xy + 3xz^2$. The direction in which the function $f(x, y, z)$ increases most rapidly at point $P = (1, 0, 2)$ is

(A) $20\hat{i} + 7\hat{j}$ (B) $20\hat{i} + 7\hat{j} + 12\hat{k}$
(C) $2\hat{i} + 12\hat{k}$ (D) $20\hat{i}$

Solution: Scalar function $f(x, y, z)$ rapidly increases in the direction of its normal vector at P.

$$\vec{a} = \nabla f(x,y,z)$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) f(x,y,z)$$

$$= \hat{i}(8x + 7y + 3z^2) + \hat{j}(7x) + \hat{k}(6xz)$$

At point P = (1, 0, 2), we get

$$\nabla f(1,0,2) = 20\hat{i} + 7\hat{j} + 12\hat{k}$$

Ans. (B)

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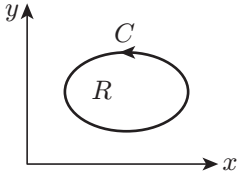
41. Let R be a region in the first quadrant of the xy plane enclosed by a closed curve C considered in counter-clockwise direction. Which of the following expressions does not represent the area of the region R?



(A) $\iint_R dxdy$ (B) $\oint_C xdy$

(C) $\oint_C ydx$ (D) $\frac{1}{2}\oint_C (xdy - ydx)$

Solution: Check every option with the given conditions

Option (A):

$$\iint_R dxdy = \text{Area of region } R \text{ by } C$$

Option (B) By Green's theorem:

$$\oint_C xdy = \iint_R (1-0)dxdy = \text{Area of region } R \text{ by } C$$

Option (C) By Green's theorem:

$$\oint_C ydx = \iint_R (0-1)dxdy = -\text{Area of region } R \text{ by } C$$

Option (D) By Green's theorem

$$\frac{1}{2}\oint_C (xdy - ydx) = \frac{1}{2}\iint_R (1-(-1))dxdy$$
$$= \text{Area of region } R \text{ by } C$$

Ans. (C)

42. Let $\vec{E}(x,y,z) = 2x\hat{i} + 5y\hat{j} + 3z\hat{k}$. The value of $\iiint_V (\vec{\nabla}\cdot\vec{E})dV$, where V is the volume enclosed by the unit cube defined by $0 \le x \le 1, 0 \le y \le 1$, and $0 \le z \le 1$, is

(A) 3 (B) 8

(C) 10 (D) 5

Solution: Given that,

$$\vec{E}(x,y,z) = 2x\hat{i} + 5y\hat{j} + 3z\hat{k}$$

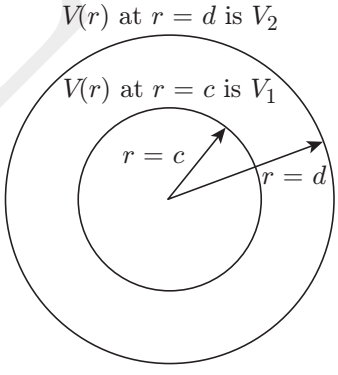
$$\text{So, } \vec{\nabla}\cdot\vec{E} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(2x^2\hat{i} + 5y\hat{j} + 3z\hat{k})$$
$$= 4x + 5 + 3$$

Now,

$$\iiint_V (\vec{\nabla}\cdot\vec{E})dV = \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} (4x+5+3)\,dzdydx$$
$$= \int_{x=0}^{1}\int_{y=0}^{1} (4x+5+3)\,dzdydx$$
$$= \int_{x=0}^{1}\int_{y=0}^{1} (4xz+8z)_0^1\,dydx$$
$$= (2x^2+8x)_0^1 = 10$$

Ans. (C)

43. As shown in the figure below, two concentric conducting spherical shells, centered at $r = 0$ and having radii $r = c$ and $r = d$ are maintained at potentials such that the potential $V(r)$ at $r = c$ is V_1 and $V(r)$ at $r = d$ is V_2. Assume that $V(r)$ depends only on r, where r is the radial distance. The expression for $V(r)$ in the region between $r = c$ and $r = d$ is



