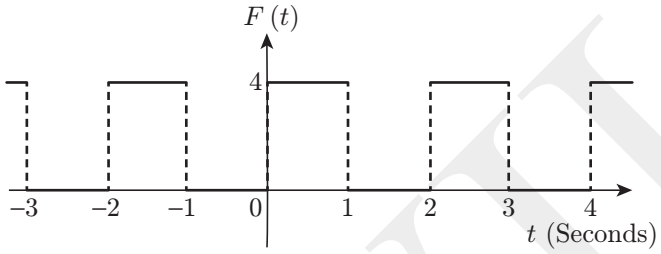
SOLVED GATE
(MECHANICAL ENGINEERING SET-II) 2022

Engineering Mathematics and Technical Section

Q. No. 1–25 Multiple Choice Question (MCQ) Carry One Mark Each

1. $F(t)$ is a periodic square wave function as shown. It takes only two values, 4 and 0, and stays at each of these values for 1 second before changing. What is the constant term in the Fourier series expansion of $F(t)$?



(A) 1 (B) 2
(C) 3 (D) 4

Solution: The constant term in Fourier series is given by,

$$\frac{1}{2L}\int_0^2 f(x)dx = \frac{1}{2(1)}(\text{Area of rectangle})$$
$$= \frac{1}{2(1)}(4\times 1) = 2$$

Here, L is the period.

Ans. (B)

2. Consider a cube of unit edge length and sides parallel to co-ordinate axes, with its centroid at the point (1, 2, 3). The surface integral $\int_A \vec{F}\cdot\overrightarrow{dA}$ of a vector field $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$ over the entire surface A of the cube is _____.

(A) 14 (B) 27
(C) 28 (D) 31

Solution: Given vector field $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$

Applying the Gauss divergence theorem, we have

$$\int_A FdA = \iiint_V \nabla\cdot\vec{F}dV$$
$$= \iiint_V \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\cdot(3x\hat{i} + 5y\hat{j} + 6z\hat{k})dV$$
$$= \iiint_V (3+5+6)dV = 14\iiint_V dV = 14\times a^3$$
$$= 14\times(1)^3 = 14$$

Ans. (A)

3. Consider the definite integral

$$\int_1^2 (4x^2 + 2x + 6)dx.$$

Let I_e be the exact value of the integral. If the same integral is estimated using Simpson's rule with 10 equal subintervals, the value is I_s. The percentage error is defined as $e = 100 \times (I_e - I_s)/I_e$. The value of e is

(A) 2.5 (B) 3.5
(C) 1.2 (D) 0

Solution: The answer for the value $e = 100 \times (I_e - I_s)/I_e$ is zero, because the value of I_e and I_s is same. The given function is quadratic equation and the Simpson's rule also uses a quadratic equation. Thus, I_e and I_s are same.

Ans. (D)

4. Given $\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$.

If a and b are positive integers, the value of $\int_{-\infty}^{\infty} e^{-a(x+b)^2}dx$ is _____.

(A) $\sqrt{\pi a}$ (B) $\sqrt{\dfrac{\pi}{a}}$

(C) $b\sqrt{\pi a}$ (D) $b\sqrt{\dfrac{\pi}{a}}$

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Solution: The given function is,

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$$

Substituting, $x+b=t; dx=dt$

The integration will be,

$$\int_{-\infty}^{\infty} e^{-at^2} dt$$

Substituting,

$$at^2 = y^2 \Rightarrow t = \frac{y}{\sqrt{a}}$$

$$dt = \frac{ydy}{at} = \frac{ydy}{a\frac{y}{\sqrt{a}}} = \frac{dy}{\sqrt{a}}$$

The integration will be,

$$\int_{-\infty}^{\infty} e^{-y^2} \frac{dy}{\sqrt{a}} = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$= \frac{1}{\sqrt{a}} \sqrt{\pi} = \sqrt{\frac{\pi}{a}}$$

Ans. (B)

5. A polynomial $\phi(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0$ of degree $n > 3$ with constant real coefficients $a_n, a_{n-1}, \ldots a_0$ has triple roots at $s = -\sigma$. Which one of the following conditions must be satisfied?

(A) $\phi(s) = 0$ at all the three values of s satisfying $s^3 + \sigma^3 = 0$

(B) $\phi(s) = 0, \frac{d\phi(s)}{ds} = 0,$ and $\frac{d^2\phi(s)}{ds^2} = 0$ at $s = -\sigma$

(C) $\phi(s) = 0, \frac{d^2\phi(s)}{ds^2} = 0,$ and $\frac{d^4\phi(s)}{ds^4} = 0$ at $s = -\sigma$

(D) $\phi(s) = 0,$ and $\frac{d^3\phi(s)}{ds^3} = 0$ at $s = -\sigma$

Solution: It is given that the value of n is greater than 3, therefore the three roots of the polynomials is $s = -\sigma\ (s+\sigma)^3$. The function at $s = -\sigma$ would be zero, that is, $\phi(s) = 0$ therefore it is one of its roots.

The first and second order derivative of the given polynomial is would be zero because any polynomial has zero slope at its maximum.

Ans. (B)

6. Which one of the following is the definition of ultimate tensile strength (UTS) obtained from a stress-strain test on a metal specimen?

(A) Stress value where the stress-strain curve transitions from elastic to plastic behavior

(B) The maximum load attained divided by the original cross-sectional area

(C) The maximum load attained divided by the corresponding instantaneous crosssectional area

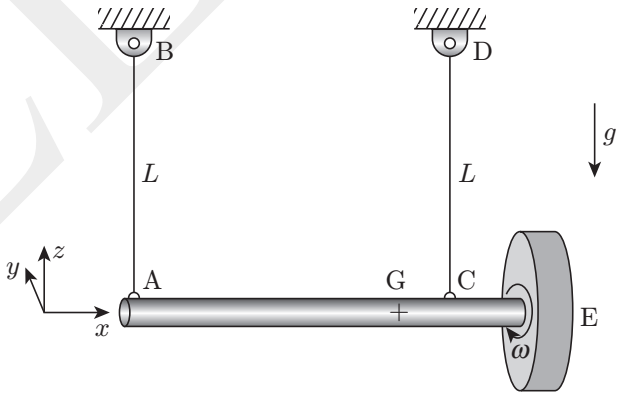
(D) Stress where the specimen fractures

Solution: The formula for UTS is,

$$\text{UTS} = \frac{F_{\max}(\text{Maximum load})}{A_{\text{original}}(\text{Original area})}$$

Ans. (B)

7. A massive uniform rigid circular disc is mounted on a frictionless bearing at the end E of a massive uniform rigid shaft AE which is suspended horizontally in a uniform gravitational field by two identical light inextensible strings AB and CD as shown, where G is the center of mass of the shaft-disc assembly and g is the acceleration due to gravity. The disc is then given a rapid spin ω about its axis in the positive x-axis direction as shown, while the shaft remains at rest. The direction of rotation is defined by using the right-hand thumb rule. If the string AB is suddenly cut, assuming negligible energy dissipation, the shaft AE will



(A) rotate slowly (compared to ω) about the negative z-axis direction

(B) rotate slowly (compared to ω) about the positive z-axis direction

(C) rotate slowly (compared to ω) about the positive y-axis direction

(D) rotate slowly (compared to ω) about the negative y-axis direction

Solution: The direction of rotation can be found by analyzing the direction of gyroscope effect.

Since the direction comes out to be negative z, therefore it rotates slowly in negative z-direction.

Ans. (A)

8. A structural member under loading has a uniform state of plane stress which in usual notations is given by $\sigma_x = 3P, \sigma_y = -2P$ and $\tau_{xy} = \sqrt{2}P$, where $P > 0$. The yield strength of the material

is 350 MPa. If the member is designed using the maximum distortion energy theory, then the value of P at which yielding starts (according to the maximum distortion energy theory) is

(A) 70 MPa (B) 90 MPa
(C) 120 MPa (D) 75 MPa

Solution: The principal stress can be calculated as,

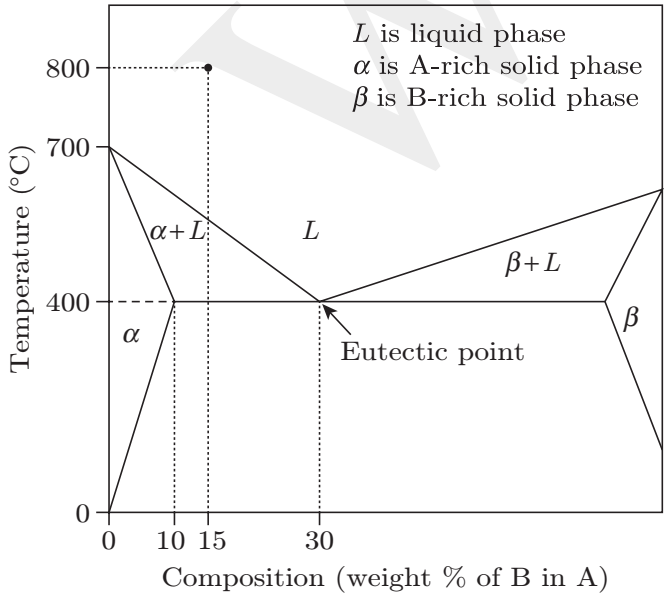
$$\sigma_{1,2} = \frac{1}{2}\left[(\sigma_x - \sigma_y) \pm \sqrt{(\sigma_x + \sigma_y) + 4\tau_{xy}^2}\right]$$
$$= \frac{1}{2}\left[(3P - 2P) \pm \sqrt{(3P - 2P)^2 + 4(\sqrt{2}P)^2}\right]$$
$$= 3.375P, -2.375P$$

