SOLVED GATE
(CIVIL ENGINEERING SET–II) 2022

Engineering Mathematics and General Aptitude

ENGINEERING MATHEMATICS

Q. No. 1–5 Carry One Mark Each

1. The function $f(x, y)$ satisfies the Laplace equation

$$\nabla^2 f(x,y) = 0$$

on a circular domain of radius $r = 1$ with its center at point P with coordinates $x = 0$, $y = 0$. The value of this function on the circular boundary of this domain is equal to 3. The numerical value of $f(0, 0)$ is:

(A) 0 (B) 2
(C) 3 (D) 1

Solution: As the function has a constant value of 3 at all points on the circular boundary, therefore $f(0, 0) = 3$.

Ans. (C)

2. $\int\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots\right)dx$ is equal to

(A) $\frac{1}{1+x}$ + Constant (B) $\frac{1}{1+x^2}$ + Constant

(C) $-\frac{1}{1-x}$ + Constant (D) $-\frac{1}{1-x^2}$ + Constant

Solution: Let

$$\int\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots\right)dx=\int f(x)dx, \text{ where } f(x)$$

is the infinite series of x.

Given function $f(x)$ is Maclaurin series expansion of $\ln(x + 1)$.

$$\ln(x+1) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$

So, the integral become:

$$\int \ln(x+1)dx$$

$$\int \ln(x+1)dx = (x+1)[\ln(x+1)-1]+C$$

Ans. (*)

3. P and Q are two square matrices of the same order. Which of the following statement(s) is/are correct?

(A) If P and Q are invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$.
(B) If P and Q are invertible, then $[QP]^{-1} = P^{-1}Q^{-1}$.
(C) If P and Q are invertible, then $[PQ]^{-1} = P^{-1}Q^{-1}$.
(D) If P and Q are not invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$.

Solution: Properties of Matrix:
If P and Q are two square matrix and are invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$,

And similarly: $[QP]^{-1} = P^{-1}Q^{-1}$.

Ans. (A, B)

4. The error in measuring the radius of a 5 cm circular rod was 0.2%. If the cross-sectional area of the rod was calculated using this measurement, then the resulting absolute percentage error in the computed area is _____. (round off to two decimal places)

Solution: The absolute percentage error in cross-section area is given by:

$$\text{Error\%} = \frac{dA}{A}\times 100$$

$$\frac{dA}{A}\times 100 = \frac{\frac{d(\pi r^2)}{dr}}{\pi r^2}\times 100$$

$$= \frac{2\pi r dr}{\pi r^2}\times 100 = 2\times\left(\frac{dr}{r}\right)\times 100$$

WILEY

Given that, $r = 5$ cm and $dr/r = 0.2\%$. Then,

$$\text{Error\%} = 2 \times \frac{0.2}{100} \times 100 = 0.40$$

Ans. (0.40)

5. The components of pure shear strain in a sheared material are given in the matrix form:

$$\varepsilon = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Here, $\text{Trace}(\varepsilon) = 0$. Given, $P = \text{Trace}(\varepsilon^8)$ and $Q = \text{Trace}(\varepsilon^{11})$. The numerical value of $(P + Q)$ is _____. (in integer)

Solution: Express the eigen vector of the matrix as follows:

$$|A - \lambda I| = 0$$

$$\Rightarrow |\varepsilon - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (\lambda^2 - 1) - 1 = 0$$

$$\Rightarrow \lambda = \pm\sqrt{2}$$

So, the eigen values for matrix ε are $+\sqrt{2}$ and $-\sqrt{2}$.

Determine the values of P and Q with the help of trace of the matrix ε as follows:

$$P = \text{Trace}(\varepsilon^8) = (\sqrt{2})^8 + (-\sqrt{2})^8 = 32$$

$$Q = \text{Trace}(\varepsilon^{11}) = (\sqrt{2})^{11} + (-\sqrt{2})^{11} = 0$$

Therefore, the value of $P + Q$ is

$$P + Q = 32 + 0 = 32$$

Ans. (32)

Q. No. 6–9 Carry Two Marks Each

6. Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ on the domain S given by $1 \le x \le 3$. The first and second derivatives are $f'(x)$ and $f''(x)$. Consider the following statements:

I. The given polynomial is zero at the boundary points $x = 1$ and $x = 3$.
II. There exists one local maxima of $f(x)$ within the domain S.
III. The second derivative $f''(x) > 0$ throughout the domain S.
IV. There exists one local minima of $f(x)$ within the domain S.

