JEE Main 2019 Paper 1
January 9, Shift 1
Section: Physics

1. Distance of convex lens from source of light, $u = -10$ cm
Distance of convex lens from screen, $v = 10$ cm
Refractive index, $\mu = 1.5$
Thickness of glass, $t = 1.5$ cm
As we know that, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \frac{1}{f} = \frac{1}{10} - \frac{1}{-10} = \frac{1}{10}(1+1) = \frac{2}{10}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{5}$$

Therefore, $f = 5$ cm

Now, the slab is shifted in the direction of incident ray $= t\left(1 - \frac{1}{\mu}\right)$

$$= 1.5\left(1 - \frac{1}{1.5}\right) = 1.5\left(1 - \frac{2}{3}\right) = 0.5$$

Distance of convex lens from source of light, $u = -(10 - 0.5) = -9.5$ cm
Again, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \frac{1}{5} = \frac{1}{v} - \frac{1}{-9.5} \Rightarrow \frac{1}{v} = \frac{1}{5} - \frac{1}{9.5}$$

$$\Rightarrow \frac{1}{v} = \frac{9}{95}$$

$$v = \frac{95}{9} \Rightarrow v = 10.55 \text{ cm}$$

Hence, the screen is shifted 0.55 cm away from the lens.
Answer: (4).

2. This resistor, read from left to right, has the colored bands of RED, VIOLET, ORANGE and SILVER.
The resistance is,
R = (1st Digit × 10 + 2nd Digit) × Multiplier
R = (RED × 10 + VIOLET) × ORANGE = (2 × 10 + 7) × 1,000
= 27,000 Ω = 27 kΩ
Since, the last band is silver, the tolerance is 10%.
Answer: (2).

3. Cross section of copper wire, $A = 5\text{ mm}^2 = 5 \times 10^{-6}\text{ m}^2$
Current flows in a copper wire, $I = 1.5$ A
Charge, $q = 1.6 \times 10^{-19}$ C
$h = 9 \times 10^{28}\text{ /m}^3$
Now, $I = hAVq$

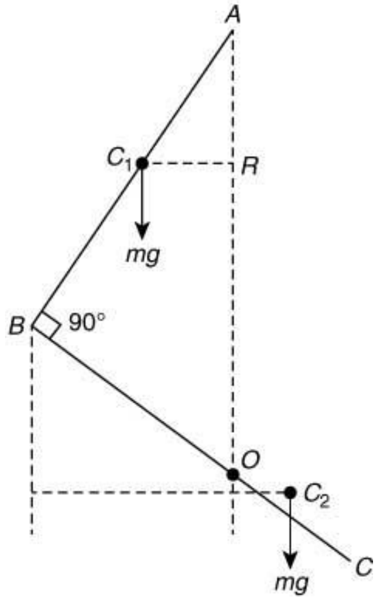
$$\Rightarrow V = \frac{I}{neA}$$

$$= \frac{1.5}{9\times10^{28} \times 1.6\times10^{-19} \times 5\times10^{-6}} = 0.02 \text{ m/s}$$

Answer: (1).

4. Let m be the mass of the rod and L be length of segment AB or BC.

So, $AB = BC = L$



Balancing torque about suspended point

$$mg(C_1R) = mg(C_2O) \qquad [\tau = \vec{F} \times \vec{r}]$$

$$\Rightarrow mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$

$$\Rightarrow \frac{L}{2} mg\sin\theta = \frac{L}{2} mg\cos\theta - L\,mg\sin\theta$$

$$\Rightarrow \frac{L}{2} mg\sin\theta + L\,mg\sin\theta = \frac{L}{2} mg\cos\theta$$

$$\Rightarrow \frac{3}{2} mg\ L\sin\theta = \frac{L}{2} mg\cos\theta$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{3} \Rightarrow \tan\theta = \frac{1}{3}$$

Answer: (4).

5. Equation for velocity of the particle is

$$\vec{v} = k(y\hat{i} + x\hat{j})$$

$$v_x\hat{i} + v_y\hat{j} = k(y\hat{i} + x\hat{j})$$

Thus,

$$v_x = ky \Rightarrow \frac{dx}{dt} = ky \qquad (1)$$

$$v_y = kx \Rightarrow \frac{dy}{dt} = kx \qquad (2)$$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{d\lambda}{dt}} = \frac{kx}{ky}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

$$\Rightarrow y\,dy = x\,dx$$

Integrating both the sides, we get

$$y^2 = x^2 + \text{constant}$$

Answer: (3).

6. rms velocity is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{2NM}} \qquad (1)$$

Since, both the gases are at same temperature,

$$v_{\text{rms}} = \sqrt{\frac{1}{M}}$$

where M is molar mass of the gas.

Therefore, $\frac{v_{\text{rms}}(\text{Helium})}{v_{\text{rms}}(\text{Argor})} = \sqrt{\frac{M_{(\text{Argon})}}{M_{(\text{Helium})}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$

Answer: (1).

7. Let B be the magnetic field at point O due to are of radius r.

$$\vec{B} = \frac{\mu_0 I}{4\pi r}\theta \qquad (1)$$

Given r_1 = 3 cm = 3 × 10^{-2} m, r_2 = 5 cm = 5 × 10^{-2} cm,

$\theta = 45° = \frac{\pi}{4}$ rad, I = 10 A

Therefore,

$$\vec{B} = \frac{\mu_0 I}{4\pi}\theta\left[\frac{1}{r_1} - \frac{1}{r_2}\right]\hat{k}$$

Since, both arcs have opposite direction of current

$$\vec{B} = \frac{4\pi \times 10^{-7}}{16} \times 10\left[\frac{1}{3\times 10^{-2}} - \frac{1}{5\times 10^{-2}}\right]\hat{k} = \frac{\pi}{3}\times 10^{-5}\,\text{T}$$

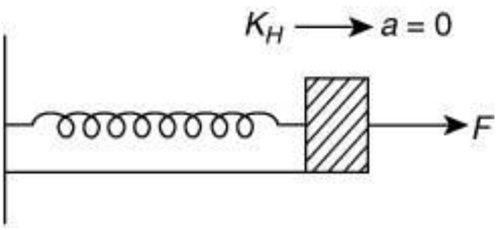
$$|\vec{B}| \approx 1.0 \times 10^{-5}\ \text{T}$$

Answer: (4).

8. The maximum speed of block is at mean position

$$F = kx \qquad \text{(equilibrium)}$$

$$\Rightarrow x = \frac{F}{k}$$



According to work energy theorem,

Work done by external force + work done by spring = Change in kinetic energy

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv_{\max}^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv_{\max}^2$$

$$\Rightarrow v_{\max}^2 = \frac{F^2}{mk}$$

$$\Rightarrow v_{\max} = \frac{F}{\sqrt{mk}}$$

Answer: (4).

9. Let the total charge on the ring is Q then, electric field at the axis of a charged ring is given by

$$E = \frac{kQA}{(R^2 + h^2)^{3/2}}$$

For maximum electric field, $\frac{dE}{dh} = 0$

$$\Rightarrow \frac{1}{(R^2 + h^2)^{3/2}} + \frac{\left(-\frac{3}{2}\right) \times 2h^2}{(h^2 + R^2)^{5/2}} = 0$$

$$\Rightarrow \frac{1}{(R^2 + h^2)^{3/2}} = \frac{3h^2}{(h^2 + r^2)^{5/2}}$$

$$\Rightarrow 3h^2 = h^2 + r^2 \;\Rightarrow\; h = \frac{R}{\sqrt{2}}$$

Answer: (2).

10. The ratio of the maximum intensity to the minimum intensity

$$\frac{I_{\max}}{I_{\min}} = \frac{16}{1}$$

We know that

$$\frac{I_{\max}}{I_{\min}} = \frac{A_{\max}^2}{A_{\min}^2}$$

where, $A_{\max}$ and $A_{\min}$ are the maximum and minimum amplitudes of the wave.

$$\Rightarrow \frac{A_{\max}}{A_{\min}} = \sqrt{\frac{16}{1}} = \frac{4}{1}$$

$$\Rightarrow \frac{A_1 + A_2}{A_1 - A_2} = \frac{4}{1}$$

Using componendo and dividendo rule, we have

$$\frac{A_1}{A_2} = \frac{5}{3}$$

Therefore,

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{5}{3}\right)^2$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{25}{9}$$

Thus, the required ratio is 25 : 9.

Answer: (2).