Applying MDET,

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \left(\frac{S_{yt}}{N}\right)^2$$
$$\Rightarrow (3.375P)^2 + (-2.375P)^2 - (3.375P)(-2.375P)$$
$$= \left(\frac{350}{1}\right)^2$$
$$\Rightarrow P = 70.06 \text{ N}$$

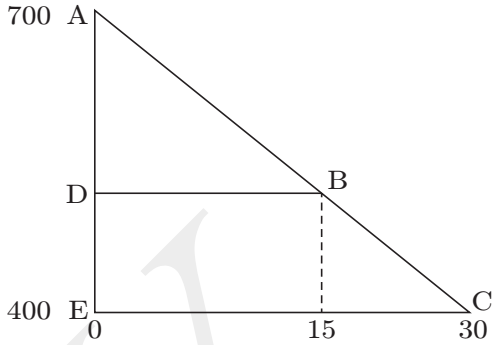
Ans. (A)

9. Fluidity of a molten alloy during sand casting depends on its solidification range. The phase diagram of a hypothetical binary alloy of components A and B is shown in the figure with its eutectic composition and temperature. All the lines in this phase diagram, including the solidus and liquidus lines, are straight lines. If this binary alloy with 15 weight % of B is poured into a mould at a pouring temperature of 800 °C, then the solidification range is



(A) 400 °C (B) 250 °C
(C) 800 °C (D) 150 °C

Solution:



Applying the similar angle triangle,

$$\frac{AD}{DE} = \frac{DB}{CE}$$
$$\Rightarrow \frac{AD}{300} = \frac{15}{30}$$
$$\Rightarrow AD = 150$$

Further solving,

$$AD + DE = AE$$
$$\Rightarrow 150 + DE = 300$$
$$\Rightarrow DE = 150\,^{\circ}C$$

Thus, the solidification range is 150 °C.

Ans. (D)

10. A shaft of diameter $25^{-0.04}_{-0.07}$ mm is assembled in a hole of diameter $25^{+0.02}_{-0.00}$ mm. Match the allowance and limit parameter in Column I with its corresponding quantitative value in Column II for this shaft-hole assembly.

Column I (Allowance and limit parameter)	**Column II (Quantitative value)**
P. Allowance	1. 0.09 mm
Q. Maximum clearance	2. 24.96 mm
R. Maximum material limit for hole	3. 0.04 mm
	4. 25.0 mm

(A) P-3, Q-1, R-4 (B) P-1, Q-3, R-2
(C) P-1, Q-3, R-4 (D) P-3, Q-1, R-2

Solution: The allowance is,

$$\text{Allowance} = \text{Maximum material limit}$$
$$= 25 - 24.96 = 0.04 \text{ mm}$$

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The clearance is,

$$\text{Maximum clearence} = \text{Hole}_s - \text{Shaft}_{\min}$$
$$= 25.02 - 24.93 = 0.09\,\text{mm}$$

Maximum material limit of hole is equal to minimum hole size = 25 mm.

Ans. (A)

11. Match the additive manufacturing technique in Column I with its corresponding input material in Column II.

Column I (Additive manufacturing technique)	**Column II (Input material)**
P. Fused deposition modelling	1. Photo sensitive liquid resin
Q. Laminated object manufacturing	2. Heat fusible powder
R. Selective laser sintering	3. Filament of polymer
	4. Sheet of thermoplastic or green compacted metal sheet

(A) P-3, Q-4, R-2 (B) P-1, Q-2, R-4
(C) P-2, Q-3, R-1 (D) P-4, Q-1, R-4

Solution: Fused deposition modelling use filament of polymer as input material.
Laminated object manufacturing use sheet of thermoplastic or green compacted metal sheet.
Selective laser sintering use heat fusible powder.

Ans. (A)

12. Which one of the following CANNOT impart linear motion in a CNC machine?
(A) Linear motor
(B) Ball screw
(C) Lead screw
(D) Chain and sprocket

Solution: Chain and sprocket does not impart any linear motion in CNC.

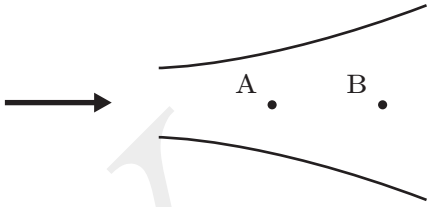
Ans. (D)

13. Which one of the following is an intensive property of a thermodynamic system?
(A) Mass (B) Density
(C) Energy (D) Volume

Solution: Density depends on mass therefore it is intensive property.

Ans. (B)

14. Consider a steady flow through a horizontal divergent channel, as shown in the figure, with supersonic flow at the inlet. The direction of flow is from left to right. Pressure at location B is observed to be higher than that at an upstream location A. Which among the following options can be the reason?



(A) Since volume flow rate is constant, velocity at B is lower than velocity at A
(B) Normal shock
(C) Viscous effect
(D) Boundary layer separation

Solution: There will be a normal shock as the pressure at B is more than the pressure at A because the velocity at B is more than the velocity A.

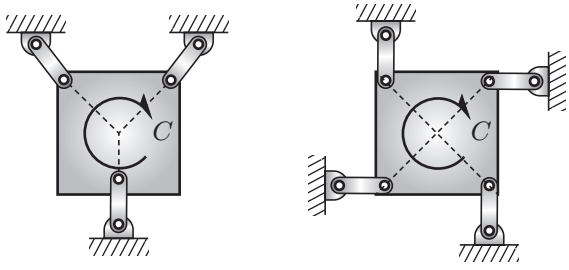
Ans. (B)

15. Which of the following non-dimensional terms is an estimate of Nusselt number?
(A) Ratio of internal thermal resistance of a solid to the boundary layer thermal resistance
(B) Ratio of the rate at which internal energy is advected to the rate of conduction heat transfer
(C) Non-dimensional temperature gradient
(D) Non-dimensional velocity gradient multiplied by Prandtl number

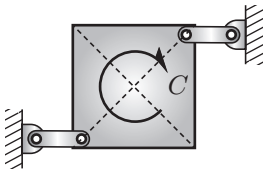
Solution: Nusselt number is a non-dimensional temperature gradient.

Ans. (C)

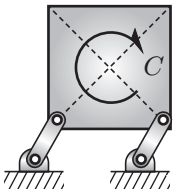
16. A square plate is supported in four different ways (configurations (P) to (S) as shown in the figure). A couple moment C is applied on the plate. Assume all the members to be rigid and mass-less, and all joints to be frictionless. All support links of the plate are identical.



(P) (Q)



(R)



(S)

The square plate can remain in equilibrium in its initial state for which one or more of the following support configurations?

(A) Configuration (P) (B) Configuration (Q)
(C) Configuration (R) (D) Configuration (S)

Solution: The plate will remain in equilibrium for the P, Q and R configuration.

Ans. (B, C, D)

17. Consider sand casting of a cube of edge length a. A cylindrical riser is placed at the top of the casting. Assume solidification time, $t_S \propto V/A$, where V is the volume and A is the total surface area dissipating heat. If the top of the riser is insulated, which of the following radius/radii of riser is/are acceptable?

(A) $\frac{a}{3}$ (B) $\frac{a}{2}$
(C) $\frac{a}{4}$ (D) $\frac{a}{6}$

Solution: Applying the optimum condition as,

$$t_{S,r} \geq t_{S,c}$$
$$\Rightarrow \left(\frac{V}{A}\right)_r \geq \left(\frac{V}{A}\right)_c$$
$$\Rightarrow \left(\frac{\pi r^2 h}{2\pi r h}\right) \geq \frac{a}{6}$$
$$\Rightarrow r \geq \frac{a}{3}$$

Therefore, all the values above $a/3$ will be acceptable.

Ans. (A, B)

18. Which of these processes involve(s) melting in metallic workpieces?

(A) Electrochemical machining
(B) Electric discharge machining
(C) Laser beam machining
(D) Electron beam machining

Solution: Only ECM does involve any melting of workpiece.

Ans. (B, C, D)

19. The velocity field in a fluid is given to be $\vec{v} - = (4xy)\hat{i} + 2(x^2 - y^2)\hat{j}$. Which of the following statement(s) is/are correct?

(A) The velocity field is one-dimensional.
(B) The flow is incompressible.
(C) The flow is irrotational.
(D) The acceleration experienced by a fluid particle is zero at $(x = 0, y = 0)$.

