SOLVED GATE (ELECTRONICS AND COMMUNICATION ENGINEERING) 2022

Engineering Mathematics and General Aptitude

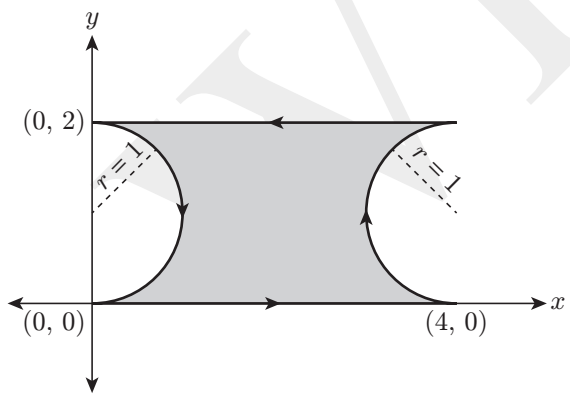
ENGINEERING MATHEMATICS

Q. No. 1–5 Carry One Mark Each

1. Consider the two-dimensional vector field $\vec{F}(x,y) = x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ denote the unit vectors along the x-axis and the y-axis, respectively. A contour C in the x-y plane, as shown in the figure, is composed of two horizontal lines connected at the two ends by two semicircular arcs of unit radius. The contour is traversed in the counter-clockwise sense. The value of the closed path integral

$$\oint_C \vec{F}(x,y)\cdot(dx\hat{i} + dy\hat{j})$$

is _____.



(A) 0 (B) 1
(C) $8 + 2\pi$ (D) -1

Solution: Given that,

$$\vec{F}(x,y) = x\hat{i} + y\hat{j}$$

Now,

$$\oint_C \vec{F}(x,y)(dx\hat{i} + dy\hat{j})$$

$$= \oint_C (x\hat{i} + y\hat{j})(dx\hat{i} + dy\hat{j})$$

$$= \oint_C xdx + ydy$$

Applying Green's theorem, we get

$$\oint_C xdx + ydy = \iint_R \left[\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(x)\right] dxdy$$

$$= \iint_R (0-0)dxdy = 0$$

Ans. (A)

2. Consider a system of linear equations $Ax = b$, where

$$A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

This system of equations admits _____.
(A) a unique solution for x
(B) infinitely many solutions for x
(C) no solutions for x
(D) exactly two solutions for x

Solution: Given that,

$$A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ and } Ax = b$$

So, $\begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$$x - \sqrt{2}y + 3z = 1 \quad (1)$$

$$-x + \sqrt{2}y - 3z = 3$$

$$\Rightarrow \quad x - \sqrt{2}y + 3z = -3 \quad (2)$$

Since both the equations represents parallel lines to each other, therefore this system of equations admits no solution for x.

Ans. (C)

3. Consider the following partial differential equation (PDE)

$$a\frac{\partial^2 f(x,y)}{\partial x^2}+b\frac{\partial^2 f(x,y)}{\partial y^2}=f(x,y)$$

where a and b are distinct positive real numbers. Select the combination(s) of values of the real parameters ξ and η such that $f(x,y)=e^{(\xi x+\eta y)}$ is a solution of the given PDE.

(A) $\xi=\frac{1}{\sqrt{2a}},\eta=\frac{1}{\sqrt{2b}}$ (B) $\xi=\frac{1}{\sqrt{a}},\eta=0$

(C) $\xi=0,\eta=0$ (D) $\xi=\frac{1}{\sqrt{a}},\eta=\frac{1}{\sqrt{b}}$

Solution: Given that $f(x,y)=e^{(\xi x+\eta y)}$ is a solution of partial differential equation.

$$a\frac{\partial^2 f(x,y)}{\partial x^2}+b\frac{\partial^2 f(x,y)}{\partial y^2}=f(x,y)$$

