SOLVED GATE
(CIVIL ENGINEERING SET–I) 2022

Engineering Mathematics and General Aptitude

ENGINEERING MATHEMATICS

Q. No. 1–5 Carry One Mark Each

1. Consider the following expression:

$$z = \sin(y + it) + \cos(y - it)$$

where z, y, and t are variables, and $i = \sqrt{-1}$ is a complex number. The partial differential equation derived from the above expression is

(A) $\dfrac{\partial^2 z}{\partial t^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$ (B) $\dfrac{\partial^2 z}{\partial t^2} - \dfrac{\partial^2 z}{\partial y^2} = 0$

(C) $\dfrac{\partial z}{\partial t} - i\dfrac{\partial z}{\partial y} = 0$ (D) $\dfrac{\partial z}{\partial t} + i\dfrac{\partial z}{\partial y} = 0$

Solution: We have the equation as follows:

$$z = \sin(y + it) + \cos(y - it)$$

The independent variables in the above equation are y and t and the dependent variable is z.
Therefore, to derive a differential equation, we must differentiate the equation by t and y.
Differentiate the above equation by t, we get

$$\frac{dz}{dt} = \frac{d}{dt}(\sin(y + it) + \cos(y - it))$$
$$= \cos(y + it)(i) - \sin(y - it)(-i)$$

Again, differentiate the above equation by t, we get

$$\frac{d^2 z}{dt^2} = \frac{d^2}{dt^2}(\cos(y + it)(i) - \sin(y - it)(-i))$$

$$\frac{d^2 z}{dt^2} = \sin(y + it) + \cos(y - it) \qquad (1)$$

Differentiate the above equation by y, we get

$$\frac{dz}{dy} = \frac{d}{dy}(\sin(y + it) + \cos(y - it))$$
$$= \cos(y + it) - \sin(y - it)$$

Again, differentiate the above equation by y, we get

$$\frac{d^2 z}{dy^2} = \frac{d^2}{dy^2}(\sin(y + it) + \cos(y - it))$$

$$\frac{d^2 z}{dy^2} = -\sin(y + it) - \cos(y - it) \qquad (2)$$

Adding Eqs. (1) and (2), we get

$$\frac{d^2 z}{dt^2} + \frac{d^2 z}{dy^2} = 0$$

Ans. (A)

2. For the equation

$$\frac{d^3 y}{dx^3} + x\left(\frac{dy}{dx}\right)^{3/2} + x^2 y = 0$$

the correct description is

(A) an ordinary differential equation of order 3 and degree 2.
(B) an ordinary differential equation of order 3 and degree 3.
(C) an ordinary differential equation of order 2 and degree 3.
(D) an ordinary differential equation of order 3 and degree 3/2.

Solution: The order of a differential equation is the highest derivative of that equation, and the degree is the power of the highest derivation in the equation.
The equation is:

$$\frac{d^3 y}{dx^3} + x\left(\frac{dy}{dx}\right)^{3/2} + x^2 y = 0$$

The highest derivative in the above equation is 3 and it's power is 1.
But to get the correct values of degree and order, the equation must have only integers powers.

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As the equation is having power 3/2, it should be converted into integer value.
Therefore, squaring the above equation would give:

$$\left(\frac{d^3y}{dx^3}+x\left(\frac{dy}{dx}\right)^{3/2}+x^2y=0\right)^2$$

$$\left(\frac{d^3y}{dx^3}\right)^2+x\left(\frac{dy}{dx}\right)^3+\left(x^2y\right)^2=0$$

From the above equation, it can be concluded that the equation is an ordinary differential equation of order 3 and degree 2.

Ans. (A)

3. The matrix M is defined as

$$M=\begin{bmatrix}1 & 3\\ 4 & 2\end{bmatrix}$$

and has eigenvalues 5 and -2. The matrix Q is formed as

$$Q=M^3-4M^2-2M$$

Which of the following is/are the eigenvalue(s) of matrix Q?

