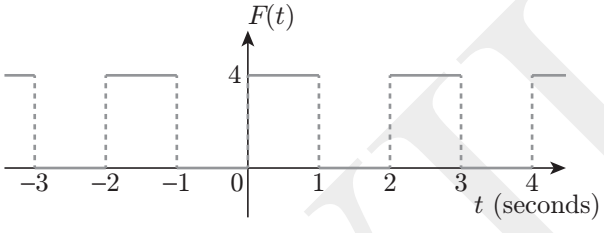
SOLVED GATE
(MECHANICAL ENGINEERING SET–II) 2022

Engineering Mathematics and General Aptitude

ENGINEERING MATHEMATICS

Q. No. 1–5 Carry One Mark Each

1. $F(t)$ is a periodic square wave function as shown. It takes only two values, 4 and 0, and stays at each of these values for 1 second before changing. What is the constant term in the Fourier series expansion of $F(t)$?



(A) 1 (B) 2
(C) 3 (D) 4

Solution: The constant term in Fourier series is given by,

$$\frac{1}{2L}\int_0^2 f(x)dx = \frac{1}{2(1)}(\text{Area of rectangle})$$
$$= \frac{1}{2(1)}(4\times 1) = 2$$

Here, L is the period.

Ans. (B)

2. Consider a cube of unit edge length and sides parallel to co-ordinate axes, with its centroid at the point (1, 2, 3). The surface integral $\int_A \vec{F}\cdot\overrightarrow{dA}$ of a vector field $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$ over the entire surface A of the cube is _____.

(A) 14 (B) 27
(C) 28 (D) 31

Solution: Given vector field $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$

Applying the Gauss divergence theorem, we have

$$\int_A FdA = \iiint_V \nabla\cdot\vec{F}dV$$
$$= \iiint_V \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)(3x\hat{i} + 5y\hat{j} + 6z\hat{k})dV$$
$$= \iiint_V (3+5+6)dV = 14\iiint_V dV = 14\times a^3$$
$$= 14\times(1)^3 = 14$$

Ans. (A)

3. Consider the definite integral

$$\int_1^2 (4x^2 + 2x + 6)dx.$$

Let I_e be the exact value of the integral. If the same integral is estimated using Simpson's rule with 10 equal subintervals, the value is I_s. The percentage error is defined as $e = 100 \times (I_e - I_s)/I_e$. The value of e is

(A) 2.5 (B) 3.5
(C) 1.2 (D) 0

Solution: The answer for the value $e = 100 \times (I_e - I_s)/I_e$ is zero, because the value of I_e and I_s is same. The given function is quadratic equation and the Simpson's rule also uses a quadratic equation. Thus, I_e and I_s are same.

Ans. (D)

4. Given $\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$.

If a and b are positive integers, the value of $\int_{-\infty}^{\infty} e^{-a(x+b)^2}dx$ is _____.

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(A) $\sqrt{\pi a}$ (B) $\sqrt{\dfrac{\pi}{a}}$

(C) $b\sqrt{\pi a}$ (D) $b\sqrt{\dfrac{\pi}{a}}$

Solution: The given function is,

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$$

Substituting, $x + b = t; dx = dt$

The integration will be,

$$\int_{-\infty}^{\infty} e^{-at^2} dt$$

Substituting,

$$at^2 = y^2 \Rightarrow t = \frac{y}{\sqrt{a}}$$

$$dt = \frac{ydy}{at} = \frac{ydy}{a\dfrac{y}{\sqrt{a}}} = \frac{dy}{\sqrt{a}}$$

The integration will be,

$$\int_{-\infty}^{\infty} e^{-y^2} \frac{dy}{\sqrt{a}} = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$= \frac{1}{\sqrt{a}} \sqrt{\pi} = \sqrt{\frac{\pi}{a}}$$

Ans. (B)

5. A polynomial $\phi(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0$ of degree $n > 3$ with constant real coefficients $a_n, a_{n-1}, \ldots a_0$ has triple roots at $s = -\sigma$. Which one of the following conditions must be satisfied?

(A) $\phi(s) = 0$ at all the three values of s satisfying $s^3 + \sigma^3 = 0$

(B) $\phi(s) = 0, \dfrac{d\phi(s)}{ds} = 0,$ and $\dfrac{d^2\phi(s)}{ds^2} = 0$ at $s = -\sigma$

(C) $\phi(s) = 0, \dfrac{d^2\phi(s)}{ds^2} = 0,$ and $\dfrac{d^4\phi(s)}{ds^4} = 0$ at $s = -\sigma$

(D) $\phi(s) = 0,$ and $\dfrac{d^3\phi(s)}{ds^3} = 0$ at $s = -\sigma$

Solution: It is given that the value of n is greater than 3, therefore the three roots of the polynomials is $s = -\sigma, (s + \sigma)^3$. The function at $s = -\sigma$ would be zero, that is, $\phi(s) = 0$ therefore it is one of its roots.