Now,

$$f(x,y)=e^{(\xi x+\eta y)}$$

Differentiating $f(x,\ y)$ two times with respect to x:

$$\frac{\partial f(x,y)}{\partial x}=\xi e^{(\xi x+\eta y)}$$

$$\frac{\partial^2 f(x,y)}{\partial x^2}=\xi^2 e^{(\xi x+\eta y)}$$

Similarly, differentiating $f(x,\ y)$ two times with respect to y:

$$\frac{\partial f(x,y)}{\partial y}=\eta e^{(\xi x+\eta y)}$$

$$\frac{\partial^2 f(x,y)}{\partial y^2}=\eta^2 e^{(\xi x+\eta y)}$$

So,

$$a\frac{\partial^2 f(x,y)}{\partial x^2}+b\frac{\partial^2 f(x,y)}{\partial y^2}$$
$$=[a\xi^2 e^{(\xi x+\eta y)}+b\eta^2 e^{(\xi x+\eta y)}]$$
$$=(a\xi^2+b\eta^2)e^{(\xi x+\eta y)}=(a\xi^2+b\eta^2)f(x,y)$$

Therefore,

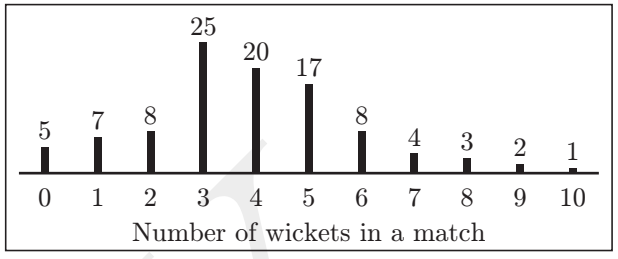
$$a\xi^2+b\eta^2=1$$

From options, it is clear that this condition is stratified when

$$\xi=\frac{1}{\sqrt{2a}},\ \ \eta=\frac{1}{\sqrt{2b}}\quad\text{or}\quad \xi=\frac{1}{\sqrt{a}},\eta=0.$$

Ans. (A, B)

4. The bar graph shows the frequency of the number of wickets taken in a match by a bowler in her career. For example, in 17 of her matches, the bowler has taken 5 wickets each. The median number of wickets taken by the bowler in a match is _____ (rounded off to one decimal place).



Solution: Total frequency of numbers of wickets is

$$5+7+8+25+20+17+8+4$$
$$+3+2+1=100$$

Now, median median number of wickets is

$$\text{Median}=\frac{\left(\frac{n}{2}\right)\text{th match's wicket}+\left(\frac{n}{2}+1\right)\text{th match's wickets}}{2}$$

$$=\text{Average of the }50^{\text{th}}\text{ and }51^{\text{st}}\text{ matches}$$

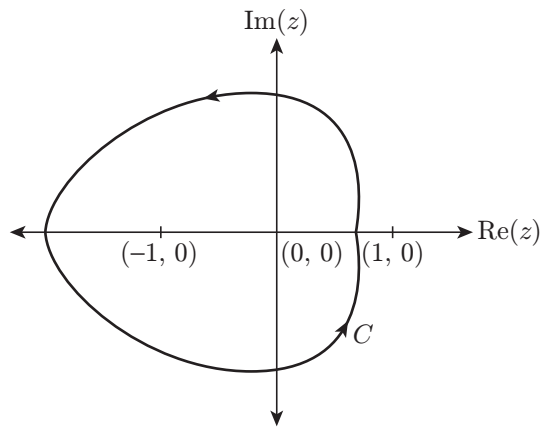
$$=\frac{4+4}{2}=4$$

Ans. (4.0)

5. A simple closed path C in the complex plane is shown in the figure. If

$$\oint_C \frac{2^z}{z^2-1}dz=-i\pi A,$$

where $i=\sqrt{-1}$, then the value of A is ____ (rounded off to two decimal places).

