JEE Main 2019 Paper 1
January 10, Shift 1
Section: Physics

1. Wavelength is given by

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$$

Therefore, required energy is

$$\text{K.E.} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\frac{p^2}{m} = \frac{(h/\lambda)^2}{2m}$$

$$= \frac{(6.6\times10^{-34})^2}{(7.5\times10^{-12})(2\times9.1\times10^{-31})} = 25\,\text{keV}$$

Answer: (4).

2. Consider a strip of r and thickness dr. Force for strip is given as

$$dF = \frac{F}{\pi R^2}\times 2\pi r\, dr$$

$$\Rightarrow dF = \frac{2Fr\, dr}{R^2}$$

Now, $F_\mu = \mu dF$

$$\Rightarrow F_\mu = \mu \times \frac{2Fr}{R^2}dr$$

$$\Rightarrow d\tau = F\mu \times r$$

$$\Rightarrow d\tau = \frac{2F\mu}{R^2}r^2 dr$$

Integrating both the sides, we get

$$\tau = \int_0^R \frac{2F\mu}{R^2}r^2 dr$$

$$\Rightarrow \tau = \frac{2F\mu}{R^2}\int_0^R r^2 dr$$

$$\Rightarrow \tau = \frac{2F\mu}{R^2}\left[\frac{r^3}{3}\right]_0^R$$

$$= \frac{2F\mu}{R^2}\frac{R^3}{3}$$

$$\Rightarrow \tau = \frac{2F\ \mu R}{3}$$

Answer: (4).

3. Let at x distance it will show no deflection then current

$$i_{AB} = \frac{\varepsilon}{13r}$$

In case of no deflection $i\left(\frac{x}{L}\cdot 12r\right) = \frac{\varepsilon}{2}$

$$\Rightarrow \frac{\varepsilon}{13r}\left[\frac{x}{L}\cdot 12r\right] = \frac{\varepsilon}{2}$$

$$\Rightarrow x = \frac{13L}{24}$$

Answer: (3).

4. Let h_r and h_R be the height of transmitter tower and height of receiver respectively.
Maximum distance upto which signal can be broadcasted is

$$d_{max} = \sqrt{2Rh_r} + \sqrt{2Rh_R}$$

$$\Rightarrow d_{max} = \sqrt{2R}(\sqrt{h_r} + \sqrt{h_R})$$

$$= \sqrt{2 \times 6.4 \times 10^6}(\sqrt{104} + \sqrt{40})$$

$$\Rightarrow d_{max} = 65 \text{ km}$$

Answer: (1).

5. Side of a cube is a

Coordinate of point $m\left(\frac{a}{2}, 0, \frac{a}{2}\right)$

$$\vec{r}_m = \frac{a}{2}\hat{i} + 0\hat{j} + \frac{a}{2}\hat{k}$$

Coordinate of point $n\left(0, \frac{a}{2}, \frac{a}{2}\right)$

Thus, $$\vec{r}_n = 0 + \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$$

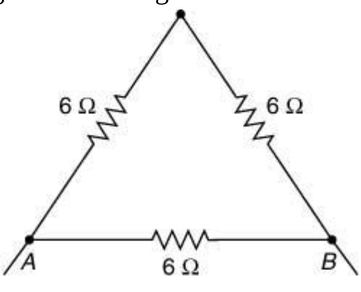
$$\vec{r}_m - \vec{r}_n = \left(\frac{a}{2}\hat{i} + \frac{a}{2}\hat{k}\right) - \left(\frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}\right)$$

$$= \frac{a}{2}\hat{i} + \frac{a}{2}\hat{k} - \frac{a}{2}\hat{j} - \frac{a}{2}\hat{k}$$

$$= \frac{a}{2}(\hat{i} - \hat{j})$$

Answer: (3).

6. Given resistance of metallic wire = 18 Ω
It is bended into equilateral triangle and the length of each side is same with resistance 6 Ω.



Now, R_{eq} between any two vertex is

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{6} = \frac{1+2}{12}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{3}{12}$$

$$\Rightarrow R_{eq} = \frac{12}{3} \Rightarrow R_{eq} = 4\ \Omega$$

Answer: (1).

7. According to the Newton second law

$$F = ma$$

$$\Rightarrow R - mg = \frac{mg}{2}$$

$$\Rightarrow R = \frac{mg}{2} + mg$$

$$\Rightarrow R = \frac{3mg}{2}$$

Work done is

$$W = \vec{R} \cdot \vec{S}$$

$$= \left(\frac{3mg}{2}\right)\left(\frac{1}{2}gt^2\right)$$

$$\Rightarrow W = \frac{3mg^2t^2}{4}$$

Answer: (*).

8. Path difference is given by $d \sin \theta = n\lambda$

and $\theta = \frac{1}{40}$ rad

$$\Rightarrow \sin\theta = \theta = \frac{1}{40}$$

$$\Rightarrow d \times \theta = n\lambda$$

Now, $n = \frac{d\theta}{\lambda}$ $\Rightarrow n = \frac{0.1 \times \frac{1}{40}}{\lambda}$

When $\lambda = 380$ nm

$$n_1 = \frac{0.1}{40 \times 380} = \frac{2500}{380} = 6.578$$

When $\lambda_2 = 740$ nm

$$n_2 = \frac{0.1}{40 \times 740} = \frac{2500}{740}$$

$= 3.378$

Thus, $n = 4, 5, 6$

For $n = 4$, $\lambda = 625$ nm

For $n = 5$, $\lambda = 500$ nm

Answer: (1).

