JEE Main 2019 Paper 1
January 9, Shift 2
Section: Physics

1. We know that,

$$N = N_0 e^{-\lambda t}$$

If $T_{1/2}$ be the half-life period, then at $t = T_{1/2}$ and $N = N_0/2$

$$\frac{N_0}{L} = N_0 e^{-\lambda T_{1/2}}$$

$$\Rightarrow e^{\lambda T/2} = 2 \Rightarrow \lambda T_{1/2} = \log_e 2$$

$$\Rightarrow T_{1/2} = \frac{\ln 2}{\lambda}$$

Half-life of $A = \ln 2$ [ln = $\log_e$]

$\lambda_A = 1$

At $t = 0$, $R_A = R_B$

Then, $N_A e^{-\lambda_A T} = N_B e^{-\lambda_B T}$

$N_A = N_B$ at $t = 0$

At $t = t$,

$$\Rightarrow \frac{R_B}{R_A} = \frac{N_0 e^{-\lambda_B t}}{N_0 e^{-\lambda_A t}} \Rightarrow e^{-(\lambda_B - \lambda_A)t} = e^{-3t}$$

$$\Rightarrow -(\lambda_B - \lambda_A) = -3 \Rightarrow \lambda_B - \lambda_A = 3$$

$$\Rightarrow \lambda_B = 3 + \lambda_A = 3 + 1 = 4$$

$$\Rightarrow t_{1/2} = \frac{\ln 2}{\lambda_B} = \frac{\ln 2}{4}$$

Answer: (3).

2. Efficiency $\eta = \frac{V_s i_s}{V_p i_p} \times 100$

$$\Rightarrow 90 = \frac{230 \times i_s}{2300 \times 5} \times 100$$

$$\Rightarrow 0.9 = \frac{i_s}{20 \times 5} \Rightarrow 50 \times 0.9 = i_s$$

$$\Rightarrow i_s = 45 \text{ A}$$

Answer: (2).

3. The average electric energy density is

$$U_E = \frac{1}{2}\varepsilon_0 E^2$$

The average magnetic energy density is

$$U_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

In electromagnetic wave, the electric and magnetic field vary sinusoidally in free space so, in above expression E and B are replaced by their rms values.

Therefore,

$$U_E = \frac{1}{2}\varepsilon_0 E_{rms}^2 \quad \text{and} \quad U_B = \frac{1}{2}\frac{B_{rms}^2}{\mu_0}$$

$$\Rightarrow U_B = \frac{B_{rms}^2}{2\mu_0} = \frac{1}{2\mu_0}\frac{E_{rms}^2}{c^2} \qquad \left[B_{rms} = \frac{E_{rms}}{c}\right]$$

$$\Rightarrow U_B = \frac{1}{2\mu_0} \times \mu_0 \varepsilon_0 \times E_{rms}^2 \qquad \left[c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right]$$

$$\Rightarrow U_B = \frac{1}{2} \varepsilon_0 E_{rms}^2$$

Since energy density of electric and magnetic field is equal.

Hence, $U_B = U_E$

Answer: (4).

4. The position of object as a function of time is given by

$$x = 3t^2 + 5 \qquad (1)$$

We know that

$$\text{velocity, } v = \frac{dx}{dt}$$

Differentiating w.r.t. t Eq. (1) becomes

$$\Rightarrow \frac{dx}{dt} = 6t + 0$$

$$\Rightarrow v = 6t$$

At $t = 0$, $v = 0$

At $t = 5$, $v = 30$ m/s

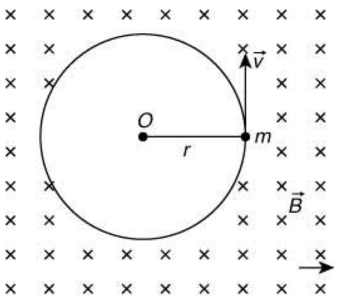
Therefore, work done by the force = change in K.E.

$$W = \Delta \text{KE} = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 2 \times (30)^2 = 900 \text{ J}$$

Answer: (4).

5. Suppose a particle of mass m and charge q, entering a uniform magnetic field induction $\vec{B}$ at O with velocity v.



The force acting on the charge particle due to magnetic field is

$$F = q\vec{v}\vec{B} \qquad (1)$$

The force F on the charged particle due to magnetic field provides the required centripetal force necessary for motion along the circular path of radius r

$$F = \frac{mv^2}{r} \qquad (2)$$

From Eq. (1) and Eq. (2), we get

$$\frac{mv^2}{r} = Bqv$$

$$\Rightarrow \frac{mv}{r} = Bq$$

$$\Rightarrow mv = Bqr \qquad (3)$$

Force due to electric field is

$$F = qE$$

Since, both are in straight line, then

$$qvB = qE$$

$$\Rightarrow E = vB \qquad (4)$$

From Eq. (3) and Eq. (4), we have

$$m = \frac{qB^2vr}{vE}$$

$$\Rightarrow m = \frac{qB^2r}{E}$$

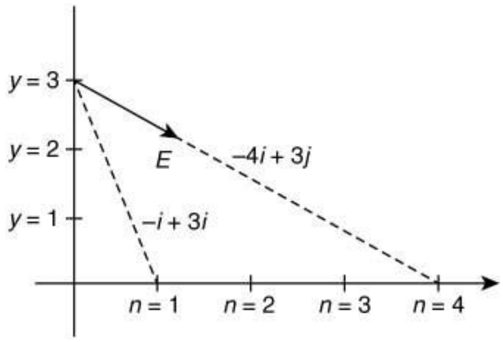
$$\Rightarrow m = \frac{1.6\times10^{-19}\times(0.5)^2\times0.5\times10^{-2}}{100}$$

$$= 2.0 \times 10^{-24}\ \text{kg}$$

Answer: (4).

6. We know that electric field vector is

$$\vec{E} = \frac{kq(\vec{r})}{|\vec{r}|^3}$$



Let E_1 and E_2 are the electric field of q_1 and q_2.

For charge $q = \sqrt{10}$ μC

$$E_1 = \frac{k\sqrt{10}\times10^{-6}(-\hat{i}+3\hat{j})}{\left|\sqrt{10}\right|^3}$$

$$= \frac{9\times10^9\times10^{-6}(-\hat{i}+3\hat{j})}{10}$$

$$= (-9\hat{i}+27\hat{j})\times10^2\ \text{V/m}$$

For charge $q_2 = -25$ μC

$$E_2 = \frac{k(-25)\times10^{-6}(-4\hat{i}+3\hat{j})}{|5|^3}$$

$$= \frac{-9\times10^9\times10^{-6}(-4\hat{i}+3\hat{j})}{5}$$

$$= -9\times10^2(-8\hat{i}+6\hat{j})$$

$$= (72\hat{i}-54\hat{j})$$

Therefore, total electric field vector is

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= (-9\hat{i}+27\hat{j})\times10^2 + (72\hat{i}-54\hat{j})\times10^2$$

$$= 10^2(-9\hat{i} + 27\hat{j} + 72\hat{i} - 54\hat{j})$$

$$= (63\hat{i} - 27\hat{j}) \times 10^2 \text{ V/m}$$

Answer: (1).