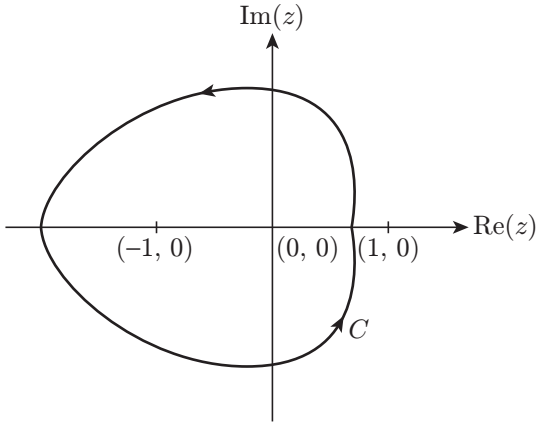


WILEY

Solution: The poles of function $f(z) = \dfrac{2^z}{(z^2-1)}dz$ is

$$(z^2 - 1) = 0 \Rightarrow z = \pm 1$$

The given closed path C in the complex plane is



Since, $z = -1$ lies inside the contour, then the residue at $z = -1$ is

$$\text{Residue (at } z = -1) = \lim_{z \to -1} (z+1) \cdot f(z)$$

$$= \lim_{z \to -1} (z+1) \cdot \frac{2^z}{(z+1)(z-1)} = \frac{2^z}{(z-1)} = \frac{-1}{4}$$

Therefore,

$$\oint_C \frac{2^2}{(z^2-1)} dz = 2\pi i \qquad \text{(Residues at } z = -1)$$

$$= 2\pi i\left(\frac{-1}{4}\right) = \frac{-1}{2}\pi i$$

On comparing with with $-i\pi A$, we have

$$A = \frac{1}{2} = 0.50$$

Ans. (0.50)

Q. No. 6–10 Carry Two Marks Each

6. The function $f(x) = 8 \log_e x - x^2 + 3$ attains its minimum over the interval $[1, e]$ at $x =$ _____. (Here $\log_e x$ is the natural logarithm of x.)

(A) 2 (B) 1

(C) e (D) $\dfrac{1+e}{2}$

Solution: Given function, $f(x) = 8\log_e x - x^2 + 3$ attains its minimum when $x \in [1, e]$

Differentiating with respect to x, we have

$$f'(x) = \frac{8}{x} - 2x$$

For maximum and minimum value,

$$f'(x) = 0 = \frac{8}{x} - 2x$$

$$\Rightarrow \quad \frac{8}{x} - 2x = 0 \Rightarrow 8 - 2x^2 = 0$$

$$\Rightarrow \quad 2x^2 = 8,\ x^2 = 4,\ x = \pm 2$$

Only $x = 2$ lies in the interval $[1, e]$.

Again differentiating with respect to x, we get

$$f''(x) = -\frac{8}{x^2} - 2$$

$$\Rightarrow \quad f''(2) = -\frac{8}{4} - 2 = -4 < 0$$

$$\Rightarrow \quad f''(2) < 0$$

Hence, $f(x)$ is maximum. Now, value of $f(x)$ at $x = 1$ and $x = e$

$$f(1) = 8\log_e 1 - 1 + 3 = 2$$

$$f(e) = 8\log_e e - e^2 + 3 = 8 - e^2 + 3 = 3.61$$

Hence, the minimum value of $f(x)$ occurs at $x = 1$.

Ans. (B)

7. Let α, β be two non-zero real numbers and v_1, v_2 be two non-zero real vectors of size 3×1. Suppose that v_1 and v_2 satisfy $v_1^T v_2 = 0, v_1^T v_1 = 1$, and $v_2^T v_2 = 1$. Let A be the 3×3 matrix given by:

$$A = \alpha v_1 v_1^T + \beta v_2 v_2^T$$

The eigenvalues of A are _____.