The correct option is:
(A) Only statements I, II and III are correct.
(B) Only statements I, II and IV are correct.
(C) Only statements I and IV are correct.
(D) Only statements II and IV are correct.

Solution: Given function:

$$f(x) = x^3 - 6x^2 + 11x - 6$$

At $x = 1$, we get

$$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6$$
$$= 1 - 6 + 11 - 6 = 0$$

At $x = 3$, we get

$$f(3) = (3)^3 - 6(3)^2 + 11(3) - 6$$
$$= 27 - 54 + 33 - 6 = 0$$

Therefore, statement I is correct.
Now, differentiating the equation with respect to x, we get

$$f'(x) = 3x^2 - 12x + 11$$

Put $f'(x) = 0$

$$f'(x) = 0 = 3x^2 - 12x + 11$$

$$\Rightarrow \quad x = 2 \pm \frac{1}{\sqrt{3}}$$

Here, point of maxima is $2 - \frac{1}{\sqrt{3}}$ and point of minima is $2 + \frac{1}{\sqrt{3}}$.

And the point of maxima and minima exist in the domain $1 \le x \le 3$. Therefore, statement II and IV are also correct.
Now, differentiate the equation again with respect to x, we get

$$f''(x) = 6x - 12$$

Put $f''(x) > 0$

$$6x - 12 > 0$$

$$\Rightarrow \quad x > 2$$

For III statement to be true, x should always have a value greater than 2. Therefore, statement III is incorrect.

Ans. (B)

7. Let y be a non-zero vector of size 2022×1. Which of the following statement(s) is/are **true**?

(A) yy^T is a symmetric matrix.
(B) y^Ty is an eigenvalue of yy^T.
(C) yy^T has a rank of 2022.
(D) yy^T is invertible.

Solution: Let us assume as matrix as follows:

$$y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{3\times 1}$$

Perform the transpose of the above matrix:

$$y^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}_{1\times 3}$$

Now,

$$yy^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3\times 3}$$

And,

$$y^T y = \left[1^2 + 1^2 + 1^2\right]$$

$$y^T y = [3]_{1\times 1}$$

Therefore, Rank of the above matrix = 1,

$y^T y$ is an eigen value of yy^T,

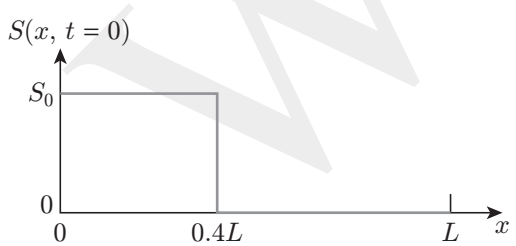
yy^T is a symmetrical matrix, and $\det(yy^T) = 0$, therefore yy^T is not invertible.

Ans. (A, B)

8. The concentration $s(x,\ t)$ of pollutants in a one-dimensional reservoir at position x and time t satisfies the diffusion equation

$$\frac{\partial s(x,t)}{\partial t} = D\frac{\partial^2 s(x,t)}{\partial x^2}$$

on the domain $0 \le x \le L$, where D is the diffusion coefficient of the pollutants. The initial condition $s(x,\ 0)$ is defined by the step-function shown in the figure.