(A) 15 (B) 25
(C) -20 (D) -30

Solution: Substitute the eigenvalues of the matrix M in the matrix Q equation, we get

$$Q=M^3-4M^2-2M$$

For $\lambda_1=5$

$$Q=5^3-4\times5^2-2\times5=15$$

For $\lambda_2=-2$

$$Q=(-2)^3-4\times(-2)^2-2\times(-2)=-20$$

Ans. (A, C)

4. Consider the following recursive iteration scheme for different values of variable P with the initial guess $x_1=1$:

$$x_{n+1}=\frac{1}{2}\left(x_n+\frac{P}{x_n}\right),\qquad n=1,2,3,4,5$$

For $P=2$, x_5 is obtained to be 1.414, rounded-off to three decimal places. For $P=3$, x_5 is obtained to be 1.732, rounded-off to three decimal places. If $P=10$, the numerical value of x_5 is _____. (round off to three decimal places)

Solution: **Short approach:**
From the results of x_5 when substituting the value of P as 2 and 3, it can be concluded that, the relation is to calculate the square root of P.

For $P=2$, $x_5=1.414$ and for $P=3$, $x_5=1.732$.

So, for $P=10$, $x_5=3.162$

Long Approach:

Substitute, $n=\infty$ and $x_{n+1}=x_n=a$ in the given equation, we get

$$x_{n+1}=\frac{1}{2}\left(x_n+\frac{P}{x_n}\right)$$

$$\Rightarrow\quad a=\frac{1}{2}\left(a+\frac{P}{a}\right)$$

$$\Rightarrow\quad a=\sqrt{P}$$

For P = 10, we get

$$a=\sqrt{10}=3.162$$

Ans. (3.162)

5. The Fourier cosine series of a function is given by:

$$f(x)=\sum_{n=0}^{\infty}f_n\cos nx$$

For $f(x)=\cos^4x$, the numerical value of (f_4+f_5) is _____. (round off to three decimal places)

Solution: For $f(x)=\cos^4x$, we have

$$\cos^4x=(\cos^2x)^2=\left(\frac{1+\cos2x}{2}\right)^2$$

$$=\frac{1}{4}(1+2\cos2x+\cos^2 2x)$$

$$=\frac{1}{4}+\frac{\cos2x}{2}+\frac{1+\cos4x}{8}$$

$$=\frac{3}{8}+\frac{\cos2x}{2}+\frac{\cos4x}{8}$$

In exponential form, it can also be written as:

$$\cos^4x=\frac{3}{8}+\frac{1}{4}e^{(-2ix)}+\frac{1}{4}e^{(2ix)}+\frac{1}{16}e^{(-4ix)}+\frac{1}{16}e^{(4ix)}$$

So, $f_0=\frac{3}{8},f_1=0,f_2=\frac{1}{2},f_3=0,f_4=\frac{1}{8}$,

and $f_5=0$.

Therefore, f_4+f_5 is

$$f_4+f_5=0.125$$

Ans. (0.125)

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Q. No. 6–9 Carry Two Marks Each

6. The Cartesian coordinates of a point P in a right-handed coordinate system are (1, 1, 1). The transformed coordinates of P due to a 45° clockwise rotation of the coordinate system about the positive x-axis are

(A) $(1, 0, \sqrt{2})$ (B) $(1, 0, -\sqrt{2})$

(C) $(-1, 0, \sqrt{2})$ (D) $(-1, 0, -\sqrt{2})$

Solution: The 3×3 rotation matrix is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

For the transport coordinated of the above matrix, substitute $\theta = 45°$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45° & -\sin 45° \\ 0 & \sin 45° & \cos 45° \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Determine the transformed coordinates as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \sqrt{2} \end{bmatrix}$$

Ans. (A)

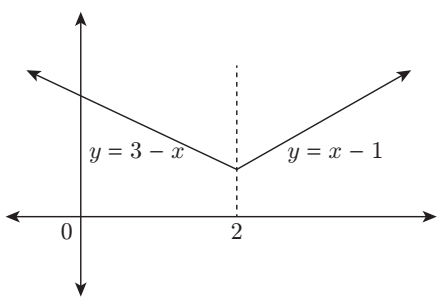
7. Let $\max\{a, b\}$ denote the maximum of two real numbers a and b. Which of the following statement(s) is/are **true** about the function $f(x) = \max\{3 - x, x - 1\}$?