(A) $V(r) = \frac{cd(V_2 - V_1)}{(d-c)r} - \frac{V_1 c + V_2 d - 2V_1 d}{d-c}$

(B) $V(r) = \frac{cd(V_1 - V_2)}{(d-c)r} - \frac{V_2 d - V_1 c}{d-c}$

(C) $V(r) = \frac{cd(V_1 - V_2)}{(d-c)r} - \frac{V_1 c - V_2 c}{d-c}$

(D) $V(r) = \frac{cd(V_2 - V_1)}{(d-c)r} - \frac{V_2 c - V_1 c}{d-c}$

Solution: From Laplace equation, we know that

$$\nabla^2 V = 0$$

$$\text{where } \nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2 V}{\partial\phi^2}\right)$$

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Given that, V only have radial component. So,

$$\nabla^2 V = \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}\left(\frac{r^2 \sin\theta}{1}\cdot\frac{\partial V}{\partial r}\right)\right] = 0$$

$$\Rightarrow \quad \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

Integrating both sides, we get

$$r^2 \frac{dV}{dr} = C_1$$

where C_1 is constant.

$$\frac{dV}{dr} = \frac{C_1}{r^2}$$

Now again integrating both sides, we get

$$V = -\frac{C_1}{r} + C_2 \tag{1}$$

where C_2 is another constant.
At $r = c$, we have $V = V_1$. So,

$$V_1 = -\frac{C_1}{c} + C_2 \tag{2}$$

At $r = d$, we have $V = V_2$. So,

$$V_2 = -\frac{C_1}{d} + C_2 \tag{3}$$

Subtracting Eq. (2) by Eq. (3), we get

$$V_1 - V_2 = \frac{-C_1}{c} + \frac{C_1}{d}$$

$$\Rightarrow V_1 - V_2 = C_1\left(\frac{c-d}{cd}\right)$$

$$\Rightarrow C_1 = \left(\frac{V_1 - V}{c-d}\right)cd$$

Put this value of C_1 in Eq. (2), we get

$$C_2 = V_1 + \left(\frac{V_1 - V}{c-d}\right)d$$

Now put values of C_1 and C_2 in Eq. (1), we get

$$V(r) = \frac{-\left(\frac{V_1 - V}{c-d}\right)cd}{r} + V_1\left(\frac{V_1 - V_2}{c-d}\right)$$

$$= \left(\frac{V_1 - V}{d-c}\right)\frac{cd}{r} + \frac{V_2 d - V_1 c}{d-c}$$

Ans. (B)

44. Let the probability density function of a random variable x be given as

$$f(x) = ae^{-2|x|}$$

The value of a is _____.

Solution: Given that, probability density function is

$$f(x) = ae^{-2|x|}$$

If $$\int_{-\infty}^{\infty} f(x)dx = 1$$

So, $$\int_{-\infty}^{0} ae^{2x}dx + \int_{-\infty}^{0} ae^{-2x}dx = 1$$

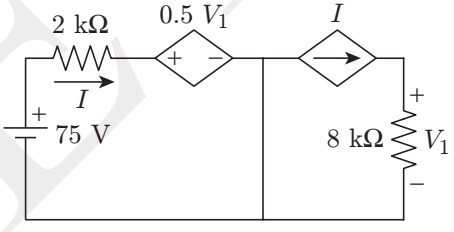
$$\left|a\frac{e^{2x}}{2}\right|_{-\infty}^{0} + \left|a\frac{e^{-2x}}{-2}\right|_{-\infty}^{0} = 1$$

$$\Rightarrow \frac{a}{2} + \frac{a}{2} = 1$$

$$\Rightarrow a = 1$$

Ans. (1)

45. In the circuit shown below, the magnitude of the voltage V_1 in volts, across the 8 kΩ resistor is _____. (round off to nearest integer)



Solution: The voltage across 8 kΩ is

$$V_1 = (8\times10^3)I \tag{1}$$

Applying kVL in left hand side loop, we get

$$75 = (2\times10^3)I + 0.5V_1$$

$$\Rightarrow I = \frac{75 - 0.5V_1}{2\times10^3} \tag{2}$$

Put value of I from Eq. (2) in Eq. (1), we get

$$V_1 = 8\times10^3\left(\frac{75 - 0.5V_1}{2\times10^3}\right) = 300 - 2V_1$$

$$\Rightarrow \quad V_1 = 100\,\text{V}$$

Ans. (100)

46. Two generating units rated for 250 MW and 400 MW have governor speed regulations of 6% and 6.4%, respectively, from no load to full load. Both the generating units are operating in parallel to share a load of 500 MW. Assuming free governor action, the load shared in MW, by the 250 MW generating unit is _____. (round off to nearest integer)