The boundary conditions of the problem are given by $\dfrac{\partial s(x,t)}{\partial x} = 0$ at the boundary points $x = 0$ and $x = L$ at all times. Consider $D = 0.1\ \text{m}^2/\text{s}$, $s_0 = 5\ \mu\text{mol/m}$, and $L = 10$ m. The steady-state concentration $\tilde{s}\left(\dfrac{L}{2}\right) = s\left(\dfrac{L}{2}, \infty\right)$ at the center $x = \dfrac{L}{2}$ of the reservoir (in μmol/m) is _____. (in integer)

Solution: From the given step function graph, the following values can be derived:

$$s_0 = s(0,t) = 5, s(0.4L,t) = 5, s(L,t) = 0$$

Calculate the concentration of the pollutants as follows:

When $0 > x > 0.4L$

$$\text{Concentration} = s_0 \times 0.4 \times L = 5 \times 0.4 \times 10$$
$$= 20\ \mu\text{mol}$$

When $0.4L > x > L$,

$$\text{Concentration} = 0$$

So, the total concentration of the pollutants is

$$20\ \mu\text{mol} + 0\ \mu\text{mol} = 20\ \mu\text{mol}$$

Determine the concentration per meter length as follows:

$$\frac{20\ \mu\text{mol}}{10\ \text{m}} = 2\ \ \mu\text{mol/m}$$

Ans. (2)

9. A pair of six-faced dice is rolled thrice. The probability that the sum of the outcomes in each roll equals 4 in exactly two of the three attempts is _____. (round off to three decimal places)

Solution: To get the sum as 4 when a six faced dice is rolled twice, the possible events are as follows:

$$X = [(1,3),(2,2),(3,1)]$$

The probability of the above events happening:

$$P(X) = \frac{3}{36} = \frac{1}{12}$$

The probability of above events not happening:

$$P(\bar{X}) = 1 - \frac{1}{12} = \frac{11}{12}$$

Determine the probability of the sum of the outcomes in each roll equals 4 in exactly two of the three attempts:

$$P(A) = {}^3C_2 P(X)^2 P(\bar{X})^1$$
$$= \frac{3!}{2! \times 1!} \times \left(\frac{1}{12}\right)^2 \times \left(\frac{11}{12}\right)^1 = 0.019$$

Ans. (0.019)

GENERAL APTITUDE

Q. No. 1–5 Carry One Mark Each

1. The movie was funny and I _____.
(A) could help laughing
(B) couldn't help laughed
(C) couldn't help laughing
(D) could helped laughed

WILEY

Solution: The movie was funny and I couldn't help laughing.

Ans. (C)

2. $x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.

What is the value of $\frac{x+z-y}{y}$?

(A) 0.75 (B) 1.25
(C) 2.25 (D) 3.25

Solution: Given that,

$$x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$$

$$\Rightarrow \quad x = \frac{k}{2}, y = \frac{k}{3}, z = \frac{k}{4}$$

Then, $$\frac{x+z-y}{y} = \frac{\frac{k}{2} + \frac{k}{4} - \frac{k}{3}}{\frac{k}{3}}$$

$$= \frac{\frac{6k+3k-4k}{12}}{\frac{k}{3}} = \frac{5k}{12} \times \frac{3}{k} = \frac{5}{4} = 1.25$$

Therefore, $\frac{x+z-y}{y} = 1.25$

Ans. (B)

3. Both the numerator and the denominator of 3/4 are increased by a positive integer, x, and those of 15/17 are decreased by the same integer. This operation results in the same value for both the fractions. What is the value of x?

(A) 1 (B) 2
(C) 3 (D) 4

Solution: From the given data, we get

$$\frac{3+x}{4+x} = \frac{15-x}{17-x}$$

If $x = 1$, $$\frac{3+1}{4+1} = \frac{15-1}{17-1}$$

$$\Rightarrow \quad \frac{4}{5} \neq \frac{14}{16}$$

If $x = 2$, $$\frac{3+2}{4+2} = \frac{15-2}{17-2}$$

$$\Rightarrow \quad \frac{5}{6} \neq \frac{13}{15}$$

If $x = 3$, $$\frac{3+3}{4+3} = \frac{15-3}{17-3}$$

$$\Rightarrow \quad \frac{6}{7} = \frac{12}{14} = \frac{6}{7}$$

So, $x = 3$ satisfy the equation

Ans. (C)

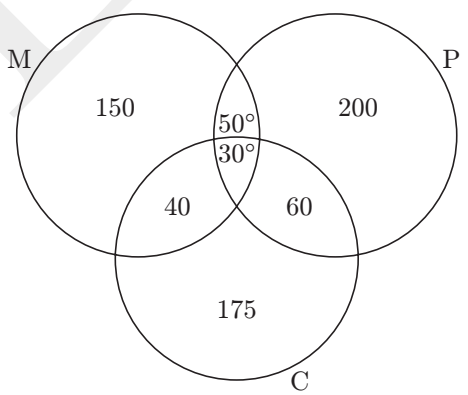
4. A survey of 450 students about their subjects of interest resulted in the following outcome.