(A) $0, \alpha, \beta$ (B) $0, \alpha+\beta, \alpha-\beta$

(C) $0, \dfrac{\alpha+\beta}{2}, \sqrt{\alpha\beta}$ (D) $0, 0, \sqrt{\alpha^2+\beta^2}$

Solution: Given that,

$$A = \alpha v_1 v_1^T + \beta v_2 v_2^T$$

Multiplying both sides by matrix v_1, we get

$$Av_1 = \alpha v_1 v_1^T v_1 + \beta v_2 v_2^T v_1$$

$$= \alpha(1)v_1 + \beta(0) = \alpha v_1$$

Now, multiplying both sides by matrix v_2, we get

$$Av_2 = \alpha v_1 v_1^T v_2 + \beta v_2 v_2^T v_2$$

$$= \alpha(0) + \beta(1)v_2 = \beta v_2$$

Since $v_1 v_1^T$ and $v_2 v_2^T$ both are singular matrices, A is also a singular matrix.

$$|A| = 0 \Rightarrow \lambda_A = 0$$

Hence, $\lambda_A = 0, \alpha, \beta$

Ans. (A)

8. Consider an even polynomial $p(s)$ given by

$$p(s) = s^4 + 5s^2 + 4 + K,$$

where K is an unknown real parameter. The complete range of K for which $p(s)$ has all its roots on the imaginary axis is _____.

(A) $-4 \le K \le \frac{9}{4}$ (B) $-3 \le K \le \frac{9}{2}$

(C) $-6 \le K \le \frac{5}{4}$ (D) $-5 \le K \le 0$

Solution: The even polynomial $p(s)$ is given by

$$p(s) = s^4 + 5s^2 + 4 + K$$

By applying Routh-Hurwitz criteria, we get

s^4	1	5	$(4+K)$
s^3	0	0	
s^2			
s^1			
s^0			

The auxiliary equation is

$$A(s) = s^4 + 5s^2 + (4+K)$$

$$\frac{dA(s)}{ds} = 4s^3 + 10s$$

Now,

s^4	1	5	$(4+K)$
s^3	4	10	0
s^2	$\frac{5}{2}$	$(4+K)$	
s^1	$\frac{25-4(4+K)}{5/2}$		
s^0	$(4+K)$		

Hence,

$$(4+K) > 0 \Rightarrow K > -4$$

$$\Rightarrow \quad \frac{25-4(4+K)}{5/2} > 0$$

$$\Rightarrow \quad 25 - 16 - 4K > 0 \Rightarrow 9 - 4K > 0$$

$$\Rightarrow \quad K < \frac{9}{4}$$

Hence, the complete range of K for which $p(s)$ has all its roots on the imaginary axis is $-4 \le k \le \frac{9}{4}$.

Ans. (A)

9. Consider the following series:

$$\sum_{n=1}^{\infty} \frac{n^d}{c^n}$$

For which of the following combinations of c, d values does this series converge?

(A) $c = 1$, $d = -1$
(B) $c = 2$, $d = 1$
(C) $c = 0.5$, $d = -10$
(D) $c = 1$, $d = -2$

Solution: The given series is

$$S_n = \sum_{n=1}^{\infty} \frac{n^d}{c^n}$$

Considering the options one by one, we get

(A) $c = 1$, $d = -1$

$$S_n = \sum \frac{n^{-1}}{1^n} = \sum \frac{1}{n}$$

Hence, series is divergent by p-test.

(B) $c = 2$, $d = 1 \Rightarrow S_n = \sum \frac{n}{2^n}$

Applying Ratio's test

$$\lim_{n\to\infty} \frac{S_n + 1}{S_n} = \lim_{n\to\infty} \frac{n+1}{2^n + 1} \cdot \frac{2^n}{n} = \frac{1}{2} < 2$$

Hence, the series is convergent.

(C) $c = 0.5$, $d = -10$

Applying Ratio test again, we get

$$\lim_{n\to\infty} \frac{S_n + 1}{S_n} = \lim_{n\to\infty} \frac{(n+1)^{-10}}{(0.5)^{n+1}} \cdot \frac{(0.5)^n}{n^{10}}$$

$$= 1/0.5 = 2 > 1$$

Hence, the series is divergent.