(A) It is continuous on its domain.
(B) It has a local minimum at $x = 2$.
(C) It has a local maximum at $x = 2$.
(D) It is differentiable on its domain.

Solution: Determine the point where both the lines intersect with each other.

$$3 - x = x - 1$$
$$x = 2$$

Plot the line as follows:



Now, we check for the continuity of the function by substitute $x = 2$ in both the equations:

$$3 - x = x - 1$$
$$3 - 2 = 2 - 1$$
$$1 = 1$$

Therefore, the function is continuous.
Now, check for the differentiability of the function:

$$\frac{d}{dx}(3 - x) = \frac{d}{dx}(x - 1)$$
$$-1 \neq 1$$

Therefore, the function is not differentiable. But from the graph it has a local minimum at $x = 2$.

Ans. (A, B)

8. Consider the differential equation

$$\frac{dy}{dx} = 4(x + 2) - y$$

For the initial condition $y = 3$ at $x = 1$, the value of y at $x = 1.4$ obtained using Euler's method with a step-size of 0.2 is _____. (round off to one decimal place)

Solution: Given that,

$$f(x) = \frac{dy}{dx} = 4(x + 2) - y$$

Euler equation is given by

$$y_n = y_{n-1} + hf(x_n, y_n)$$

First Iteration $x_1 = 1, y_1 = 3$:

$$y_2 = y_1 + hf(x_1, y_1)$$
$$= 3 + 0.2(4(1 + 2) - 3) = 4.8$$

Second Iteration $x_2 = 1.2, y_2 = 4.8$:

$$y_3 = y_2 + hf(x_2, y_2)$$
$$= 4.8 + 0.2(4(1.2 + 2) - 4.8) = 6.4$$

Ans. (6.4)

9. A set of observations of independent variable (x) and the corresponding dependent variable (y) is given below.

x	5	2	4	3
y	16	10	13	12

Based on the data, the coefficient a of the linear regression model

$$y = a + bx$$

is estimated as 6.1.
The coefficient b is _____. (round off to one decimal place)

Solution: As per the linear regression method:

$$\sum y = na + b\sum x$$
$$(16+10+13+12) = (4\times 6.1) + b(5+2+4+3)$$
$$b = 1.9$$

Ans. (1.9)

GENERAL APTITUDE

Q. No. 1–5 Carry One Mark Each

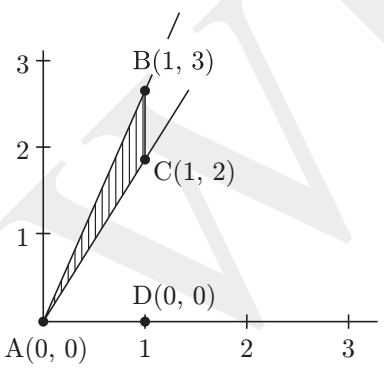
1. You should _____ when to say _____.
(A) no/no (B) no/know
(C) know/know (D) know/no

Solution: You should know when to say no.
So, know/no are correct words to file the blanks.