11. For λ_1:

$$\frac{hc}{350} - \phi = \frac{1}{2} m(2v)^2$$

$$\Rightarrow \frac{hc}{350} - \phi = \frac{1}{2} m(4v)^2 \qquad (1)$$

For λ_2:

$$\frac{hc}{540} - \phi = \frac{1}{2} mv^2 \qquad (2)$$

Now, dividing Eq. (1) by Eq. (2), we get

$$\frac{\frac{hc}{350} - \phi}{\frac{hc}{540} - \phi} = 4$$

$$\Rightarrow \frac{hc}{350} - \phi = 4\left[\frac{hc}{540} - \phi\right]$$

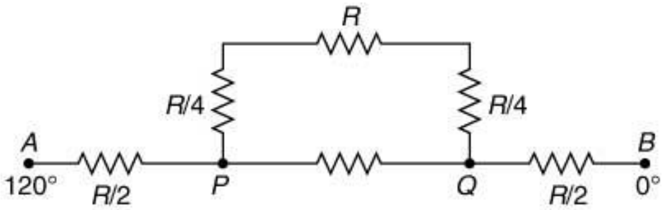
$$\Rightarrow \frac{4hc}{540} - \frac{hc}{350} = 3\phi$$

$$\phi = \frac{1}{3}hc\left[\frac{4}{540} - \frac{1}{350}\right] = \frac{1}{3}\times 1240\left[\frac{4\times 350 - 540}{350\times 540}\right]$$

$$\phi = 1.85\ \text{eV}$$

Answer: (1).

12. Suppose the thermal conductivity is uniform for all rods.



Now, total resistance $= \frac{8R}{5}$

Thus, $i = \frac{\Delta T}{R_T} = \frac{120}{\frac{8R}{5}}$

$$\Rightarrow T_P - T_Q = i \times \frac{3R}{5} = \frac{120\times 5}{81^2}\times\frac{3R}{5} = 45\ °\text{C}$$

Answer: (1).

13. Heat flow in path ACB, ΔQ = 60 J
Work done by system ACB, ΔW = 30 J
So,
$\Delta U_{\text{ACB}} = \Delta Q_{\text{ACB}} - \Delta W_{\text{ACB}}$ = 60 – 30 = 30 J
$\Delta U_{\text{ACB}} = \Delta U_{\text{ADB}}$ = 30 J
Now, work done by path ADB, ΔW = 10 J
$\Rightarrow \Delta U_{\text{ADB}} = \Delta Q_{\text{ADB}} - \Delta W_{\text{ADB}}$
$\Delta Q_{\text{ADB}} = \Delta U_{\text{ADB}} + \Delta W_{\text{ADB}}$ = 30 + 10 = 40 J
Answer: (1).

14. As we know that $i_p > i_c$
where i_p = Angle of Brewster
i_c = Critical angle
And $\sin i_p > \sin i_c$ (1)
According to Brewester's law,

$$\tan i_p = \mu_0 = \frac{\mu_g}{\mu_\omega}$$

$$\Rightarrow \frac{\sin i_p}{\cos i_p} = \frac{1.5}{\mu}$$

$$\Rightarrow \sin i_p = \frac{1.5}{\sqrt{\mu^2 + (1.5)^2}}$$

Putting this value in Eq. (1), we get

$$\sin i_c < \sin i_p$$

$$\Rightarrow \frac{1}{\mu} < \frac{1.5}{\sqrt{\mu^2 + (1.5)^2}}$$

$$\Rightarrow \sqrt{\mu^2 + (1.5)^2} < 1.5\mu$$

Squaring both the sides, we get

$$\Rightarrow \mu^2 + (1.5)^2 < (1.5\mu)^2$$
$$\Rightarrow \mu^2 + 2.25 < 2.25\mu^2$$
$$\Rightarrow \mu^2 > \frac{2.25}{1.25} \Rightarrow \mu > \frac{3}{5}$$
$$\Rightarrow \mu > \sqrt{\frac{9}{5}} \Rightarrow \mu > \frac{3}{\sqrt{5}}$$

Thus, the minimum value of μ is $\frac{3}{\sqrt{5}}$.

Answer: (2).

15. Current in the semiconductor is,

$$I = n_e\rho A v_\text{d} + n_n\rho A v_n$$
$$I = n_e\rho A v_\text{d} \qquad [n_n\rho A v_n = 0]$$

We know that,

$$I = JA$$
$$\Rightarrow JA = n_e e A v_\text{d}$$
$$\Rightarrow J = n_e e v_\text{d}$$
$$\Rightarrow \frac{E}{\rho} = n_e e v_\text{d}$$
$$\Rightarrow \frac{1}{\rho} = n_e e \frac{v_\text{d}}{E} \qquad \left[\frac{1}{\rho} = \sigma, \frac{v_\text{d}}{E} = \mu_e\right]$$

Thus, $\sigma = n_e e \mu_e$

$= 10^{19} \times 1.6 \times 10^{-19} \times 1.6 = 2.56$

Therefore, $\rho = \frac{1}{\sigma} = \frac{1}{2.56} = 0.4\ \Omega$ m

Answer: (3).

16. Let $\vec{B}$ be the magnetic field, at a particular point $\vec{E} = 6.3\hat{j}$ V/m

So, $|\vec{B}| = \frac{|\vec{E}|}{c}$,

where c is the speed of light.

$$|\vec{B}| = \frac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8}\ \text{T}$$

Unit vector

$$\hat{E} \times \hat{B} = \hat{c} \Rightarrow \hat{j} \times \hat{B} = \hat{i}$$
$$\Rightarrow \hat{B} = \frac{\hat{i}}{\hat{j}} \Rightarrow \hat{B} = \hat{k}$$
$$\Rightarrow \vec{B} = |\vec{B}|\hat{B} = 2.1 \times 10^{-8}\hat{k}\ \text{T}$$

Answer: (2).

17. Charge q is placed at center so, it is equilibrium condition for $+Q$ placed at $x = 0$

For equilibrium,

$\vec{F}_\text{a} + \vec{F}_\text{b} = 0$

$$\Rightarrow \frac{KQQ}{d^2} + \frac{KQq}{\left(\frac{d}{2}\right)^2} = 0$$
$$\Rightarrow \frac{kQ^2}{d^2} = -\frac{kQq}{\frac{d^2}{4}}$$

$$\Rightarrow Q = -4q \Rightarrow q = \frac{-Q}{4}$$

Answer: (1).

18. Resistance is given by,

$$R = \frac{\rho l}{A}$$

We know that Al = Volume (constant)

$$\Rightarrow A = \frac{V}{l}$$

$$\Rightarrow R = \frac{\rho l^2}{V}$$

Now, percentage change is

$$\frac{\Delta R}{R}\% = 2 \cdot \frac{\Delta l}{l}\%$$

$$= 2 \times 0.5 = 1\%$$

Answer: (3).

19. Activity is given as,

$A = \lambda N$

for A $\quad \lambda_A N_A = 10 \quad$ (1)

for B $\quad \lambda_B N_B = 20 \quad$ (2)

Dividing Eq. (1) by Eq. (2), we get

$$\frac{\lambda_A N_A}{\lambda_B N_B} = \frac{1}{2}$$

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{1}{2}\frac{N_B}{N_A} = \frac{1}{2} \times \frac{1}{2} \qquad [N_A = 2N_B]$$

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{1}{4}$$

$\Rightarrow \lambda_B = 4\lambda_A$

$(T_{1/2})_A = 4(T_{1/2})_B$

If $T_{1/2} = 5$

$(T_{1/2})_A = 20$ days

$(T_{1/2})_B = 5$ days

Answer: (3).