7. Dimension of time, $t = [T]$

Now, $t \propto G^k h^q c^r$

$t = kG^k h^q c^r$ (1)

Dimension of Gravitational constant, $G = [M^{-1}L^3T^{-2}]$

Dimension of Planck's constant $h = [ML^2T^{-1}]$

Dimension of speed of light $c = [LT^{-1}]$

Put the Dimensions in Eq.(1), we get

$[T] = [M^{-1}L^3T^{-2}]^p\ [ML^2T^{-1}]^q\ [LT^{-1}]^r$

$[T] = [M^{-p+q}]\ [L^{3p+2q+r}]\ [T^{-2p-q-r}]$

On comparing the power of both sides

$-p + q = 0 \Rightarrow p = q$ (2)

$3p + 2q + r = 0 \Rightarrow 5p + r = 0$ (3)

$-2p - q - r = 1 \Rightarrow -3p - r = 1$ (4)

On solving the above Eq. (2), (3) and (4), we get

$p = q = \frac{1}{2}$ and $r = \frac{-5}{2}$

Put these value in Eq. (1), we get

$t = G^{1/2}\ h^{1/2}\ c^{-5/2}$

$t = k\sqrt{\frac{Gh}{c^5}}$ where, k = constant

Answer: (3).

8. In the given circuit, voltage $V = 12$ V

Resistance, $R = 5\ \text{k}\Omega$

Current, $I = \frac{12 - 0.3}{5\ \text{k}\Omega}$

$= \frac{11.7}{5 \times 10^3 \Omega}$

$= 2.34\ \mu\text{A}$

Output voltage

$V_0 = RI$

$= (5 \times 10^3) \times (2.34 \times 10^{-3})$

$= 11.7$ V

When the connection of Ge diode are reversed then the current will be through Si.

Current $I = \frac{12 - 0.7}{5\ \text{k}\Omega}$

$= \frac{11.3}{5 \times 10^{-3}} = 2.26\ \mu\text{A}$

Therefore, output voltage

$V = IR$

$= (2.26 \times 10^{-3}) \times (5 \times 10^3)$

$= 11.3$ V

$V_0 = (11.7 - 11.3)$ V

$= 0.4$ V

Answer: (4).

9. Volume of water inflow = Volume of water out flow

$$\frac{0.74}{60} = \text{Area} \times \text{velocity}$$

$$= \pi r^2 \times v$$

$$\Rightarrow \frac{0.74}{60} = \pi \times (2 \times 10^{-2})^2 \times \sqrt{2gh}$$

$$\Rightarrow \frac{0.74}{60} = \pi \times 4 \times 10^{-4} \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{0.74}{60 \times \pi \times 4 \times 10^{-4}}$$

$$\Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \left(\frac{740}{24\pi}\right)^2$$

$$\Rightarrow h = \frac{740 \times 740}{24\pi \times 24\pi \times 2 \times g}$$

$$\Rightarrow h = \frac{547600}{113582.6} = 4.8 \text{ m}$$

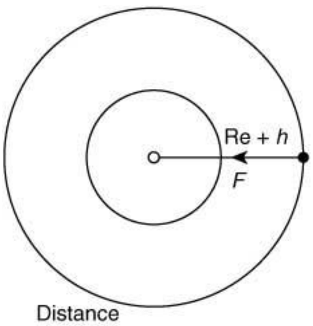
Answer: (2).

10. The energy required to take satellite at height h above earth surface is equal to change in potential energy, that is, $E_1 = U_f - U_i$

$$E_1 = \frac{-GMm}{R_E + h} + \frac{GMm}{R_E}$$

$$= GMm\left[\frac{-1}{R_E + h} + \frac{1}{R_E}\right]$$

$$= \frac{GMm\,h}{R_E(R_E + h)}$$



If satellite revolve around the earth at a distance ($R_E + h$), the required centripetal acceleration will be provided by Gravitational attraction force. Thus,

$$E_2 = \frac{mv^2}{(R_E + h)} = \frac{GMm}{(R_E + h)^2}$$

$$\Rightarrow mv^2 = \frac{GMm}{(R_E + h)}$$

$$\Rightarrow E_2 = \frac{mv^2}{2} = \frac{GMm}{2(R_E + h)}$$

Therefore, $E_1 = E_2$

$$\frac{Gmh}{R_E(R_E+h)} = \frac{GMm}{2(R_E+h)}$$

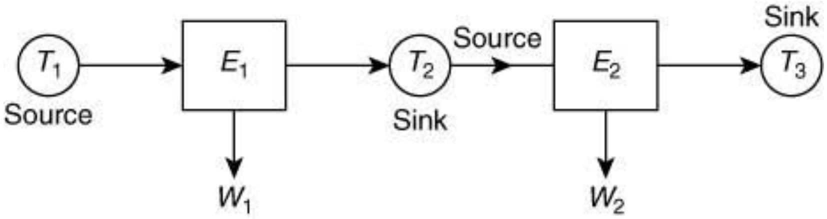
$$\Rightarrow \frac{h}{R_E} = \frac{1}{2}$$

$$\Rightarrow h = \frac{R_E}{2} = \frac{6400}{2}\text{km} \qquad (R_E = 6400 \text{ km})$$

$$= 3200 \text{ km} = 3.2 \times 10^3 \text{ km}$$

Answer: (2).

11. Let E_1 and E_2 be the two engines.



E_1 engine absorb the T_1 heat from source and exhausts it to sink at T_2. Sink of Engine E_1 is source for engine E_2 at T_2.

Let W_1 and W_2 be the work of Engine E_1 and E_2

Thus,

$$W_1 = W_2$$

$$\Rightarrow \Delta V_1 = \Delta V_2$$

$$\Rightarrow T_1 - T_2 = T_2 - T_3$$

$$\Rightarrow T_1 + T_3 = 2T_2$$

$$\Rightarrow 2T_2 = 600 + 400 = \frac{1000}{2}$$

Therefore, $T_2 = 500$ K

Answer: (4).

12. Given $R = 60\ \Omega$, $f = 50$ Hz, $C = 120\ \mu\text{F} = 120 \times 10^{-6}$ F

$$X_C = \frac{1}{\omega C}$$

$$= \frac{1}{2\pi fC} \qquad (\omega = 2\pi f)$$

$$= \frac{1}{2\times\pi\times 60\times 120\times 10^{-6}} = 26.52\ \Omega$$

Inductive Reactance

$$X_L = \omega L$$

$$= 2\pi fL$$

$$= 2 \times \pi \times 50 \times 20 \times 10^{-3}$$

$$= 6.28\ \Omega$$

Now, $X_C - X_L = 26.52 - 6.28$

$$= 20.24$$

$$X_C - X_L \approx 20$$

Impedance

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

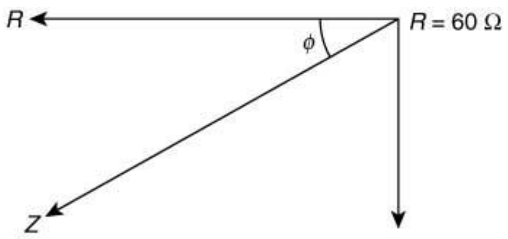
$$= \sqrt{(60)^2 + (20)^2}$$

$$= \sqrt{3600 + 400}$$

$$= \sqrt{4000}$$

$$= 20\sqrt{10}\ \Omega$$

$$\cos\phi = \frac{R}{Z} = \frac{3}{\sqrt{10}}$$



Power consumed by the AC

$$P = VI\cos\phi \qquad \left[I = \frac{V}{2}\right]$$

$$= V.\frac{V}{2}\cos\phi$$

$$= \frac{V^2}{2}\cos\phi$$

$$= \frac{(24)^2}{20\sqrt{10}} \times \frac{3}{\sqrt{10}}$$

$$= 8.64$$

Hence, energy dissipated in the circuit in 60s = 5.17×10^2 J

Answer: (3).