The first and second order derivative of the given polynomial is would be zero because any polynomial has zero slope at its maximum.

Ans. (B)

Q. No. 6–9 Carry Two Marks Each

6. For the exact differential equation,

$$\frac{du}{dx} = \frac{-xu^2}{2 + x^2 u},$$

which one of the following is the solution?

(A) $u^2 + 2x^2 =$ constant

(B) $xu^2 + u =$ constant

(C) $\dfrac{1}{2}x^2u^2 + 2u =$ constant

(D) $\dfrac{1}{2}ux^2 + 2x =$ constant

Solution: The given equation can be written as,

$$(2 + x^2 u)du + (xu^2)dx = 0$$

Checking the solution is exact or not,

$$M = (2 + x^2 u)du + (xu^2)dx$$

$$\frac{\partial M}{\partial u} = 2xu$$

Further solving,

$$N = (2 + x^2 u)du + (xu^2)dx$$

$$\frac{\partial N}{\partial x} = 2xu$$

Since $\dfrac{\partial M}{\partial u} = \dfrac{\partial N}{\partial x}$.

Therefore, the solution of the given differential equation is,

$$\int xu^2 dx + \int 2du = C$$

$$\Rightarrow \quad \frac{x^2u^2}{2} + 2u = C$$

Ans. (C)

7. A is a 3×5 real matrix of rank 2. For the set of homogeneous equations $Ax = 0$, where 0 is a zero vector and x is a vector of unknown variables, which of the following is/are true?

(A) The given set of equations will have a unique solution.

(B) The given set of equations will be satisfied by a zero vector of appropriate size.

(C) The given set of equations will have infinitely many solutions.

(D) The given set of equations will have many but a finite number of solutions.

Solution: There will be infinitely many solution when the rank of the matrix is less than the number of variables.

Ans. (B, C)

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8. If the sum and product of eigenvalues of a 2×2 real matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4 and -1 respectively, then $|p|$ is _____ (in integer).

Solution:

Property 1: The trace of matrix is equal to sum of eigen values,

$$4 = 3 + q$$

$$\Rightarrow \quad q = 1$$

Property 2: The product of eigen values in equal to determinant of matrix,

$$-1 = 3q - p^2$$

$$\Rightarrow \quad -1 = 3(1) = p^2$$

$$\Rightarrow \quad p = 2$$

Ans. (2)

9. Given $z = x + iy,\ i = \sqrt{-1}$. C is a circle of radius 2 with the centre at the origin. If the contour C is traversed anticlockwise, then the value of the integral $\dfrac{1}{2\pi}\int_C \dfrac{1}{(z-i)(z+4i)}dz$ is _____ (round off to one decimal place).

Solution: The singular points will be,

$$z = i, -4i$$

The residue can be calculated as,

$$\text{Res}\big|_{z=i} = \lim(z-i)f(z)$$

$$= \lim(z-i)\frac{1}{z+4i} = \frac{1}{5i}$$

Applying residue theorem as,

$$\oint_C f(z)dz = 2\pi i\,\text{Res}\big|_{z=i} = 2\pi i\left(\frac{1}{5i}\right)$$

$$\Rightarrow \quad \frac{1}{2\pi}\oint_C f(z)dz = \frac{1}{5} = 0.2$$

Ans. (0.2)

GENERAL APTITUDE

Q. No. 1–5 Carry One Mark Each

1. Writing too many things on the _____ while teaching could make the students get _____.

(A) bored / board (B) board / bored

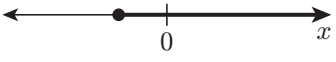
(C) board / board (D) bored / bored

Solution: Writing too many things on the board while teaching could make the students get bored.

Ans. (B)

2. Which one of the following is a representation (not to scale and in bold) of all values of x satisfying the inequality $2 - 5x \le -\dfrac{6x-5}{3}$ on the real number line?