20. We know that when car is at rest velocity of wave is

$$v = \sqrt{\frac{mg}{\mu}} \qquad (1)$$

When the car has accelerated velocity is

$$v' = \sqrt{\frac{m(g^2 + a^2)^{1/2}}{\mu}} \qquad (2)$$

Dividing Eq. (2) by Eq. (1), we have

$$\frac{v'}{v} = \frac{\sqrt{\frac{m(g^2 + a^2)^{1/2}}{\mu}}}{\sqrt{\frac{mg}{\mu}}}$$

$$\Rightarrow \frac{v'}{v} = \left(\frac{g^2 + a^2}{g^2}\right)^{1/4}$$

$$\Rightarrow \frac{60.5}{60} = \left(\frac{g^2 + a^2}{g^2}\right)^{1/4}$$

$$\Rightarrow \frac{60 + 0.5}{60} = \left(\frac{g^2 + a^2}{g^2}\right)^{1/4}$$

$$\Rightarrow 1 + \frac{0.5}{60} = \left(\frac{g^2 + a^2}{g^2}\right)^{1/4}$$

Using binomial expansion, we have

$$1 + \frac{1}{120} = 1 + \frac{a^2}{4g^2}$$

$$\Rightarrow \frac{1}{120} = \frac{a^2}{4g^2} \Rightarrow a^2 = \frac{4g^2}{120}$$

$$\Rightarrow a^2 = \frac{g^2}{30} \Rightarrow a = \frac{g}{\sqrt{30}}$$

$$\Rightarrow a = \frac{g}{5.47} \Rightarrow a \approx \frac{g}{5}$$

Answer: (2).

21. We have $A = 3.5 \times 10^{-3}$ m^2, $R = 10\ \Omega$

Now, $Q = \frac{\Delta\phi}{R} = \frac{1}{10} \times A \times (B_{10} - B)$

$$= \frac{1}{10} \times 3.5 \times 10^{-3} \times \left(0.4 \sin\frac{\pi}{2} - 0\right)$$

$$= \frac{1}{10} \times 3.5 \times 10^{-3} \times 0.4$$

$= 1.4 \times 10^{-4} = 14$ mC

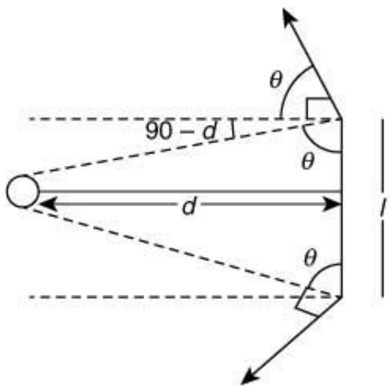
Answer: (1).

22. Let the effective length of magnet be l.

So, magnetic moment is

$M = i_2 \pi a^2 \times n$ (number of turns, $n = 1$)

$= i_2 \pi a^2 = ml$



Suppose i_w is the current flowing through the wire. The force on north pole of magnet = mB and the force on south pole of magnet = mB. Then, resultant force is

$$F = 2mB\cos\theta$$

$$= 2mB \frac{\frac{l}{2}}{\sqrt{d^2 + \frac{\rho^2}{4}}} = 2mB \frac{l}{2\sqrt{d^2 + \frac{l^2}{4}}}$$

$$= 2 \times i_2 \pi a^2 \times \frac{\mu_0 I_w}{2\pi \sqrt{\left(d^2 + \frac{l^2}{4}\right)^2}} = \frac{i_2 a^2 \times \mu_0 I_w}{2\left(\sqrt{d^2 + \frac{l^2}{4}}\right)^2}$$

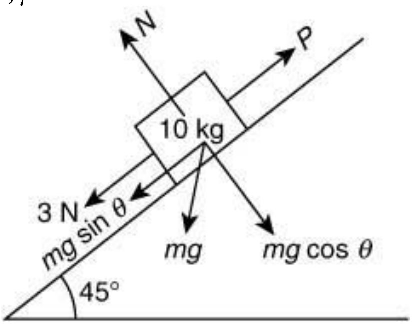
$$F \propto \frac{a^2}{\left(d^2 + \frac{l^2}{4}\right)}$$

$l << d$, so l can be neglected.

Therefore, $F \propto \frac{a^2}{d^2}$

Answer: (4).

23. The coefficient of static friction, $\mu = 0.6$



The resultant force is

$$P + \mu mg\cos\theta = 3 + mg\sin\theta$$

$$\Rightarrow P + \frac{\mu mg}{\sqrt{2}} = 3 + \frac{mg}{\sqrt{2}}$$

Given $m = 10$ kg, $\mu = 0.6$ and $g = 10$ m/s^2

$$\Rightarrow P + 0.6 \times \frac{100}{\sqrt{2}} = 3 + \frac{100}{\sqrt{2}}$$

$$\Rightarrow P + 0.6 \times 50\sqrt{2} = 3 + 50\sqrt{2}$$

$$\Rightarrow P + 42.42 = 73.71$$

$$\Rightarrow P = 73.71 - 42.42 = 31.29$$

Therefore, $P \approx 32$ N

Answer: (1).

24. Consider a small element dx at a distance x.

Thus,

$$\frac{y}{x} = \frac{d}{a}$$

$$\Rightarrow y = \frac{d}{a}x$$

$$\Rightarrow dy = \frac{d}{a}(dx)$$

$$\Rightarrow \frac{1}{dC} = \frac{y}{KEadx} + \frac{d-y}{\varepsilon_0 adx}$$

$$\Rightarrow \frac{1}{dC} = \frac{1}{\varepsilon_0 adr}\left(\frac{V}{K} + d - y\right)$$

$$\Rightarrow dC = \frac{\varepsilon_0 a\, dx}{\left(\frac{y}{K} + d - y\right)}$$

Integrating both the sides, we get

$$\int dC = \int \frac{\varepsilon_0 a\, dx}{\frac{y}{K} + d - y}$$

$$\Rightarrow C = \varepsilon_0 a \cdot \frac{a}{d} \int_0^{\phi} \frac{dy}{d + y\left(\frac{1}{K} - 1\right)}$$

$$\Rightarrow C = \frac{\varepsilon_0 a^2}{\left(\frac{1}{K} - 1\right) d} \left[\ln\left(d + y\left(\frac{1}{K} - 1\right)\right)\right]_0^d$$

$$= \frac{K\varepsilon_0 a^2}{(1-K)d} \ln\left[\frac{d + d\left(\frac{1}{K} - 1\right)}{d}\right]$$

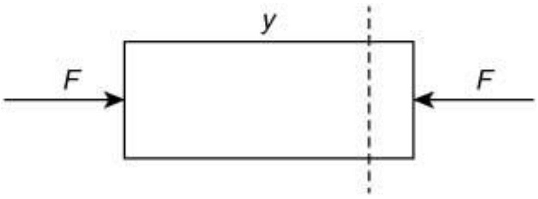
$$= \frac{K\varepsilon_0 a^2}{(1-K)d} \ln\left(\frac{1}{K}\right) = \frac{K\varepsilon_0 a^2 \ln K}{(K-1)d}$$

Answer: (2).

25. Young modulus $Y = \frac{\text{Stress}}{\text{Strain}}$

$$\Rightarrow Y = \frac{F/A}{\frac{\Delta l}{l}}$$

$$\Rightarrow Y = \frac{F}{A} \times \frac{l}{\Delta l} \qquad (1)$$



The force acts on both the end of rod. So, the length will decrease but temperature will increase.
Thus, $\Delta l = l\alpha\Delta T$

$$\Rightarrow \frac{\Delta l}{l} = \alpha\Delta T$$

Put this value in Eq. (1), we get

$$Y = \frac{F}{A\alpha\Delta T}$$

Answer: (1).

26. Bar magnet requires a magnetic intensity μ.
The current through the solenoid is,

$$i = \frac{\mu}{n}$$

$$\Rightarrow \mu = i \times n \qquad (1)$$

Let n be the turns per unit length.

$$n = \frac{N}{l}$$

Put this value in Eq. (1), we get

$$\mu = \frac{N}{l} i \Rightarrow \mu = \frac{100}{0.2} \times 5.2$$

Coercivity, μ = 2600 A/m

Answer: (2).

27. From the linear momentum, we have

$$mv = (2m + M)v_f \Rightarrow v_f = \frac{mv}{2m+M}$$

Initial kinetic energy $K_i = \frac{1}{2}\frac{P^2}{m}$

Final kinetic energy $K_f = \frac{1}{(2m+M)}P^2$

Now, according to the question,

$$K_f = \frac{1}{6}K_i$$

$$\Rightarrow \frac{1}{2}\frac{P^2}{(2m+M)} = \frac{1}{6}\cdot\frac{1}{2}\frac{P^2}{m}$$

$$\Rightarrow \frac{1}{(2m+M)} = \frac{1}{6m}$$

$$\Rightarrow 6m = 2m + M \Rightarrow 4m = M$$

$$\Rightarrow \frac{M}{m} = 4$$

Answer: (3).

28. Let V be the voltage at C.
According to the Kirchhoff's current law, we have

$$i_1 + i_2 - i = 0$$

$$\Rightarrow \frac{20-V}{2} + \frac{10-V}{2} - \frac{0-V}{2} = 0 \Rightarrow V = 10$$

Current, $i = \frac{V}{R} = \frac{10}{2} = 5$ A

Answer: (2).