13. Potential energy $(U) = \frac{1}{2}kx^2$ $\qquad (K = -m\omega^2)$

Kinetic energy $K = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

It is given that $U = K$

$$\frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$\Rightarrow kx^2 = \frac{1}{2}kA^2$$

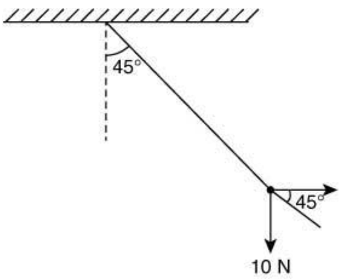
$$\Rightarrow x^2 = \frac{A^2}{2}$$

$$\Rightarrow x = \pm\frac{A}{\sqrt{2}}$$

Answer: (3).

14. It is given that, m = 10 kg and g = 10 m/s^2

In equilibrium position, tension in the string has two components.



Horizontal component = $T\sin\theta$

Vertical component = $T\cos\theta$

horizontal force is balanced by the 10 × 10 = 100 N forceand vertical component is balanced by $F = mg$

$T \sin \theta = 10 \times 10$

$T \cos \theta = mg$ or F

$$\Rightarrow \frac{T\sin\theta}{T\cos\theta} = \frac{10\times10}{mg}$$

$$\Rightarrow \tan\theta = \frac{100}{F}$$

$$\Rightarrow \tan 45° = \frac{100}{F}$$

$$\Rightarrow F = \frac{100}{\tan 45°} \qquad [\tan 45° = 1]$$

Therefore, $F = 100$ N

Answer: (4).

15. Molar heat is given by

$Q = n\,C_V\Delta T$

$$h = \frac{\text{mass of Nitrogen}}{\text{molecular weight of Nitrogen}} = \frac{15}{28}$$

Nitrogen is diatomic gas $C_V = \frac{5}{2}R$

Since rms of velocity of gas particle is

$V_{\text{rms}} \propto T^{1/2}$

Thus,

$$Q = \frac{15}{28}\times\frac{5\times R}{2}\times(4T - T)$$

$$\Rightarrow Q = \frac{15}{28}\times\frac{5\times8.3}{2}\times(3T)$$

$\Rightarrow Q = 10000$ J = 10 kJ

Answer: (3).

16. Given $\lambda = 500$ nm, $d = 0.32$ mm, $\theta = 30°$

Path difference

$$d\sin\theta = n\lambda$$

$$0.32\times10^{-3}\sin30° = n\times500\times10^{-9}$$

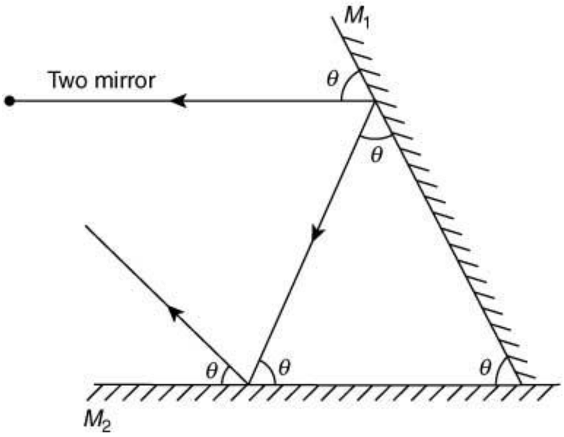
$$n = \frac{0.32\times10^{-3}\times\frac{1}{2}}{500\times10^{-9}}$$

$$\Rightarrow n = \frac{0.32\times10^{-3}}{1000\times10^{-9}}$$

Therefore, $n = 320$

Answer: (2).

17. Let θ be the angle between two mirrors.



As we know that, sum of angle of triangle = 180°

$$\theta + \theta + \theta = 180°$$
$$\Rightarrow 3\theta = 180°$$
$$\Rightarrow \theta = 60°$$

Answer: (2).

18. Let m be the mass of rod and ω be the angular speed.
Work done by gravity on rod from initial stale to final state.

$$W = mg\frac{l}{2}\sin 30°$$
$$= mg \times \frac{l}{4} = \frac{mgl}{4} \quad (1)$$

According to work energy theorem

$$W = \frac{1}{2}I\omega^2$$
$$= \frac{1}{2} \times \frac{1}{3}ml^2 \times \omega^2 \quad (2) \left[\text{Since } I = \frac{1}{3}ML^2\right]$$

From Eq. (1) and (2), we get

$$\frac{mgl}{4} = \frac{1}{2}\frac{ml^2}{3} \times \omega^2$$
$$\Rightarrow \omega^2 = \frac{3g}{2l}$$
$$\Rightarrow \omega = \sqrt{\frac{3g}{2l}}$$
$$= \sqrt{\frac{3 \times 10}{2 \times 0.5}}$$
$$\Rightarrow \omega = \sqrt{30} \text{ rad/sec}$$

Answer: (2).

19. Resistor code for Green = 5
Resistor code for Orange = 3
Resistor code for Yellow = 10^4
Tolerance for Golden = ±5%
Therefore, G O Y Golden = 5 × 10 + × 10^4 ± 5%
= 53 × 10^4 ± 5%

$= 530\ \text{k}\Omega \pm 5\%$

Answer: (1).

20. Let R be the radius of loop and r be the radius of circular coil of N turn.

For loop,

$$L = 2\pi R \qquad (1)$$

For coil,

$$L = N \times 2\pi r \qquad (2)$$

Fron Eq.(1) and Eq. (2), we get

$$R = Nr$$

Magnetic filed at the centre of loop is

$$B_L = \frac{\mu_0 i}{2R}$$

Magnetic field at the centre of coil is

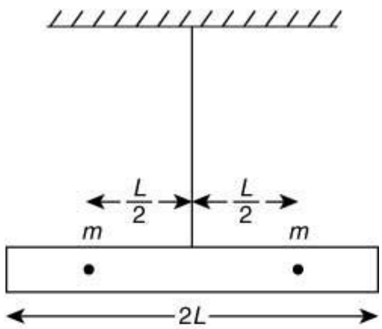
$$B_C = \frac{\mu Ni}{2r}$$

$$\Rightarrow R\frac{B_L}{B_C} = \frac{\frac{\mu_0 i}{2R}}{\frac{\mu_0 Ni}{2r}} = \frac{2r}{2RN} = \frac{r}{rN.N}$$

$$\Rightarrow \frac{B_L}{B_C} = \frac{1}{N^2}$$

Answer: (4).

21. Let ω be the frequency of torsional oscillation.



$$\omega = \sqrt{\frac{K}{I}}$$

$$\omega_1 = \sqrt{\frac{K}{\frac{M(2L)^2}{12}}} \Rightarrow \sqrt{\frac{3K}{ML^2}}$$

$$\omega_2 = \frac{\sqrt{k}}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}} = \sqrt{\frac{k}{2^2\left(\frac{M}{3} + \frac{m}{2}\right)}}$$

$\omega_2 = 0.8\ \omega_1$

$$\Rightarrow \frac{m}{M} = \frac{3}{8} = 0.37$$

Answer: (3).