Solution: Condition for incompressible,

$$\nabla \cdot v = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}$$
$$= 4y - 4y = 0$$

Thus, the fluid is incompressible.
Condition for irrotational flow,

$$\omega_2 = 0$$
$$\Rightarrow \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$
$$\Rightarrow \frac{1}{2}(4x - 4x) = 0$$
$$\Rightarrow 0 = 0$$

Thus, the fluid is incompressible.
Acceleration at $x = 0$ and $y = 0$.

$$a_x = \frac{\Delta u}{\Delta t} = \left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial y}\right) = 0$$
$$a_y = \frac{\Delta v}{\Delta t} = \left(u\frac{\partial v}{\partial x}v + v\frac{\partial v}{\partial y}\right) = 0$$

Thus, acceleration at $x = 0$ and $y = 0$ is zero.
The given fluid flow is a 2-D flow.

Ans. (B, C, D)

20. A rope with two mass-less platforms at its two ends passes over a fixed pulley as shown in the figure. Discs with narrow slots and having equal weight of 20 N each can be placed on the platforms. The number of discs placed on the left side platform is n and that on the right side platform is m.

It is found that for $n = 5$ and $m = 0$, a force $F = 200$ N (refer to part (i) of the figure) is just sufficient to initiate upward motion of the left side platform. If the force F is removed then the minimum value of m (refer to part (ii) of the figure) required to prevent downward motion of the left side platform is _____ (in integer).

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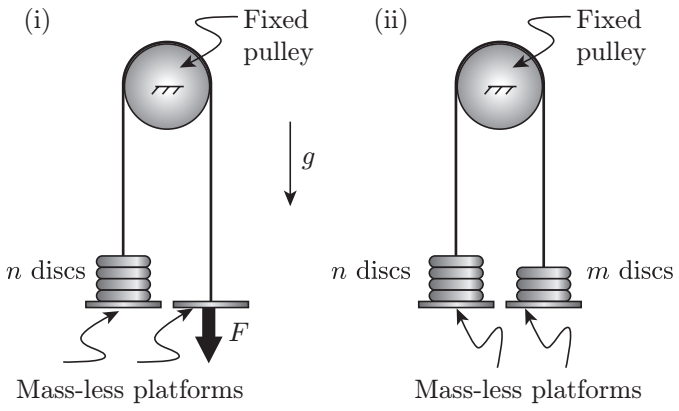


Solution: The ratio of tension for diagram (i) is,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\Rightarrow \frac{200}{100} = e^{\mu\left(180^\circ \times \frac{\pi}{180^\circ}\right)}$$

$$\Rightarrow \mu = 0.22$$

The ratio of tension for diagram (ii) is,

$$\frac{T_1}{T_3} = e^{\mu\theta}$$

$$\Rightarrow \frac{200}{T_3} = e^{0.22\left(180^\circ \times \frac{\pi}{180^\circ}\right)}$$

$$\Rightarrow T_3 = 50\,\text{N}$$

Total number of discs will be,

$$n = \frac{50}{20} \approx 3$$

Ans. (3)

21. For a dynamical system governed by the equation,

$$\ddot{x}(t) = 2\xi\omega_n \dot{x} \cdot (t) + \omega_n^2 x(t) = 0,$$

the damping ratio ξ is equal to $\frac{1}{2\pi}\log_e 2$. The displacement x of this system is measured during a hammer test. A displacement peak in the positive displacement direction is measured to be 4 mm. Neglecting higher powers (>1) of the damping ratio, the displacement at the next peak in the positive direction will be _____ mm (in integer).

Solution: The damping ratio is given by,

$$\xi = \frac{1}{2\pi}\log 2 = 0.1103$$

The logarithmic decrement will be,

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$= \frac{2\pi(0.1103)}{\sqrt{1-(0.1103)^2}} = 0.6974$$

The logarithmic decrement is given by,

$$\frac{x_0}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} \ldots\ldots\ldots = e^{\delta}$$

$$\Rightarrow \frac{x_0}{x_1} = e^{\delta}$$

$$\Rightarrow x_1 = \frac{x_0}{e^{\delta}} = \frac{4}{e^{0.6974}} = 2\,\text{mm}$$

Ans. (2)

22. An electric car manufacturer underestimated the January sales of car by 20 units, while the actual sales was 120 units. If the manufacturer uses exponential smoothing method with a smoothing constant of $\alpha = 0.2$, then the sales forecast for the month of February of the same year is _____ units (in integer).

Solution: Since the demand for the month of January is 120 and the error is 20, then the forecast for the month of January will be $F_{t-1} = 100$.
Applying the formula for the forecast,

$$F_t = F_{t-1} + \alpha e_{t-1}$$

$$= 100 + (0.2 \times 20) = 104\,\text{unit}$$

Ans. (104)

23. The demand of a certain part is 1000 parts/year and its cost is ₹1000/part. The orders are placed based on the economic order quantity (EOQ). The cost of ordering is ₹100/order and the lead time for receiving the orders is 5 days. If the holding cost is ₹20/part/year, the inventory level for placing the orders is _____ parts (round off to the nearest integer).

Solution: The inventory level for placing the orders is,

$$\text{Inventory level} = \text{Lead time} \times \text{Demand per day}$$

$$= 5 \times \frac{1000}{365} \approx 14$$

Ans. (14)

24. Consider 1 kg of an ideal gas at 1 bar and 300 K contained in a rigid and perfectly insulated container. The specific heat of the gas at constant volume c_v is equal to 750 J/(kg·K). A stirrer performs 225 kJ of work on the gas. Assume that the container does not participate in the thermodynamic interaction. The final pressure of the gas will be _____ bar (in integer).

Solution: Since the container is insulated, therefore we can write,

$$Q = mc_v(T_2 - T_1) + W$$

$$\Rightarrow 0 = [1 \times 0.75 \times (T_2 - 300)] + (-225)$$

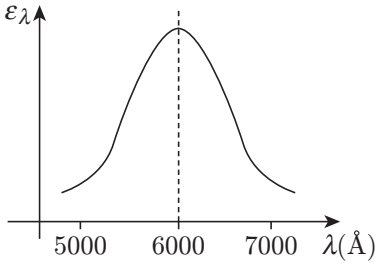
$$\Rightarrow T_2 = 600\,\text{K}$$

For constant volume we can write,

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$
$$\Rightarrow \frac{P_2}{1} = \frac{600}{300}$$
$$\Rightarrow P_2 = 2\,\text{bar}$$

Ans. (2)

25. Wien's law is stated as follows: $\lambda_m T = C$, where C is 2898 μm·K and λ_m is the wavelength at which the emissive power of a black body is maximum for a given temperature T. The spectral hemispherical emissivity (ε_λ) of a surface is shown in the figure below (1 Å = 10−10 m). The temperature at which the total hemispherical emissivity will be highest is _____ K (round off to the nearest integer).



Solution: Applying Wein's law, we get

$$\lambda_m T = C$$
$$\Rightarrow \frac{6000\times10^{-10}}{10^{-6}} \times T = 2898$$
$$\Rightarrow T = 4830\,\text{K}$$

Ans. (4830)

Q. No. 26–55 Multiple Choice Question (MCQ) Carry Two Marks Each

26. For the exact differential equation,

$$\frac{du}{dx} = \frac{-xu^2}{2+x^2u},$$

which one of the following is the solution?

(A) $u^2 + 2x^2 =$ constant

(B) $xu^2 + u =$ constant

(C) $\frac{1}{2}x^2u^2 + 2u =$ constant

(D) $\frac{1}{2}ux^2 + 2x =$ constant

Solution: The given equation can be written as,

$$(2+x^2u)du + (xu^2)dx = 0$$

Checking the solution is exact or not,

$$M = (2+x^2u)du + (xu^2)dx$$
$$\frac{\partial M}{\partial u} = 2xu$$

Further solving,

$$N = (2+x^2u)du + (xu^2)dx$$
$$\frac{\partial N}{\partial x} = 2xu$$

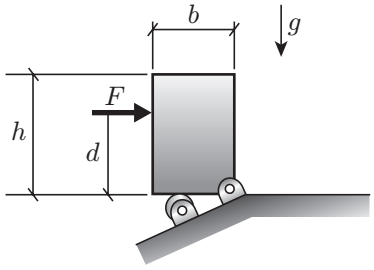
Since $\frac{\partial M}{\partial u} = \frac{\partial N}{\partial x}$.