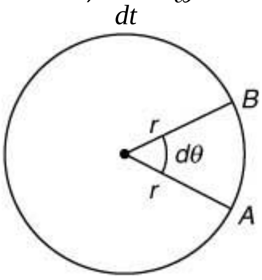
29. Consider a planet moving in an elliptical orbit from point A to B and traces small area dA at the focus in time dt. Let the angle traced by radius vector be $d\theta$. The area of sector is given by

$$dA = \frac{1}{2}r^2 d\theta$$

$$\Rightarrow \frac{dA}{dt} = \frac{1}{2}\frac{r^2 d\theta}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{1}{2}\omega r^2 \qquad (1)$$

$$\Rightarrow \frac{d\theta}{dt} = \omega$$



Now the instantaneous angular momentum is given by

$$L = mvr$$

$$= m((r\omega)r) \qquad (v = r\omega)$$

$$\Rightarrow L = mr^2\omega$$

$$\Rightarrow r^2\omega = \frac{L}{m}$$

Substitute the value in Eq. (1), we have

$$\Rightarrow \frac{dA}{dt} = \frac{L}{2m}$$

Answer: (3).

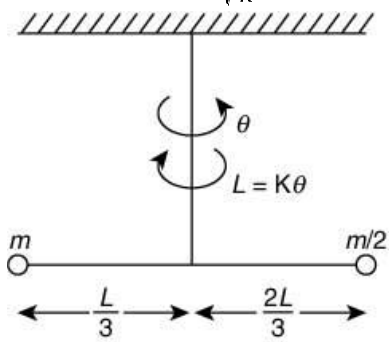
30. If the rod is rotating through an angle θ then

$$\tau = k\theta = I\ddot{\theta}$$

where $\ddot{\theta} =$ restoring angular acceleration

Time period of S.H.M is

$$T = 2\pi\sqrt{\frac{T}{k}}$$



$\Omega = \omega\theta$ (average velocity)

Thus,

$$\Rightarrow \Omega = \sqrt{\frac{k}{I}} \Rightarrow \Omega^2 = \frac{k}{I}$$

Velocity of rotation is

$$v_{\max} = \frac{2}{3} l\theta_0\,\Omega$$

Tension towards the centre is

$$T = \frac{mv^2}{r}$$

$$\Rightarrow T = \frac{\frac{1}{2}mv_{\max}^2}{r}$$

$$\Rightarrow T = \frac{\frac{m}{2}\times\frac{4}{9}l^2\theta_0^2\Omega^2}{\frac{2l}{3}}$$

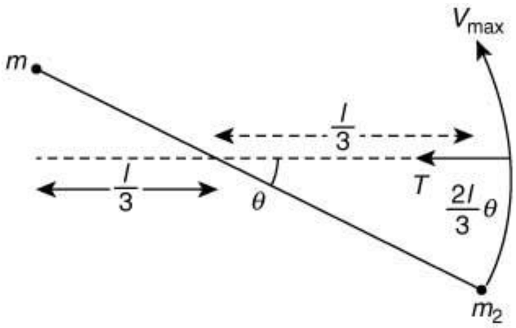
$$\Rightarrow T = \frac{m}{2}\times\frac{2}{3}\theta_0^2\Omega^2 l$$

$$T = \frac{m}{3}\theta_0^2\Omega^2 l \Rightarrow T = \frac{m}{3}\theta_0^2\frac{k}{I}l \qquad (1)$$

Moment of inertia is given by

$$I = \mu l^2 = \frac{\frac{m^2}{2}}{\frac{3m}{2}}l^2$$

$$\Rightarrow I = \frac{m}{3}l^2$$



Put these values in Eq. (1), we get

$$T = m \times \frac{3k}{ml^2}\theta_0^2 \frac{l}{3} \qquad \left[r = \frac{l}{3}\right]$$

$$T = \frac{k\theta_0^2}{l}$$

Answer: (3).

Section: Chemistry

31. Considering the spectrochemical series, the Δ_o follows the order $[Cr(H_2O)_6]Cl_3 < [Cr(NH_3)_6]Cl_3$ as H_2O is a weak field ligand compared to NH_3. This order is nothing to do with the energies of violet and yellow light.
Answer: (1).

32. The acidity of the acids depends upon stability of the conjugate base so formed. In given case, more electronegative group at α–position stabilizes the conjugates base at more extent. Hence, the correct order is $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$.
Answer: (4).

33. DIBAL-H is a reducing reagent used for selective reduction of substituted nitriles to the corresponding aldehydes.

$$R-C\equiv N \xrightarrow{AlH(iBu)_2} RCH=N-Al(iBu)_2 \xrightarrow{H_2O} RCHO$$

Answer: (3).

34. The spin only magnetic moment is given by $\mu_{spin} = \sqrt{n(n+2)}$
The maximum no. of electrons a transition element can have in its *d*–orbital is 5 (*e.g.*, Mn^{2+}). So, the maximum spin only moment is $\mu = \sqrt{5(5+2)} = 5.92$ BM .
Answer: (1).

35. Assuming ideal gas behaviour of gas A.
We have $$PV = (n_1 + n_2)RT \qquad (1)$$
Substituting the given values in Eq. (1), we get

$$200 \times 10 = (0.5 + x)1000 \times R$$

$$2 = (0.5 + x)R$$

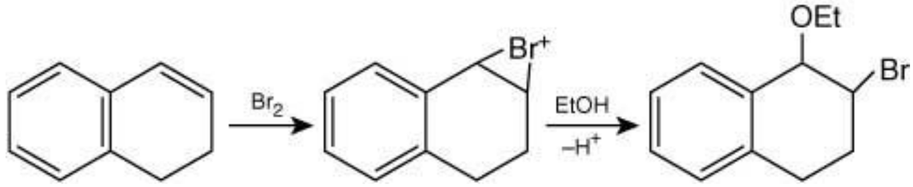
$$x = \frac{4-R}{2R}$$

Answer: (4).

36. Quartz produces electric current when placed under mechanical stress.
Answer: (3).

37. In general, silicones have high thermal stability and high dielectric constant.
Answer: (4).

38. This reaction will proceed with the formation of cyclic bromonium ion followed by stability of carbocation. The reaction involved is



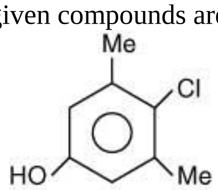
Answer: (1).

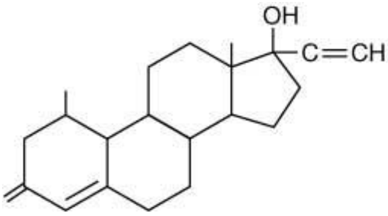
39. Electronegativity is the tendency of an atom to attract a shared pair of electrons towards itself. Electron gain enthalpy is the amount of energy released when electron is added to an isolated gaseous atom. Electronegativity decreases, atomic size increases and electron gain enthalpy becomes less negative as we move down a group in periodic table.
Answer: (3).

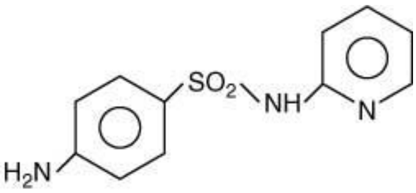
40. Molality is defined as the number of moles of solute dissolved per kg of solvent. So, for Na^+ as solute and water as solvent, we have $\text{Molality} = \frac{(92/23)\,\text{mol of Na}^+\text{ ions}}{1\text{ kg of water}} = 4\text{ mol kg}^{-1}$

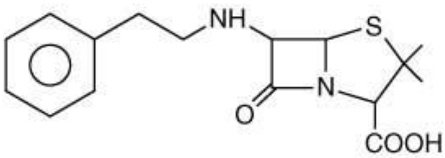
Answer: (2).

41. The structures of the given compounds are as follows

(A) Chloroxylenol 

(B) Norethindrone 

(C) Sulphapyridine 



(D) Penicillin

So, the correct match is A $\rightarrow$ R; B $\rightarrow$ S; C $\rightarrow$ P; D $\rightarrow$ Q.
Chloroxylenol will give test for phenolic –OH group. (Ferric chloride test)
Norethindrone will give test for the unsaturation, that is, carbon-carbon triple bond. (Baeyer's test)
Sulphapyridine will give test for aromatic primary amine, that is, $-NH_2$ group. (Carbylamine test)
Penicillin will give test for carboxylic acid, that is, –COOH group. (Sodium carbonate test)
Answer: (3).