22. Let Q be charge is distribution within a sphere of radius R

$$Q = \int \rho dV$$

$$= \int \rho(r)(4\pi r^2 dr)$$

$$= \int_0^R \frac{A}{r^3} e^{\frac{-2r}{a}} 4\pi r^2 dr$$

$$= 4\pi A \int_0^R e^{-2r/a} dr \quad = 4\pi a\left(-\frac{a}{2}\right)[e^{-2R/G} - e^0]$$

$$= 4\pi a\left(\frac{-a}{2}\right)[e^{-2R/a} - 1]$$

$$\Rightarrow Q = 2\pi a A[1 - e^{-2R/a}]$$

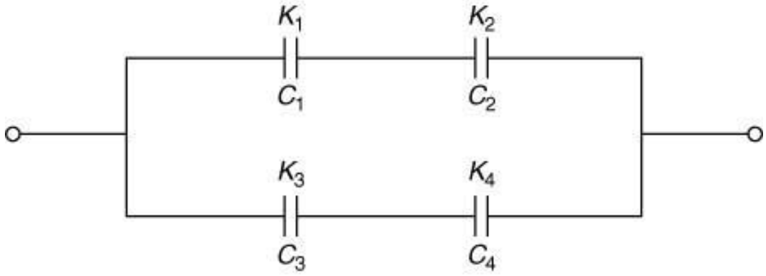
$$\Rightarrow Q = 2\pi Aa - 2\pi aAe^{-2R/a}$$

$$\Rightarrow \frac{2R}{a} = \log \frac{1}{1 - \frac{Q}{2\pi aA}}$$

$$\Rightarrow R = \frac{a}{2} \log\left[\frac{1}{1 - \frac{Q}{2\pi aA}}\right]$$

Answer: (2).

23. The arrangement of capacitor shown in the figure



$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\dfrac{K_1 \varepsilon_0 \frac{L}{2} \times 2}{d/2} \cdot \dfrac{K_2\left[\varepsilon_0 \frac{L}{2} \times L\right]}{d/2}}{(K_1 + K_2)\left[\dfrac{\varepsilon_0 \frac{L}{2} \times L}{d/2}\right]}$$

$$C_{12} = \frac{K_1 K_2}{K_1 + K_2} \frac{\varepsilon_0 L^2}{d}$$

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{\dfrac{K_3\left[\varepsilon_0 \frac{L}{2} \times L\right]}{d/2} \cdot \dfrac{K_4\left[\varepsilon_0 \frac{L}{2} \times L\right]}{d/2}}{(K_3 + K_4)\left[\dfrac{\varepsilon_0 \frac{L}{2} \times L}{d/2}\right]}$$

$$= \frac{K_3 K_4}{K_3 + 4n} \frac{\varepsilon_0 L^2}{d}$$

$C_{eq} = C_{12} + C_{34}$

$$= \left[\frac{K_1K_2}{K_1+K_2} + \frac{K_3K_4}{K_3+K_4}\right]\frac{\varepsilon_0 L^2}{d} \qquad (1)$$

If $K_{eq} = k$, then $C_{eq} = \dfrac{k\varepsilon_0 L^2}{d}$ (2)

$$K_{eq} = \frac{K_1K_2(K_3+K_4) + K_3K_4(K_1+K_2)}{(K_1+K_2)(K_3+K_4)}$$

This Answer does not match with any given options.

Answer: (*).

24. Least count $= \dfrac{\text{Pitch}}{\text{Number of division}}$

$= \dfrac{0.5}{100}$

$= 0.5 \times 10^{-2}$ mm

Positive error $= 3 \times 0.5 \times 10^{-2}$

$= 1.5 \times 10^{-2}$

$= 0.015$ mm

Reading = MSR + CSR – Positive Error

$= 5.5 \times (48 \times 0.5 \times 10^{-2}) - 0.015$

$= 5.5 + 0.24 - 0.015$

$= 5.725$ mm

Answer: (3).

25. f is frequency of sound produced by a flute

$$f = 2\left(\frac{v}{2l}\right)$$

$$= 2\left(\frac{330}{2\times 0.5}\right) \qquad \text{(speed of sound = 330 m/s)}$$

$= 660$

Velocity of observer, $v_o = 10$ km/h

$$= 10 \times \frac{5}{18}$$

$$= \frac{25}{9} \text{ m/s}$$

Frequency absorb by observer $f' = \left[\dfrac{v + v_o}{v}\right] f$

$$v' = \left[\frac{\frac{25}{9} + 330}{330}\right] 660$$

$= 335.56 \times 2 = 671.12 \approx 666$ Hz

Answer: (1).

26. Let time taken by car A is t'

$v_{\text{B}} = a_2(t' + t)$

$v_{\text{A}} - v_{\text{B}} = (a_1 - a_2)t' - a_2 t$

$v = (a_1 - a_2)t' - a_2 t \qquad (1)$

$S_{\text{B}} = S_{\text{A}}$

$\Rightarrow ut + \frac{1}{2}a_1 t'^2 = ut - \frac{1}{2}a_2(t' + t)^2$

$\Rightarrow 0 + \frac{1}{2}a_1 t'^2 = 0 + \frac{1}{2}a_2(t' + t)^2$

$\Rightarrow \frac{1}{2}a_1 t'^2 = \frac{1}{2}a_2(t' + t)^2$

$\Rightarrow \sqrt{a_1 t'} = \sqrt{a_2}(t' + t)$

$\Rightarrow \sqrt{a_1 t'} = \sqrt{a_2}t' + \sqrt{a_2}t$

$\Rightarrow \sqrt{a_2}t = (\sqrt{a_1} - \sqrt{a_2})t'$

Put this value in Eq. (1), we get

$$v = (a_1 - a_2)\frac{\sqrt{a_2 t}}{\sqrt{a_1} - \sqrt{a_2}} - a_2 t$$

$$= (\sqrt{a_2} + \sqrt{a_2})\sqrt{a_2}t - a_2 t$$

$$\Rightarrow v = \sqrt{a_1 a_2}t + a_2 t - a_2 t$$

Therefore, $v = \sqrt{a_1 a_2}t$

Answer: (3).

27. Given, magnetic field associated with light wave is

$B = B_0[\sin(3.14 \times 10^7 c)t + \sin(6.28 \times 10^7 c)t] \qquad (1)$

where c is the speed of light

In above wave equation, there are two electromagnetic waves with different frequency.

To get maximum kinetic energy consider the photon with higher frequency

$B_1 = B_0\sin(\pi \times 10^7\, c)t$

$v_1 = \frac{10^7}{2}c$

$B_2 = B_0\sin(2\pi \times 10^7\, c)t$

$v_2 = 10^7 c$

$v_2 > v_1$

Kinetic energy of photoelectron will be a maximum for photon of higher energy.

$E = \phi + \text{KE} \qquad [E_{\text{Ph}} = hf]$

$hf = \phi + \text{KE}_{\text{max}}$

$E_{\text{Ph}} = hf = 6.6 \times 10^{-34} \times 10^7 \times 3 \times 10^6$

$= 6.6 \times 3 \times 10^{-19}$

$= \frac{6.6 \times 3 \times 10^{-19}}{1.6 \times 10^{-19}}\text{eV} = 12.37 \text{ eV}$

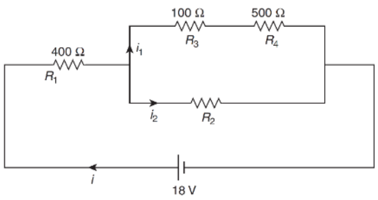
$K.E_{\text{max}} = E_{\text{Ph}} - \phi$

$= 12.375 - 4.7 = 7.675$

≈ 7.7 eV

Answer: (4).

28.



The voltage across resistance R_4 is $V_4 = 5$V

Thus, current $i_1 = 5/500 = 0.01$ A

Voltage across resistance R_3 is

$$V_3 = 0.01 \times 100 = 1 \text{ V}$$

Thus, total voltage = $V_1 + V_3 + V_4$

(since voltage across R_2 is same as voltage across R_3 and R_4)

$\Rightarrow 18 = 1 + 5 + V_1 \Rightarrow V_1 = 12$ V

$\Rightarrow i = 12/400 = 0.03$ A

$\Rightarrow i_2 = i - i_1 = 0.03 - 0.01 = 0.02$ A

Therefore, $R_2 = \dfrac{V_2}{i_2} = \dfrac{6}{0.02} = 300\,\Omega$

Answer: (1).