- 150 students are interested in Mathematics.
- 200 students are interested in Physics.
- 175 students are interested in Chemistry.
- 50 students are interested in Mathematics and Physics.
- 60 students are interested in Physics and Chemistry.
- 40 students are interested in Mathematics and Chemistry.
- 30 students are interested in Mathematics, Physics and Chemistry.
- Remaining students are interested in Humanities.

Based on the above information, the number of students interested in Humanities is

(A) 10 (B) 30
(C) 40 (D) 45

Solution: Using venn diagram



Students interested in humanities

$$= 450 - n(M \cup P \cup C)$$

$$= 450 - [m(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(C \cap M) + n(M \cap P \cap C)$$

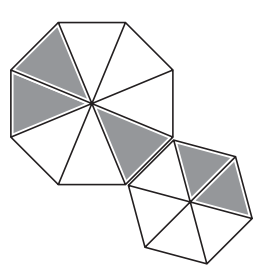
$$= 450 - [150 + 200 + 175 - 50 - 40 - 60 + 30]$$

$$= 450 - [405] = 45$$

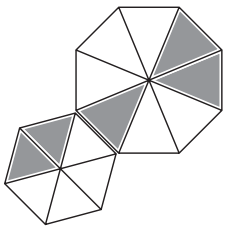
Therefore, students interested in humanities = 45

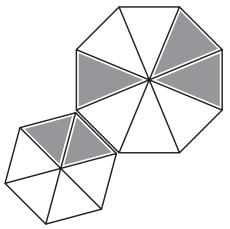
Ans. (D)

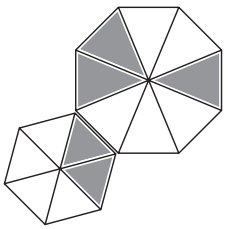
5.

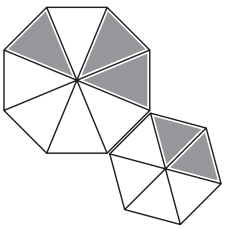


For the picture shown above, which one of the following is the correct picture representing reflection with respect to the mirror shown as the dotted line?

(A)


(B)


(C)


(D)


Solution: Option (A) represents the reflection with respect to the mirror.

Ans. (A)

Q. No. 6–10 Carry Two Marks Each

6. In the last few years, several new shopping malls were opened in the city. The total number of visitors in the malls is impressive. However, the total revenue generated through sales in the shops in these malls is generally low. Which one of the following is the **correct** logical inference based on the information in the above passage?

(A) Fewer people are visiting the malls but spending more
(B) More people are visiting the malls but not spending enough
(C) More people are visiting the malls and spending more
(D) Fewer people are visiting the malls and not spending enough

Solution: Based on the information in given passage, correct logical inference is more people are visiting the malls but not spending enough.

Ans. (B)

7. In a partnership business the monthly investment by three friends for the first six months is in the ratio 3 : 4 : 5. After six months, they had to increase their monthly investments by 10%, 15% and 20%, respectively, of their initial monthly investment. The new investment ratio was kept constant for the next six months. What is the ratio of their shares in the total profit (in the same order) at the end of the year such that the share is proportional to their individual total investment over the year?

(A) 22 : 23 : 24 (B) 22 : 33 : 50
(C) 33 : 46 : 60 (D) 63 : 86 : 110

Solution: Monthly investment for first six months is 3 : 4 : 5.
Monthly investment for next six months with ratio being constant is

$$\left(3+3\times\frac{10}{100}\right):\left(4+4\times\frac{15}{100}\right):\left(5+5\times\frac{20}{100}\right)$$

$$=\frac{330}{100}:\frac{460}{100}:\frac{600}{100}=3.3:4.6:6$$

Total investment over the year is

$$(3+3.3):(4+4.6):(5+6)$$

$$=6.3:8.6:11$$

Then total profit at the end of year in same ratio will be 63 : 86 : 110

Ans. (D)

8. Consider the following equations of straight lines:
Line L1: $2x - 3y = 5$
Line L2: $3x + 2y = 8$
Line L3: $4x - 6y = 5$
Line L4: $6x - 9y = 6$
Which one among the following is the correct statement?