42. The permissible limits for the presence of metal ions in drinking water (in ppm) are Fe = 0.2; Mn = 0.05; Cu = 3.0; Zn = 0.2.
Answer: (2).

43. The anodic half-cell equations are

$$PbSO_4 + 2OH^- \rightarrow PbO_2 + H_2SO_4 + 2e^-$$

$$PbSO_4 + 2e^- + 2H^+ \rightarrow Pb(s) + H_2SO_4$$

2 F of electricity will electrolyze 1 mol of $PbSO_4$ or 303 g.

Therefore, 0.05 F of electricity will electrolyze $\frac{303\times0.05}{2}=7.6$ g of $PbSO_4$.

Answer: (3).

44. According to Henry's law $p_{gas}=K_H\times x_{gas}$. So, as K_H increases, the solubility of the gas will decrease in solution.

Answer: (1).

45. For reaction $2A+B\rightarrow \text{Products}$, we have the following rate law equations

$$R_A=k[A]^x[B]^y\Rightarrow 6.93\times10^{-3}=k[0.1]^x[0.2]^y \quad (1)$$

$$R_B=k[A]^x[B]^y\Rightarrow 6.93\times10^{-3}=k[0.1]^x[0.25]^y \quad (2)$$

$$R_C=k[A]^x[B]^y\Rightarrow 13.86\times10^{-3}=k[0.2]^x[0.3]^y \quad (3)$$

Dividing Eq. (1) by Eq. (2), we get

$$\frac{6.93\times10^{-3}}{6.93\times10^{-3}}=\frac{k[0.1]^x[0.2]^y}{k[0.1]^x[0.25]^y}\Rightarrow 1=\left(\frac{4}{5}\right)^y\Rightarrow \log1=y\log\left(\frac{4}{5}\right)\Rightarrow y=0 \text{ as } \log1=0$$

Substituting y = 0 in Eq. (2) and Eq. (3), and dividing Eq. (2) by Eq. (3), we get

$$\frac{6.93\times10^{-3}}{13.86\times10^{-3}}=\frac{k[0.1]^x}{k[0.2]^x}\Rightarrow\frac{1}{2}=\left(\frac{1}{2}\right)^x\Rightarrow x=1$$

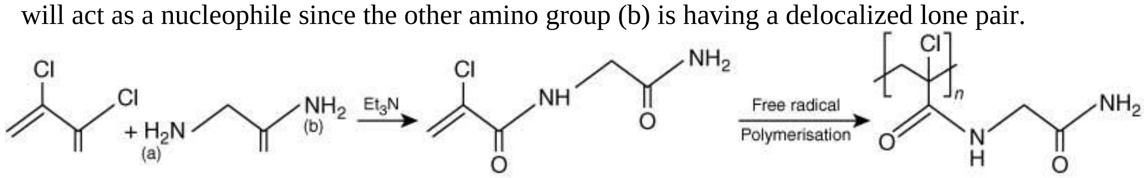
Substituting x = 1 and y = 0 in Eq. (1), we get

$$6.93\times10^{-3}=k[0.1]\Rightarrow k=6.93\times10^{-2}\ \text{min}^{-1}$$

$$t_{1/2}=\frac{0.693}{k}=\frac{0.693}{6.93\times10^{-2}}=10\ \text{min}$$

Answer: (2).

46. Et_3N is a strong base and elimination will take place, where Cl will leave, NH_2 group (a) attached to CH_2 will act as a nucleophile since the other amino group (b) is having a delocalized lone pair.



Answer: (4).

47. Among alkaline earth metals, Ba^{2+} has large ionic radius, therefore, less number of water molecules will surround it, thus, it does not get crystallized.

Answer: (4).

48. Number of millimoles of H_2SO_4 = 4 mmol (as it is dibasic), and no. of moles of NH_4OH = 6 mmol. We have the equation:

$$\underset{\substack{4\text{ mmol}\\0}}{H_2SO_4}+\underset{\substack{6\text{ mmol}\\2}}{2NH_4OH}\rightarrow\underset{\substack{0\\4}}{(NH_4)_2SO_4}+\underset{\substack{0\\4}}{2H_2O}$$

Since the solution is basic, we have according to Henderson-Hasselbalch equation,

$$pOH=pK_b+\log\frac{NH_4^+}{NH_4OH}$$

$$=4.7+\log\frac{4}{2}=4.7+0.3=5$$

Now, pH of the solution is $pH=14-pOH=14-5=9$

Answer: (2).

49. According to Freundlich isotherm,

$$\frac{x}{m}=kp^{1/n}$$

$$\log\left(\frac{x}{m}\right)=\log k+\frac{1}{n}\log p$$

From the graph, slope = 2/4 = $1/n\Rightarrow n$ = 2

Therefore, $\frac{x}{m}$ is proportional to $p^{1/2}$.
Answer: (3).

50. $CH(CN)_3$ is the strongest acid among the given compounds, because its conjugate base, that is, $C(CN)_3^-$ is stabilized by the three nitriles groups through delocalization of negative charge.
Answer: (3).

51. Copper pyrites is $CuFeS_2$; malachite is $Cu(OH)_2 \cdot CuCO_3$; azurite is $Cu(OH)_2 \cdot 2CuCO_3$; dolomite is $CaCO_3 \cdot MgCO_3$.
Answer: (1).

52. We know

$$\bar{\nu} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For emission from $n_1 = 8$ to $n_2 = n$, we have

$$\bar{\nu} = R_H \left(\frac{1}{64} - \frac{1}{n^2} \right) = \frac{R_H}{64} - \frac{R_H}{n^2}$$

This equation is of the form of straight-line equation $y = mx + c$, with $y = \bar{\nu}$, $x = 1/n^2$, slope, $m = -R_H$ and intercept, $c = R_H/64$.
Answer: (4).

53. Isotopes of H are ${}_1^3H$ (tritium), ${}_1^2H$ (deuterium), ${}_1^1H$ (protium).
Answer: (3).

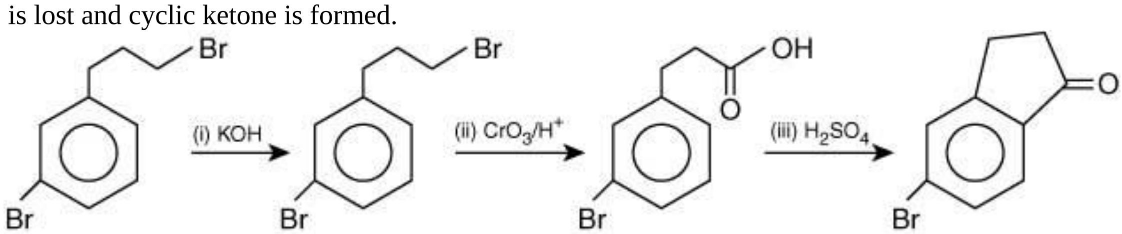
54. According to MO theory,

$$Li_2^+(5e^-) = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^1 \Rightarrow \text{bond order } = \frac{3-2}{2} = 0.5$$

$$Li_2^-(7e^-) = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^1 \Rightarrow \text{bond order } = \frac{4-3}{2} = 0.5$$

As both are having bond order 0.5, they are both stable.
Answer: (3).

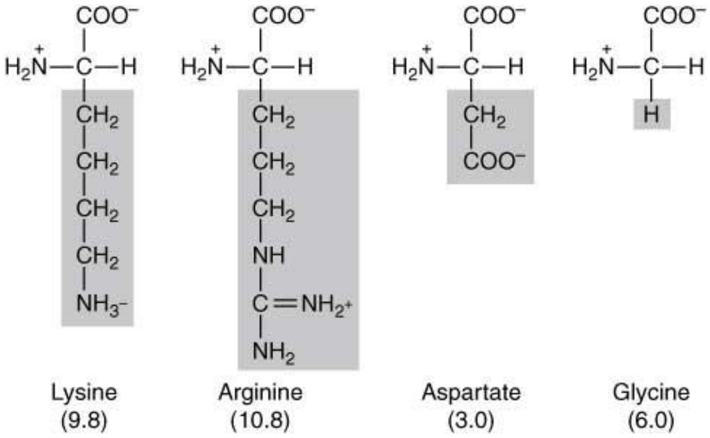
55. First step is S_N2 reaction, in which –OH group gets substituted and –Br leaves; second step is oxidation reaction in which –OH gets oxidized to –COOH by CrO_3/H^+; third step is dehydration step in which H_2O is lost and cyclic ketone is formed.