29. Frequency (f) $= \dfrac{\text{velocity}}{\text{wavelength}}$

$$= \frac{c}{\lambda} = \frac{3\times10^8}{800\times10^{-9}}$$

$$= 3.75 \times 10^{14} \text{ Hz}$$

Usable frequency = 1% of f

$$= \frac{3.75\times10^{14}\times1}{100} = 3.75\times10^{12}\,\text{Hz}$$

Therefore, required number of channel $= \dfrac{3.75\times10^{12}}{6\times10^6}$

$$= 6.25 \times 10^5$$

Answer: (3).

30. Velocity co-ordinates of particle is obtained by differentiating the position co-ordinate with respect to time, Thus,

$$v_x = \frac{dx}{dt} = -a\omega\sin\omega t$$

$$v_y = \frac{dy}{dt} = a\omega\cos\omega t$$

$$v_z = \frac{dz}{dt} = a\omega$$

Hence, velocity of particle is

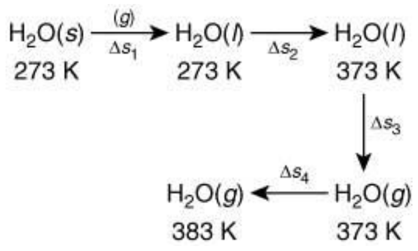
$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$= \sqrt{a^2\omega^2(\sin^2\omega t + \cos^2\omega t) + a^2\omega^2} = \sqrt{2}a\omega$$

Answer: (1).

Section: Chemistry

31.



$$\Delta S_1 = \frac{\Delta H_{\text{helium}}}{273} = \frac{334}{273} = 1.22$$

$$\Delta S_2 = 4.2\ln\left(\frac{363}{273}\right) = 1.19$$

$$\Delta S_3 = \frac{\Delta H_{\text{vap}}}{373} = \left(\frac{2491}{373}\right) = 6.67$$

$$\Delta S_4 = 2\ln\left(\frac{383}{373}\right) = 0.05$$

Therefore, $\Delta S_{\text{Total}} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4$

$\Delta S_{\text{Total}} = 9.14\ \text{kJ kg}^{-1}\text{K}^{-1}$

Answer: (4).

32. $2C_{57}H_{110}O_6(s) + 163\,O_2(g) \rightarrow 114\,CO_2(g) + 110\,H_2O(l)$

Form the above reaction, 2 moles of $C_{57}H_{110}O_6$ produces 110 moles of water.

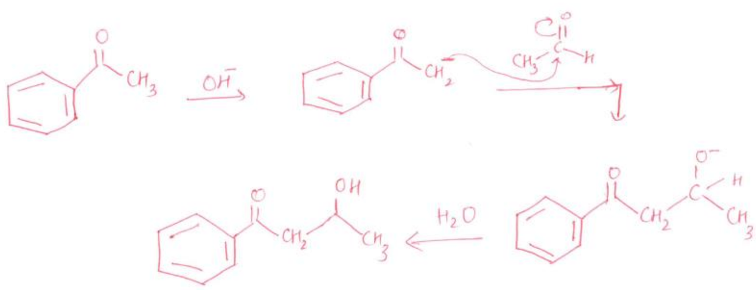
Moles of $C_{57}H_{110}O_6(s) = \frac{445}{890} = 0.5$ moles

Therefore, 0.5 mol of $C_{57}H_{110}O_6$ produces $n_{H_2O} = \frac{110}{4} = \frac{55}{2}$ moles of water.

Or $m_{H_2O} = \frac{55}{2} \times 18 = m_{H_2O} = 49.5$ g

Answer: (3).

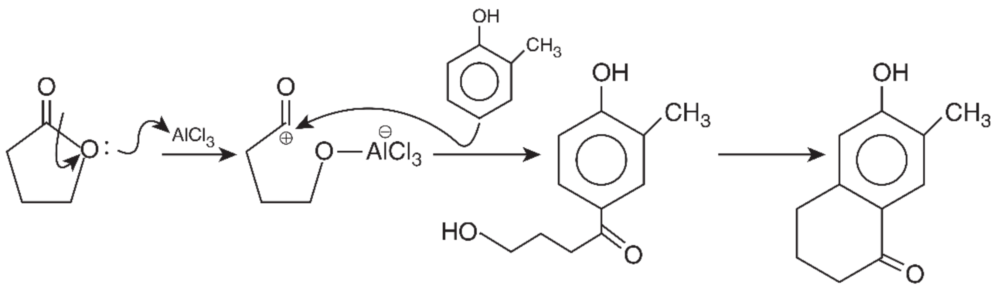
33.



Answer: (3).

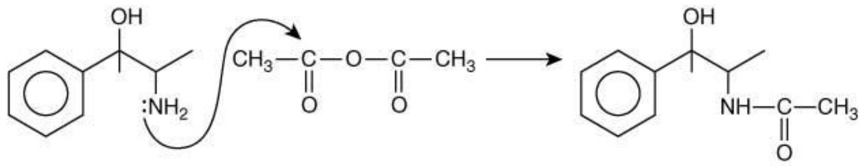
34. Concentration of nitrate > 50 ppm in drinking water causes methemoglobinemia.
Answer: (3).

35. The reaction involved is



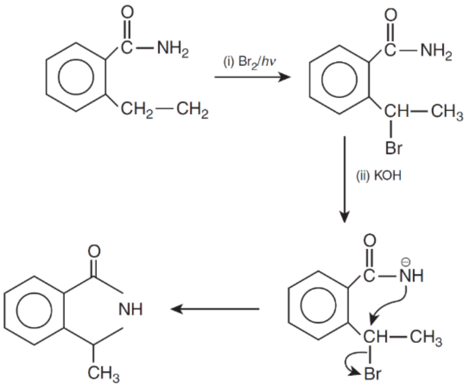
Answer: (3).

36. The reaction involved is



Answer: (4).

37.



Answer: (3).

38. Correct match of the above is

Item I	Item II
(A) Benzaldehyde	(R) Adsorbate
(B) Alumina	(Q) Adsorbent
(C) Acetonitrile	(P) Mobile phase

Answer: (2).

39. The metal that forms nitride by reacting directly with N_2 of air is Lithium.
$6Li + N_2 \rightarrow 2Li_3N$
Answer: (2).

40. Sulphide sol is negative charged colloid, so cation with maximum charge will be most effective for coagulation $Al^{3+} > Ba^{2+} > Na^{+}$ (coagulating power).
Therefore, aluminium will be the most effective for coagulation of sulphide sol
Answer: (2).

41. Since CN^{-} is a strong field ligand, therefore, $K_3[Co(CN)_6]$ is a low spin complex compounds and have highest crystal field splitting energy (Δ_o).
Answer: (4).

42. The pH of rain water normally is 5.6 approximately.
Answer: (1).

43. $$A_2(g) + B_2(g) \underset{}{\overset{K_1}{\rightleftharpoons}} 2AB(g) \qquad (1)$$

Reversing and multiplying Eq. (1) by 3, we get

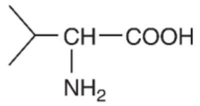
$$6AB(g) \overset{K_2}{\rightleftharpoons} 3A_2(g) + 3B_2(g) \qquad (2)$$

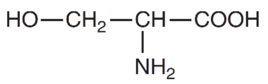
Equilibrium of this equation is $K_2 = \left(\frac{1}{K_1}\right)^3$

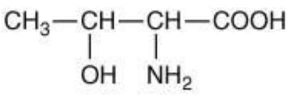
Therefore, $K_2 = K_1^{-3}$

Answer: (3).

44.