(D) $c = 1$, $d = -2$

$$S_n = \sum \frac{(n)^{-2}}{(1)^n} = \sum \frac{1}{n^2}$$

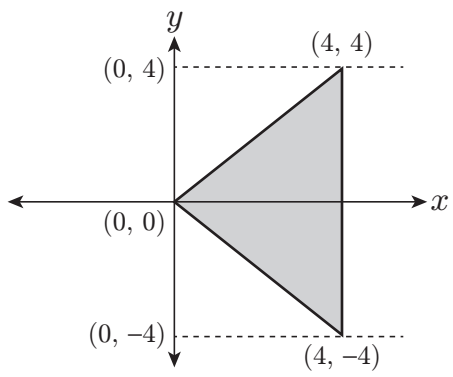
Hence, series is convergent by p-test.

Ans. (B, D)

10. The value of the integral

$$\iint_D 3(x^2 + y^2)dxdy,$$

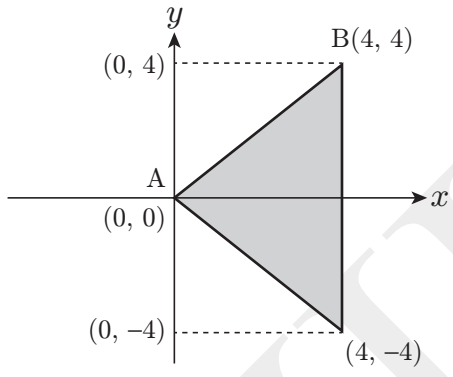
where D is the shaded triangular region shown in the diagram, is _____ (rounded off to the nearest integer).



Solution: Let

$$I = \iint_D 3(x^2 + y^2)dxdy$$

where D is the shaded triangular region shown below



The slope of line AB is

$$m = \frac{4-0}{4-0} = 1$$

The equation of line AB is

$$y - 0 = m(x - 0)$$

$$\Rightarrow \qquad y = 1 \times x \Rightarrow y = x$$

Now,

$$I = \int_0^4 \int_{-x}^{x} (3x^2 + 3y^2)dydx$$

$$= \int_0^4 [3x^2y + y^3]_{-x}^{x} dx$$

$$= \int_0^4 [6x^3 + 2x^3]dx = \int_0^4 8x^3 dx$$

$$= 8\int_0^4 x^3 dx = 8\left[\frac{x^4}{4}\right]_0^4$$

$$= 8 \times \frac{1}{4} \times (4)^4 = 2 \times 256 = 512$$

Ans. (512)

GENERAL APTITUDE

Q. No. 1–5 Carry One Mark Each

1. Mr. X speaks _____ Japanese _____ Chinese.
(A) neither / or (B) either / nor
(C) neither / nor (D) also / but

Solution: Mr. X speaks <u>neither</u> Japanese <u>nor</u> Chinese.

Ans. (C)

2. A sum of money is to be distributed among P, Q, R, and S in the proportion 5 : 2 : 4 : 3, respectively. If R gets ₹1000 more than S, what is the share of Q (in ₹)?
(A) 500 (B) 1000
(C) 1500 (D) 2000

Solution: Money is distributed in ratio

$$P : Q : R : S = 5 : 2 : 4 : 3$$

Let the sum of money received by P be $5x$, then share of Q = $2x$, share of R = $4x$ and share of S = $3x$.

Given that, $\quad R = S + 1000$

$$\Rightarrow \qquad 4x = 3x + 1000$$

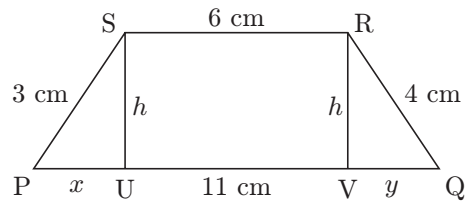
$$\Rightarrow \qquad x = 1000$$

Then Q's share = $2x = 2 \times 1000 = 2000$

Ans. (D)

3. A trapezium has vertices marked as P, Q, R and S (in that order anticlockwise). The side PQ is parallel to side SR. Further, it is given that, PQ = 11 cm, QR = 4 cm, RS = 6 cm and SP = 3 cm. What is the shortest distance between PQ and SR (in cm)?
(A) 1.80 (B) 2.40
(C) 4.20 (D) 5.76

