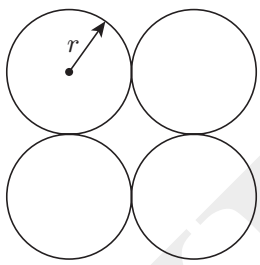
SOLVED GATE (ELECTRICAL ENGINEERING) 2022

Engineering Mathematics and General Aptitude

ENGINEERING MATHEMATICS

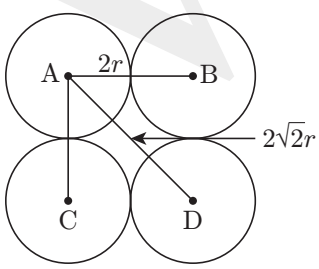
Q. No. 1–2 Carry One Mark Each

1. The geometric mean radius of a conductor, having four equal strands with each strand of radius 'r', as shown in the figure below, is



(A) $4r$ (B) $1.414r$
(C) $2r$ (D) $1.723r$

Solution: Let mark centres of four equal strands be A, B, C, D.
So, the distance between AB = AC = $2r$, AD = $2\sqrt{2}r$.



The geometric mean radius of a conductor is same as the geometric mean radius of one strand.
The geometric mean radius of one strand is given by

$$\text{GMR} = [r_{\text{AA}} \times r_{\text{AB}} \times r_{\text{AC}} \times r_{\text{AD}}]^{1/4}$$
$$= [r \times 2r \times 2r \times 2\sqrt{2}r]^{1/4} = 1.723r$$

Ans. (D)

2. Consider a 3×3 matrix A whose (i, j)-th element, $a_{i,\,j} = (i - j)^3$. Then the matrix A will be
(A) symmetric. (B) skew-symmetric.
(C) unitary. (D) null.

Solution: Given, 3×3 matrix A.

$$A = [a_{i,j}]_{3\times 3}$$

where $$a_{i,j} = (i - j)^3$$

For $i = j$, we have

$$a_{i,j} = (i - i)^3 = 0$$

For $i \neq j$, we have

$$a_{i,j} = (i - j)^3 = -(j - i)^3 = -a_{i,j}$$

Hence, $A_{3\times 3}$ is a skew symmetric matrix

Ans. (B)

Q. No. 3–10 Carry Two Marks Each

3. e^A denotes the exponential of a square matrix A. Suppose λ is an eigenvalue and v is the corresponding eigen-vector of matrix A. Consider the following two statements:
Statement 1: e^λ is an eigenvalue of e^A.
Statement 2: v is an eigen-vector of e^A.
Which one of the following options is correct?
(A) Statement 1 is true and statement 2 is false.
(B) Statement 1 is false and statement 2 is true.
(C) Both the statements are correct.
(D) Both the statements are false.

Solution: Given λ is an eigen value of A. Then eigen value of e^A is given as:

$$e^A = 1 + A + \frac{A^2}{2} + \frac{A^3}{6} + \ldots\ldots \text{ is}$$

$$e^\lambda = 1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6} + \ldots\ldots$$

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So, statement 1 is true.
We know that eigen value of A and polynomial matrix in A is same. Eigen vector of A and e^A is same.
So, statement 2 is true.

Ans. (C)

4. Let $f(x) = \int_0^x et(t-1)(t-2)dt$. Then $f(x)$ decreases in the interval

(A) $x \in (1,2)$ (B) $x \in (2,3)$
(C) $x \in (0,1)$ (D) $x \in (0.5,1)$

Solution: The given function

$$f(x) = \int_0^x e^t(t-1)(t-2)dt$$

is decreasing if $f'(x) = 0$.

By Leibnitz rule

$$f'(x) = \frac{d}{dx}f(x) < 0$$
$$= e^x(x-1)(x-2)\frac{d}{dx}(x) - e^0(0-1)(0-2)\frac{d}{dx}(0)$$
$$= e^x(x-1)(x-2) - 0 < 0$$
$$= (x-1)(x-2) < 0$$
$$= x > 1 \text{ and } x < 2$$

Hence x belongs to (1, 2).

Ans. (A)

5. Consider a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars and I is the identity matrix. Then $(c + d)$ is equal to

(A) 5 (B) 17
(C) -6 (D) 11

Solution: Given matrix is

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

Now, eigen values of matrix A are

$$A - \lambda I = 0$$
$$\Rightarrow \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 4-\lambda & -2 \\ 0 & 1 & 1-\lambda \end{bmatrix} = 0$$
$$\Rightarrow (1-\lambda)[(4-\lambda)(1-\lambda)+2] = 0$$
$$\Rightarrow (1-\lambda)[\lambda^2 - 5\lambda + 4 + 2] = 0$$
$$\Rightarrow (1-\lambda)[\lambda^2 - 5\lambda + 6] = 0$$
$$\Rightarrow (1-\lambda)[\lambda^2 - 2\lambda - 3\lambda + 6] = 0$$
$$\Rightarrow (1-\lambda)(\lambda-2)(\lambda-3) = 0$$
$$\Rightarrow \lambda = 1, 2, 3$$