Valine 

Serine 

Threonine 

Therefore, the above given amino acid sequence is Val – Ser – Thar.

Answer: (1).

45. For the reaction 2A + B → products
Rate equation is $R = k\,[A]^x[B]^y$
On doubling the concentrations of both the reactants, the rate will be increased 8 times, that is

$$8 = k\,[2A]^x[2B]^y$$

Or $$8 = k\,2^{x+y}\,[A]^x[B]^y \qquad (1)$$

On doubling the concentration of A alone, the rate will be increased 2 times, that is

$$2 = k\,2^x\,[A]^x[B]^y \qquad (2)$$

Dividing Eq. (1) by (2), we get

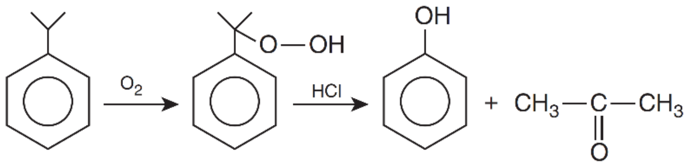
$$x + y = 3$$
$$x = 2$$

$\Rightarrow \quad x = 1, y = 2$
Therefore,

Order w.r.t. A = 1
Order w.r.t B = 2

Answer: (2).

46. This method is an industrial method for preparation of phenol starting from cumene.



Answer: (3).

47. 2,4-DNP test is given by aldehyde or ketone. Iodoform test is given by compound having $-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$ or $-\overset{\overset{\displaystyle OH}{|}}{CH}-CH_3$ group, and ethanal is the only aldehydes which respond to iodoform test. Therefore, none of the above aldehydes will respond to iodoform test. In option (2), nitrogen does not have any hydrogen atom attached to it, therefore this compound is the correct structural formula of compounds X.

Answer: (2).

48. We know

$$\Delta G° = -RT \ln K \qquad (1)$$

$$\Delta G° = -nFE°_{\text{cell}} \qquad (2)$$

From Eq. (1) and (2), we get

$$\ln K = \frac{nFE°}{R \times T} = \frac{2 \times 96000 \times 2}{8 \times 300}$$

$$\ln K = 160$$

or

$$K = e^{160}$$

Answer: (4).

49. Temporary hardness of water is due to the bicarbonates of calcium and magnesium.

Answer: (3).

50. Bond order of NO is 2.5 and paramagnetic in nature and NO^+ has bond order of 3 and diamagnetic in nature. Whereas N_2 and O_2 are paramagnetic in nature.

Answer: (1).

51. (a) An electron in an orbital of higher angular momentum stays away from the nucleus than an electron in the orbital of lower angular momentum.

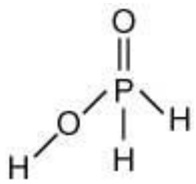
(c) According to wave mechanics, the ground state angular momentum $\frac{h}{2\pi}$.

Answer: (3).

52. The compound which do not have $(4n + 2)\pi$ electrons is not an aromatic compound. Compound in option (1) has $4n\pi$ electrons. Hence it is not aromatic.

Answer: (1).

53. H_3PO_2 is good reducing agent due to presence of two P-H bonds. Oxidation state of P is +1 and +3, which can further be oxidized to a higher oxidation state.



Answer: (3).

54. According to the Ellingham diagram Al can reduce ZnO.

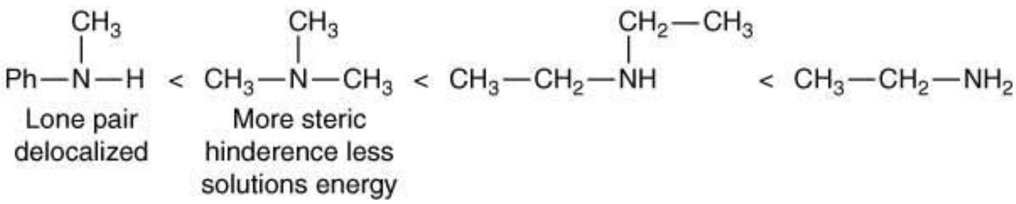
$$3ZnO + 2Al \rightarrow 3Zn + Al_2O_3$$

Answer: (1).

55. Since Zn is not a considered as transition element, therefore, transition element having lowest atomization energy out of Cu, V and Fe is Cu.

Answer: (4).

56.



Answer: (2).

57. Second electron given enthalpy is always positive for every element due to the repulsion experience by the upcoming electron.

$$O^{-}(g) + e^{-} \rightarrow O^{2-}(g)$$

$$\Delta_{eg}H = +ve.$$

Answer: (4).

58. For FCC unit cell $Z = 4$

$$\rho = \frac{63.5 \times 4}{6 \times 10^{33} \times x^3 \times 10^{-24}} \text{ g/cm}^3$$

$$\rho = \frac{63.5 \times 4 \times 10}{6} \text{g/cm}^3$$

$$\rho = \frac{423.33}{x^3} \cong \left(\frac{422}{x^3}\right)$$

Answer: (4).

59. We know

$$\Delta T_f = K_f m$$

$$10 = 1.86 \times \frac{62/62}{W \text{ kg}}$$

$$W = 0.186 \text{ kg}$$

Therefore, the amount of water separated as ice is $(\Delta W) = (250 - 186) = 64$ g

Answer: (3).

60. Order of λ_{abs} is $L_3 > L_1 > L_2$

So Δ_o order will be $L_2 > L_1 > L_3$

We know

$$\Delta_o \propto \frac{1}{\lambda_{abs}}$$

Therefore, order of ligand strength will be $L_2 > L_1 > L_3$.

Answer: (1).

Section: Mathematics

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61. $T_n = \frac{(3+(n-1)3)(1^2+2^2+\cdots+n^2)}{(2n+1)}$

$$T_n = \frac{3\times\frac{n(n+1)(2n+1)}{6}}{(2n+1)} = \frac{n^2(n+1)}{2}$$

$$S_{15} = \frac{1}{2}\sum_{n=1}^{15}(n^3+n^2)$$

$$= \frac{1}{2}\left[\frac{(15(15+1))}{2}\right]^2 + \frac{15\times16\times31}{6}$$

$$= \frac{1}{2}\left[\frac{225+15}{2}\right]^2 + \frac{7440}{6}$$

$$= \frac{1}{2}\left[\frac{57600}{4}\right] + \frac{7440}{6}$$

= 7200 + 620

= 7820

Answer: (4).

62. $\lim_{x\to0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$

$x \to 0^\infty$

$[x] = -1 \Rightarrow \lim_{x\to0^-}\frac{x(-x-1)\sin(-1)}{-x} = -\sin 1$

Answer: (1).

63. Given $f(xy) = f(x).\ f(y)$

Since, $f(0) \neq 0$, thus $f(0) = 1$

So, $f(x) = 1$

Now, $\frac{dy}{dx} = f(x) = 1 \Rightarrow y = x + c$

At $x = 0$, $y = 1$, $c = 1$

Therefore, $y = x + c$

Hence, $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \frac{1}{4}+1+\frac{3}{4}+1$

$$= \frac{1}{4}+\frac{3}{4}+2$$

= 1 + 2 = 3

Answer: (1).