(A)

(B) 

(C)

(D)

Solution: Given that,

$$2 - 5x \le \frac{-(6x-5)}{3}$$

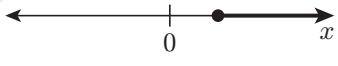
$$\Rightarrow \quad 6 - 15x \le -6x + 5$$

$$\Rightarrow \quad 6 - 5 \le 15x - 6x$$

$$\Rightarrow \quad 1 \le 9x$$

$$\Rightarrow \quad x \ge \frac{1}{9}$$

So, representation of inequality is



Ans. (C)

3. If $f(x) = 2\ln(\sqrt{e^x})$, what is the area bounded by $f(x)$ for the interval [0, 2] on the x-axis?

(A) $\dfrac{1}{2}$ (B) 1

(C) 2 (D) 4

Solution: Given that,

$$f(x) = 2\ln\sqrt{e^x}$$

$$= 2\ln(e^x)^{1/2} = 2\times\frac{1}{2}\ln(e^x) = x$$

Area bounded by $f(x)$ for interval [0, 2] is

$$A = \int_0^2 f(x)dx = \int_0^2 x\,dx = \frac{x^2}{2}\Big|_0^2$$

$$= \frac{(2)^2 - (0)^2}{2} = \frac{4}{2} = 2$$

Ans. (C)

4. A person was born on the fifth Monday of February in a particular year. Which one of the following statements is correct based on the above information?

(A) The 2nd February of that year is a Tuesday

(B) There will be five Sundays in the month of February in that year

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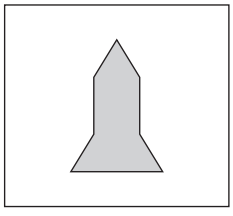
(C) The 1st February of that year is a Sunday
(D) All Mondays of February in that year have even dates

Solution: Among the given statements, statement A is correct, that is, the 2nd February of that year is a Tuesday.

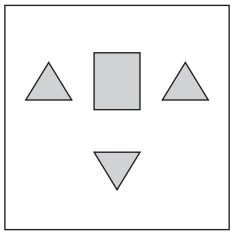
Ans. (A)

5.

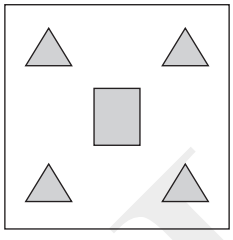


Which one of the groups given below can be assembled to get the shape that is shown above using each piece only once without overlapping with each other? (rotation and translation operations may be used).

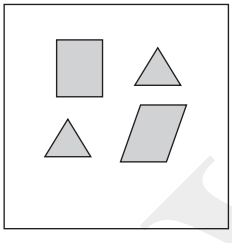
(A)



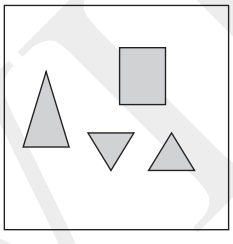
(B)



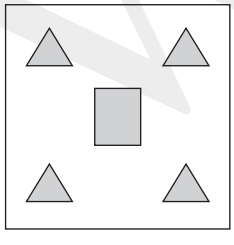
(C)



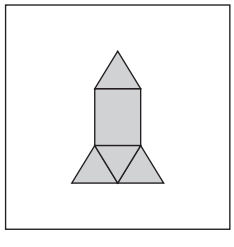
(D)



Solution: Let us take option (B)



These can be assembled to get the shape shown



Ans. (B)

Q. No. 6–10 Carry Two Marks Each

6. Fish belonging to species S in the deep sea have skins that are extremely black (ultra-black skin). This helps them not only to avoid predators but also sneakily attack their prey. However, having this extra layer of black pigment results in lower collagen on their skin, making their skin more fragile. Which one of the following is the **correct** logical inference based on the information in the above passage?
(A) Having ultra-black skin is only advantageous to species S
(B) Species S with lower collagen in their skin are at an advantage because it helps them avoid predators
(C) Having ultra-black skin has both advantages and disadvantages to species S
(D) Having ultra-black skin is only disadvantageous to species S but advantageous only to their predators

Solution: Based on the information in above passage, correct logical inference is "Having ultra-black skin has both advantages and disadvantages to species S".

Ans. (C)

7. For the past m days, the average daily production at a company was 100 units per day. If today's production of 180 units changes the average to 110 units per day, what is the value of m?
(A) 18 (B) 10
(C) 7 (D) 5

Solution: Given that,

Average production for m days = 100 units per day

Today's production = 180 units

Then, Average of $(m+1)$ days $= \dfrac{100\times m+180}{m+1}$

New average = 110 units per day

$$\Rightarrow \quad \frac{100\times m+180}{(m+1)} = 110$$

$$\Rightarrow \quad 100m+180 = 110(m+1)$$

$$\Rightarrow \quad 100m+180 = 110m+110$$

$$\Rightarrow \quad 110m-100m = 180-110$$

$$\Rightarrow \quad 10m = 70 \Rightarrow m = 7$$

Ans. (C)

8. Consider the following functions for non-zero positive integers, p and q.

$$f(p,q) = \underbrace{p\times p\times p\times \ldots\ldots\ldots\times p}_{q \text{ terms}} = p^q; \qquad f(p,1) = p$$

$$g(p,q) = p^{p^{p^{p^{p^{\cdots}}}}} \text{ (upto } q \text{ terms)} \quad ; \quad g(p,1) = p$$

Which one of the following options is correct based on the above?