Ans. (D)

2. Two straight lines pass through the origin $(x_0, y_0) = (0, 0)$. One of them passes through the point $(x_1, y_1) = (1, 3)$ and the other passes through the point $(x_2, y_2) = (1, 2)$. What is the area enclosed between the straight lines in the interval [0, 1] on the x-axis?
(A) 0.5 (B) 1.0
(C) 1.5 (D) 2.0

Solution: Given, two straight lines one passes through (0, 0) and $(x_1, y_1) = (1, 3)$ other passes through (0, 0) and $(x_2, y_2) = (1, 2)$



Area enclosed by the two lines in the interval [0, 1] is the shaded region in the graph figure.
So, the required area is given by

$$A = (\text{area of triangle ABD}) - (\text{area of triangle ACD})$$
$$= \frac{1}{2}\times \text{AD}\times\text{DB} - \frac{1}{2}\times\text{AD}\times\text{AC}$$
$$= \frac{1}{2}\times 1\times 3 - \frac{1}{2}\times 1\times 2 = \frac{3}{2} - \frac{2}{2} = \frac{1}{2} = 0.5$$

Therefore, required area is 0.5.

Ans. (A)

3. If $p : q = 1 : 2$, $q : r = 4 : 3$, $r : s = 4 : 5$ and u is 50% more than s, what is the ratio $p : u$?
(A) 2 : 15 (B) 16 : 15
(C) 1 : 5 (D) 16 : 45

Solution: Given that,

$$p : q = 1 : 2 \quad (1)$$
$$q : r = 4 : 3 \quad (2)$$
$$r : s = 4 : 5 \quad (3)$$

Now, $p : q = (1 \times 2) : (2 \times 2) = 2 : 4$
From Eq. (2), we get

$$p : q : r = 2 : 4 : 3$$

So, $p : q : r = (2 \times 4) : (4 \times 4) : (3 \times 4)$

$$p : q : r = 8 : 16 : 12$$

and from Eq. (3), we get

$$r : s = (4 \times 3) : (5 \times 3) = 12 : 15$$

Therefore, $p : q : r : s = 8 : 16 : 12 : 15$
Given u is 50% more than s. So,

$$u = s + 50\% \text{ of } s = s + \frac{1}{2}s = \frac{3}{2}s$$

Now, $p : u = p : \frac{3}{2}s$

$$p : u = 2p : 3s = (8 \times 2) : (3 \times 15)$$
$$\Rightarrow \quad p : u = 16 : 45$$

Ans. (D)

4. Given the statements:
- P is the sister of Q.
- Q is the husband of R.
- R is the mother of S.
- T is the husband of P.

Based on the above information, T is _____ of S.
(A) the grandfather
(B) an uncle
(C) the father
(D) a brother

Solution: Given that,
P = sister of Q
Q = husband of R
R = mother of S
T = husband of P
From given statements we conclude P is Q's sister or Q is P's brother and brother Q is R's husband or R is Q's wife and wife R is S's mother and T is P's husband or P is T's wife. So, based on information T is uncle of S.

Ans. (B)

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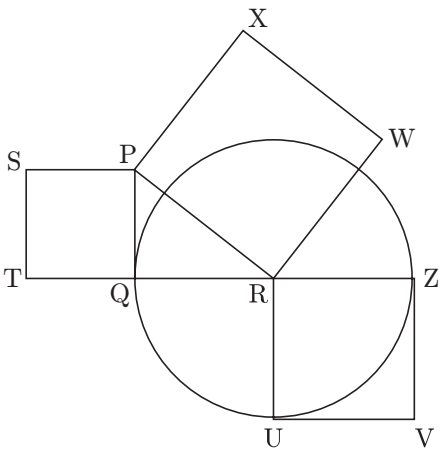
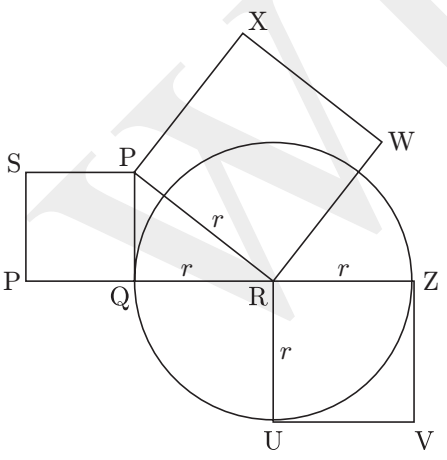
5. In the following diagram, the point R is the center of the circle. The lines PQ and ZV are tangential to the circle. The relation among the areas of the squares, PXWR, RUVZ and SPQT is