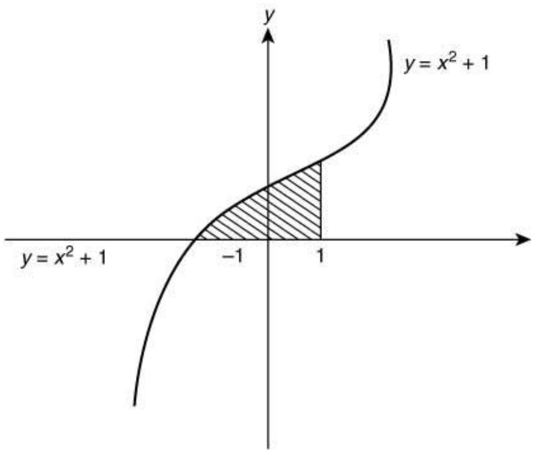
64. Given,

$x = \sin^{-1}(\sin 10) = 3\pi - 10$
$y = \cos^{-1}(\cos 10) = 4\pi - 10$
Therefore,
$y - x = 4\pi - 10 - 3\pi + 10$
$\Rightarrow y - x = \pi$
Answer: (4).

65. We have,
$\sin x - \sin 2x + \sin 3x = 0$
$\Rightarrow (\sin x + \sin 3x) - \sin 2x = 0$
$\Rightarrow 2\sin x \cos x = \sin 2x = 0$
$\Rightarrow \sin 2x (2 \cos x - 1) = 0$
$\Rightarrow \sin 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$
$\Rightarrow x = 0, \frac{\pi}{3}$
Hence, two values of x are possible.
Answer: (4).

66. $z_0 = \omega$ or ω^2 (where ω is non-real cube roof of unify)
$\Rightarrow z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93} \Rightarrow \omega^{81} = \omega^{93} = 1$
$\Rightarrow z = 3 + 3i$
Therefore, $\text{Arg}(z) = \frac{\pi}{4}$
Answer: (1).

67. The graph is as follows:



Required area = $\int_{-1}^{0} (-x^2 + 1)dx + \int_{0}^{1} (x^2 + 1)dx = 2$
Answer: (2).

68. $P_1 = x - 4y + 72 - g = 0$
$P_2 = 3x - 5y - h = 0$
$P_3 = -2x + 5y - 9z - k = 0$
Here $\Delta = 0$
Therefore, $2P_1 + P_2 + P_3 = 0$ when $2g + h + k = 0$
Answer: (3).

69. We have, $(1 + t^6)^3 (1 - t)^{-3}$

$= (1 - t^{18} - 3t^6 + 3t^{12}) (1 - t)^{-3}$

Therefore, the coefficient of t^4 in $(1 - t)^{-3} = {}^{3+4-1}C_4 = {}^{6}C_4 = 15$

Answer: (2).

70. We have,

$$m^2 - 16 > 0 \Rightarrow m \in (-\infty, -4) \cup (4, \infty)$$

$$\Rightarrow 1 < \frac{-(-m)}{2} < 5 \Rightarrow 2 < m < 10$$

$$\Rightarrow 1 - m + 4 > 0 \quad \text{and} \quad 25 - m(5) + 4 > 0$$

$$\Rightarrow m < 5,\ m < \frac{29}{5}$$

$$\Rightarrow\ m \in (4, 5)$$

Answer: (2).

71. Let A(α, 0) and B(0, β) be the vectors of the given Δ AOB.

where, $|\alpha\beta| = 100$

Therefore, number of triangles = 4 × (Number of divisors of 100)

= 4 × 9 = 36

Answer: (3).

72. It is given that,

$$a = A + 6d$$
$$b = A + 10d$$
$$c = A + 12d$$

Since, a, b, c are in G.P.

$$r = \frac{b}{a} = \frac{c}{b} = \frac{A+10d}{A+6d} = \frac{A+12d}{A+10d} = \frac{2d}{4d} = \frac{1}{2}$$

$$\Rightarrow\ r = \frac{1}{2} = \frac{1}{r^2} = 4$$

Answer: (4).

73. $S[\sim(\sim p \vee q) \wedge (p \wedge r)] \cap (\sim p \wedge r)$

$\equiv [(p \wedge \sim p) \vee (p \wedge r)] \wedge (\sim q \vee r)$

$\equiv [p \wedge (\sim q \vee r)] \wedge (\sim p \wedge r)$

$\equiv p \wedge (\sim q \wedge r)$

$\equiv (p \wedge r) \wedge \sim q$

Answer: (3).

74. Perpendicular to plane containing straight lines

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{2} \text{ and } \frac{x}{4} = \frac{y}{2} = \frac{z}{3}$$

By considering matrix,

$$\begin{vmatrix} i & j & k \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix}$$

and, vector perpendicular to the vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $8\hat{i}-\hat{j}-10\hat{k}$ is $26\hat{i}-52\hat{j}+26\hat{k}$

Hence, required plane is $26x - 52y + 26z = 0 \Rightarrow x-2y+z=0$

Answer: (1).

75. $$\sum(x_i+1)^2 = 9n \qquad (1)$$

$$\sum(x_i-1)^2 = 5n \qquad (2)$$

From Eq. (1) and (2), we get

$$1+11 \Rightarrow \sum(x_i^2+1) = 7n \Rightarrow \frac{\sum x_i^2}{x} = 6$$

Now, subtracting Eq. (2) from Eq. (1), we have

$$4\Sigma xi = 4n \Rightarrow \frac{\sum x_i}{n} = 1$$

So, deviation (σ) = 6 – 1 = 5

Therefore, standard deviation $(\sigma^2) = \sqrt{5}$

Answer: (2).

76. Given,

$$A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

$$|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$= e^{-t}[5\cos^2 t + 5\sin^2 t]\ \forall\ t \in R$$

$$= 5e^{-t} \neq 0\ \forall\ t \in R$$

Hence, A is invertible for all $t \in R$.

Answer: (1).

77. Given,

$$f(x) = \int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}\,dx$$

$$= \int \frac{5x^8+7x^6}{x^{14}\left(\frac{1}{x^5}+\frac{1}{x^7}+2\right)^2}\,dx$$

$$= \int \frac{\left(\frac{5}{x^6}+\frac{7}{x^8}\right)}{\left[\frac{1}{x^5}+\frac{1}{x^7}+2\right]^2}\,dx$$

Suppose $\frac{1}{x^5}+\frac{1}{x^7}+2 = t$

$$\Rightarrow \left(-\frac{5}{x^6}-\frac{7}{x^8}\right)dx = -dt$$

Thus,

$$f(x) = \int \frac{-dt}{t^2} = \frac{1}{t} + C$$

$$\Rightarrow f(x) = \frac{1}{\left(\frac{1}{x^5} + \frac{1}{x^7} + 2\right)} + C$$

$$= \frac{x^7}{x^2 + 1 + 2x^2} + C$$

Since, $f(0) = 0 \Rightarrow C = 0$

Hence, $f(1) = \frac{(1)^7}{1^2 + 1 + 2 \times 1^2} = \frac{1}{4}$

Answer: (4).

78. Given,

$$|f(x) - f(y)| \leq 2|x - y|^{3/2}$$

Dividing both sides by $|x - y|$, we have

$$\frac{|f(x) - f(y)|}{|x - y|} \leq \frac{2|x - y|^{3/2}}{|x - y|}$$

$$\Rightarrow \left|\frac{f(x) - f(y)}{x - y}\right| \leq 2.|x - y|^{1/2}$$

By Applying limit $x \to y$

$$|f'(y)| \leq 0$$

$$\Rightarrow f'(y) = 0$$

$$\Rightarrow f(y) = c$$

$$\Rightarrow f(x) = 1$$

Hence,

$$\int_0^1 f^2(x)dx = \int_0^1 1.dx = 1$$

Answer: (1).