(A) $f(2,2) = g(2,2)$

(B) $f(g(2,2), 2) < f(2, g(2,2))$

(C) $g(2,1) \neq f(2,1)$

(D) $f(3,2) > g(3,2)$

Solution: Given that,

$$f(p,q) = p^q\,,\ f(p,1) = p\,,\ g(p,q) = p^{p^p}$$

and $g(p,1) = p$

For $p = q = 2$, we get

$$f(2,2) = 2^2 = 4$$

and $$g(2,2) = 2^2 = 4$$

Therefore, $$f(2,2) = g(2,2)$$

Now, $f(g(2,2),2) = f(4,2) = 4^2 = 16$

and $f(2, g(2,2)) = f(2,4) = 2^4 = 16$

Therefore, $f(g(2,2),2)) = f(2, g(2,2))$

For $p = 2$, $q = 1$ we get

$$g(2,1) = 2$$

and $$f(2,1) = 2$$

Therefore, $$g(2,1) = f(2,1)$$

For $p = 3$, $q = 2$ we get

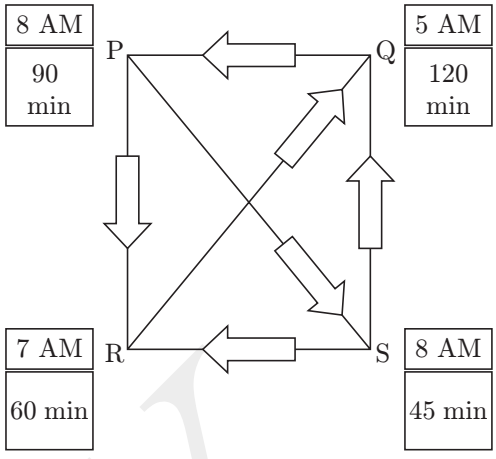
$$f(3,2) = 3^2 = 9$$

and $$g(3,2) = 3^3 = 9$$

$$f(3,2) = g(3,2)$$

Ans. (A)

9. Four cities P, Q, R and S are connected through one-way routes as shown in the figure. The travel time between any two connected cities is one hour. The boxes beside each city name describe the starting time of first train of the day and their frequency of operation. For example, from city P, the first trains of the day start at 8 AM with a frequency of 90 minutes to each of R and S. A person does not spend additional time at any city other than the waiting time for the next connecting train. If the person starts from R at 7 AM and is required to visit S and return to R, what is the minimum time required?



(A) 6 hours 30 minutes (B) 3 hours 45 minutes

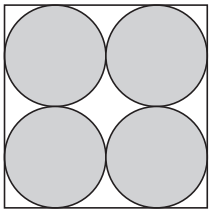
(C) 4 hours 30 minutes (D) 5 hours 15 minutes

Solution: Given that, person take train from R at 7 AM reaches Q at 8 AM. Then, train starts from Q at 9 AM and reaches P at 10 AM. Now, train starts from P at 11 AM and reaches S at 12 PM. Then, train starts from S at 12:30 PM and reaches R at 1:30 PM.

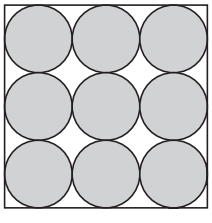
Therefore, minimum time = 6 hours 30 min

Ans. (A)

10. Equal sized circular regions are shaded in a square sheet of paper of 1 cm side length. Two cases, case M and case N, are considered as shown in the figures below. In the case M, four circles are shaded in the square sheet and in the case N, nine circles are shaded in the square sheet as shown. What is the ratio of the areas of unshaded regions of case M to that of case N?



Case M



Case N

(A) 2 : 3 (B) 1 : 1

(C) 3 : 2 (D) 2 : 1

Solution: Given that,

Case M:

Length of square = 1 cm

Radius of 1 circle $= \dfrac{1}{4}$ cm

Area of square = 1 cm^2

Area of shaded circles $= 4 \times \pi \left(\frac{1}{4}\right)^2$

$= 4 \times \pi \times \frac{1}{16} = \frac{\pi}{4} \text{cm}^2$

Area of unshaded region $= \left(1 - \frac{\pi}{4}\right) \text{cm}^2$

Case N:

Length of square = 1 cm

Radius of circle $= \frac{1}{6} \text{cm}$

Area of square = 1 cm^2

Area of shaded circles $= 9 \times \pi \times \left(\frac{1}{6}\right)^2$

$= 9 \times \frac{\pi}{36} = \frac{\pi}{4} \text{cm}^2$

Area of unshaded region $= \left(1 - \frac{\pi}{4}\right) \text{cm}^2$

Then ratio of areas of unshaded regions of case M to that of case N is 1 : 1

Ans. (B)

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