SOLVED GATE (MECHANICAL ENGINEERING SET–I) 2022

Engineering Mathematics and General Aptitude

ENGINEERING MATHEMATICS

Q. No. 1–5 Carry One Mark Each

1. The limit

$$p = \lim_{x \to \pi} \left(\frac{x^2 + \alpha x + 2\pi^2}{x - \pi + 2\sin x} \right)$$

has a finite value for a real α. The value of α and the corresponding limit p are
(A) $\alpha = -3\pi$, and $p = \pi$
(B) $\alpha = -2\pi$, and $p = 2\pi$
(C) $\alpha = \pi$, and $p = \pi$
(D) $\alpha = 2\pi$, and $p = 3\pi$

Solution: The denominator becomes 0 for $x = \pi$, therefore the value of constant α can be calculated by equating the numerator with zero,

$$x^2 + \alpha x + 2\pi^2 = 0$$
$$\pi^2 + \alpha\pi + 2\pi^2 = 0$$
$$\alpha = -3\pi$$

Applying L'Hospital rule on the given limit as,

$$p = \lim_{x \to p} \left(\frac{\frac{d}{dx}(x^2 + \alpha x + 2\pi^2)}{\frac{d}{dx}(x - \pi + 2\sin x)} \right)$$
$$p = \lim_{x \to p} \left(\frac{2x + \alpha}{1 + 2\cos x} \right)$$
$$p = \lim_{x \to p} \left(\frac{2\pi - 3\pi}{1 + 2\cos \pi} \right)$$
$$p = \pi$$

Ans. (A)

2. Solution of $\nabla^2 T = 0$ in a square domain ($0 < x < 1$ and $0 < y < 1$) with boundary conditions:

$$T(x,0) = x; T(0,y) = y; T(x,1) = 1 + x; T(1,y) = 1 + y$$

is

(A) $T(x,y) = x - xy + y$ (B) $T(x,y) = x + y$
(C) $T(x,y) = -x + y$ (D) $T(x,y) = x + xy + y$

Solution: Given that,

$$T(x,0) = x;\ T(0,y) = y;$$
$$T(x,1) = 1 + x;\ T(1,y) = 1 + y$$

This question can be solved by checking the options,
Option (A)

$$\left.\begin{array}{l} T(x,y) = x - xy + y \\ T(x,0) = x \\ T(0,y) = y \\ T(x,1) = y \\ T(1,y) = x \end{array}\right\} \text{Not satisfied}$$

Option (B)

$$\left.\begin{array}{l} T(x,y) = x + y \\ T(x,0) = x \\ T(0,y) = y \\ T(x,1) = x + 1 \\ T(1,y) = 1 + y \end{array}\right\} \text{satisfied}$$

Ans. (B)

3. Given a function $\phi = \frac{1}{2}(x^2 + y^2 + z^2)$ in three-dimensional Cartesian space, the value of the surface integral

$$\oiint_S \hat{n} \cdot \nabla\phi \, dS,$$

where S is the surface of a sphere of unit radius and $\hat{n}$ is the outward unit normal vector on S, is

(A) 4π (B) 3π
(C) $\frac{4\pi}{3}$ (D) 0

WILEY

Solution: The divergence of the given function will be,

$$\begin{aligned}\nabla\phi &= \frac{\partial\phi}{\partial x}\hat{i}+\frac{\partial\phi}{\partial y}\hat{j}+\frac{\partial\phi}{\partial z}\hat{k}\\ &= \frac{\partial}{\partial x}\frac{1}{2}(x^2+y^2+z^2)\hat{i}+\frac{\partial}{\partial y}\frac{1}{2}(x^2+y^2+z^2)\hat{j}\\ &\quad+\frac{\partial}{\partial z}\frac{1}{2}(x^2+y^2+z^2)\hat{k}\\ &= x\hat{i}+y\hat{j}+z\hat{k}\end{aligned}$$

Applying the Gauss integral theorem to calculate the surface integral,

$$\begin{aligned}\oiint_S \hat{n}\cdot\nabla\phi ds &= \iiint_V \nabla\cdot(\nabla\phi)dV\\ &= \iiint_V\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\cdot(x\hat{i}+y\hat{j}+z\hat{k})dV\\ &= \iiint_V (1+1+1)dV = 3\iiint_V dV\\ &= 3\left(\frac{4}{3}\pi(1)^3\right)=4\pi\end{aligned}$$

Ans. (A)

4. The Fourier series expansion of x^3 in the interval $-1\le x<1$ with periodic continuation has

(A) only sine terms
(B) only cosine terms
(C) both sine and cosine terms
(D) only sine terms and a non-zero constant

Solution: The given function is,

$$y = x^3$$

Check for odd function:

$$y = (-x)^3 = -x^3$$

Hence, the above function is odd function, therefore it will only contains sine terms in the expansion.