79. Given $x = 3 \tan t, y = 3 \sec t$

Differentiating both the sides w.r.t. t, we get

$$\frac{dx}{dt} = 3\sec^2 t \text{ and } \frac{dy}{dt} = 3\sec t \tan t$$

Thus,

$$\frac{dy}{dx} = \frac{3\sec t \tan t}{3\sec^2 t} = \frac{\tan t}{\sec t} = \sin t \qquad (1)$$

Now, again differentiating both the sides of Eq. (1), we get

$$\frac{d^2 y}{dx^2} = \cos t \frac{dt}{dx}$$

$$= \frac{\cos t}{3\sec^2 t} \qquad \left(\frac{dx}{dt} = 3\sec^2 t\right)$$

$$= \frac{\cos^3 t}{3}$$

$$\left(\frac{d^2 y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{3 \times 2\sqrt{2}} = \frac{1}{6\sqrt{2}}$$

Hence, $$\left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{6\sqrt{2}}$$

Answer: (2).

80.

a_1	a_2	a_3

Number of numbers = $5^3 - 1$ (000 is not included)

a_4	a_1	a_2	a_3

2 ways for a_4

Total numbers which can be formed = 2×5^3

Hence, required number = $5^3 + 2 \times 5^3 - 1$

$= 125 + 2 \times 125 - 1$

$= 125 + 250 - 1$

$= 125 + 249$

$= 374$

Answer: (1).

81. We have,

$x^2 + y^2 - 16x - 20y + 164 = r^2$ (1)

$(x - 8)^2 + (y - 10)^2 = r^2$

$(x - 4)^2 + (y - 7)^2 = (6)^2$

Since, they intersect at two point

A(8, 10), $R_1 = r$

B(4, 7), $R_2 = 6$

$|R_1 - R_2| < AB < R_1 + R_2$

$|r - 6| < 5 < r + 6$

$r + 6 > 5 \Rightarrow r > -1$ (1)

$|r - 6| < 5 \Rightarrow r - 6 \in (-5, 5)$

$r \in (1, 11)$ (2)

From Eq. (1) and Eq. (2), we have

$r \in (1, 11)$

Therefore, $1 < r < 11$

Answer: (4).

82. The equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Here, $2a = 4 \Rightarrow a = 2$

$$\frac{x^2}{4} - \frac{y^2}{b^2} = 1$$

Since, line passes through (4, 2), $\frac{16}{4} - \frac{4}{b^2} = 1$

$$\Rightarrow 4 - \frac{4}{b^2} = 1 \Rightarrow b^2 = \frac{4}{3}$$

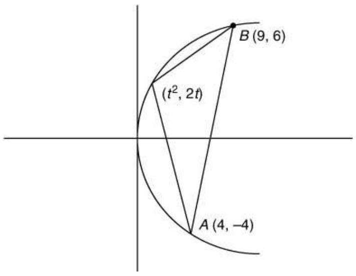
Hence,

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{\frac{4}{3}}{4}}$$

$$=\sqrt{1+\frac{1}{3}}=\frac{2}{\sqrt{3}}$$

Answer: (4).

83.



Area = $5|t^2 - t - 6|$

$$=5\left|\left(t-\frac{1}{2}\right)^2-\frac{25}{4}\right| \text{ is maximum if } t=\frac{1}{2}$$

Hence, $5\left|\left(\frac{1}{2}-\frac{1}{2}\right)^2-\frac{25}{4}\right|$

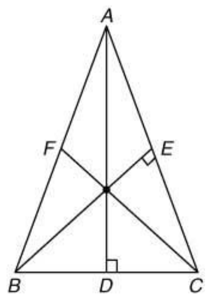
$$=5\left|\frac{1}{4}+\frac{1}{4}-\cancel{2}.\frac{1}{\cancel{2}}.\frac{1}{2}-\frac{25}{4}\right|$$

$$=5\left|\frac{2}{4}-\frac{1}{2}-\frac{25}{4}\right|$$

$$=5\left|\frac{2-2-25}{4}\right|=5\left|\frac{-25}{4}\right|=31\frac{1}{4} \text{ sq. units}$$

Answer: (1).

84.



Equation of AB is

$3x + 2y + 6 = 0$ (1)

Equation of AC is

$4x + 5y - 20 = 0$ (2)

Orthocenter is (1, 1).

Equation of CF is,

Line perpendicular to 4x + 5y – 20 = 0 and passes through (1, 1) is,

$5x - 4y = 1$ (3)

Similarly,

Equation of BC is,

Line perpendicular to $3x - 2y + 6 = 0$ and passes through (1, 1) is

$$2x + 3y = 5 \qquad (4)$$

Solving Eq. (1) and Eq. (4), we get

$$(35/2, -10)$$

Solving Eq. (2) and Eq. (3), we get

$$(-13, -33/2)$$

Therefore, side BC is $y + 10 = \frac{13}{61} \times (-35/2) = 26x - 122y - 1675 = 0$

Answer: (4).

85. Let,

E = Event of drawing a red ball in second draw

E_1 = Event of drawing a red ball and adding of green ball in urn

E_2 = Event of drawing a green ball and adding of red ball in urn.

Hence,

Probability that the second ball is red is

$$P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$$

$$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7}$$

$$= \frac{20}{49} + \frac{12}{49} = \frac{32}{49}$$

Answer: (4).

86. Line $x = ay + b$, $z = cy + d \Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$

Line $x = a'z + b'$, $y = c' + d' = \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1}$

Given both the lines are perpendicular, we have

$aa' + c' + c' = 0$

Answer: (4).

87. We have,

$$\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$$

$$\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$$

Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = |\vec{a}|$

$$\Rightarrow b_1 + b_2 = 2 \qquad (1)$$

Since, $(\vec{a} + \vec{b})$ is perpendicular to $\vec{c}$

Hence, $(\vec{a} + \vec{b}).\vec{c} = 0$

$$\Rightarrow 5b_1 + b_2 = -10 \qquad (2)$$

From Eq. (1) and Eq. (2), we get

$b_1 = -3$ and $b_2 = 5$

Hence,

$$\left|\vec{b}\right| = \sqrt{b_1^2 + b_2^2 + 2}$$
$$= \sqrt{(-3)^2 + (5)^2 + 2}$$
$$= \sqrt{36} = 0$$

Answer: (2).

88. Given roots are rational, so D must be perfect square.

$D = 121 - 24\alpha = k^2$

$$\left.\begin{array}{l} \alpha = 1 \quad \Rightarrow \quad k \notin I \\ \alpha = 2 \quad \Rightarrow \quad k \notin I \\ \alpha = 3 \quad \Rightarrow \quad k \in I \\ \alpha = 4 \quad \Rightarrow \quad k \in I \\ \alpha = 5 \quad \Rightarrow \quad k \in I \end{array}\right\}$$

Therefore, the number of all possible values of α is 3.

Answer: (1).

89. Given,

$$f(x) = \frac{2x}{x-1}$$
$$\Rightarrow f(x) = 2\left(1 + \frac{1}{x-1}\right)$$
$$f'(x) = -\frac{2}{(x-1)^2}$$

Hence, f is injective but not surjective.

Answer: (4).

90. Given,

$$\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}}\, d\theta = 1 - \frac{1}{\sqrt{2}}$$

It can be also written as

$$\frac{1}{\sqrt{2K}} \int_0^{\pi/3} \frac{\sin\theta}{\sqrt{\cos\theta}}\, d\theta = 1 - \frac{1}{\sqrt{2}}$$

Let, $\cos\theta = t$

$\Rightarrow -\sin\theta d\theta = dt$

Hence,

$$\frac{1}{\sqrt{2k}} \int_0^{\pi/3} \frac{-dt}{\sqrt{t}} = 1 - \frac{1}{\sqrt{2}}$$
$$\Rightarrow \left(2.\frac{2}{\sqrt{2}}\right) = \sqrt{2k}\left(1 - \frac{1}{\sqrt{2}}\right)$$

Hence, $k = 2$

Answer: (4).