(A) L1 is parallel to L2 and L1 is perpendicular to L3
(B) L2 is parallel to L4 and L2 is perpendicular to L1
(C) L3 is perpendicular to L4 and L3 is parallel to L2
(D) L4 is perpendicular to L2 and L4 is parallel to L3

Solution: Given that,

$$\text{L1}: 2x - 3y = 5 \Rightarrow y = \frac{2}{3}x - \frac{5}{3}$$

Comparing with equation of line

$$y = mx + C$$

Slope of line L1 is $m_1 = \frac{2}{3}$

Similarly,

$$\text{L2}: 3x + 2y = 8 \Rightarrow y = \frac{-3x}{2} + 4$$

Slope of line L2 is $m_2 = \frac{-3}{2}$

and $\text{L3}: 4x - 6y = 5 \Rightarrow y = \frac{4x}{6} - \frac{5}{6}$

Slope of line L3 is $m_3 = \frac{2}{3}$

and $\quad$ L4 : $6x - 9y = 6 \Rightarrow y = \dfrac{6x}{9} - \dfrac{6}{9}$

Slope of line L4 is $m_4 = \dfrac{2}{3}$

Here L1 || L3 || L4 as slope of all the lines are equal. So, L4 is parallel to L3 also L4 is perpendicular to L2. Then,

$$\text{Slope of L4} \times \text{Slope of L2} = -1$$

$$\Rightarrow \quad +\frac{2}{3} \times \left(\frac{-3}{2}\right) = -1$$

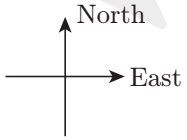
Ans. (D)

9. Given below are two statements and four conclusions drawn based on the statements.
Statement 1: Some soaps are clean.
Statement 2: All clean objects are wet.
Conclusion I: Some clean objects are soaps.
Conclusion II: No clean object is a soap.
Conclusion III: Some wet objects are soaps.
Conclusion IV: All wet objects are soaps.
Which one of the following options can be logically inferred?
(A) Only conclusion I is correct
(B) Either conclusion I or conclusion II is correct
(C) Either conclusion III or conclusion IV is correct
(D) Only conclusion I and conclusion III are correct

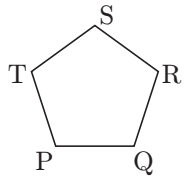
Solution: Among the given conclusions only conclusion I and conclusion III can be inferred

Ans. (D)

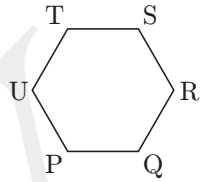
10. An ant walks in a straight line on a plane leaving behind a trace of its movement. The initial position of the ant is at point P facing east. The ant first turns 72° anticlockwise at P, and then does the following two steps in sequence exactly FIVE times before halting.
1. moves forward for 10 cm.
2. turns 144° clockwise.



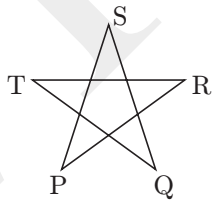
The pattern made by the trace left behind by the ant is

(A)


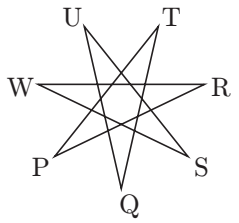
PQ = QR = RS = ST = TP = 10 cm

(B)


PQ = QR = RS = ST = TU = UP = 10 cm

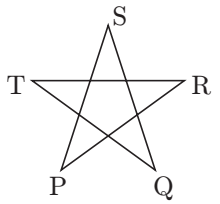
(C)


SQ = QT = TR = RP = PS = 10 cm

(D)


SW = WR = RP = PT = TQ = QU = US = 10 cm

Solution: As directed, the patterns made by ant is



PQ = QT = TR = RP = PS = 10 cm

Ans. (C)

WILEY