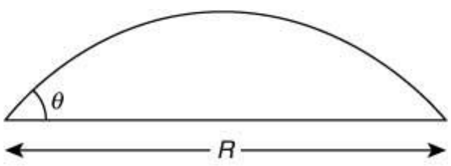
9. Area covered = πR^2

$$A \propto R^2$$

Thus, range

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R \propto u^2$$



So, area covered $\propto u^4$

$$A \propto u^4$$

Therefore, required ratio is $\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4}$

$$\Rightarrow \frac{A_1}{A_2} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$$

Answer: (1).

10. Density $= \frac{\text{Mass}}{\text{Volume}}$

$$= \frac{[M]}{[L^3]}$$

$$= [ML^{-3}]$$

$$128 \text{ kg/m}^2 = h\left[\frac{50}{1000}\right] \times \left[\frac{25}{100}\right]^{-3}$$

$$\Rightarrow 128 = h\left[\frac{1}{20}\right]\left[\frac{1}{4}\right]^{-3}$$

$$= h \times \frac{1}{20} \times (4)^3$$

$$\Rightarrow h = \frac{128 \times 20}{64} = 40 \text{ unit}$$

Answer: (1).

11. Magnetic moment, $\mu = 10^{-2}\hat{i}$ A m^2

Magnetic field $B = B\hat{i}(\cos \omega t)$

$$= 1\cos(0.125t)\hat{i}$$

At t = 1 second

B = cos(0.125)

Therefore, work done is

$$W = (\Delta\mu) \cdot \vec{B}$$

$$= 2 \times 10^{-2} \cos(0.125)$$

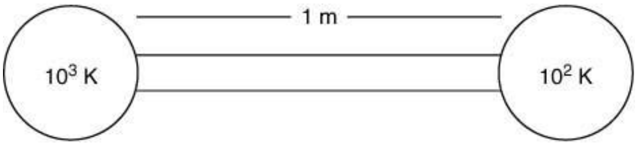
$$= 0.019$$

$$= 0.02$$

Answer: (3).

12. Heat current is

$$\frac{dH}{dt} = \frac{kA\Delta T}{L}$$



Heat flux is

$$\frac{1}{A}\frac{dH}{dt} = \frac{k\Delta T}{L}$$

$$= \frac{(0.1)(900)}{1} = 90 \text{ W/m}^2$$

Answer: (1).

13. Let K be the dielectric constant of material

$$C_{\text{eq}} = \frac{10\varepsilon_0 A/3}{d} + \frac{12\varepsilon_0 A/3}{d} + \frac{14\varepsilon_0 A/3}{d}$$

$$C = \frac{K\varepsilon_0 A}{d}$$

$$\frac{K\varepsilon_0 A}{d} = \frac{10\varepsilon_0 A/3}{d} + \frac{12\varepsilon_0 A/3}{d} + \frac{14\varepsilon_0 A/3}{d}$$

$$\Rightarrow \frac{K\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d}\left[\frac{10}{3} + \frac{12}{3} + \frac{14}{3}\right]$$

$$\Rightarrow K = \frac{36}{3} = 12$$

Answer: (3).

14. Let potential at distance r from then common center is

$$V = \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c} \qquad (1)$$

Since, $Q_a : Q_b : Q_c = a^2 : b^2 : c^2$

But surface change density for all are same.

$$\Rightarrow Q_a = \left[\frac{a^2}{a^2 + b^2 + c^2}\right]Q$$

$$\Rightarrow Q_1 = \left[\frac{b^2}{a^2 + b^2 + c_1^2}\right]Q$$

$$\Rightarrow Q_i = \left[\frac{c^2}{a^2 + b^2 + c_1^2}\right]Q$$

Put these value in Eq. (1), we get

$$V = \frac{K}{a}\left[\frac{a^2}{a^2 + b^2 + c^2}\right]Q + \frac{k}{b}\left[\frac{b^2}{a^2 + b^2 + c^2}\right]Q + \frac{k}{c}\left[\frac{c^2}{a^2 + b^2 + c^2}\right]$$

$$= kQ\left[\frac{a + b + c}{a^2 + b^2 + c^2}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{a + b + c}{a^2 + b^2 + c^2}\right]$$

Answer: (4).

15. Efficiency of Carnot engine is $\eta = 1 - \frac{T_2}{T_1}$

Let η_1, η_2 and η_3 are efficiency of three Carnot engines.

Thus,

$\eta_1 = 1 - \frac{T_2}{T_1}$; $\eta_2 = 1 - \frac{T_3}{T_2}$; $\eta_3 = 1 - \frac{T_4}{T_3}$

According to the question

$$\eta = \eta_1 = \eta_2 = \eta_3$$

$$= 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$$\Rightarrow T_2 = \sqrt{T_3 T_1} = \sqrt{T_1\sqrt{T_2 T_4}}$$

For T_3:

$$\sqrt{T_1 T_3} = \sqrt{T_1\sqrt{T_2 T_4}}$$

$$\Rightarrow \sqrt{T_3} = \sqrt{\sqrt{T_2 T_4}}$$

$$\Rightarrow T_3 = \sqrt{T_2 T_4}$$

For T_2:

$$T_2^{3/4} = \sqrt{T_1^{1/2} T_4^{1/4}}$$

$$\Rightarrow T_2 = T_1^{2/3} T_4^{1/3}$$

$$= (T_1^2 T_4)^{1/3}$$

Answer: (2).