Solution: Let us consider no load frequency be 50 Hz. Now we draw the dropping curve (see in the following figure)

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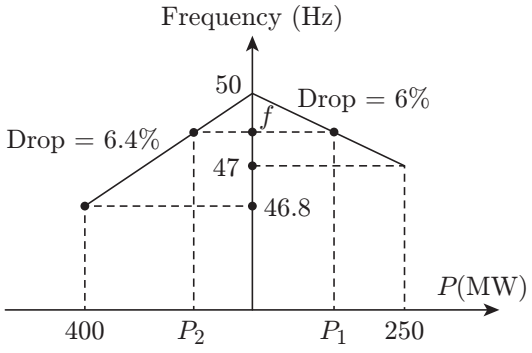


From curve, we get two slopes equations one for left side line other from right side line

$$\frac{3}{50} = \frac{50 - f}{P_1} \quad (1)$$

$$\frac{3.2}{400} = \frac{50 - f}{P_2} \quad (2)$$

Divide Eq. (1) divide by Eq. (2), we get

$$\frac{3/250}{3.2/400} = \frac{P_2}{P_1}$$

$$\Rightarrow \qquad 1.5 = \frac{P_2}{P_1}$$

Given that,

$$P_1 + P_2 = 500 \Rightarrow P_2 = 500 - P_1$$

So, $$1.5P_1 = 500 - P_1$$

$$\Rightarrow P_1 = 200 \text{ MW}$$

Ans. (200)

47. A 20 MVA, 11.2 kV, 4-pole, 50 Hz alternator has an inertia constant of 15 MJ/MVA. If the input and output powers of the alternator are 15 MW and 10 MW, respectively, the angular acceleration in mechanical degree/s^2 is _____. (round off to nearest integer)

Solution: Swing equation is given by

$$\frac{2Hs}{\omega_s} \cdot \frac{d^2\delta}{dt^2} = P_a$$

where $P_a = P_{\text{in}} - P_{\text{out}} = 15 \text{ MW} - 10 \text{ MW} = 5 \text{ MW}$

Given that, $H = 15$, $s = 20$ and $\omega_s = 360 \times 50 = 18000$.

Putting these values, we get

$$\frac{2 \times 15 \times 20}{360 \times 50} \cdot \frac{d^2\delta}{dt^2} = 5$$

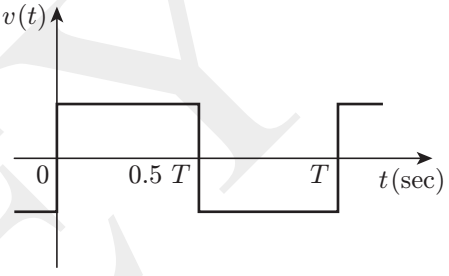
$$\Rightarrow \frac{d^2\delta}{dt^2} = 150 \text{ electric degree/s}^2$$

The angular acceleration is given by

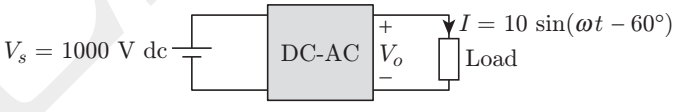
$$\alpha = \frac{1}{2} \cdot \frac{d^2\delta}{dt^2} = 75 \text{ mechanical degree/s}^2$$

Ans. (75)

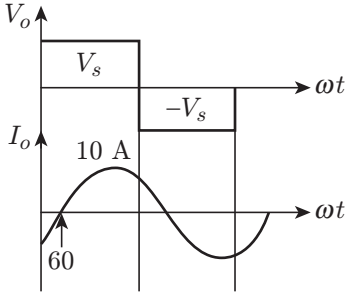
48. Consider an ideal full-bridge single-phase DC-AC inverter with a DC bus voltage magnitude of 1000 V. The inverter output voltage $v(t)$ shown below, is obtained when diagonal switches of the inverter are switched with 50% duty cycle. The inverter feeds a load with a sinusoidal current given by, $i(t) = 10\sin(\omega t - \pi/3)$ A, where $\omega = 2\pi/T$. The active power, in watts, delivered to the load is _____. (round off to nearest integer)



Solution: An ideal full-bridge single-phase DC-AC inverter with a DC bus voltage is shown in the following figure:



The input and output waveform is shown in the following figure.