If the matrix A satisfies the given equation, so its eigen value too. So,

$$6A^{-1} = A^2 + cA + dI$$
$$\Rightarrow \quad 6\lambda^{-1} = \lambda^2 + c\lambda + d$$

Taking $\lambda = 1$, we get

$$\frac{6}{1} = 1 + c + d$$
$$\Rightarrow \quad c + d = 5$$

Ans. (A)

6. The fuel cost functions in rupees/hour for two 600 MW thermal power plants are given by

Plant 1: $C_1 = 350 + 6P_1 + 0.004P_1^2$

Plant 2: $C_2 = 450 + aP_2 + 0.003P_2^2$

where P_1 and P_2 are power generated by plant 1 and plant 2, respectively, in MW and a is constant. The incremental cost of power (λ) is 8 rupees per MWh. The two thermal power plants together meet a total power demand of 550 MW. The optimal generation of plant 1 and plant 2 in MW, respectively, are

(A) 200, 350 (B) 250, 300
(C) 325, 225 (D) 350, 200

Solution: Given that,

Plant 1: $C_1 = 350 + 6P_1 + 0.004P_1^2$

Plant 2: $C_2 = 450 + aP_2 + 0.003P_2^2$

The incremental cost of power is

$$\lambda = 8 \text{ rupees per MWh.}$$

Incremental equation od power plant 1 is

$$\lambda = \frac{dC_1}{dP_1} = 6 + 0.008P_1$$

and $\quad 6 + 0.008P_1 = 8$

$$\Rightarrow \quad 0.008P_1 = 2$$
$$\Rightarrow \quad P_1 = 250 \text{ MWh}$$

Total power demand is

$$P_T = 550\,\text{MWh}$$

and $P_T = P_1 + P_2 = 550\,\text{MWh}$

$$P_2 = 550\,\text{MWh} - P_1 = 550 - 250 = 300\,\text{MWh}$$

Ans. (B)

7. Let, $f(x,y,z) = 4x^2 + 7xy + 3xz^2$. The direction in which the function $f(x, y, z)$ increases most rapidly at point $P = (1, 0, 2)$ is

(A) $20\hat{i} + 7\hat{j}$ (B) $20\hat{i} + 7\hat{j} + 12\hat{k}$

(C) $2\hat{i} + 12\hat{k}$ (D) $20\hat{i}$

Solution: Scalar function $f(x, y, z)$ rapidly increases in the direction of its normal vector at P.

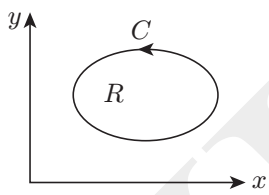
$$\vec{a} = \nabla f(x,y,z)$$
$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) f(x,y,z)$$
$$= \hat{i}(8x + 7y + 3z^2) + \hat{j}(7x) + \hat{k}(6xz)$$

At point P = (1, 0, 2), we get

$$\nabla f(1,0,2) = 20\hat{i} + 7\hat{j} + 12\hat{k}$$

Ans. (B)

8. Let R be a region in the first quadrant of the xy plane enclosed by a closed curve C considered in counter-clockwise direction. Which of the following expressions does not represent the area of the region R?



(A) $\iint\limits_R dxdy$ (B) $\oint\limits_C xdy$

(C) $\oint\limits_C ydx$ (D) $\frac{1}{2}\oint\limits_C (xdy - ydx)$

Solution: Check every option with the given conditions

Option (A):

$$\iint\limits_R dxdy = \text{Area of region } R \text{ by } C$$

Option (B) By Green's theorem:

$$\oint\limits_C xdy = \iint\limits_R (1-0)dxdy = \text{Area of region } R \text{ by } C$$

Option (C) By Green's theorem:

$$\oint\limits_C ydx = \iint\limits_R (0-1)dxdy$$
$$= -\text{Area of region } R \text{ by } C$$

Option (D) By Green's theorem

$$\frac{1}{2}\oint\limits_C (xdy - ydx) = \frac{1}{2}\iint\limits_R (1-(-1))dxdy$$
$$= \text{Area of region } R \text{ by } C$$

Ans. (C)

9. Let $\vec{E}(x,y,z) = 2x\hat{i} + 5y\hat{j} + 3z\hat{k}$. The value of $\iiint_V (\vec{\nabla}\cdot\vec{E})dV$, where V is the volume enclosed by the unit cube defined by $0 \le x \le 1, 0 \le y \le 1$, and $0 \le z \le 1$, is