Ans. (A)

5. If $A=\begin{bmatrix}10 & 2k+5\\ 3k-3 & k+5\end{bmatrix}$ is a symmetric matrix, the value of k is _____.

(A) 8 (B) 5

(C) -0.4 (D) $\dfrac{1+\sqrt{1561}}{12}$

Solution: Applying the condition for symmetry,

$$3k-3 = 2k+5$$
$$k = 8$$

Thus, the value of k is 8.

Ans. (A)

Q. No. 6–10 Carry Two Marks Each

6. The value of the integral

$$\oint\left(\frac{6z}{2z^4-3z^3+7z^2-3z+5}\right)dz$$

evaluated over a counter-clockwise circular contour in the complex plane enclosing only the pole $z=i$, where i is the imaginary unit, is

(A) $(-1+i)\pi$ (B) $(1+i)\pi$

(C) $2(1-i)\pi$ (D) $(2+i)\pi$

Solution: Checking for singularity at $z=i$.

$$\begin{aligned}f(z) &= 2z^4-3z^3+7z^2-3z+5\\ &= 2(i)^4-3(i)^3+7(i)^2-3(i)+5=0\end{aligned}$$

Thus, we have singular point at $z=i$.
Applying residue theorem,

$$\oint\frac{6z}{2z^4-3z^3+7z^2-3z+5}dz = 2\pi i(\text{Res}|_{z=i})$$

The residue can be calculated as,

$$\begin{aligned}\text{Res}|_{z=i} &= \lim_{z\to i}(z-i)\frac{6z}{2z^4-3z^3+7z^2-3z+5}\\ &= \lim_{z\to i}\frac{6z^2-6zi}{2z^4-3z^3+7z^2-3z+5}\\ &= \lim_{z\to i}\frac{12z-6i}{8z^3-9z^2+14z-3}\\ &= \frac{12(i)-6i}{8(i)^3-9(i)^2+14(i)-3}=\frac{i}{i+1}\end{aligned}$$

The final solution will be,

$$\begin{aligned}\oint\frac{6z}{2z^4-3z^3+7z^2-3z+5}dz &= 2\pi i\left(\frac{i}{i+1}\right)\\ &= (-1+i)\pi\end{aligned}$$

Ans. (A)

7. The system of linear equations in real (x, y) given by

$$\begin{pmatrix}x & y\end{pmatrix}\begin{bmatrix}2 & 5-2\alpha\\ \alpha & 1\end{bmatrix}=\begin{pmatrix}0 & 0\end{pmatrix}$$

involves a real parameter α and has infinitely many non-trivial solutions for special value(s) of α. Which one or more among the following options is/are non-trivial solution(s) of (x, y) for such special value(s) of α?

(A) $x=2, y=-2$ (B) $x=-1, y=4$
(C) $x=1, y=1$ (D) $x=4, y=-2$

Solution: The determinate for non-trivial solution is zero,

$$|A| = 0$$

WILEY

$$\Rightarrow \begin{vmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{vmatrix} = 0$$

$$\Rightarrow 2-\alpha(5-2\alpha)=0$$

$$\Rightarrow 2-5\alpha+2\alpha^2=0$$

$$\Rightarrow \alpha = 2, \frac{1}{2}$$

For $\alpha = 2$, the eigen vector will be:

$$[x \quad y]\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = [0 \quad 0]$$

$$\Rightarrow 2x+2y=0$$

$$\Rightarrow x+y=0$$

$$\Rightarrow x=-y$$

For $\alpha = \frac{1}{2}$, the eigen vector will be:

$$[x \quad y]\begin{bmatrix} 2 & 4 \\ \frac{1}{2} & 1 \end{bmatrix} = [0 \quad 0]$$

$$\Rightarrow 2x+\frac{1}{2}y=0$$

$$\Rightarrow 4x+y=0$$

$$\Rightarrow x=\frac{-y}{4}$$

Option (A) and (B) satisfy the given condition.