Solution: From the given data we have the following figure of trapezium:



Shorter distance between PQ and SR is perpendicular distance. Now, in a triangle PSU, we have

$$(3)^2 = h^2 + x^2$$

$$\Rightarrow \qquad h^2 + x^2 = 9 \qquad (1)$$

In triangle RVQ

$$(4)^2 = h^2 + y^2$$

$$\Rightarrow \quad h^2 + y^2 = 16 \qquad (2)$$

Subtracting Eq. (1) from Eq. (2), we get

$$y^2 - x^2 = 7$$

$$\Rightarrow \quad (y - x)(y + x) = 7 \qquad (3)$$

From figure, we have

$$x + 6 + y = 11$$

$$\Rightarrow \quad x + y = 5 \qquad (4)$$

From Eqs. (4) and (3), we get

$$(y - x) = \frac{7}{5} \qquad (5)$$

Adding Eqs. (4) and (5), we get

$$2y = 5 + \frac{7}{5} \Rightarrow y = \frac{32}{10}$$

Put $y = \frac{32}{10}$ in Eq. (2), we get

$$h^2 + \left(\frac{32}{10}\right)^2 = 16$$

$$\Rightarrow \quad h^2 + (3.2)^2 = 16$$

$$\Rightarrow \quad h^2 = 16 - 10.24$$

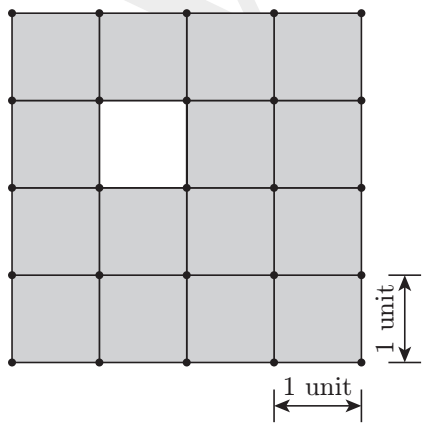
$$\Rightarrow \quad h^2 = 5.76$$

$$\Rightarrow \quad h = \sqrt{5.76} = 2.4$$

Therefore, shortest distance between PQ and SR is 2.4 cm

Ans. (B)

4. The figure shows a grid formed by a collection of unit squares. The unshaded unit square in the grid represents a hole.



What is the maximum number of squares without a "hole in the interior" that can be formed within the 4 × 4 grid using the unit squares as building blocks?

(A) 15 (B) 20
(C) 21 (D) 26

Solution: In the given grid, we have

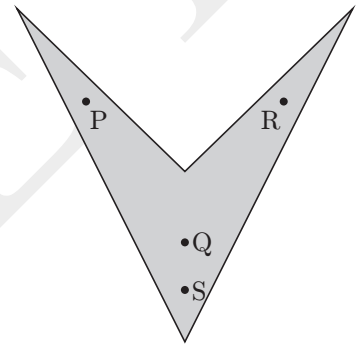
Number of 1 × 1 squares = 15

Number of 2 × 2 squares = 5

Therefore, number of squares without a hole = 15 + 5 = 20

Ans. (B)

5. An art gallery engages a security guard to ensure that the items displayed are protected. The diagram below represents the plan of the gallery where the boundary walls are opaque. The location the security guard posted is identified such that all the inner space (shaded region in the plan) of the gallery is within the line of sight of the security guard.



If the security guard does not move around the posted location and has a 360° view, which one of the following correctly represents the set of ALL possible locations among the locations P, Q, R and S, where the security guard can be posted to watch over the entire inner space of the gallery.

(A) P and Q (B) Q
(C) Q and S (D) R and S

Solution: If person will stand on P, then he can not see the visibility side of R and vice versa. So, security guard positing at Q and S can watch entire space of gallery.

Ans. (C)

Q. No. 6–10 Carry Two Marks Each

6. Mosquitoes pose a threat to human health. Controlling mosquitoes using chemicals may have undesired consequences. In Florida, authorities have used genetically modified mosquitoes to control the overall mosquito population. It remains to be seen if this novel approach has unforeseen consequences.