(A) Area of SPQT = Area of RUVZ = Area of PXWR
(B) Area of SPQT = Area of PXWR − Area of RUVZ
(C) Area of PXWR = Area of SPQT − Area of RUVZ
(D) Area of PXWR = Area of RUVZ − Area of SPQT

Solution: Let radius of circle = r and PR = r_1.
Redraw the figure with given additional values, we have



From right angled triangle PQR, we get

$$PQ = \sqrt{r_1^2 - r^2}$$

Area of square PXWR is r_1^2

Area of square RUVZ is r^2

Area of square SPQT $(\sqrt{r_1^2 - r^2})^2 = r_1^2 - r^2$

Therefore,

Area of SPQT = Area of PXWR − Area of RUVZ

Ans. (B)

Q. No. 6–10 Carry Two Marks Each

6. Healthy eating is a critical component of healthy aging. When should one start eating healthy? It turns out that it is never too early. For example, babies who start eating healthy in the first year are more likely to have better overall health as they get older.
Which one of the following is the **correct** logical inference based on the information in the above passage?
(A) Healthy eating is important for those with good health conditions, but not for others
(B) Eating healthy can be started at any age, earlier the better
(C) Eating healthy and better overall health are more correlated at a young age, but not older age
(D) Healthy eating is more important for adults than kids

Solution: Based on information in above passage, correct logical inference is "eating healthy can be started at any age, earlier the better".

Ans. (B)

7. P invested ₹5000 per month for 6 months of a year and Q invested ₹x per month for 8 months of the year in a partnership business. The profit is shared in proportion to the total investment made in that year.
If at the end of that investment year, Q receives 4/9 of the total profit, what is the value of x (in ₹)?
(A) 2500 (B) 3000
(C) 4687 (D) 8437

Solution: Given that, investment of P = ₹5000 per month, period of investment = 6 months
So, the total investment of P = ₹5000 × 6 = ₹30000
Investment of Q = ₹x per month, period of investment = 8 months
So, the total investment Q = ₹x × 8 = ₹8x
Since profit is shared in proportion of total investment, that is, if Q receives 4/9 of the total profit. Then P receives 1 − 4/9 = 5/9 of the total profit.
Therefore,
Profit of P : Profit of Q = Investment of P : Investment of Q

$$\Rightarrow \quad \frac{\text{profit of P}}{\text{profit of Q}} = \frac{\text{investment of P}}{\text{investment of Q}}$$

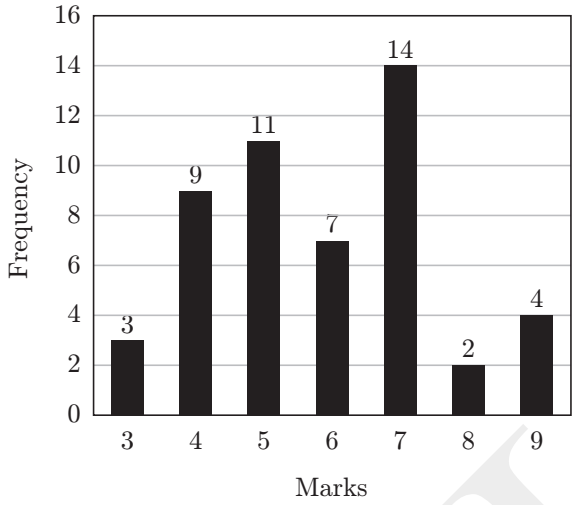
$$\Rightarrow \quad \frac{5/9}{4/9} = \frac{5000\times 6}{x\times 8} \Rightarrow \frac{5}{4} = \frac{5000\times 6}{8\times x}$$

$$\Rightarrow \quad x = \frac{5000\times 6\times 4}{8\times 5} = 3000$$

So, the value of x = ₹3000

Ans. (B)

8. The frequency chart (figure) shows the frequency distribution of marks obtained by a set of students in an exam.



From the data presented above, which one of the following is **correct**?