Ans. (A, B)

8. Let a random variable X follow Poisson distribution such that Prob$(X = 1)$ = Prob$(X = 2)$. The value of Prob$(X = 3)$ is _____ (round off to 2 decimal places).

Solution: Given that,

$$P(X=1)=P(X=2)$$

The Poisson distribution can be applied as,

$$\frac{\lambda e^{-\lambda}}{1!}=\frac{\lambda^2 e^{-\lambda}}{2!}$$

$$\Rightarrow 2\lambda=\lambda^2$$

$$\Rightarrow \lambda=2$$

The probability at X= 3 will be,

$$P(X=3)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{(2)^3 e^{-2}}{3!}=0.18$$

Ans. (0.18)

9. Consider two vectors:

$$\vec{a}=5\hat{i}+7\hat{j}+2\hat{k}$$

$$\vec{b}=3\hat{i}-\hat{j}+6\hat{k}$$

Magnitude of the component of $\vec{a}$ orthogonal to $\vec{b}$ in the plane containing the vectors $\vec{a}$ and $\vec{b}$ is _____ (round off to 2 decimal places).

Solution: The angle between the two vectors will be,

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$=\frac{\left(5\hat{i}+7\hat{j}+2\hat{k}\right)\cdot\left(3\hat{i}-\hat{j}+6\hat{k}\right)}{\left|\sqrt{5^2+7^2+2^2}\right|\left|\sqrt{3^2+(-1)^2+6^2}\right|}$$

$$\Rightarrow \theta=70.495^\circ$$

The required answer is,

$$\left||a|\sin\theta\right|=\left|\sqrt{78}\sin(70.495^\circ)\right|=8.32$$

Ans. (8.32)

10. A steady two-dimensional flow field is specified by the stream function

$$\phi = kx^3y,$$

where x and y are in meter and the constant $k = 1\ \text{m}^{-2}\text{s}^{-1}$. The magnitude of acceleration at a point $(x, y) = (1\ \text{m}, 1\ \text{m})$ is _____ m/s^2 (round off to 2 decimal places).

Solution: The U-component of velocity is

$$U=\frac{\partial\psi}{\partial y}=\frac{\partial(kx^3y)}{\partial y}=kx^3$$

Where $k = 1\ \text{m}^{-2}\,\text{s}^{-1}$, we get

$$U=x^3$$

The V-component of velocity is

$$V=-\frac{\partial\psi}{\partial x}=-\frac{\partial(kx^3y)}{\partial x}=-k3x^2y$$

Where $k = 1\ \text{m}^{-2}\,\text{s}^{-1}$, we get

$$V=-3x^2y$$

The x-component of acceleration is,

$$a_x=U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}$$

$$=x^3\frac{\partial x^3}{\partial x}+(-3x^2y)\frac{\partial x^3}{\partial y}=3x^5$$

At $x = 1$, $y = 1$ we have

$$a_x=3(1)^5=3$$

The y-component of acceleration is,

$$a_y=U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}$$

$$= x^3 \frac{\partial(-3x^2y)}{\partial x} + (-3x^2y)\frac{\partial(-3x^2y)}{\partial y}$$

$$= x^3(6xy) + (-3x^2y)(-3x^2)$$

At $x = 1$, $y = 1$ we have

$$a_y = 3$$

The net acceleration will be,

$$a = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{3^2 + 3^2} = 4.24$$

Ans. (4.24)

GENERAL APTITUDE

Q. No. 1–5 Carry One Mark Each

1. After playing _____ hours of tennis, I am feeling _____ tired to walk back.
(A) too / too (B) too / two
(C) two / two (D) two / too

Solution: After playing two hours of tennis, I am feeling too tired to walk back.