(A) 3 (B) 8

(C) 10 (D) 5

Solution: Given that,

$$\vec{E}(x,y,z) = 2x\hat{i} + 5y\hat{j} + 3z\hat{k}$$

So,

$$\vec{\nabla}\cdot\vec{E} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(2x^2\hat{i} + 5y\hat{j} + 3z\hat{k})$$
$$= 4x + 5 + 3$$

Now,

$$\iiint_V (\vec{\nabla}\cdot\vec{E})dV = \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} (4x+5+3)\, dzdydx$$
$$= \int_{x=0}^{1}\int_{y=0}^{1} (4x+5+3)\, dzdydx$$
$$= \int_{x=0}^{1}\int_{y=0}^{1} (4xz + 8z)_0^1\, dydx$$
$$= (2x^2 + 8x)_0^1 = 10$$

Ans. (C)

10. Let the probability density function of a random variable x be given as

$$f(x) = ae^{-2|x|}$$

The value of 'a' is _____.

Solution: Given that, probability density function is

$$f(x) = ae^{-2|x|}$$

If $$\int_{-\infty}^{\infty} f(x)dx = 1$$

So, $$\int_{-\infty}^{0} ae^{2x}dx + \int_{-\infty}^{0} ae^{-2x}dx = 1$$

$$\left|a\frac{e^{2x}}{2}\right|_{-\infty}^{0} + \left|a\frac{e^{-2x}}{-2}\right|_{-\infty}^{0} = 1$$

$$\Rightarrow \quad \frac{a}{2}+\frac{a}{2}=1$$

$$\Rightarrow \quad a=1$$

Ans. (1)

GENERAL APTITUDE

Q. No. 1–5 Carry One Mark Each

1. As you grow older, an injury to your _____ may take longer to _____.
(A) heel / heel
(B) heal / heel
(C) heal / heal
(D) heel / heal

Solution: As you grow older, an injury to your heel may take longer to heal.

Ans. (D)

2. In a 500 m race, P and Q have speeds in the ratio of 3 : 4. Q starts the race when P has already covered 140 m. What is the distance between P and Q (in m) when P wins the race?
(A) 20 (B) 40
(C) 60 (D) 140

Solution: Given that, speeds of P and Q are in ratio 3 : 4 and Q starts race when P has covered 140 m.
Since, total distance = 500 m
Let speed of P be $3v$ and speed of Q be $4v$.
Starting from 140 m point time taken by P to reach end is

$$t=\frac{\text{distance}}{\text{speed}}=\frac{500-140}{3v}=\frac{360}{3v}$$

Distance covered by Q in this time is

$$t\times 4v=\frac{360}{3v}\times 4v=480\,\text{m}$$

Distance between P and Q when P wins the race is 500 m − 480 m = 20 m

Ans. (A)

3. Three bells P, Q, and R are rung periodically in a school. P is rung every 20 minutes; Q is rung every 30 minutes and R is rung every 50 minutes. If all the three bells are rung at 12:00 PM, when will the three bells ring together again the next time?
(A) 5:00 PM (B) 5:30 PM
(C) 6:00 PM (D) 6:30 PM

Solution: Given that, bell P ring every 20 min, bell Q ring every 30 min and bell R ring every 50 min.
Now, LCM of 20 min, 30 min, 50 min is 300 min or 5 hr.
So, the three bells ring again after 5 hr, that is, at 5:00 PM.

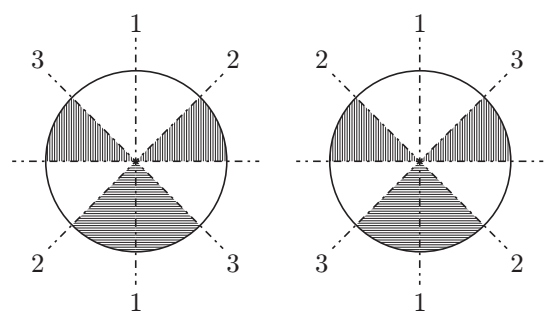
Ans. (A)

4. Given below are two statements and four conclusions drawn based on the statements.
Statement 1: Some bottles are cups.
Statement 2: All cups are knives.
Conclusion I: Some bottles are knives.
Conclusion II: Some knives are cups.
Conclusion III: All cups are bottles.
Conclusion IV: All knives are cups.
Which one of the following options can be logically inferred?
(A) Only conclusion I and conclusion II are correct
(B) Only conclusion II and conclusion III are correct
(C) Only conclusion II and conclusion IV are correct
(D) Only conclusion III and conclusion IV are correct

Solution: Based on statements 1 and 2 conclusions I and II are correct, that is, same bottles are knives, and some knives are cups.