The active power is given by

$$P_a = V_{01\text{rms}} I_{01\text{rms}} \times \cos\phi$$

Here,

$$V_{01\text{rms}} = \frac{2\sqrt{2}V_s}{\pi} = \frac{2\sqrt{2}}{\pi} \times 1000,$$

$$I_{01\text{rms}} = \frac{I_o}{\sqrt{2}} = \frac{10}{\sqrt{2}},$$

$$\phi = \frac{\pi}{3}$$

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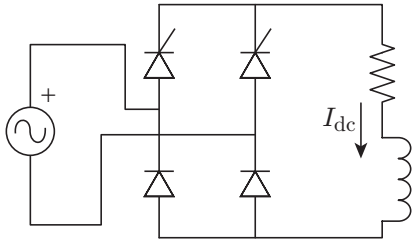
So,

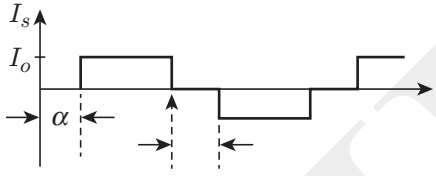
$$P_a = \left(\frac{2\sqrt{2}}{\pi}\times 1000\right)\left(\frac{10}{\sqrt{2}}\right)\times 0.5 = 3182\,\text{W}$$

Ans. (3182)

49. For the ideal AC-DC rectifier circuit shown in the figure below, the load current magnitude is $I_{\text{dc}} = 15$ A and is ripple free. The thyristors are fired with a delay angle of 45°. The amplitude of the fundamental component of the source current, in amperes, is _____. (round off to two decimal places).



Solution: Given rectifier circuit is a single phase semi-converter. The waveform of source current:



The source current is given by

$$I_{sn} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4I_o}{n\pi}\cos\left(\frac{n\alpha}{2}\right)\sin(n\omega t + \phi_n)$$

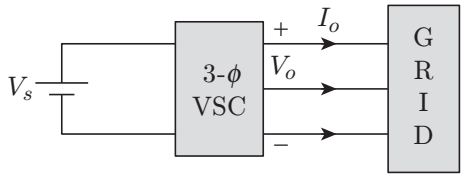
So, the fundamental component of source current is

$$I_{s1} = \frac{4I_0}{\pi}\cos\left(\frac{\alpha}{2}\right) = \frac{4\times 15}{\pi}\cos\left(\frac{45^\circ}{2}\right)$$
$$= \frac{4\times 15}{\pi}\cos(22.5^\circ) = 17.65\,\text{A}$$

Ans. (17.6)

50. A 3-phase grid-connected voltage source converter with DC link voltage of 1000 V is switched using sinusoidal Pulse Width Modulation (PWM) technique. If the grid phase current is 10 A and the 3-phase complex power supplied by the converter is given by $(-4000 - j3000)$ VA, then the modulation index used in sinusoidal PWM is _____. (round off to two decimal places)

Solution: A 3-phase grid-connected voltage source converter with DC link voltage is shown in the following figure:



Given that,

$S = (-4000 - j3000)\,\text{VA} = 5000\angle -143.13^\circ\,\text{VA}$,

$V_s = 1000$ V and $I_L = 10$ A.

Apparent power is given by

$$S = \sqrt{3}V_L I_L$$

So, $$V_L = \frac{S}{\sqrt{3}I_L} = \frac{5000}{\sqrt{3}\times 10}\,\text{V}$$

Now, peak load voltage is given by

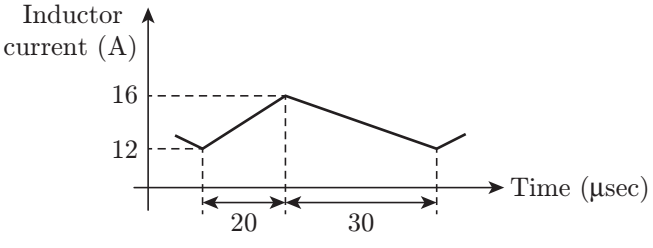
$$V_{L,p} = \sqrt{3}M_A\frac{V_s}{2} = \sqrt{2}V_L$$

where M_A is modulation index.
Now putting values, we get

$$288.675 = M_A\left(\frac{\sqrt{3}}{2\sqrt{2}}\times 1000\right)$$
$$\Rightarrow M_A = 0.47$$