Therefore, the solution of the given differential equation is,

$$\int xu^2dx + \int 2du = C$$
$$\Rightarrow \frac{x^2u^2}{2} + 2u = C$$

Ans. (C)

27. A rigid homogeneous uniform block of mass 1 kg, height h = 0.4 m and width b = 0.3 m is pinned at one corner and placed upright in a uniform gravitational field (g = 9.81 m/s^2), supported by a roller in the configuration shown in the figure. A short duration (impulsive) force F, producing an impulse I_F, is applied at a height of d = 0.3 m from the bottom as shown. Assume all joints to be frictionless. The minimum value of I_F required to topple the block is



(A) 0.953 Ns (B) 1.403 Ns
(C) 0.814 Ns (D) 1.172 Ns

Solution: The inertia about point A is given by,

$$I_A = \frac{mh^2}{3} + \frac{mb^2}{3}$$
$$= \frac{1\times0.4^2}{3} + \frac{1\times0.3^2}{3} = 0.08334$$

From conservation of energy we have, potential energy will be equal to the kinetic energy

$$KE = PE$$
$$\Rightarrow \frac{1}{2}I_A\omega^2 = mg\left(r - \frac{h}{2}\right)$$

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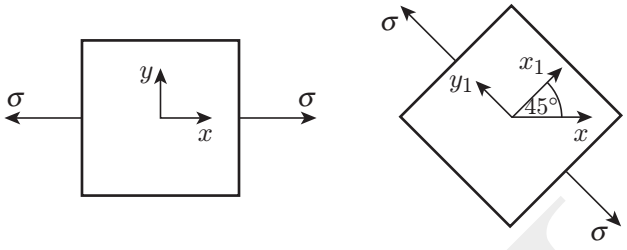
$$\Rightarrow \frac{1}{2}(0.08334)\omega^2 = 1\times 9.81\left(0.25-\frac{0.4}{2}\right)$$
$$\Rightarrow \omega = 3.43\,\text{rad/s}$$

Applying the conservation of angular momentum,

$$I_F d = I_A \omega$$
$$\Rightarrow I_F \times 0.3 = 0.08334\times 3.43$$
$$\Rightarrow I_F = 0.953\,\text{N}\cdot\text{s}$$

Ans. (A)

28. A linear elastic structure under plane stress condition is subjected to two sets of loading, I and II. The resulting states of stress at a point corresponding to these two loadings are as shown in the figure below. If these two sets of loading are applied simultaneously, then the net normal component of stress σ_{xx} is _____.



Loading - I Loading - II

(A) $3\sigma/2$ (B) $\sigma(1+1/\sqrt{2})$

(C) $\sigma/2$ (D) $\sigma(1-1/\sqrt{2})$

Solution: The normal stress on the element can be calculated as,

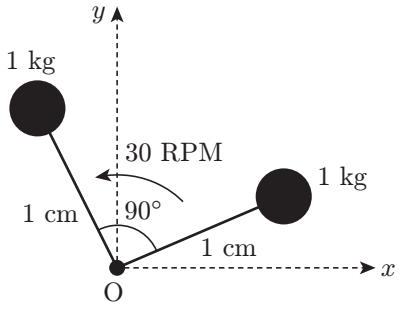
$$\sigma_x = \left(\frac{\sigma_{x1}+\sigma_{y1}}{2}\right)+\left(\frac{\sigma_{x1}+\sigma_{y1}}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \left(\frac{0+\sigma}{2}\right)+\left(\frac{0+\sigma}{2}\right)\cos(2\times 45)+\tau_{xy}\sin(2\times 45)$$
$$= \frac{\sigma}{2}$$

The net stress on the element in x-direction will be,

$$\sigma_{\text{net}} = \sigma_x + \sigma$$
$$= \frac{\sigma}{2}+\sigma = \frac{3}{2}\sigma$$

Ans. (A)

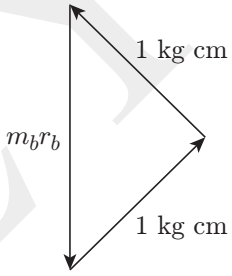
29. A rigid body in the x-y plane consists of two point masses (1 kg each) attached to the ends of two massless rods, each of 1 cm length, as shown in the figure. It rotates at 30 RPM counter-clockwise about the z-axis passing through point O. A point mass of $\sqrt{2}$ kg, attached to one end of a third massless rod, is used for balancing the body by attaching the free end of the rod to point O. The length of the third rod is _____ cm.



(A) 1 (B) $\sqrt{2}$

(C) $1/\sqrt{2}$ (D) $1/2\sqrt{2}$

Solution: The resultant balancing force is $m_b r_b$. Now, the force polygon is shown in the following figure,

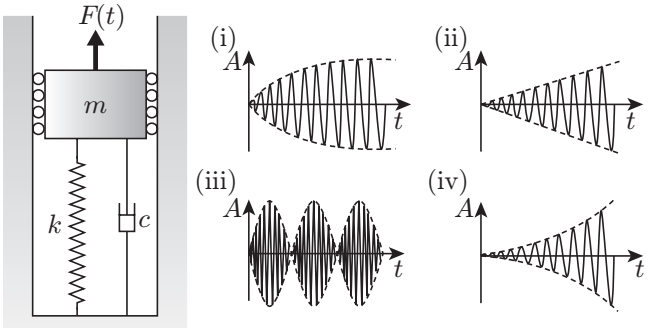


The resultant force is given by,

$$m_b r_b = \sqrt{1^2+1^2}$$
$$\Rightarrow \sqrt{2} r_b = \sqrt{2}$$
$$\Rightarrow r_b = 1\,\text{cm}$$

Ans. (A)

30. A spring mass damper system (mass m, stiffness k, and damping coefficient c) excited by a force $F(t) = B\sin\omega t$, where B, ω and t are the amplitude, frequency and time, respectively, is shown in the figure. Four different responses of the system (marked as (i) to (iv)) are shown just to the right of the system figure. In the figures of the responses, A is the amplitude of response shown in red color and the dashed lines indicate its envelope. The responses represent only the qualitative trend and those are not drawn to any specific scale.



Four different parameter and forcing conditions are mentioned below.

(P) $c > 0$ and $\omega = \sqrt{k/m}$

(Q) $c < 0$ and $\omega \neq 0$

(R) $c = 0$ and $\omega = \sqrt{k/m}$

(S) $c = 0$ and $\omega \cong \sqrt{k/m}$

Which one of the following options gives correct match (indicated by arrow $\rightarrow$) of the parameter and forcing conditions to the responses?

(A) (P) $\rightarrow$ (i), (Q) $\rightarrow$ (iii), (R) $\rightarrow$ (iv), (S) $\rightarrow$ (ii)
(B) (P) $\rightarrow$ (ii), (Q) $\rightarrow$ (iii), (R) $\rightarrow$ (iv), (S) $\rightarrow$ (i)
(C) (P) $\rightarrow$ (i), (Q) $\rightarrow$ (iv), (R) $\rightarrow$ (ii), (S) $\rightarrow$ (iii)
(D) (P) $\rightarrow$ (iii), (Q) $\rightarrow$ (iv), (R) $\rightarrow$ (ii), (S) $\rightarrow$ (i)

Solution: The natural frequency is given by,

$$\omega_n = \sqrt{\frac{k}{m}}$$

At resonance the given frequency is equal to natural frequency, and systems displacements keep on increasing,

$$\omega = \omega_n$$

$$\Rightarrow \frac{\omega}{\omega_n} = 1$$

When the system becomes critically damped then the displacement keeps on increasing.

The displacement for forced damped system is given by,

$$x = (A\sin\omega_n t + B\cos\omega_n t) + \left(\frac{F_0/k}{2m\omega_n}\right)\sin\omega_n t$$

The correct option for forced system with $c = 0$ and $\dfrac{\omega}{\omega_n} = 1$ is option (ii) i.e. R-(ii).

Ans. (C)

31. Parts P1-P7 are machined first on a milling machine and then polished at a separate machine. Using the information in the following table, the minimum total completion time required for carrying out both the operations for all 7 parts is _____ hours.

Part No.	Milling (hours)	Polishing (hours)
P1	8	6
P2	3	2
P3	3	4
P4	4	6
P5	5	7
P6	6	4
P7	2	1

(A) 31 (B) 33
(C) 30 (D) 32

Solution: Applying Johnson's Algorithm

		Milling machine	Polishing machine	
	A	8	6	
	~~B~~	~~3~~	~~2~~✓	X
X	C	3✓	4	
	D	4✓	6	
	E	5✓	7	
	F	6	4✓	
	G	2	1✓	X

The sequence is given by,

C – D – E – A – F – B – G

	Milling machine		Polishing machine	
Sequence	**In**	**Out**	**In**	**Out**
C	0	3	3	7
D	3	7	7	13
E	7	12	13	20
A	12	20	20	26
F	20	26	26	30
B	26	29	30	32
G	29	31	32	33

Thus, the minimum total competition time is 33 min.

Ans. (B)

32. A manufacturing unit produces two products P1 and P2. For each piece of P1 and P2, the table below provides quantities of materials M1, M2, and M3 required, and also the profit earned. The maximum quantity available per day for M1, M2 and M3 is also provided. The maximum possible profit per day is ₹ _____.