Ans. (D)

2. The average of the monthly salaries of M, N and S is ₹4000. The average of the monthly salaries of N, S and P is ₹5000. The monthly salary of P is ₹6000. What is the monthly salary of M as a percentage of the monthly salary of P?
(A) 50% (B) 75%
(C) 100% (D) 125%

Solution: Given that, average monthly salary of M, N and S = ₹4000

$$\frac{M + N + S}{3} = 4000 \quad (1)$$

Average monthly salary of N, S and P = ₹5000

$$\frac{N + S + P}{3} = 5000 \quad (2)$$

Monthly salary of P = ₹6000 (3)

From Eqs. (2) and (3), we get

$$N + S + 6000 = 3 \times 5000$$

$$\Rightarrow N + S = 15000 - 6000 = 9000 \quad (4)$$

From Eqs. (1) and (4), we get

$$\frac{M + 9000}{3} = 4000$$

$$\Rightarrow M = 3 \times 4000 - 9000$$

$$= 12000 - 9000 = 3000$$

Therefore, percentage of monthly salary of M from the monthly salary of P is

$$\frac{3000}{6000} \times 100\% = 50\%$$

Ans. (A)

3. A person travelled 80 km in 6 hours. If the person travelled the first part with a uniform speed of 10 kmph and the remaining part with a uniform speed of 18 kmph. What percentage of the total distance is travelled at a uniform speed of 10 kmph?
(A) 28.25 (B) 37.25
(C) 43.75 (D) 50.00

Solution: Given that, total distance travelled = 80 km and total time taken = 6 hours
Let distance travelled at speed of 10 kmph = x
So, distance travelled at speed of 18 kmph = $80 - x$
Using speed, distance and time relation, we have

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\Rightarrow \quad \frac{x}{10} + \frac{80 - x}{18} = 6$$

$$\Rightarrow \quad 8x + 10(80 - x) = 6 \times 10 \times 18$$

$$\Rightarrow \quad 18x + 800 - 10x = 1080$$

$$\Rightarrow \quad 8x = 1080 - 800 \Rightarrow x = 35$$

So, distance travelled at speed of 10 kmph = 35
Therefore, percentage of total distance travelled at uniform speed of 10 kmph is

$$\frac{35}{80} \times 100\% = 43.75\%$$

Ans. (C)

4. Four girls P, Q, R and S are studying languages in a University. P is learning French and Dutch. Q is learning Chinese and Japanese. R is learning Spanish and French. S is learning Dutch and Japanese.
Given that: French is easier than Dutch; Chinese is harder than Japanese; Dutch is easier than Japanese, and Spanish is easier than French. Based on the above information, which girl is learning the most difficult pair of languages?
(A) P (B) Q
(C) R (D) S

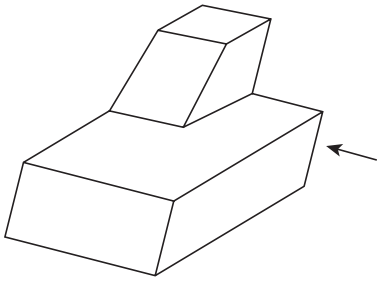
Solution: From the given data, we have
P → French + Dutch
Q → Chinese + Japanese
R → Spanish + French
S → Dutch + Japanese

MTEX

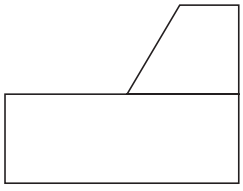
French is easier than Dutch
Chinses is harder than Japanese
Dutch is easier than Japanese
Spanish is easier than French
So, Spanish > French < Dutch < Japanese < Chinese
Therefore, Q is learning Chinese and Japanese which is most difficult pair of languages.

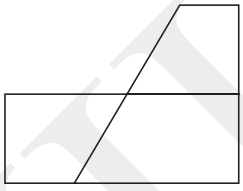
Ans. (B)

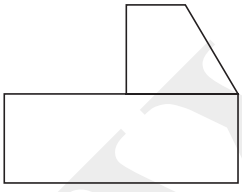
5.

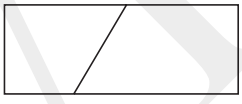


A block with a trapezoidal cross-section is placed over a block with rectangular cross section as shown above.
Which one of the following is the correct drawing of the view of the 3D object as viewed in the direction indicated by an arrow in the above figure?

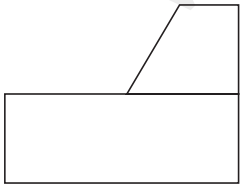
(A) 

(B) 

(C) 

(D) 

Solution: Correct drawing of 3D object when viewed in direction of arrow is



Ans. (A)

Q. No. 6–10 Carry Two Marks Each

6. Humans are naturally compassionate and honest. In a study using strategically placed wallets that appear "lost", it was found that wallets with money are more likely to be returned than wallets without money. Similarly, wallets that had a key and money are more likely to be returned than wallets with the same amount of money alone. This suggests that the primary reason for this behavior is compassion and empathy.
Which one of the following is the CORRECT logical inference based on the information in the above passage?