Ans. (0.47)

51. The steady state current flowing through the inductor of a DC-DC buck boost converter is given in the figure below. If the peak-to-peak ripple in the output voltage of the converter is 1 V, then the value of the output capacitor, in μF, is _____. (round off to nearest integer)



Solution: The peak-to-peak ripple in the output voltage is given by

$$\Delta V_C = \frac{\alpha I_o}{fC}$$

where $$\alpha = \frac{T_{\text{on}}}{T_{\text{tot}}} = \frac{20}{20+30} = \frac{20}{50} = \frac{2}{5}$$

and $$I_o = I_L(1-\alpha)$$

where $$I_L = \frac{I_{\max} + I_{\min}}{2} = \frac{16+12}{2} = 14\,\text{A}$$

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So, $I_o = 14 \times \dfrac{30}{50} = 8.4\,\text{A}$

Given that, $\Delta V_C = 1$

Now, putting all values in first equation, we get

$$1 = \frac{(2/5)\times 8.4}{\dfrac{1}{50\times 10^{-6}}\times C}$$

$$\Rightarrow C = 168\,\mu\text{F}$$

Ans. (168)

52. A 280 V, separately excited DC motor with armature resistance of 1 Ω and constant field excitation drives a load. The load torque is proportional to the speed. The motor draws a current of 30 A when running at a speed of 1000 rpm. Neglect frictional losses in the motor. The speed, in rpm, at which the motor will run, if an additional resistance of value 10 Ω is connected in series with the armature, is _____. (round off to nearest integer)

Solution: Given that,

$$V = 280,\ R_a = 1\,\Omega\ N = 1000\,\text{rpm}, R_{\text{ext}} = 10\,\Omega$$

and $I_{a1} = 30\,\text{A}.$

Back emf is given by

$$E_b = V - I_a R_a$$

For first condition, we have

$$E_{b1} = 280 - (30\times 1) = 250\,\text{V}$$

When additional 10 Ω resistance connected in series with armature.

$$E_{b2} = 280 - (10+1)I_{a2} = 280 - 11I_{a2}$$

Given condition is

$$\tau \propto N$$

We know that

$$\tau \propto \phi I_a$$

Here ϕ constant. So,

$$\tau \propto I_a$$

So, $I_a \propto N$

Then, $\dfrac{I_{a1}}{I_{a2}} \propto \dfrac{N_1}{N_2}$

$$\Rightarrow I_{a2} \propto \left(\frac{I_{a1}}{N_1}\right)N_2 = \left(\frac{30}{1000}\right)N_2 = 0.03N_2$$

We also say that,

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

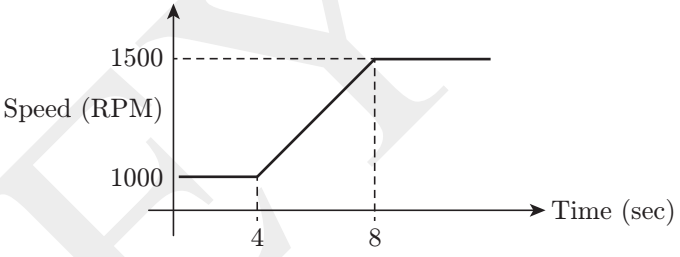
$$\Rightarrow \frac{N_2}{N_1} = \frac{V - I_{a2}(R_a + R_{\text{ext}})}{V - I_a R_a}$$

$$\Rightarrow \frac{N_2}{1000} = \frac{280 - 0.03N_2(10+1)}{280 - 30\times 1}$$

$$\Rightarrow \quad N_2 = 482.76\,\text{rpm} \approx 483\,\text{rpm}$$

Ans. (483)

53. A 4-pole induction motor with inertia of 0.1 kg-m^2 drives a constant load torque of 2 Nm. The speed of the motor is increased linearly from 1000 rpm to 1500 rpm in 4 seconds as shown in the figure below. Neglect losses in the motor. The energy, in joules, consumed by the motor during the speed change is _____. (round off to nearest integer)