16. Orbital velocity, $v = \sqrt{\frac{Gm}{r}}$

By conservation of energy, we have

KE of Particle having mass $m + \left(\frac{-GMm}{r}\right) = 0 + 0$ (PE = KE = 0)

KE of particle having mass $m = \frac{GMm}{r}$

$$= \left(\frac{GM}{r}\right)m$$

$$= v^2 m$$

Therefore, KE of particle having mass $m = mv^2$

Answer: (2).

17. Height of water column is constant.

Flow of water in tank = Area of hole = 1 cm^2 = 10^{-4} m

Thus, rate of water flow in the tank = rate of water out flow

10^{-4} = Area × U

$$\Rightarrow 10^{-4} = 10^{-4} \times \sqrt{2gh}$$

$$\Rightarrow 1 = \sqrt{2 \times 10 \times h}$$

$$\Rightarrow 1 = 20 \times h$$

$$\Rightarrow h = \frac{1}{20} m$$

$$\Rightarrow h = \frac{100}{20} \text{cm} = 5 \text{ cm}$$

Answer: (1).

18. Velocity of wave, $v = \sqrt{\frac{T}{\mu}}$

$$= \sqrt{\frac{8}{\frac{5 \times 10^{-3}}{1}}}$$

$$= \sqrt{\frac{8 \times 10}{5}} = 40 \text{ m/s}$$

Wavelength of wave $v = f\lambda$

$$\lambda = \frac{v}{f} = \frac{40}{100}\,\text{m}$$

Therefore, distance between two consecutive nodes $= \frac{\lambda}{2}$

$$= \frac{40}{100 \times 2} = \frac{20}{100}$$

$$= 20\ \text{cm}$$

Answer: (4).

19. As we know that,

$$f_2 = f_0\left(\frac{v}{v-17}\right) \qquad (1)$$

$$f_1 = f_0\left(\frac{v}{v-34}\right) \qquad (2)$$

Dividing Eq. (2) by Eq. (1), we get

$$\frac{f_2}{f_1} = \frac{f_0\left(\frac{v}{v-17}\right)}{f_0\left(\frac{v}{v-34}\right)}$$

$$= \frac{v-34}{v-17}$$ [speed of sound v = 340 m/s]

$$= \frac{340-34}{340-17}$$

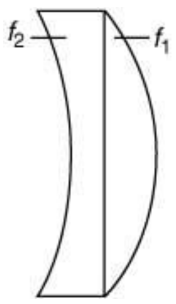
$$\Rightarrow \frac{f_2}{f_1} = \frac{306}{323} = \frac{18}{19}$$

$$\Rightarrow \frac{f_1}{f_2} = \frac{19}{18}$$

Answer: (2).

20. We have,

$$f_1 = 2f_2 \qquad (1)$$



We know that

$$\frac{1}{f_1} = \left(\frac{\mu_1 - 1}{1}\right)\frac{1}{R} \Rightarrow f_1 = \frac{R}{\mu_2 - 1}$$

$$\frac{1}{f_2} = \left(\frac{\mu_2 - 1}{1}\right)\left(\frac{-1}{R}\right) \Rightarrow f_2 = \frac{-R}{\mu_2 - 1}$$

Put these values in Eq. (1), we get

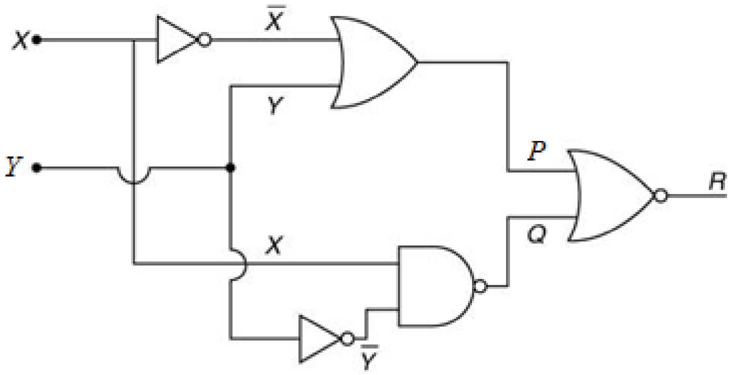
$$\frac{R}{\mu_1 - 1} = \frac{2R}{\mu_2 - 1}$$

$$\Rightarrow \mu_2 - 1 = 2\mu_1 - 2$$

$$\Rightarrow 2\mu_1 - \mu_2 = 1$$

Answer: (2).

21.



$$P = \overline{X} + Y$$

$$Q = \overline{\overline{Y} \cdot X}$$

$$= Y + \overline{X}$$

$$\text{Output} = \overline{P + Q}$$

$P + Q$ must be 0

Therefore, $Y = 0$, $X = 1$

Answer: (3).

22. Speed of light is $c = 3 \times 10^8$ m/s

Magnetic field is $B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^5 \left(t - \frac{x}{c}\right)\right]$

Maximum magnetic field, $B_0 = 100 \times 10^{-6}$ T

Therefore, electric field is

$$E_0 = c \times B_0$$

$$= 3 \times 10^8 \times 100 \times 10^{-6}$$

$$= 3 \times 10^4 \text{ N/C}$$

Answer: (2).