	M1	M2	M3	Profit per piece (₹)
P1	2	2	0	150
P2	3	1	2	100
Maximum quantity available per day	70	50	40	

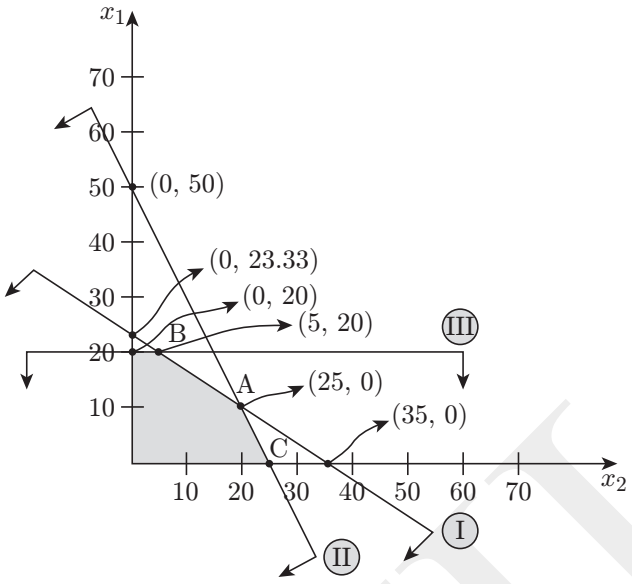
MTEP

(A) 5000 (B) 4000
(C) 3000 (D) 6000

Solution: The equations of constraints are,

$$2x_1 + 3x_2 \le 70;$$
$$2x_1 + x_2 \le 50;$$
$$2x_2 \le 40;$$
$$\max = 150x_1 + 100x_2$$

The graph is draw with the given constraints is shown in the following figure,



At point B, we have

$$2x_1 + (3 \times 20) = 70$$
$$\Rightarrow x_1 = 5 \text{ and } x_2 = 20$$

At point A, we have

$$(50 - x_2) + 2x_2 = 70$$
$$\Rightarrow x_2 = 10$$
$$\Rightarrow x_1 = \frac{50 - 10}{2} = 20$$

At point (5, 20), we get

$$Z = 150 \times 5 + 100 \times 20 = 2750$$

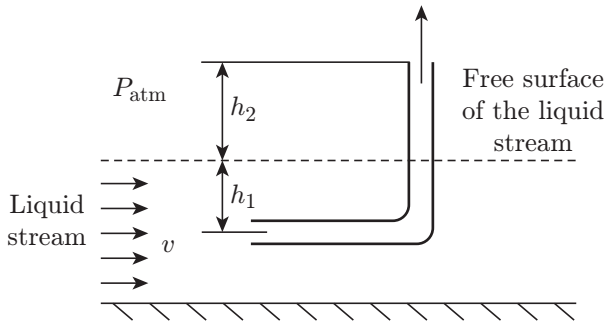
At point (20, 10), we get

$$Z = 150 \times 20 + 100 \times 10 = 4000$$

So, the maximum profit is ₹4000.

Ans. (B)

33. A tube of uniform diameter D is immersed in a steady flowing inviscid liquid stream of velocity v, as shown in the figure. Gravitational acceleration is represented by g. The volume flow rate through the tube is _____.



(A) $\frac{\pi}{4} D^2 v$ (B) $\frac{\pi}{4} D^2 \sqrt{2gh_2}$

(C) $\frac{\pi}{4} D^2 \sqrt{2g(h_1 + h_2)}$ (D) $\frac{\pi}{4} D^2 \sqrt{v^2 - 2gh_2}$

Solution: The pressure at inlet of pipe is,

$$P_1 = \rho g(h_1 + h_2)$$

The free stream pressure is,

$$P_0 = \rho g h_1$$

Applying Bernoulli's equation, between free stream and inlet of pipe is,

$$\frac{P_0}{\rho g} + \frac{v^2}{2g} = \frac{P_1}{\rho g} + \frac{v_1^2}{2g}$$
$$\Rightarrow \frac{\rho g h_1}{\rho g} + \frac{v^2}{2g} = \frac{\rho g(h_1 + h_2)}{\rho g} + \frac{v_1^2}{2g}$$
$$\Rightarrow \frac{v^2}{2g} - h_2 = \frac{v_1^2}{2g}$$
$$\Rightarrow v_1 = \sqrt{v^2 - 2gh_2}$$

The flow rate will be,

$$Q = Av_1 = \frac{\pi}{4} d^2 (\sqrt{v^2 - 2gh_2})$$

Ans. (D)

34. The steady velocity field in an inviscid fluid of density 1.5 is given to be $\vec{v} = (y^2 - x^2)\hat{i} + (2xy)\hat{j}$. Neglecting body forces, the pressure gradient at $(x = 1, y = 1)$ is _____.

(A) $10\hat{j}$ (B) $20\hat{i}$

(C) $-6\hat{i} - 6\hat{j}$ (D) $-4\hat{i} - 4\hat{j}$

Solution: Applying Navier strokes equation in x-direction is,

$$\frac{\partial P}{\partial x} = -\rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right]$$
$$= -1.5[(y^2 - x^2)(-2x) + (2xy)(2y)]$$

WILEY

At point $x = 1$, $y = 1$, we get

$$\left.\frac{\partial P}{\partial x}\right|_{(1,1)} = -1.5\left[(1^2 - 1^2)(-2(1)) + (2(1)(1))(2(1))\right]$$
$$= -6$$

Applying Navier strokes equation in y-direction,

$$\frac{\partial P}{\partial y} = -\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right]$$
$$= -1.5[(y^2 - x^2)(2y) + (2xy)(2x)]$$

At point $x = 1$, $y = 1$, we get

$$\left.\frac{\partial P}{\partial x}\right|_{(1,1)} = -1.5[(1^2 - 1^2)(2(1)) + (2(1)(1))(2(1))] = -6$$

Thus, the pressure gradient is $(-6\hat{i} - 6\hat{j})$.

Ans. (C)

35. In a vapour compression refrigeration cycle, the refrigerant enters the compressor in saturated vapour state at evaporator pressure, with specific enthalpy equal to 250 kJ/kg. The exit of the compressor is superheated at condenser pressure with specific enthalpy equal to 300 kJ/kg. At the condenser exit, the refrigerant is throttled to the evaporator pressure. The coefficient of performance (COP) of the cycle is 3. If the specific enthalpy of the saturated liquid at evaporator pressure is 50 kJ/kg, then the dryness fraction of the refrigerant at entry to evaporator is _____.

(A) 0.2 (B) 0.25
(C) 0.3 (D) 0.35

Solution: The coefficient of performance (COP) of the cycle is given by,

$$\text{COP} = \frac{h_1 - h_4}{h_2 - h_1}$$
$$\Rightarrow 3 = \frac{250 - h_4}{300 - 250}$$
$$\Rightarrow h_4 = 100\,\text{kJ/kg}$$

Here, h_4 is the point in wet region, h_1 is the point on saturated vapor, and h_2 is the superheat point. So, the dryness fraction is,

$$h_4 = h_f + x(h_1 - h_2)$$
$$\Rightarrow 100 = 50 + x(250 - 50)$$
$$\Rightarrow x = 0.25$$

Ans. (B)

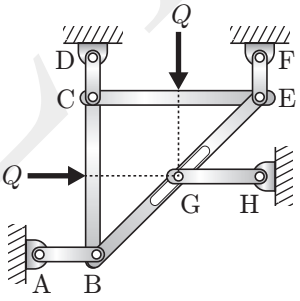
36. A is a 3×5 real matrix of rank 2. For the set of homogeneous equations $Ax = 0$, where 0 is a zero vector and x is a vector of unknown variables, which of the following is/are true?

(A) The given set of equations will have a unique solution.
(B) The given set of equations will be satisfied by a zero vector of appropriate size.
(C) The given set of equations will have infinitely many solutions.
(D) The given set of equations will have many but a finite number of solutions.

Solution: There will be infinitely many solution when the rank of the matrix is less than the number of variables.

Ans. (B, C)

37. The lengths of members BC and CE in the frame shown in the figure are equal. All the members are rigid and lightweight, and the friction at the joints is negligible. Two forces of magnitude $Q > 0$ are applied as shown, each at the mid-length of the respective member on which it acts.



Which one or more of the following members do not carry any load (force)?

(A) AB (B) CD
(C) EF (D) GH

Solution: When two colinear members are meeting on a point along with one more member, then this member will experience no force.

Thus, member CD and GH will experience no force.

Ans. (B, D)

38. If the sum and product of eigenvalues of a 2×2 real matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4 and -1 respectively, then $|p|$ is _____ (in integer).

Solution: **Property 1:** The trace of matrix is equal to sum of eigen values,

$$4 = 3 + q$$
$$\Rightarrow q = 1$$

Property 2: The product of eigen values in equal to determinant of matrix,

$$-1 = 3q - p^2$$

WILEY

$$\Rightarrow -1 = 3(1) = p^2$$
$$\Rightarrow p = 2$$

Ans. (2)

39. Given $z = x + iy,\ i = \sqrt{-1}$. C is a circle of radius 2 with the centre at the origin. If the contour C is traversed anticlockwise, then the value of the integral $\frac{1}{2\pi}\int_C \frac{1}{(z-i)(z+4i)}dz$ is _____ (round off to one decimal place).