Solution: Equation of speed vs time from 4 s to 8 s is given by

$$N = 125t + 500$$

where $4\,\text{s} \le t \le 8\ \text{s}$

Now, we know that

$$J\frac{d\omega}{dt} = \tau_e - \tau_L$$

$$\Rightarrow\ \tau_e = J\frac{d\omega}{dt} + \tau_L$$

$$\Rightarrow \omega\tau_e = J\omega\frac{d\omega}{dt} + \omega\tau_L$$

We know that,

$$P_e = \tau_e\omega$$

So,

$$P_e = J\omega\frac{d\omega}{dt} + \omega\tau_L$$

We also know that,

$$P_e = \frac{dE}{dt}$$

Then,

$$\frac{dE}{dt} = J\omega\frac{d\omega}{dt} + \omega\tau_L$$

$$\Rightarrow dE = J\omega d\omega + \omega\tau_L dt$$

Integrating both sides, we get

$$E = J\int \omega d\omega + \tau_L \int \omega dt$$
$$= J\left(\frac{2\pi}{60}\right)^2 \int NdN + \tau_L \left(\frac{2\pi}{60}\right)\int Ndt$$
$$= J\left(\frac{2\pi}{60}\right)^2 \int NdN + \tau_L \left(\frac{2\pi}{60}\right)\int (125t + 500)dt$$
$$= J\left(\frac{2\pi}{60}\right)^2 \left(\frac{N^2}{2}\right)\Bigg|_{N_1}^{N_2} + \tau_L \left(\frac{2\pi}{60}\right)\left(\frac{125t^2}{2} + 500t\right)\Bigg|_{t_1}^{t_2}$$

Putting the given values, we get

$$E = \left[0.1 \times \left(\frac{2\pi}{60}\right)^2 \left(\frac{1500^2 - 1000^2}{2}\right)\right] + \left[2 \times \left(\frac{2\pi}{60}\right)\left(\frac{125t^2}{2} + 500t\right)\Bigg|_4^8\right]$$
$$= 1732.5865\,\text{J}$$

Ans. (1732)

54. A star-connected 3-phase, 400 V, 50 kVA, 50 Hz synchronous motor has a synchronous reactance of 1 ohm per phase with negligible armature resistance. The shaft load on the motor is 10 kW while the power factor is 0.8 leading. The loss in the motor is 2 kW. The magnitude of the per phase excitation emf of the motor, in volts, is _____. (round off to nearest integer).

Solution: Given that, $P_{\text{sh}} = 10$ kW, $P_L = 2$ kW, $V_{\text{in}} = 400$ V, $Z_s = 1\ \Omega$/phase and pf $= \cos\phi = 0.8$. Excitation emf or back emf is given by

$$E_b = \sqrt{(V\cos\phi - I_a R_a)^2 + (V\sin\phi \pm I_a X_s)^2}$$

Input power is

$$P_{\text{in}} = P_{\text{sh}} + P_L = 10 + 2 = 12\ \text{kW}$$

Armature current is

$$I_a = \frac{P_{\text{in}}}{\sqrt{3}V_{\text{in}}\cos\phi} = \frac{12000}{\sqrt{3}\times 400 \times 0.8} = 21.65\,\text{A}$$

Therefore, back emf per phase is

$$E_b = \sqrt{\left(\frac{400}{\sqrt{3}} \times 0.8\right)^2 + \left[\left(\frac{400}{\sqrt{3}} \times 0.6 + 21.65\right)^2\right]}$$
$$= 244.55\,\text{V} \approx 245\ \text{V}$$

Ans. (255)

55. A 3-phase, 415 V, 4-pole, 50 Hz induction motor draws 5 times the rated current at rated voltage at starting. It is required to bring down the starting current from the supply to 2 times of the rated current using a 3-phase autotransformer. If the magnetizing impedance of the induction motor and no load current of the autotransformer is neglected, then the transformation ratio of the autotransformer is given by _____. (round off to two decimal places).

Solution: Given that,

$$I_{\text{sc}} = 5I_{\text{rated}}$$
$$\Rightarrow I_{\text{sc}} = 5I_f$$

It is required to make starting current to $2I_f$ using auto transformer, that is, $I_L = 2I_f$. Line current with auto transformer is

$$I_L = x^2 I_{\text{sc}}$$

So,

$$x^2 I_L = 2I_f$$
$$\Rightarrow x^2 = \frac{2I_f}{I_L} = \frac{2\times 1}{5}$$
$$\Rightarrow x = 0.63$$

Ans. (0.63)

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