Answer: (1).

56. Tl is the 5th member of boron family, therefore, it shows inert pair effect, due to which it exists in +1 and +3 oxidation states, out of which it is more stable in +1 oxidation state.
Answer: (1).

57. The order of pK_a of the given amino acids is Asp < Gly < Lys < Arg. This can be deduced based on their pI values and the fact that Asp is acidic, Gly is neutral, Arg and Lys are basic in nature due to the presence of –COOH and $-NH_2$ groups in their side chain.



Answer: (3).

58. For an isothermal process, we have

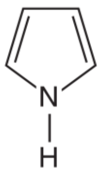
$$w = -nRT \ln \frac{V_2}{V_1}$$

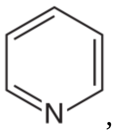
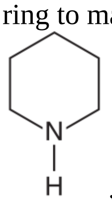
$$\text{or} \quad |w| = nRT \ln V_2 - nRT \ln V_1$$

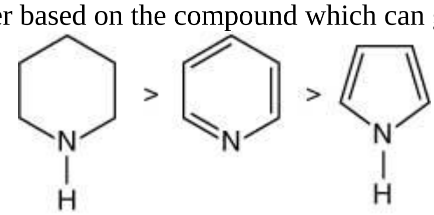
Since, the above equation is of the form $y = mx - c$, so the slope will be positive and more for T_2 than T_1 and intercept of T_2 curve is more negative than that of T_1.

Answer: (2).

59.

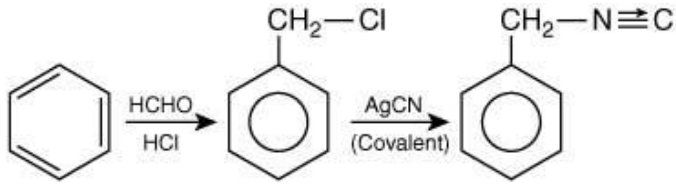
In  , the lone pair on N is delocalized in the ring to make pyrrole an aromatic compound.

While in  , it is delocalized in sp^2 orbital. In  , it is delocalized in sp^3 orbital. Hence, the basicity follows the order based on the compound which can give up lone pair easily.



Answer: (2).

60. The reaction involved is



AgCN is a covalent compound, therefore, it will bind with N side rather than C side, so it forms isocyanide.

Answer: (4).

Section: Mathematics

61. $\int_0^{\pi} |\cos x|^3 \, dx = \int_0^{\pi/2} \cos^3 x \, dx - \int_{\pi/2}^{\pi} \cos^3 x \, dx$

$$= \int_0^{\pi/2} \left(\frac{\cos 3x + 3\cos x}{4} \right) dx - \int_{\pi/2}^{\pi} \left(\frac{\cos 3x + 3\cos x}{4} \right) dx$$

$$= \frac{1}{4}\left[\left(\frac{\sin 3x}{3} + 3\sin x \right)_0^{\pi/2} - \left(\frac{\sin 3x}{3} + 3\sin x \right)_{\pi/2}^{\pi} \right]$$

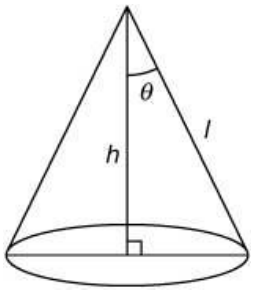
$$= \frac{1}{4}\left[\left(\frac{-1}{3} + 3 \right) - (0+0) - \left\{ (0+0) - \left(\frac{-1}{3} + 3 \right) \right\} \right]$$

$$= \frac{4}{3}$$

Answer: (2).

62. Given,

Slant height (l) = 3 m



Here, $h = 3 \cos \theta$

$r = 3 \sin \theta$

Volume of cone is $V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (9\sin^2 \theta) \times 3\cos\theta$

So, $\frac{dV}{d\theta} = 0 \Rightarrow \sin\theta = \sqrt{\frac{2}{3}}$

$$\left(\frac{d^2V}{d\theta} \right)_{\sin\theta = \sqrt{\frac{2}{3}}} = \text{Negative}$$

Since, volume will be maximum at $\sin\theta = \sqrt{\frac{2}{3}}$

Hence, $V_{\max} = 2\sqrt{3}\pi$ cu.m

Answer: (4).

63. We have

$$I = \int x \sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}} \, dx$$

$$= \int x \sqrt{\frac{(1 - \cos(x^2-1))}{(1 + \cos(x^2-1))}} dx$$

$$= \int x \frac{\sin\left(\frac{x^2-1}{2}\right)}{\cos\left(\frac{x^2-1}{2}\right)} dx = \int x \tan\left(\frac{x^2-1}{2}\right) dx$$

Let, $\frac{x^2-1}{2} = t \Rightarrow 2x\,dx = 2dt$

Thus,

$$I = \int \tan t\, dt$$

$$= \log_e \sec t + c$$

$$I = \log_e \sec\left(\frac{x^2-1}{2}\right) + c$$

Answer: (4).

64. We have

$$x\frac{dy}{dx} + 2y = x^2$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$$

Above equation is linear differential equation.

$$\text{I.f} = e^{\int \frac{2}{x}dx} = x^2$$

Solution of this differential equation will be,

$$y.x^2 = \int x.x^2 dx = \frac{x^4}{4} + c$$

$$\Rightarrow y = \frac{x^4}{4} + \frac{c}{x^2}$$

At $y(1) = 1$

$$1 = \frac{1}{4} + c \Rightarrow c = \frac{3}{4}$$

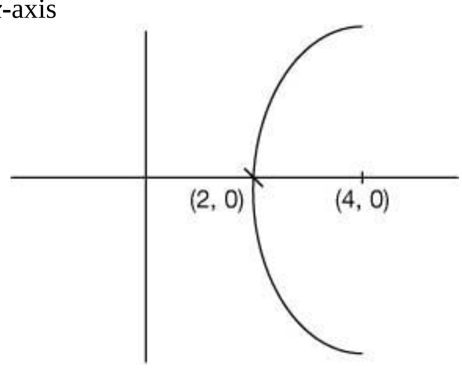
Hence, at $y\left(\frac{1}{2}\right)$ we have

$$y\left(\frac{1}{2}\right) = \frac{1}{16} + \frac{3}{4 \times \frac{1}{4}} = \frac{49}{16}$$

Answer: (3).

65. We have,

Axis of parabola lies along x-axis



Thus, equation of parabola will be

$(y - 0)^2 = 4\cdot 2(x - 2)$

$y^2 = 8(x - 2)$

Now check all options, (8, 6) does not satisfy.

Answer: (2).

66. Eccentricity of the hyperbola is $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2} = 1$

According to the question, $e > 2$ (given)

$$e^2 > 4 \Rightarrow 1 + \frac{\sin^2\theta}{\cos^2\theta} > 4$$

$$\Rightarrow 1 + \tan^2\theta > 4$$

$$\Rightarrow \tan^2\theta > 3 \qquad \because \theta \in \left(0, \frac{\pi}{2}\right)$$

$$\theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

Latus Rectum $= 2 \times \frac{\sin^2\theta}{\cos\theta} = 2\tan\theta\sin\theta$

For $\theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right), 2\tan\theta\sin\theta$ is an increasing function.

Hence, latus rectum $\in (3, \infty)$.

Answer: (1).

67. We have

$f_1(x) = \frac{1}{x},\ f_2(x) = 1 - x,\ f_3(x) = \frac{1}{1-x}$

$\Rightarrow (f_2 \circ J \circ f_1)(x) = f_3(x)$

$\Rightarrow f_2 \circ (J(f_1(x)) = f_3(x)$

$\Rightarrow f_2 \circ \left(J\left(\frac{1}{x}\right)\right) = \frac{1}{1-x}$

$\Rightarrow 1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$

$\Rightarrow J\left(\frac{1}{x}\right) = 1 - \frac{1}{1-x} = \frac{-x}{1-x} = \frac{x}{x-1}$

So, $x \to \frac{1}{x}$

$\Rightarrow J(x) = \frac{\frac{1}{x}}{\frac{1}{x} - 1} = \frac{1}{1-x} = f_3(x)$

Answer: (1).