Solution: The singular points will be,

$$z = i, -4i$$

The residue can be calculated as,

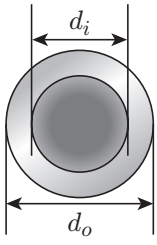
$$\text{Res}\big|_{z=i} = \lim(z-i)f(z)$$
$$= \lim(z-i)\frac{1}{z+4i} = \frac{1}{5i}$$

Applying residue theorem as,

$$\oint_C f(z)dz = 2\pi i\,\text{Res}\big|_{z=i} = 2\pi i\left(\frac{1}{5i}\right)$$
$$\Rightarrow \frac{1}{2\pi}\oint_C f(z)dz = \frac{1}{5} = 0.2$$

Ans. (0.2)

40. A shaft of length L is made of two materials, one in the inner core and the other in the outer rim, and the two are perfectly joined together (no slip at the interface) along the entire length of the shaft. The diameter of the inner core is d_i and the external diameter of the rim is d_o, as shown in the figure. The modulus of rigidity of the core and rim materials are G_i and G_o, respectively. It is given that $d_o = 2d_i$ and $G_i = 3G_o$. When the shaft is twisted by application of a torque along the shaft axis, the maximum shear stress developed in the outer rim and the inner core turn out to be τ_o and τ_i, respectively. All the deformations are in the elastic range and stressstrain relations are linear. Then the ratio τ_i/τ_o is _____ (round off to 2 decimal places).



Shaft cross-section

Solution: For the parallel shafts the angle of twist is given by,

$$\theta_1 = \theta_2$$

$$\Rightarrow \frac{32T_1L_1}{G\pi d_1^4} = \frac{32T_2L_2}{G\pi d_2^4(1-k^4)}$$

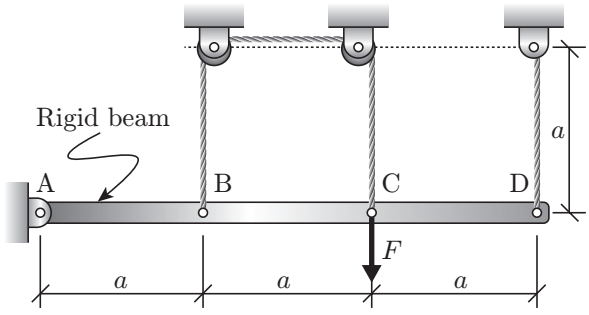
$$\Rightarrow \frac{T_1}{T_2} = \frac{G_1}{G_2}\left(\frac{d_1}{d_2}\right)^4\left(\frac{1}{1-k^4}\right)$$
$$= 3\left(\frac{1}{2}\right)^4\left(\frac{1}{1-\left(\frac{1}{2}\right)^4}\right) = \frac{3}{15}$$

The ratio of shear stress is,

$$\frac{\tau_1}{\tau_2} = \frac{16T_1}{\pi d_1^3}\times\frac{\pi d_2^3}{16T_2}(1-k^4)$$
$$= \frac{3}{15}\left(\frac{2d}{d}\right)^3\left(1-\left(\frac{1}{2}\right)^4\right) = 1.50$$

Ans. (1.50)

41. A rigid beam AD of length $3a = 6$ m is hinged at frictionless pin joint A and supported by two strings as shown in the figure. String BC passes over two small frictionless pulleys of negligible radius. All the strings are made of the same material and have equal cross-sectional area. A force $F = 9$ kN is applied at C and the resulting stresses in the strings are within linear elastic limit. The self-weight of the beam is negligible with respect to the applied load. Assuming small deflections, the tension developed in the string at C is _____ kN (round off to 2 decimal places).



Solution: Considering the tension at B and C are T, and tension at D is T'. The reaction at A is R_A. Applying force balance in vertical direction,

$$R_A + T + T + T' = 9$$
$$\Rightarrow R_A + 2T + T' = 9$$

The sum moment at A is,

$$Ta + T(2a) + T'(3a) = 9(2a)$$
$$\Rightarrow \quad T + T' = 6 \qquad (1)$$

The deflection at D will be one third of the deflection at B (by similar angle triangle).

$$\Delta_B = \frac{\Delta_D}{3}$$

$$\Rightarrow \left(\frac{Ta}{AE}\right) = \frac{\left(\frac{T'a}{AE}\right)}{3}$$

$$\Rightarrow T = \frac{T'}{3}$$

From Eq. (1), we have

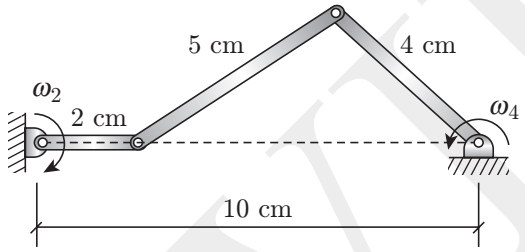
$$T + T' = 6$$

$$\Rightarrow \frac{T'}{3} + T' = 6$$

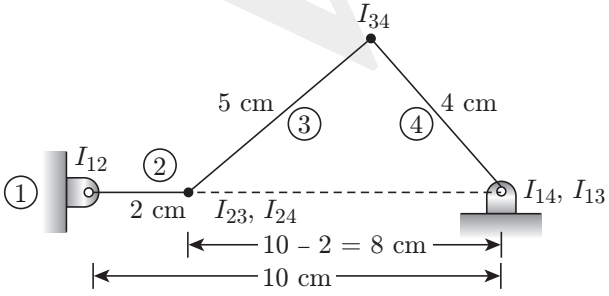
$$\Rightarrow T' = 4.5\,\text{kN and}\, T = 6 - 4.50 = 1.50\,\text{kN}$$

Ans. (1.50)

42. In the configuration of the planar four-bar mechanism at a certain instant as shown in the figure, the angular velocity of the 2 cm long link is $\omega_2 = 5$ rad/s. Given the dimensions as shown, the magnitude of the angular velocity ω_4 of the 4 cm long link is given by _____ rad/s (round off to 2 decimal places).



Solution: The *I*-center of the given mechanism are shown in the following figure,



Applying angular velocity theorem, we have

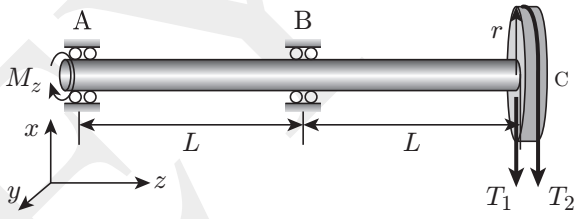
$$\omega_2(I_{24}I_{12}) = \omega_4(I_{24}I_{14})$$

$$\Rightarrow 5 \times 2 = \omega_4 \times 8$$

$$\Rightarrow \omega_4 = 1.25\,\text{rad/s}$$

Ans. (1.25)

43. A shaft AC rotating at a constant speed carries a thin pulley of radius $r = 0.4$ m at the end C which drives a belt. A motor is coupled at the end A of the shaft such that it applies a torque M_z about the shaft axis without causing any bending moment. The shaft is mounted on narrow frictionless bearings at A and B where AB = BC = $L = 0.5$ m. The taut and slack side tensions of the belt are $T_1 = 300$ N and $T_2 = 100$ N, respectively. The allowable shear stress for the shaft material is 80 MPa. The self-weights of the pulley and the shaft are negligible. Use the value of π available in the on-screen virtual calculator. Neglecting shock and fatigue loading and assuming maximum shear stress theory, the minimum required shaft diameter is _____ mm (round off to 2 decimal places).



Solution: Let reaction at A, B, and C are R_A, R_B, and R_C. Here the moment about A is zero, so

$$R_B \times L = 400 \times 2L$$

$$\Rightarrow R_B = 800\,\text{N}$$

The reaction force at A and C will be same and of magnitude 400 N. So, the net twisting moment is,

$$T = (T_1 - T_2)r = (300 - 100) \times 0.4 = 80 \times 10^3\,\text{N mm}$$

Applying the MSST,

$$T_e \le T_R$$

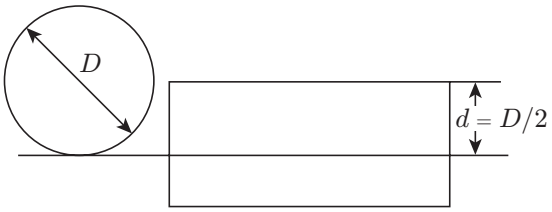
$$\Rightarrow \sqrt{(k_D M)^2 + (k_t T)^2} \le \left(\frac{\pi}{16} d^3\right)(\tau_{per})$$

$$\Rightarrow \sqrt{(1 \times 200)^2 + (1 \times 80)^2} \le \left(\frac{\pi}{16} d^3\right)(80 \times 10^6)$$

$$\Rightarrow d \ge 23.94\,\text{mm}$$

Ans. (23.94)

44. A straight-teeth horizontal slab milling cutter is shown in the figure. It has 4 teeth and diameter (D) of 200 mm. The rotational speed of the cutter is 100 rpm and the linear feed given to the workpiece is 1000 mm/minute. The width of the workpiece (w) is 100 mm, and the entire width is milled in a single pass of the cutter. The cutting force/tooth is given by $F = Kt_c w$, where specific cutting force $K = 10$ N/mm^2, w is the width of cut, and t_c is the uncut chip thickness. The depth of cut (d) is $D/2$, and hence the assumption of $\frac{d}{D} \ll 1$ is invalid. The maximum cutting force required is _____ kN (round off to one decimal place).