(A) mean > mode > median
(B) mode > median > mean
(C) mode > mean > median
(D) median > mode > mean

Solution: From the given frequency chart, we have

mode = 7,

$$\text{mean} = \frac{3\times 3+4\times 9+5\times 11+6\times 7+7\times 14+8\times 2+9\times 4}{3+9+11+7+14+2+4}$$

$$= \frac{9+36+55+42+98+16+36}{3+9+11+7+14+2+4} = \frac{292}{50} = 5.84,$$

and median = 6

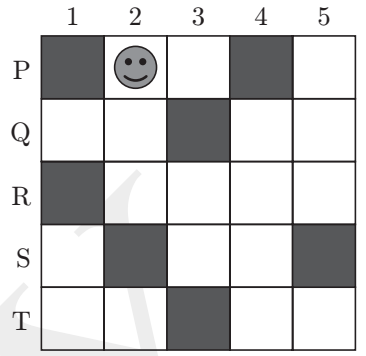
Therefore, mode > median > mean

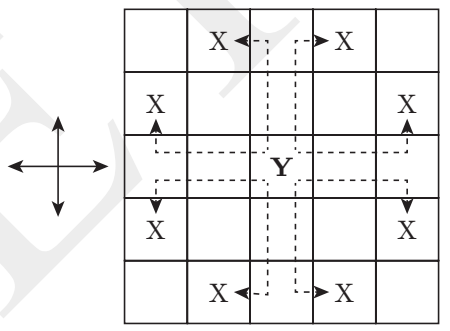
Ans. (B)

9. In the square grid shown on the left, a person standing at P2 position is required to move to P5 position.

The only movement allowed for a step involves, "two moves along one direction followed by one move in a perpendicular direction". The permissible directions for movement are shown as dotted arrows in the right.

For example, a person at a given position Y can move only to the positions marked X on the right. Without occupying any of the shaded squares at the end of each step, the minimum number of steps required to go from P2 to P5 is





Example: Allowed steps for a person at **Y**

(A) 4 (B) 5
(C) 6 (D) 7

Solution:
Step 1: Q2 → R2 → R3
Step 2: R4 → R5 → Q5
Step 3: Q4 → Q3 → P3
Step 4: Q3 → R3 → R4
Step 5: Q4 → P4 → P5
So minimum step required = 5

Ans. (B)

10. Consider a cube made by folding a single sheet of paper of appropriate shape. The interior faces of the cube are all blank. However, the exterior faces that are not visible in the above view may not be blank.

Which one of the following represents a possible unfolding of the cube?

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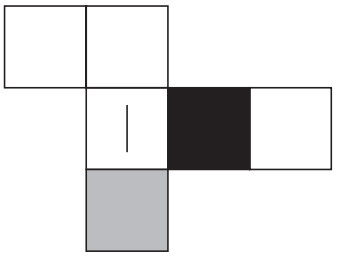
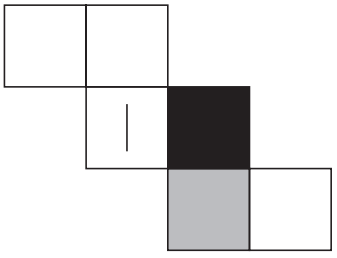
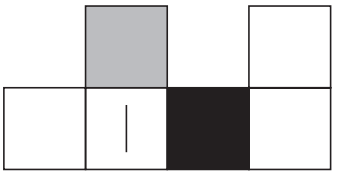
(A)



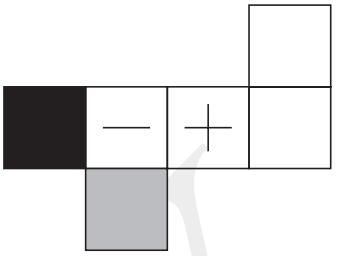
(B)



(C)



(D)



Solution: Unfolding may be represented by (A, B, D)

Ans. (A, B, D)

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