23. At $t = 0$ s

$$\frac{dN}{dt} = A_0 = 1600$$

At $t = 8$s

$$\frac{dN}{dt} = A = 100 \text{ cps}$$

$$\Rightarrow \frac{A}{A_v} = \frac{100}{1600}$$

$$\Rightarrow 2^n = 16$$

$$n = 4 = \frac{8}{T_{1/2}}$$

$$T_{1/2} = \frac{8}{4} = 2 \text{ s}$$

In $t = 6$ s number of half-life's = 3

Therefore, $A = \frac{1600}{2^3} = \frac{1600}{8} = 200$ cps

Answer: (1).

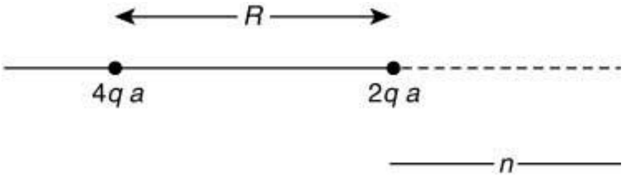
24. Potential difference between two faces = $E \times l$

$$= (v \times B)l$$
$$= 6 \times 0.1 \times 2 \times 10^{-2}$$
$$= 12 \text{ mV}$$

Answer: (1).

25. On the x axis in left on A potential due to A and B. But potential due to A is higher than that of B, between sign of potential due to A and B is opposite. So, potential can be same only in right of B on x-axis.

$$V = \frac{4qa}{(R+x)^2} = \frac{2qa}{x^2}$$



$$\Rightarrow \frac{R+x}{x} = 12$$
$$\Rightarrow R + x = x\sqrt{2}$$
$$\Rightarrow R = x\sqrt{2} - x$$
$$\Rightarrow R = x(\sqrt{2} - 1)$$
$$\Rightarrow x = \frac{R}{\sqrt{2} - 1}$$

Therefore, distance $= \dfrac{R}{\sqrt{2}-1} + R$

$$= \frac{R + R\sqrt{2} - R}{\sqrt{2} - 1} = \frac{R\sqrt{2}}{\sqrt{2} - 1}$$

Answer: (4).

26. Potential difference across

$$R_1 = 10 - 0 = 10 \text{ V}$$

Current i through $R_1 = \dfrac{10}{20} = \dfrac{1}{2}$

$$= 0.5 \text{ Ampere}$$

Potential difference across $R_2 = 0 - 0$ Volt

Therefore, current i through $R_2 = 0$

Answer: (3).

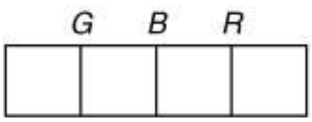
27. Resistor from color coding

$$R = 50 \times 10^2 \ \Omega = 5000 \ \Omega$$

Power = 2 W

Power is

$$P = i^2R$$
$$\Rightarrow 2 = i^2 \times 5000$$



$$\Rightarrow i^2 = \frac{2}{5000}$$

$\Rightarrow i^2 = \frac{0.4}{1000}$

$\Rightarrow i = \frac{2}{100}$ A = 20 mA

Answer: (1).

28. Bullet will collide with piece of wood at

$$t = \frac{d}{v} = \frac{100}{100} = 1\,\text{s}$$

Velocity of piece of wood u = 0 + 10 × 1
= 10 m/s

Velocity of bullet v = 100 – 10
= 90 m/s

From conservation of linear momentum,

−(0.02) (1v) + (0.02) (9v) = (0.05)v

$\Rightarrow$ 150 = 5v

$\Rightarrow$ v = 30 m/s

Maximum height reached by body is

$$h = \frac{v^2}{2g}$$

$$\Rightarrow h = \frac{30 \times 30}{2 \times 10}$$

$$\Rightarrow h = 45\text{ m}$$

At time of collision system is h distance below top of tower

$$h = \frac{1}{2} \times 10 \times 1^2 = 5$$

Therefore, height above tower = 45 – 5 = 40 m

Answer: (3).

29. Magnetic moment $M = NIA$

Charge of element $dq = \rho dx$

$$= \frac{\rho_0 x}{l} dx$$

Magnetic moment of this charge element $= \frac{dqvr}{2}$

$\Rightarrow dM = \frac{\rho_0 x\, dx}{l} \times \omega x \times \frac{x}{2}$

$\Rightarrow dM = \frac{\rho_0 \omega x^2 dx}{2l}$

Integrating both the sides, we get

$M = \int_0^l \frac{\rho_0 \omega x^3 dx}{2l}$

$= \frac{\rho_0 \omega}{2l} \int_0^l x^3 dx$

$= \frac{\rho\omega}{2l} \left[\frac{r^4}{4} \right]_0^l$

$$= \frac{\rho\omega}{2l}\left[\frac{l^4}{4} - 0\right]$$

$$= \frac{\rho_0\omega}{2l} \times \frac{l^4}{4}$$

$$= \frac{\rho_0\omega l^3}{8} \qquad [\omega = 2\pi n]$$

$$= \frac{2\pi n\rho_0 l^3}{8} = \frac{\pi}{4} n\rho_0 l^3$$

Answer: (4).

30. Relation between torque and moment of inertia

$$\tau = I\alpha$$

We also know that

$$\tau = F \times R$$

$$\Rightarrow F \times R = \left(\frac{1}{2}MR^2 - MR^2\right)\alpha$$

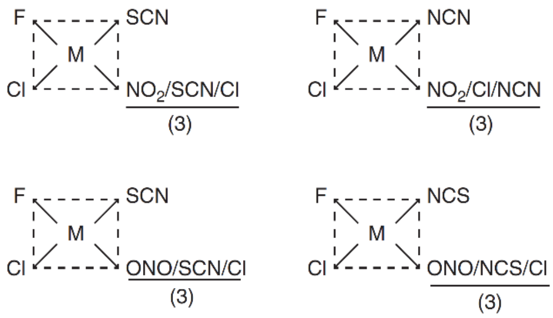
$$\Rightarrow F \times R = \frac{3}{2}MR^2\alpha$$

$$\Rightarrow \alpha = \frac{2F}{3MR}$$

Answer: (4).