Solution: The uncut chip thickness will be,

$$t_{c,\max} = \frac{2f}{n}\sqrt{\frac{d}{D}\left(1-\frac{d}{D}\right)}$$
$$= \frac{2\times 10}{4}\sqrt{\frac{D/2}{D}\left(1-\frac{D/2}{D}\right)} = 2.5 \text{ mm}$$

The maximum force will be,

$$F_{\max} = kt_{c,\max}w$$
$$= 10\times 2.5\times 100 = 2.5\,\text{kN}$$

Ans. (2.5)

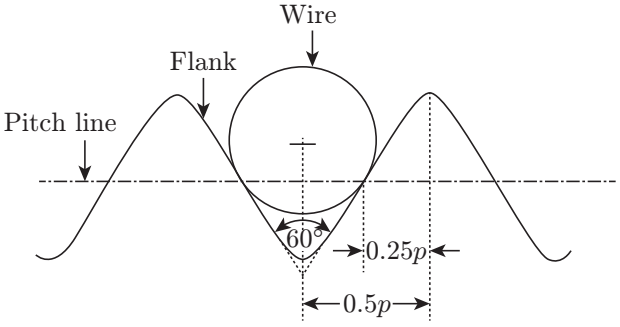
45. In an orthogonal machining operation, the cutting and thrust forces are equal in magnitude. The uncut chip thickness is 0.5 mm and the shear angle is 15°. The orthogonal rake angle of the tool is 0° and the width of cut is 2 mm. The workpiece material is perfectly plastic and its yield shear strength is 500 MPa. The cutting force is _____ N (round off to the nearest integer).

Solution: The friction angle would be $\tau = 45^\circ$ for the normal force and friction force to be equal.
The cutting force is given by,

$$F_C = \tau_s\left(\frac{bt_1}{\sin\theta}\right)\frac{\cos(\tau-\alpha)}{\cos(\theta+\tau-\alpha_0)}$$
$$= 500\times\frac{2\times 0.5}{\sin 15^\circ}\frac{\cos(45-0^\circ)}{\cos(15^\circ+45^\circ-0^\circ)} = 2732\,\text{N}$$

Ans. (2732)

46. The best size wire is fitted in a groove of a metric screw such that the wire touches the flanks of the thread on the pitch line as shown in the figure. The pitch (p) and included angle of the thread are 4 mm and 60°, respectively. The diameter of the best size wire is _____ mm (round off to 2 decimal places).



Solution: The best diameter of the wire is given by,

$$D = \frac{p}{2}\sec\left(\frac{\alpha}{2}\right) = \frac{4}{2}\sec\left(\frac{60}{2}\right)$$
$$= 2\sec 30^\circ = 2.309 \approx 2.31\,\text{mm}$$

Ans. (2.31)

47. In a direct current arc welding process, the power source has an open circuit voltage of 100 V and short circuit current of 1000 A. Assume a linear relationship between voltage and current. The arc voltage (V) varies with the arc length (l) as $V = 10 + 5l$, where V is in volts and l is in mm. The maximum available arc power during the process is _____ kVA (in integer).

Solution: The voltage and current for linear power source is,

$$V_t = \frac{V_0}{2} = \frac{100}{2} = 50\,\text{V}$$
$$I_t = \frac{I_0}{2} = \frac{1000}{2} = 500\,\text{V}$$

The maximum arc power is

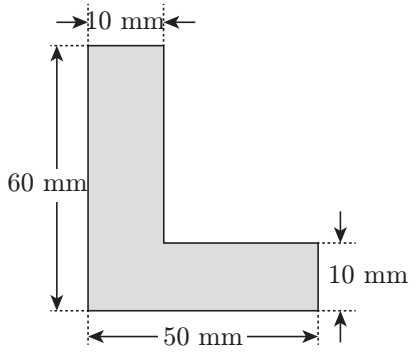
$$P = V_tI_t$$
$$= 50\times 500 = 25\,\text{kW}$$

Ans. (25)

48. A cylindrical billet of 100 mm diameter and 100 mm length is extruded by a direct extrusion process to produce a bar of L-section. The cross sectional dimensions of this L-section bar are shown in the figure. The total extrusion pressure (p) in MPa for the above process is related to extrusion ratio (r) as

$$p = K_S\sigma_m\left[0.8+1.5\ln(r)+\frac{2l}{d_0}\right],$$

where σ_m is the mean flow strength of the billet material in MPa, l is the portion of the billet length remaining to be extruded in mm, d_0 is the initial diameter of the billet in mm, and K_S is the die shape factor. If the mean flow strength of the billet material is 50 MPa and the die shape factor is 1.05, then the maximum force required at the start of extrusion is _____ kN (round off to one decimal place).



Solution: The extrusion ratio is given by,

$$r = \frac{A_{\text{initial}}}{A_{\text{final}}} = \frac{\frac{\pi}{4}d^2}{(50\times10)+(50\times10)}$$

$$= \frac{\frac{\pi}{4}(100)^2}{(50\times10)+(50\times10)} = 7.85398$$

The total extrusion pressure is given by,

$$P = k_s\sigma_m\left[0.8+1.5\ln(r)+\frac{2l}{d_0}\right]$$

$$= 1.05\times50\left[0.8+1.5\ln(7.85397)+\frac{2\times100}{100}\right]$$

$$= 309.296 \text{ MPa}$$

The force is given by,

$$F = PA_{\text{initlal}}$$

$$= 309.296\,\text{MPa}\times\frac{\pi}{4}\times(100\,\text{mm})^2$$

$$= 2429.278 \approx 2429.3\,\text{kN}$$

Ans. (2429.3)

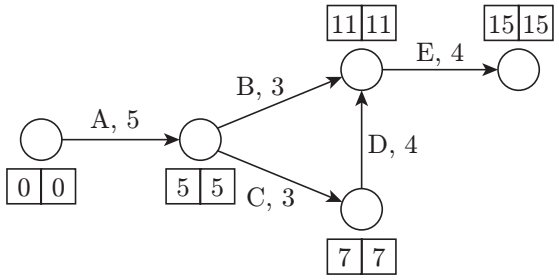
49. A project consists of five activities (A, B, C, D and E). The duration of each activity follows beta distribution. The three time estimates (in weeks) of each activity and immediate predecessor(s) are listed in the table. The expected time of the project completion is _____ weeks (in integer).

Activity	**Time estimates (in weeks)**			**Immediate predecessor(s)**
	Optimistic time	**Most likely time**	**Pessimistic time**	
A	4	5	6	None
B	1	3	5	A
C	1	2	3	A
D	2	4	6	C
E	3	4	5	B, D

Solution: The estimated time for each activity is shown below,

	$t_e = \frac{t_o + 4t_m + t_p}{6}$
A	5
B	3
C	2
D	2
E	4

The network is shown by the following figure,



Thus, the expected time for the project completion is 15 weeks.

Ans. (15)

50. A rigid tank of volume of 8 m^3 is being filled up with air from a pipeline connected through a valve. Initially the valve is closed and the tank is assumed to be completely evacuated. The air pressure and temperature inside the pipeline are maintained at 600 kPa and 306 K, respectively. The filling of the tank begins by opening the valve and the process ends when the tank pressure is equal to the pipeline pressure. During the filling process, heat loss to the surrounding is 1000 kJ. The specific heats of air at constant pressure and at constant volume are 1.005 kJ/kg.K and 0.718 kJ/kg.K, respectively. Neglect changes in kinetic energy and potential energy. The final temperature of the tank after the completion of the filling process is _____ K (round off to the nearest integer).

Solution: Applying the conservation of energy, we get

$$m_{\text{exit}}u_{\text{exit}} = m_{\text{inlet}}h_{\text{inlet}} + Q$$

Given that, $m_{\text{inlet}} = m_{\text{exit}} = m$.

$$mC_VT_{\text{exit}} = mC_VT_{\text{inlet}} + Q$$

$$\Rightarrow \frac{P_{\text{exit}}V}{RT_2}C_VT_{\text{exit}} = \frac{P_{\text{exit}}V}{RT_2}C_VT_{\text{inlet}} + Q$$

$$\Rightarrow \frac{600\times8}{0.286}\times0.718 = \left[\frac{600\times8}{0.286T_2}\times1.005\times306\right] + (-1000)$$

$$\Rightarrow T_2 = 395.388\,\text{K}$$

Ans. (395)

51. At steady state, 500 kg/s of steam enters a turbine with specific enthalpy equal to 3500 kJ/kg and specific entropy equal to 6.5 kJ/(kg·K). It expands reversibly in the turbine to the condenser pressure. Heat loss occurs reversibly in the turbine at a temperature of 500 K. If the exit specific enthalpy and specific entropy are 2500 kJ/kg and 6.3 kJ/(kg·K), respectively, the work output from the turbine is _____ MW (in integer).