Ans. (A)

5. The figure below shows the front and rear view of a disc, which is shaded with identical patterns. The disc is flipped once with respect to any one of the fixed axes 1-1, 2-2 or 3-3 chosen uniformly at random. What is the probability that the disc **does not** retain the same front and rear views after the flipping operation?



Front view Rear view

(A) 0 (B) $\frac{1}{3}$
(C) $\frac{2}{3}$ (D) 1

Solution: If disc is flipped along 1-1 axis, the front and rear views are same. If disc is flipped along 2-2 and 3-3 axes, the front and rear views are not same.

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Probability that the disc does not retain the same front and rear views after flipping is

$$P = \frac{2}{3}$$

Ans. (C)

Q. No. 6–10 Carry Two Marks Each

6. Altruism is the human concern for the wellbeing of others. Altruism has been shown to be motivated more by social bonding, familiarity and identification of belongingness to a group. The notion that altruism may be attributed to empathy or guilt has now been rejected. Which one of the following is the **correct** logical inference based on the information in the above passage?

(A) Humans engage in altruism due to guilt but not empathy
(B) Humans engage in altruism due to empathy but not guilt
(C) Humans engage in altruism due to group identification but not empathy
(D) Humans engage in altruism due to empathy but not familiarity

Solution: Based on the information in above passage, correct logical inference is "humans engage in altruism due to group identification but not empathy".

Ans. (C)

7. There are two identical dice with a single letter on each of the faces. The following six letters: Q, R, S, T, U, and V, one on each of the faces. Any of the six outcomes are equally likely. The two dice are thrown once independently at random. What is the probability that the outcomes on the dice were composed only of any combination of the following possible outcomes: Q, U and V?

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$
(C) $\frac{1}{6}$ (D) $\frac{5}{36}$

Solution: We have, total number of possibilities = 36

Number of outcomes with combination of Q, U and V are (Q, Q) (Q, U) (Q, V) (U, Q) (U, U) (U, V) (V, Q) (V, U) (V, V), that is, 9.

Therefore, the probability of outcomes with combination of Q, U and V is

$$P = \frac{9}{36} = \frac{1}{4}$$

Ans. (A)

8. The price of an item is 10% cheaper in an online store S compared to the price at another online store M. Store S charges ₹150 for delivery. There are no delivery charges for orders from the store M. A person bought the item from the store S and saved ₹100. What is the price of the item at the online store S (in ₹) if there are no other charges than what is described above?

(A) 2500 (B) 2250
(C) 1750 (D) 1500

Solution: Let price of item on store M be $100x$. So, price of item on store S is 10% cheaper, that is, $90x$.

Delivery charges by store S = ₹150
Delivery charges by store M = ₹0

Since, person saved 100 kg by buying item from store S.

$$100x - (90x + 150) = 100$$

$$\Rightarrow \quad 100x - 90x - 150 = 100$$

$$\Rightarrow \quad 10x = 250 \Rightarrow x = 25$$

Thus, price of item at store S = 90 × 25 = 2250

Ans. (B)

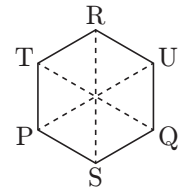
9. The letters P, Q, R, S, T and U are to be placed one per vertex on a regular convex hexagon, but not necessarily in the same order. Consider the following statements:

- The line segment joining R and S is longer than the line segment joining P and Q.
- The line segment joining R and S is perpendicular to the line segment joining P and Q.
- The line segment joining R and U is parallel to the line segment joining T and Q.

Based on the above statements, which one of the following options is **correct**?

(A) The line segment joining R and T is parallel to the line segment joining Q and S
(B) The line segment joining T and Q is parallel to the line joining P and U
(C) The line segment joining R and P is perpendicular to the line segment joining U and Q
(D) The line segment joining Q and S is perpendicular to the line segment joining R and P

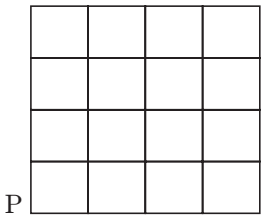
Solution: Based on the statements, the letters are placed as shown in the following figure



Thus, the line segment joining R and T is parallel to the line segment joining Q and S, that is, option (A) is correct.

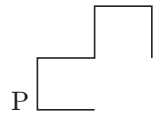
Ans. (A)

10.



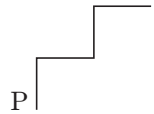
An ant is at the bottom-left corner of a grid (point P) as shown above. It aims to move to the top-right corner of the grid. The ant moves only along the lines marked in the grid such that the current distance to the top-right corner strictly decreases. Which one of the following is a part of a possible trajectory of the ant during the movement?

(A)



(B)

(C)



(D)

Solution: Part of a possible trajectory of ant can be

So, option (C) is correct.

In other options the current distance to top right corner is not strictly decreasing. So, they can not be possible trajectories.

Ans. (C)

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