Section: Chemistry

31.



Answer: (4).

32. We know $\Delta G = \Delta H - T\Delta S$

At equilibrium $\Delta G = 0$

Therefore,

$$T = \frac{\Delta H}{\Delta S} = \frac{200}{40} = 5\text{ K}$$

Answer: (3).

33. For first equation

$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$$

$$\frac{K_p}{K_C} = (RT)^{\Delta n_g} = (RT)^0 = 1$$

For second equation

$$N_2O_4(g) \rightleftharpoons 2NO_2(g)$$

$$\frac{K_p}{K_C} = (RT)^2 = 24.62 \text{ dm}^3\text{atm mol}^{-1}$$

For third equation

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

$$\frac{K_p}{K_C} = (RT)^{-2} = \frac{1}{(RT)^2} = 1.65\times10^{-3} \text{ dm}^{-6}\text{atm}^{-2}\text{mol}^2$$

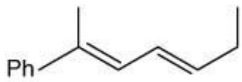
Answer: (2).

34. The total number of isotopes of hydrogen is three, that is, ${}^1_1H$, ${}^2_1H$ or ${}^2_1D$ and ${}^3_1T$ or ${}^3_1H$ and only ${}^3_1H$ or ${}^3_1T$ is an radio-active isotope of hydrogen.
Answer: (1).

35. Clean water has a BOD value 5 ppm. Since B has large BOD value than A, therefore, B is more polluted than A.
Answer: (1).

36. In triclinic unit cell, the lattice parameters are $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90°$.
Answer: (1).

37. Alcoholic KOH will give elimination product through $\beta-$ elimination mechanism. More conjugated diene will be the major product.



Therefore, the product given in option (1) will be the major product.
Answer: (1).

38. We know

Arrhenius equation $k = Ae^{-E_a/RT}$

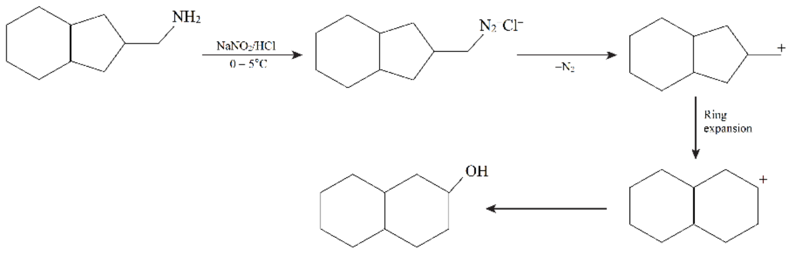
From the equation rate constant k is exponentially inversely proportional to E_a and exponentially directly proportion to T, that is,

$$k \propto e^{-E_a}$$

$$\propto T$$

Therefore, on increasing E_a, k will decrease exponentially and increasing T, k will increase exponentially.
Answer: (2).

39. The reaction involved is



The product formed does not match with the given option.

Answer: (*).

40. Wilkinson catalyst is $[(Ph_3P)_3RhCl]$
Answer: (3).

41. An alkyl halide is an absolutely water insoluble and all alkyl halide are dense as compared to water. In a separating funnel upper layer is called layer 1 and lower layer is called layer 2. Hence the upper layer will be less dense having H_2O and the lower will be denser having DCM.
Answer: (1).

42. Adipic acid will on dehydration give a seven-member ring with is unstable, therefore, adipic acid is least reactive toward anhydride formation reaction.

$$CO_2H-(CH_2)_4-CO_2H \xrightarrow{\text{Dehydrating agent}} \text{Seven member cyclic anhydride}$$

Answer: (1).

43. Electron withdrawing groups will increase the ease of alkaline hydrolysis of aromatic asters. Therefore, the correct order of ester hydrolysis is III > II > I > IV.
Answer: (2).

44. From photoelectric equation

$$h\nu = W_0 + \frac{1}{2}mv^2$$

$$\text{KE} = h\nu - h\nu_0$$

$$\text{KE} = h\nu + (-h\nu_0)$$

$$y = mx + C$$

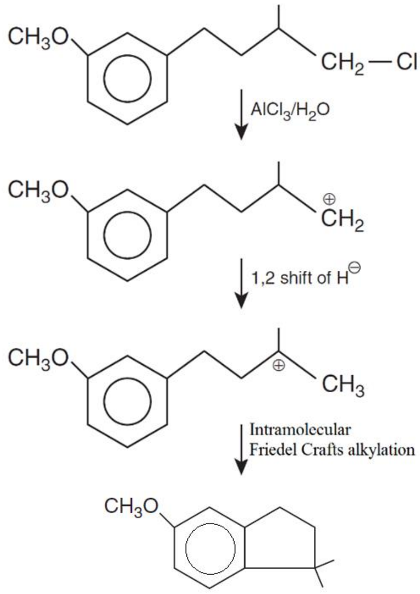
So, the straight line will be observed.
Answer: (3).