Which one of the following is the correct logical inference based on the information in the above passage?

(A) Using chemicals to kill mosquitoes is better than using genetically modified mosquitoes because genetic engineering is dangerous

(B) Using genetically modified mosquitoes is better than using chemicals to kill mosquitoes because they do not have any side effects

(C) Both using genetically modified mosquitoes and chemicals have undesired consequences and can be dangerous

(D) Using chemicals to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence

Solution: Based on information in above passage, correct logical inference is "using chemicals to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence".

Ans. (D)

7. Consider the following inequalities.

(i) $2x - 1 > 7$

(ii) $2x - 9 < 1$

Which one of the following expressions below satisfies the above two inequalities?

(A) $x \leq -4$

(B) $-4 < x \leq 4$

(C) $4 < x < 5$

(D) $x \geq 5$

Solution: We have two inequalities

$$2x - 1 > 7 \quad (1)$$

$$\Rightarrow \quad 2x > 8 \Rightarrow x > 4$$

$$2x - 9 < 1 \quad (2)$$

$$\Rightarrow \quad 2x < 10 \Rightarrow x < 5$$

Therefore, $4 < x < 5$

Ans. (C)

8. Four points P(0, 1), Q(0, −3), R(−2, −1), and S(2, −1) represent the vertices of a quadrilateral. What is the area enclosed by the quadrilateral?

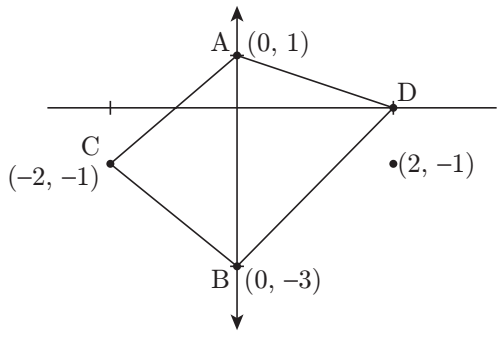
(A) 4

(B) $4\sqrt{2}$

(C) 8

(D) $8\sqrt{2}$

Solution: From given data, we have the following figure of quadrilateral



Length of side AD is $\sqrt{(2-0)^2 + (-1-1)^2} = \sqrt{8}$

Length of side DB is $\sqrt{(2-0)^2 + (-1+3)^2} = \sqrt{8}$

Length of side BC is $\sqrt{(0+2)^2 + (-3+1)^2} = \sqrt{8}$

Length of side CA is $\sqrt{(-2-0)^2 + (1+1)^2} = \sqrt{8}$

Since, AD = DB = BC = CA the quadrilateral is square.

Now, the area of quadrilateral is area of square ABCD $= (\sqrt{8})^2 = 8$

Ans. (C)

9. In a class of five students P, Q, R, S and T, only one student is known to have copied in the exam. The disciplinary committee has investigated the situation and recorded the statements from the students as given below.

Statement of P: R has copied in the exam.

Statement of Q: S has copied in the exam.

Statement of R: P did not copy in the exam.

Statement of S: Only one of us is telling the truth.

Statement of T: R is telling the truth.

The investigating team had authentic information that S never lies.

Based on the information given above, the person who has copied in the exam is

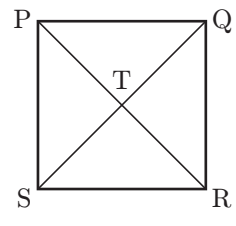
(A) R (B) P

(C) Q (D) T

Solution: Based on the given information P has copied in the exam.

Ans. (B)

10. Consider the following square with the four corners and the center marked as P, Q, R, S and T respectively.



Let X, Y and Z represent the following operations:
X: rotation of the square by 180 degree with respect to the S-Q axis.
Y: rotation of the square by 180 degree with respect to the P-R axis.
Z: rotation of the square by 90 degree clockwise with respect to the axis perpendicular, going into the screen and passing through the point T.
Consider the following three distinct sequences of operation (which are applied in the left to right order).