68. We have

$$\vec{a} \times (\vec{a} \times \vec{c}) + \vec{a} \times \vec{b} = 0$$

$$\Rightarrow (\vec{a}.\vec{c})\vec{a} - (\vec{a}.a)\vec{c} + \vec{a} \times \vec{b} = 0$$

$$\Rightarrow 4\vec{a} - 2\vec{c} + \vec{a} \times \vec{b} = 0$$

$$\Rightarrow \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= i(-1) - j(1) + k(2)$$

$$= -\hat{i} - \hat{j} + 2\hat{k}$$

Hence, $2\vec{c} = 4\hat{i} - 4\hat{j} - \hat{i} - \hat{j} + 2\hat{k}$

$$= 3\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\Rightarrow \vec{c} = \frac{3}{2}\hat{i} - \frac{5}{2}\hat{j} + \hat{k}$$

$$|\vec{c}| = \sqrt{\frac{9}{4} + \frac{25}{4} + 1} = \sqrt{\frac{38}{4}} = \sqrt{\frac{19}{2}}$$

$$|\vec{c}|^2 = \frac{19}{2}$$

Answer: (1).

69. We have $a + ar + ar^2 = xar$

since $a \neq 0$, so, $\frac{r^2 + r + 1}{r} = x$

$$1 + r + \frac{1}{r} = x$$

Therefore, $r + \frac{1}{r} \in [-\infty, -2) \cup [2, \infty] \Rightarrow x \in (\infty, -1) \cup (3, \infty)$

Answer: (4).

70. We have

$$\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{2}{3x}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{2}{3x} \times \frac{3}{4x} - \sqrt{1 - \frac{4}{9x^2}} \times \sqrt{1 - \frac{9}{16x^2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{1}{2x^2} = \frac{\sqrt{9x^2 - 4} \times \sqrt{16x^2 - 9}}{12x^2}$$

$$\Rightarrow 6 = \sqrt{9x^2 - 4} \times \sqrt{16x^2 - 9}$$

Squaring both the sides, we get

$$\Rightarrow 36 = (9x^2 - 4) \times (16x^2 - 9)$$

$$\Rightarrow x^4 = \frac{145x^2}{144}$$

$$\Rightarrow x = \pm\frac{\sqrt{145}}{12}$$

Answer: (1).

71. We have

Equation of a circle is $x^2 + y^2 - 6x = 0$

And, equation of a parabola is $y^2 = 4x$

Tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$

If it is tangent to given circle it's distance from (3, 0) is equal to 3.

Thus, $\left|\frac{3m + \frac{1}{m}}{\sqrt{1 + m^2}}\right| = 3$

$$\Rightarrow |3m^2 + 1| = 3m\sqrt{1 + m^2}$$

Squaring both the sides, we obtain

$9m^4 + 6m^2 + 1 = 9m^2 + 9m^4$

$\Rightarrow 6m^2 - 9m^2 = -1 \Rightarrow -3m^2 = -1$

$\Rightarrow m^2 = \frac{1}{3} \Rightarrow m = \pm\frac{1}{\sqrt{3}}$

Hence, common tangents are,

$y = \frac{x}{\sqrt{3}} + \sqrt{3}$ or $y = -\left(\frac{x}{\sqrt{3}} + \sqrt{3}\right)$

Answer: (2).

72. Consider matrix

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix} = a^2 - 3$$

$$D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 3 & 2 \\ a+1 & 3 & a^2 - 1 \end{vmatrix} = a^2 - a + 1$$

$$D_2 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 2 & a+1 & a^2 - 1 \end{vmatrix} = a^2 - 3$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 2 & 3 & a+1 \end{vmatrix} = a - 4$$

$D = 0$ at $(9) = \sqrt{3}$,

$D_3 \pm \sqrt{3} - 4 \neq 0$

So, the system will be inconsistent for $|a| = \sqrt{3}$

Answer: (4).

73. We have

$$\frac{2^{403}}{15} = \frac{(2^4)^{100}}{15} \times 2^3$$
$$= \frac{(16)^{100}}{15} \times 2^3$$

Remainder $\left(\frac{2^{403}}{15}\right) = 1 \times 8 = 8$

So, fractional part = 8

Therefore, the value of $k = 8$

Answer: (2).

74. Let point of intersection with line is $(-3\lambda - 1, 2\lambda + 3, -\lambda + 2)$

So, direction ratio of line will be $(-3\lambda + 3, 2\lambda, -\lambda + 1)$

This line is parallel to $x + 2y - 2 = 5$

So, $V_1.V_2 = 0$

$\Rightarrow -3\lambda + 3 + \lambda + \lambda - 1 = 0$

$\Rightarrow 2\lambda + 2 = 0 \Rightarrow \lambda = -2$

Thus, direction ratio's are 6, −2, 2

Therefore, equation of line is $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

Answer: (3).

75. Given relation is $3p + 2q + 4r = 0$

$$\Rightarrow \frac{3}{4}p + \frac{q}{2} + r = 0 \qquad (1)$$

Set of all lines $px + qy + r = 0$ (2)

Comparing Eq. (1) and Eq. (2), we get

$$x = \frac{3}{4},\ y = \frac{1}{2}$$

Hence, we can say that these lines are concurrent at $\left(\frac{3}{4}, \frac{1}{2}\right)$

Answer: (1).

76. We have $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

By rationalization

$$\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4} \times \frac{\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}}{\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}}$$

$$= \lim_{y\to 0} \frac{\sqrt{1+y^4} - \sqrt{1}}{y^4} \times \frac{1}{\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}} \times \frac{\sqrt{1+y^4}+1}{\sqrt{1+y^4}+1}$$

$$= \lim_{y\to 0} \frac{1}{\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}} \times \frac{1}{\sqrt{1+y^4}+1}$$

By applying limit

$$\frac{1}{2\sqrt{2}} \times \frac{1}{2} = \frac{1}{4\sqrt{2}}$$

By applying L Hospital's Rule, we have

$$\lim_{y\to 0} \frac{1}{2\sqrt{1+\sqrt{1+y^2}}} \times \frac{4y^3}{2\sqrt{1+y^4}} = \frac{1}{4\sqrt{2}}$$

Answer: (1).

77. Given equations are $x + y + z = 1$ and $2x + 3y - z + 4 = 0$

Thus, equation of plane will be

$$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$$

$$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z - 1 + 4\lambda = 0$$

So, direction of normal of the plane will be $1 + 2\lambda, 1 + 3\lambda, 1 - \lambda$

Since, plane is parallel to y-axis as given in question,

$$1 + 3\lambda = 0 \Rightarrow \lambda = \frac{-1}{3}$$

Therefore, equation of plane is

$$x + 4z - 7 = 0$$

Hence, point (3, 2, 1) is satisfying the equation.

Answer: (4).

78. Point of intersection is $P(2, 6)$

Also $m_1 = \left(\frac{dy}{dx}\right)_{P(2,6)} = -2x = -4$

$$m_2 = \left(\frac{dy}{dx}\right)_{P(2,6)} = 2x = 4$$

Therefore, $$|\tan\theta| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{8}{15}$$

Answer: (2).

79. Given

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\Rightarrow A^{-m} = \begin{bmatrix} \cos(m\theta) & \sin(m\theta) \\ -\sin(m\theta) & \cos(m\theta) \end{bmatrix}$$

Hence,

$$\Rightarrow A^{-50} = \begin{bmatrix} \cos(50\theta) & \sin(50\theta) \\ -\sin(50\theta) & \cos(50\theta) \end{bmatrix}$$

$$A^{-50} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

Answer: (3).

80. Check all the options repeatedly

(i) $(A \wedge B) \wedge (\vee A \vee B) \equiv A \wedge (B \wedge (\sim A \vee B))$

$\equiv A \wedge B \equiv A \wedge B$

Hence, this is correct.

(ii) $(A \wedge B) \wedge (\sim A \wedge B) \equiv (A \wedge \sim A) \wedge B$

$\equiv f \wedge B \equiv F$

(iii) $(A \vee B) \wedge (\sim A \vee B) \equiv B$

(iv) $(A \vee B) \wedge (-A \vee B)$

$\equiv B \vee (A \wedge \sim A) \equiv B \vee F \equiv F$

Thus, option (1) is correct.

Answer: (4).