45. No catalyst is required in combustion of coal.
Answer: (2).

46. Due to lanthanoid contraction both atomic and ionic radii decreases gradually in the lanthanide series.
Answer: (3).

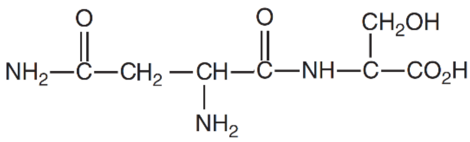
47. γ –Hydrogen is allylic, and it will form stable free radical with is stabilized by resonance, therefore, this γ-hydrogen easily replicable during examination reaction in presence of light.
Answer: (2).

48. The reaction involved is



Answer: (4).

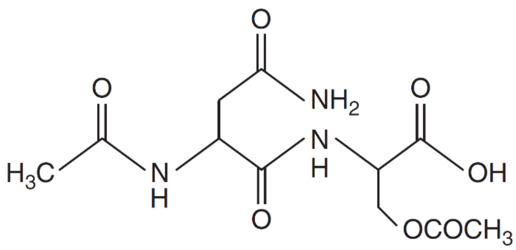
49. The given sequence is dipeptide having structural formula



The reaction involved is

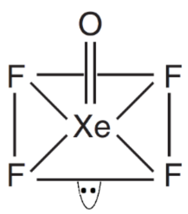
$$\text{Asn}-\text{Ser} + (CH_3CO)_2O \ \text{(Excess)} \xrightarrow{NEt_3} \text{Product } P$$

So, *P* is



Answer: (2).

50.



Hybridization of Xe in $XeOF_4$ is $sp^3d^2 \Rightarrow 5\sigma$ bonds + 1 lone pair (l.p).

Answer: (1).

51. Electronegativity of Al (EN = 1.5), which is similar to Be (EN = 1.5).
Answer: (2).

52. Higher the oxidation potential better will be the reducing power. Ca^{2+} have lowest reduction potential, that is, highest oxidation potential, therefore, Ca^{2+} have highest reducing power.
Answer: (2).

53. H_2O_2 act as oxidant

$$H_2O_2 + 2H^+ + 2e^- \longrightarrow 2H_2O \text{ (in acidic medium)}$$
$$H_2O + 2e^- \longrightarrow 2OH^- \text{ (in basic medium)}$$

H_2O_2 act as redundant

$$H_2O_2 + 2OH^- \longrightarrow 2H_2O + O_2 + 2e^- \text{ (in basic medium)}$$
$$H_2O_2 \longrightarrow 2H^+ + O_2 + 2e^- \text{ (in acidic medium)}$$

Answer: (4).

54. $\text{Millimoles of } Na_2SO_4 = \dfrac{2}{142} = 14 \text{ mmoles}$

From the stoichiometry of the reaction,

$$\underset{100\text{ mmoles}}{Ca(OH)_2} + \underset{14\text{ mmoles}}{Na_2SO_4} \rightarrow \underset{14\text{ mmoles}}{CaSO_4} + \underset{28\text{ mmoles}}{2NaOH}$$

Millimoles of $CaSO_4$ = 14 m moles.
Therefore, the mass of calcium sulphate formed (W) = $14 \times 10^{-3} \times 136 = 1.9$ g

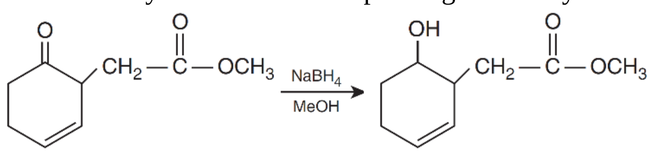
Concentration of $[OH^-] = \dfrac{28}{100} = 0.28 \text{ mol L}^{-1}$

Answer: (1).

55.
$$x_A = \frac{p_A}{p_{Total}} = \frac{p_A^o \cdot x_A}{p_A^o \cdot x_A \times p_B^o x_B}$$
$$= \frac{7 \times 10^3 \times 0.4}{7 \times 10^3 \times 0.4 + 12 \times 10^3 \times 0.6}$$
$$= \frac{280}{1000} = 0.28$$
$$x_B = 1 - 0.28 = 0.72$$

Answer: (2).

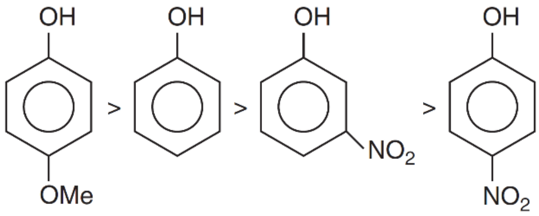
56. $NaBH_4$ only can reduced carbonyl of aldehydes and ketones and does not affect ester group and double bonds. Hence, the ketonic carbonyl reduces to corresponding secondary alcohol group.



Answer: (4).

57. Metallic Be is used to make X - ray tube window.
Answer: (3).

58. Electron withdrawing groups like $-NO_2$ increases the K_a values of the phenols that is acidity of the phenols whereas electron donating groups like –OMe will decreases acidity and pK_a value increases.



Answer: (4).

59. In Hall-Heroult's process

$$2Al_2O_3(l) \rightleftharpoons 4Al^{3+}(l) + 6O^{2-}(l)$$

At cathode: $4Al^{3+}(l) + 12e^- \rightarrow 4Al(l)$

At anode: $6O^{2-}(l) \rightarrow 2O_2(g) + 12e^-$

$$3C + 3O_2 \rightarrow 3CO_2$$

$$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$$

Answer: (3).

60. The electronic configuration of N_2^+ is $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma 2s^{2*} \pi 2p_x^2 = \pi 2p_y^2 \sigma 2p_z^1$.