Solution: The entropy at inlet and exit would be equal, so

$$S_{\text{inlet}} = S_{\text{exit}}$$

$$\Rightarrow \dot{m}s_1 = \dot{m}s_2 + \frac{\dot{Q}}{T}$$

$$\Rightarrow 500 \times 6.5 = (500 \times 6.30) + \frac{\dot{Q}}{500}$$

$$\Rightarrow \dot{Q} = 50\,\text{MW}$$

The work done is given by,

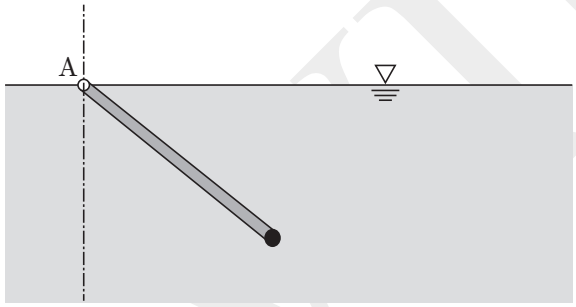
$$\dot{W} = \dot{m}(h_1 - h_2) - \dot{Q}$$

$$= 500 \times (3500 - 2500) \times \frac{1}{1000} - 50$$

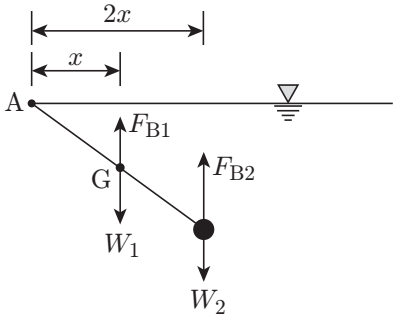
$$= 450\ \text{MW}$$

Ans. (450)

52. A uniform wooden rod (specific gravity = 0.6, diameter = 4 cm and length = 8 m) is immersed in the water and is hinged without friction at point A on the waterline as shown in the figure. A solid spherical ball made of lead (specific gravity = 11.4) is attached to the free end of the rod to keep the assembly in static equilibrium inside the water. For simplicity, assume that the radius of the ball is much smaller than the length of the rod. Assume density of water = 103 kg/m^3 and $\pi = 3.14$. Radius of the ball is _____ cm (round off to 2 decimal places).



Solution: The FBD of the given system is shown in the following figure,



The buoyance force F_{B1} is given by,

$$F_{\text{B1}} = \rho\pi r_1^2 lg$$

The buoyance force F_{B2} is given by,

$$F_{\text{B2}} = \rho\frac{4}{3}\pi r_2^3 g$$

The weight W_1 is,

$$W_1 = \rho_1 g\pi r_1^2 l$$

The weight W_2 is,

$$W_2 = \rho_2\frac{4}{3}\pi r_2^3 g$$

Specific density of 1 and 2 is,

$$S_1 = \frac{\rho_1}{\rho}$$

$$S_2 = \frac{\rho_2}{\rho}$$

The sum of moment at A will be zero, so

$$W_1 x + W_2(2x) = F_{\text{B1}}x + F_{\text{B2}}(2x)$$

$$\Rightarrow \rho_1\pi r_1^2 lg + \rho_2\frac{4}{3}\pi r_2^3 g = \rho g\left[\pi r_1^2 lg + \frac{4}{3}\pi r_2^3 g\right]$$

$$\Rightarrow \pi S_1 l r_1^2 + 2S_2 g\frac{4}{3}\pi r_2^3 = \pi r_1^2 l + \frac{4}{3}\pi r_2^3 \times 2$$

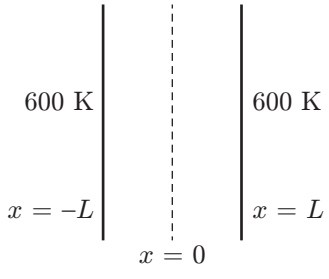
$$\Rightarrow (2S_2 - 2)\frac{4}{3}r_2^3 = (-S_1 + 1)r_1^2 l$$

$$\Rightarrow [(2 \times 11.4) - 2]\frac{4}{3}r_2^3 = (1 - 0.6) \times 8 \times (0.02)^2$$

$$\Rightarrow r_2 = 3.58\,\text{cm}$$

Ans. (3.58)

53. Consider steady state, one-dimensional heat conduction in an infinite slab of thickness $2L$ ($L = 1$ m) as shown in the figure. The conductivity (k) of the material varies with temperature as $k = CT$, where T is the temperature in K, and C is a constant equal to 2 W/(m·K^2). There is a uniform heat generation of 1280 kW/m^3 in the slab. If both faces of the slab are maintained at 600 K, then the temperature at $x = 0$ is _____ K (in integer).



Solution: The equation of heat transfer is given by,

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = -\dot{q}$$

On integrating, we get

$$k\frac{dT}{dx} = -\dot{q}x + C_1$$
$$\Rightarrow 2T\frac{dT}{dx} = -\dot{q}x + C_1$$
$$\Rightarrow TdT = \left(\frac{-\dot{q}x + C_1}{2}\right)dx$$

Again integrating, we get

$$\frac{T^2}{2} = \frac{1}{2}\left[\frac{\dot{q}x^2 + C_1}{2}\right] + C_2$$

Substituting $T = 600$ at $x = 1$, also $T = 600$ at $x = -1$. Then,

$$C_1 = 0$$
$$C_2 = 10^6$$

The equation of temperature is,

$$T^2 = -\frac{\dot{q}x^2}{2} + C_2$$

At $x = 0$, we get

$$T^2\Big|_{x=0} = -\frac{\dot{q}(0)^2}{2} + 10^6$$
$$\Rightarrow T\big|_{x=0} = 1000\,\text{K}$$

Ans. (1000)

54. Saturated vapor at 200°C condenses to saturated liquid at the rate of 150 kg/s on the shell side of a heat exchanger (enthalpy of condensation $h_{fg} =$ 2400 kJ/kg). A fluid with $c_p = 4$ kJ/(kg·K) enters at 100°C on the tube side. If the effectiveness of the heat exchanger is 0.9, then the mass flow rate of the fluid in the tube side is _____ kg/s (in integer).

Solution: The effectiveness of the hear exchange is given by,

$$\varepsilon = \frac{T_{He} - T_{Ci}}{T_{Hi} - T_{Ci}}$$
$$\Rightarrow 0.9 = \frac{T_{Ce} - 100}{200 - 100}$$
$$\Rightarrow T_{Ce} = 190\,^\circ\text{C}$$

Applying heat balance equation among the fluids, we get

$$\dot{m}_{\text{condensing}} h_{fg} = m_C C_{PC}(T_{\text{ice}} - T_{Ci})$$
$$\Rightarrow 150 \times 2400 = \dot{m}_C \times 4 \times (190 - 100)$$
$$\Rightarrow \dot{m}_C = 1000\,\text{kg/s}$$

Ans. (1000)

55. Consider a hydrodynamically and thermally fully-developed, steady fluid flow of 1 kg/s in a uniformly heated pipe with diameter of 0.1 m and length of 40 m. A constant heat flux of magnitude 15000 W/m^2 is imposed on the outer surface of the pipe. The bulk-mean temperature of the fluid at the entrance to the pipe is 200 °C. The Reynolds number (Re) of the flow is 85000, and the Prandtl number (Pr) of the fluid is 5. The thermal conductivity and the specific heat of the fluid are 0.08 W/(m·K) and 2600 J/(kg·K), respectively. The correlation $\text{Nu} = 0.023\,\text{Re}^{0.8}\text{Pr}^{0.4}$ is applicable, where the Nusselt Number (Nu) is defined on the basis of the pipe diameter. The pipe surface temperature at the exit is _____ °C (round off to the nearest integer).

Solution: Applying the energy balance equation, we get

$$q'' \times \pi DL = \dot{m}c_p(T_{b.e} - T_{b,i})$$
$$\Rightarrow 1500 \times \pi \times 0.1 \times 40 = 1 \times 2600(T_{b.e} - 200)$$
$$\Rightarrow T_{b,e} = 272.498\,^\circ\text{C}$$

Nusselt number is given by

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4}$$
$$\Rightarrow \frac{h \times 0.1}{0.08} = 0.023 \times (85000)^{0.8} \times (5)^{0.4}$$
$$\Rightarrow h = 307.56\,\text{W/(m}\cdot\text{K)}$$

Applying Newton's law of cooling,

$$q'' = h(T_{w,e} - T_{b,e})$$
$$\Rightarrow 15000 = 307.56(T_{w,e} - 272.498)$$
$$\Rightarrow T_{w.e} = 321.26 \approx 321\,^\circ\text{C}$$

Ans. (321)

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