81. Assume that 5 students are S_1, S_2, S_3, S_4 and S_5.

Given $S = \frac{\Sigma Si}{5} = 150$

$$\sum_{i=1}^{5} Si = 750 \qquad (1)$$

$$\frac{\Sigma Si^2}{5} - (S)^{-2} = 18 \Rightarrow \frac{\Sigma Si^2}{5} - (150)^2 = 18$$

$$\Rightarrow \Sigma Si^2 = (22500 + 18)5 = 112590 \qquad (2)$$

Height of new Student = 156 (let S_6)

Now $S_1 + S_2 + S_3 + S_4 + S_5 + S_6 = 750 + 156$

$$\bar{S}_{\text{New}} = \frac{S_1 + S_2 + S_3 + S_4 + S_5 + S_6}{6} = \frac{906}{6} = 151 \qquad (3)$$

$$\text{Variance (New)} = \frac{\Sigma Si^2}{6} - (\bar{S})^2$$

$$= \frac{S_1^2 + S_2^2 + S_3^2 + S_4^2 + S_5^2 + S_6^2}{6} - (151)^2$$

From Eq. (2) and (3), we have

$$\text{Variance (New)} = \frac{112590 + (156)}{6} - (151)^2 = 22821 - 22801 = 20$$

Answer: (3).

82. We have

$3(\sin \theta - \cos \theta)^4 + 6(\sin \theta + \cos \theta)^2 + 4 \sin^6\theta$

$= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4(1 - \cos^2\theta)^3$

$= 3(1 - 2 \sin 2\theta + \sin^2 2\theta) + 6(1 + \sin 2\theta) + 4(1 - 3 \cos^2\theta + 3 \cos^4\theta - \cos^6\theta)$

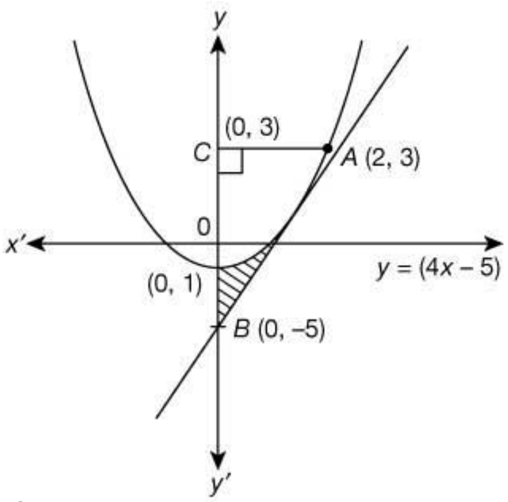
$= 13 + 3 \sin^2 2\theta + 12 \cos^2\theta (\cos^2\theta - 1) - 4 \cos^6\theta$

$= 13 + 12 \sin^2\theta \cos^2\theta - 12 \sin^2\theta \cos^2\theta - 4 \cos^6$

$= 13 - 4 \cos^6\theta$

Answer: (2).

83.



Equation of tangent on $y = x^2 - 1$ at (2, 3) is $y = (4x - 5)$ (1)

$$\text{Required shaded area} = (\Delta ABC) - \int_{-1}^{-3} \sqrt{y+1}\, dy$$

$$= \frac{1}{2} \times 8 \times 2 - \frac{2}{3}((y+1)^{3/2})_{-1}^{3}$$

$$= 8 - \frac{16}{3} = \frac{8}{3} \text{ (sq. units)}$$

Answer: (1).

84. We have,

$$S = a_1 + a_2 + \cdots + a_{30} = \frac{30}{2}(2a + 29d) = 15(2a + 29d)$$

$$T = a_1 + a_3 + a_5 \cdots + a_{29} = \frac{15}{2}(2a + 28d) = 15(a + 14d)$$

$a_5 = 27 \Rightarrow a + 4d = 27$ (1)

Thus, $S - 2T = 75$

$\Rightarrow 15(2a + 29d) - 2(15(a + 14d)) = 75$

$\Rightarrow 30a + 435d - (30a + 420d) = 75$

$\Rightarrow$ 435*d* – 420*d* = 75 $\Rightarrow$ 15*d* = 75
$\Rightarrow d = 5$
Put the value of *d* in Eq. (1), we have
$a \times 4 \times 5 = 27$
$\Rightarrow a + 20 = 27$
$\Rightarrow a = 7$
Therefore, $a_{10} = a + 9d = 7 + 9 \times 5 = 7 + 45 = 52$
Answer: (1).

85. Given

$$f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a + bx, & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$$

Since the function is to be continuous at *x* = 1
$f(1) = \lim_{h \to 0}(1+h) = \lim_{h \to 0}(1-h)$
$f(1) = 5$
$\lim_{h \to 0}(1+h) = \lim_{h \to 0} a + b(1+h) = a + b$
$\lim_{h \to 0}(1-h) = \lim_{h \to 0} 5 = 5$
So, $a + b = 5$ (1)
Similarly, at *x* = 3
$f(3) = b + 15$
$\lim_{h \to 0} f(3+h) = b + 15$
$\lim_{h \to 0} f(3-h) = a + 3b$
So, $a + 2b = 15$ (2)
Similarly, at *x* = 5
$f(5) = 30$
$\lim_{h \to 0} f(5+h) = 30$
$\lim_{h \to 0} f(5-h) = b + 25$
so, $b = 5$ (3)
From Eq. (2) and Eq. (3), we have
$a = 5$
From Eq. (1) and Eq. (3), we have
$a = 0$
Hence, *f*(*x*) is not continuous for any values of *a* and *b*.
Answer: (4).

86. Let $z = \dfrac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ $\theta \in \left(\dfrac{-\pi}{2}, \pi\right)$

For *z* to be purely imaginary

$$z + \bar{z} = 0$$

$$\Rightarrow \frac{3 + 2i\sin\theta}{1 - 2i\sin\theta} + \frac{3 - 2i\sin\theta}{1 + 2i\sin\theta} = 0$$

$$\Rightarrow \frac{3 + 6i\sin\theta + 2i\sin\theta - 4\sin^2\theta + 3 - 6i\sin\theta - 2i\sin\theta - 4\sin^2\theta}{1 + 4\sin^2\theta} = 0$$

$$3 - 4\sin^2\theta = 0 \quad \Rightarrow \sin^2\theta = \frac{3}{4}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$$

It is given that, $\theta \in \left(\frac{-\pi}{2}, \pi\right)$

Therefore, $\theta = \frac{-\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

Hence, sum of all possible values of $\theta = \frac{2\pi}{3}$.

Answer: (4).

87. Total number of ways of forming team = ${}^7C_3 \times {}^5C_2 = 350$
If two specific boys B_1 and B_2 are in the same team then, total number of team formed = ${}^5C_1 \times {}^5C_2 = 50$
Therefore, total ways = 350 – 50 = 300
Answer: (3).

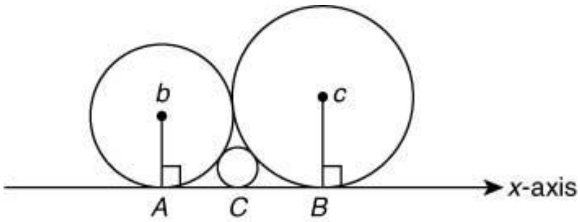
88. Roots are $(-1 + i)$ and $(-1 - i)$
Let $\alpha = -1 + i$ and $\beta = -1 - i$
Thus, $\alpha^{15} + \beta^{15} = (-1 + i)^{15} + (-1 - i)^{15}$

$$= \frac{(-1+i)^{16}}{-1+i} + \frac{(-1-i)^{16}}{-1-i}$$

$$= 2^8 \frac{(-1+i-1-i)^{16}}{1^2 - i^2} = 128(-2) = -256$$

Answer: (1).

89.



$AB = AC + CB$

$$\sqrt{(b+c)^2 - (b-c)^2}$$

$$= \sqrt{(b+a)^2 - (b-a)^2} + \sqrt{(a+c)^2 - (a-c)^2}$$

$$\Rightarrow \sqrt{bc} = \sqrt{ab} + \sqrt{ac}$$

$$\Rightarrow \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}}$$

Answer: (1).

90. We have 4 ace cards and 48 non-ace cards.

$$P(x=1) = \frac{4}{52} \times \frac{48}{52} \times 2 = \frac{24}{169}$$

$$P(x=2) = \frac{4}{52} \times \frac{4}{52} \times 2 = \frac{1}{169}$$

$$P(x = 1) + P(x = 2) = \frac{24}{169} + \frac{1}{169} = \frac{25}{169}$$

Answer: (4).