Since, one electron is removed from $\sigma 2p_z$ orbital, therefore, it left with 2 π bonds and half σ bond.

Answer: (4).

Section: Mathematics

61. Probability that the noted number is either 7 or 8 is,

$P(7 \text{ or } 8) = P(\text{Head})\, P(7 \text{ or } 8) + P(\text{Tail})\, P(7 \text{ or } 8)$

$$= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{19}{72}$$

Answer: (3).

62. Let the points be $\left(\frac{3}{2}, 0\right)$, (t^2, t) $t > 0$

$$\text{Distance} = \sqrt{t^2 + \left(t^2 - \frac{3}{2}\right)^2}$$

$$= \sqrt{t^4 - 2t^2 + \frac{9}{4}} = \sqrt{(t^2 - 1)^2 + \frac{5}{4}}$$

Therefore, the shortest distance = $\frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2}$

Answer: (1).

63. Given lines are

$$\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2} \text{ and } \frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$$

By considering matrix, we have

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

Thus, $\vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$

Therefore, equation of the plane will be $-1(x - 4) - 1(y + 1) + 1(z - 2) = 0$

$$\Rightarrow x + y - z - 1 = 0$$

Hence, the plane passes through the point (1, 1, 1).

Answer: (2).

64. Let two observations be x_1 and x_2.

$$\bar{x}(\text{mean}) = 5$$

Variance (σ^2) = 9.20

$$\text{Mean} = \frac{\sum xi}{5} = 1 + 3 + 8 + x_1 + x_2 = 25$$

$x_1 + x_2 = 13$ (1)

$$\text{Variance} = \frac{\sum xi^2}{5} - 25 = 9.20$$

$$\frac{\sum xi^2}{5} = 9.20 + 25$$

$$\sum xi^2 = 34.2 \times 5 = 171$$

$$x_1^2 + x_2^2 = 97 \quad (2)$$

From Eq. (1) and Eq. (2), we get

$(x_1 + x_2)^2 - 2x_1x_2 = 97$

$\Rightarrow 169 - 2x_1x_2 = 97$

$\Rightarrow x_1x_2 = 36$

Therefore, $x_1 : x_2 = 4 : 9$

Answer: (2).

65. $r = 1$ is obviously true and let $0 < r < 1$.

$r + r^2 > 1$

$\Rightarrow r^2 + r - 1 > 0$

$$\Rightarrow \left(r - \frac{-1-\sqrt{5}}{2}\right)\left(r - \left(\frac{-1+\sqrt{5}}{2}\right)\right)$$

$$\Rightarrow r - \frac{1-\sqrt{5}}{2} \quad \text{or} \quad r > \frac{-1+\sqrt{5}}{2}$$

$$r \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$$

$$\frac{\sqrt{5}-1}{2} < r < 1$$

When $r > 1$

$$\frac{\sqrt{5}+1}{2} > \frac{1}{r} > 1$$

$$r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$$

Therefore, $r = \frac{7}{4}$ is incorrect.

Answer: (3).

66. We have

$$\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{21}C_i}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \frac{1}{(21)^3}\left[\frac{20(21)}{2}\right]^2 = \frac{k}{21}$$

$$\Rightarrow k = 100$$

Answer: (4).

67. We have

$$\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4} \Rightarrow 1 - \cos^2\theta + \cos^4 2\theta = \frac{3}{4}$$

Let $\cos^2 2\theta = t \Rightarrow t^2 - t + \frac{1}{4} = 0 \Rightarrow \left(t - \frac{1}{2}\right)^2 = 0$

$$\Rightarrow t = \frac{1}{2} \Rightarrow \cos^2 2\theta = \frac{1}{2}$$

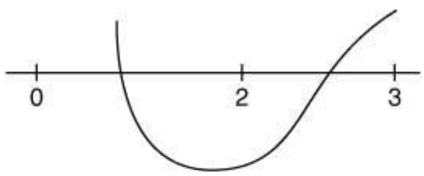
$$\Rightarrow 2\cos^2 2\theta - 1 = 0 \Rightarrow \operatorname{cosec}\theta = 0 \Rightarrow 4\theta = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{8} \Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \in \left[0, \frac{\pi}{2}\right]$$

Therefore, the sum of all values of θ is $\frac{\pi}{2}$.

Answer: (3).

68.



Let $f(x) = (c-5)x^2 - 2cx + c - 4$

Therefore, $f(0)\,f(2) < 0$ (1)

And, $f(2)\,f(3) < 0$ (2)

From Eq. (1) and Eq. (2), we have

$$(c-4)(c-24) < 0 \text{ and } (c-24)(4c-49) < 0$$

$$\Rightarrow \frac{49}{4} < c < 24$$

Thus, $S = \{13, 14, 15.....24\}$

Therefore, number of elements in set $S = 11$

Answer: (4).

69. We have $\frac{dy}{dx} + \frac{3}{\cos^2 x}y = \frac{1}{\cos^2 x}$

$$\Rightarrow \frac{dy}{dx} + 3\sec^2 x.y = \sec^2 x$$

$$\text{I.F.} = e^{3\int \sec^2 x\,dx} = e^{3\tan x}$$

$$y.e^{3\tan x} = \int \sec^2 x.e^{3\tan x}dx$$

$$\Rightarrow y.e^{3\tan x} = \frac{1}{3}e^{3\tan x} + C \qquad (1)$$

It is given that $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$

So, $\frac{4}{3} \times e^3 = \frac{1}{3}e^3 + C$

$\Rightarrow C = e^3$

For $y\left(-\frac{\pi}{4}\right)$, $x = -\frac{\pi}{4}$

Hence, $y.e^{-3} = \frac{1}{3}e^{-3} + e^3 \Rightarrow y = \frac{1}{3}e^6$

Answer: (1).