(A) Wallets with a key are more likely to be returned because people do not care about money
(B) Wallets with a key are more likely to be returned because people relate to suffering of others
(C) Wallets used in experiments are more likely to be returned than wallets that are really lost
(D) Money is always more important than keys

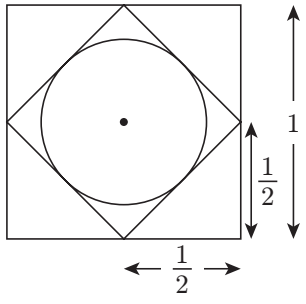
Solution: Based on the information in above passage, correct logical inference is "wallets with a key are more likely to be returned because people relate to suffering of others"

Ans. (B)

7. A rhombus is formed by joining the midpoints of the sides of a unit square. What is the diameter of the largest circle that can be inscribed within the rhombus?

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$
(C) $\sqrt{2}$ (D) $2\sqrt{2}$

Solution: From the given data we have the following figure



From the figure, we can say that

Diameter of largest circle = Side of rhombus

And side of rhombus is

$$s = \sqrt{(1/2)^2 + (1/2)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Therefore, diameter of largest circle $= 1/\sqrt{2}$

Ans. (A)

WILEY

8. An equilateral triangle, a square and a circle have equal areas. What is the ratio of the perimeters of the equilateral triangle to square to circle?

(A) $3\sqrt{3} : 2 : \sqrt{\pi}$ (B) $\sqrt{(3\sqrt{3})} : 2 : \sqrt{\pi}$

(C) $\sqrt{(3\sqrt{3})} : 4 : 2\sqrt{\pi}$ (D) $\sqrt{(3\sqrt{3})} : 2 : 2\sqrt{\pi}$

Solution: Given that,

$$\text{Area of equilateral } \Delta = \text{Area of square} = \text{Area of circle}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = x^2 = \pi r^2 \qquad (1)$$

And perimeter ratio's are

$$3a : 4x : 2\pi r \qquad (2)$$

From Eq. (1), we have

$$\frac{\sqrt{3}}{4}a^2 = x^2 = \pi r^2$$

Now, taking square root, we get

$$\frac{3^{1/4}}{2}a = x = \sqrt{\pi}r$$

Let $\dfrac{3^{1/4}a}{2} = x = \sqrt{\pi}r = k$. So,

$$a = \frac{2k}{3^{1/4}},\ x = k \text{ and } r = \frac{k}{\sqrt{\pi}}$$

Then Eq. (2) becomes,

$$3 \cdot \frac{2k}{3^{1/4}} : 4k : 2\pi\frac{k}{\sqrt{\pi}}$$

$$\Rightarrow \quad 3^{3/4} : 2 : \sqrt{\pi}$$

$$\Rightarrow \quad \sqrt{3\sqrt{3}} : 2 : \sqrt{\pi}$$

Ans. (B)

9. Given below are three conclusions drawn based on the following three statements.
Statement 1: All teachers are professors.
Statement 2: No professor is a male.
Statement 3: Some males are engineers.
Conclusion I: No engineer is a professor.
Conclusion II: Some engineers are professors.
Conclusion III: No male is a teacher.
Which one of the following options can be logically inferred?

(A) Only conclusion III is correct
(B) Only conclusion I and conclusion II are correct
(C) Only conclusion II and conclusion III are correct
(D) Only conclusion I and conclusion III are correct

Solution: Based on given statements only conclusion III is logically correct.

Ans. (A)

10. In a 12-hour clock that runs correctly, how many times do the second, minute, and hour hands of the clock coincide, in a 12-hour duration from 3 PM in a day to 3 AM the next day?

(A) 11 (B) 12
(C) 144 (D) 2

Solution: Every hour the second, minute and hour hands of clock coincide once. So, clock coincides once between 3 PM − 4 PM, 4 PM − 5 PM, 5 PM − 6 PM, 6 PM − 7 PM, & PM − 8 PM, 8 PM − 9 PM, 9 PM − 10 PM, 10 PM − 11 PM, 11 PM − 10 AM, 1 AM − 2 AM, 2 AM − 3 AM
Therefore, the hands coincide 11 times.

Ans. (A, B, D)

WILEY