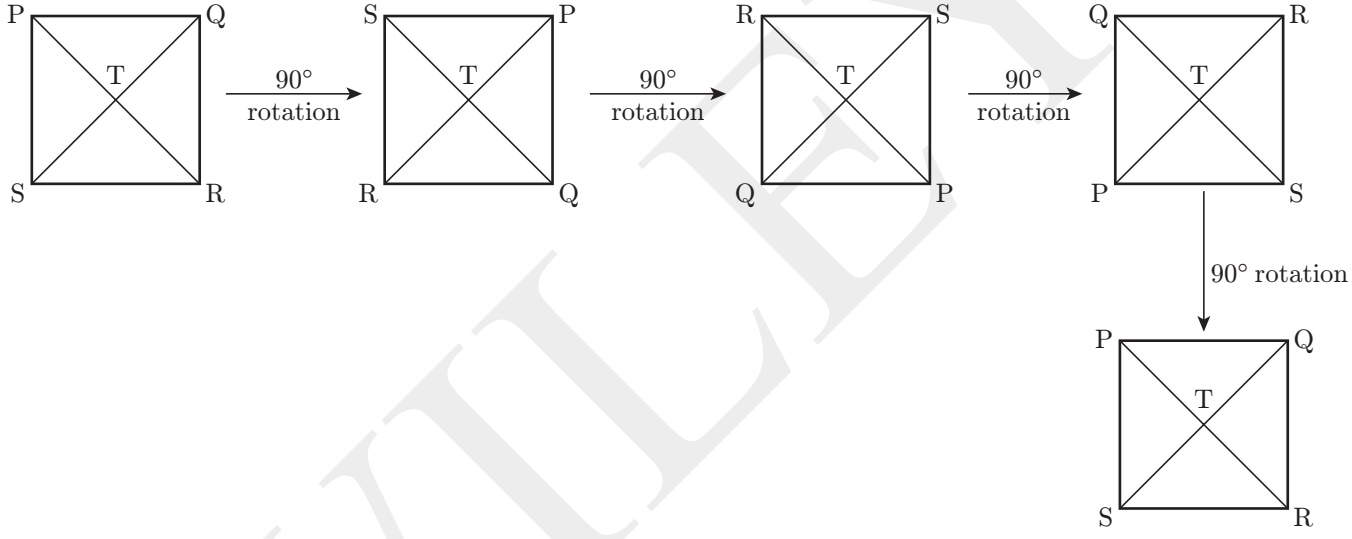
(1) XYZZ
(2) XY
(3) ZZZZ

Which one of the following statements is correct as per the information provided above?

(A) The sequence of operations (1) and (2) are equivalent
(B) The sequence of operations (1) and (3) are equivalent
(C) The sequence of operations (2) and (3) are equivalent
(D) The sequence of operations (1), (2) and (3) are equivalent

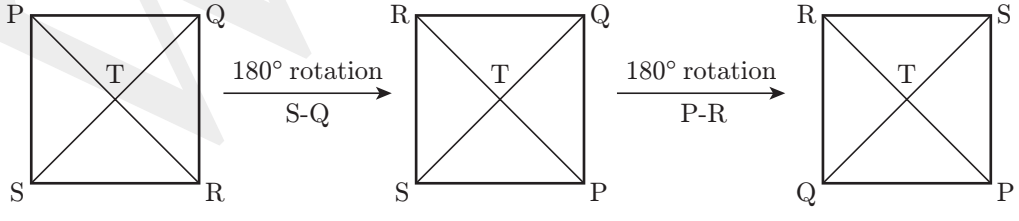
Solution: Operations:

ZZZZ



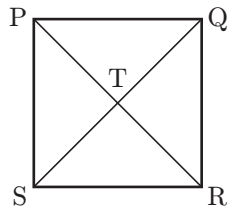
Similarly,

XY



and

XYZZ



Hence, the sequence of operations (1) and (3) are equivalent.

Ans. (B)

WILEY