70. Let $n(A)$ = Number of students opted Mathematics = 70

$n(B)$ = Number of students opted Physics = 46

$n(C)$ = Number of students opted Chemistry = 28

$n(A \cap B) = 23$

$n(B \cap C) = 9$

$n(A \cap C) = 14$

$n(A \cap B \cap C) = 4$

Now,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$$

Therefore, the number of students who did not opted for any of the three courses = 140 – 102 = 38

Answer: (4).

71. Given expression $(1 + x^{\log 2x})^2$

Its third term is

$T_3 = {}^5C_2.(x^{\log 2x})^2 = 2560$

$\Rightarrow 10.x^{2\log 2^x} = 2560$

$\Rightarrow x^{2\log 2x} = 256$

$\Rightarrow 2(\log_2 x)^2 = \log_2 256$

$\Rightarrow 2(\log_2 x)^2 = 8$

$\Rightarrow (\log_2 x)^2 = 4 \Rightarrow \log_2 x = 2 \text{ or } -2$

$\Rightarrow x = 4 \text{ or } \frac{1}{4}$

Answer: (1).

72. Normal to these two curves are

$y = m(x - c) - 2bm - bm^3$

$\Rightarrow y = mx - 4\,am - 2\,am^3$

If they have a common normal, then

$(c + 2b)m + bm^3 = 4\,am + 2\,am^3$

Now, $(4a - c - 2b)m = (b - 2a)m^3$

Therefore, all options are correct for $m = 0$

Answer: (1,2,3,4).

73. Given,

$x + y + z = 5$
$x + 2y + 3z = 9$
$x + 3y + \alpha z = \beta$
By considering matrix, we have

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \alpha - 1 \end{vmatrix} = (\alpha - 1)^{-4}$$

$D = \alpha - 5$
For infinitely many solutions, $D = 0$
Thus, $\alpha = 5$

$$\text{Now, } Dx = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 3 \\ \beta - 15 & -2 & 5 \end{vmatrix} = \beta - 13$$

Since, $D_x = 0$
$\beta - 13 = 0$
$\beta = 13$
Therefore, $\beta - \alpha = 13 - 5 = 8$
Answer: (2).

74. We have

$$\lim_{x \to 1^+} \frac{(1 - |x| + \sin|-x|)\sin\left(\frac{\pi}{2}[1 - x]\right)}{|1 - x|[1 - x]}$$

$$= \lim_{x \to 1^+} \frac{(1 - x) + \sin(x - 1)}{(x - 1)(-1)} \sin\left(\frac{\pi}{2}(-1)\right)$$

$$= \lim_{x \to 1^+} \left(1 - \frac{\sin(x - 1)}{(x - 1)}\right)(-1) = (1 - 1)(-1) = 0$$

Answer: (2).

75. We have

$$|A| = \begin{bmatrix} -2 & 4 + d & (\sin\theta - 2) \\ 1 & (\sin\theta) + 2 & d \\ 5 & (2\sin\theta) - 2 & (-\sin\theta) + 2 + 2d \end{bmatrix}$$

By applying, $R_3 \to R_3 - 2R_2 + R_1$

$$= \begin{bmatrix} -2 & 4 + d & \sin\theta - 2 \\ 1 & \sin\theta + 2 & d \\ 1 & 0 & 0 \end{bmatrix}$$

$= (4 + d)d - \sin^2\theta - 4$
$= (2 + d)^2 - \sin^2\theta$

Since, the minimum value of $|A| = 8$

$$(d + 2)^2 = 9 \Rightarrow d = -5$$

Answer: (1).

76. It is given that

$$3|z_1| = 4|z_2|$$

$$\Rightarrow \frac{|z_1|}{|z_2|} = \frac{4}{3}$$

$$\Rightarrow \frac{|3z_1|}{|2z_2|} = 2$$

Let $3z_1 = a = 2\cos\theta + 2i\sin\theta$

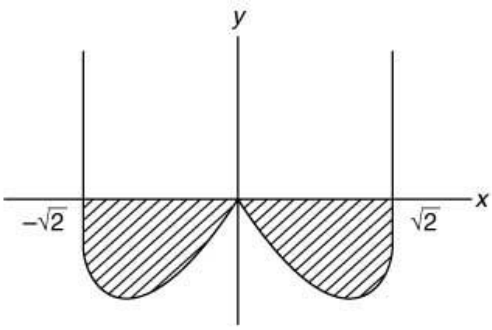
$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = a + \frac{1}{a} = \frac{5}{2}\cos\theta + \frac{3}{2}i\sin\theta$$

Answer: (*).

77. $I = \int_a^b (x^4 - 2x^2)dx$

Let, $f(x) = x^2(x^2 - 2)$

From figure, ordered pair (a, b) is $(-\sqrt{2}, \sqrt{2})$.



Answer: (4).

78. Let the centroid of ΔPQR is (h, k) and P is (x, y) then,

$\frac{x+1+3}{3} = h$ and $\frac{y+y-2}{3} = k$

$x = (3h - 4), y = (3k - 4)$

Point $P(x, y)$ lies on line $2x - 3y + 4 = 0$

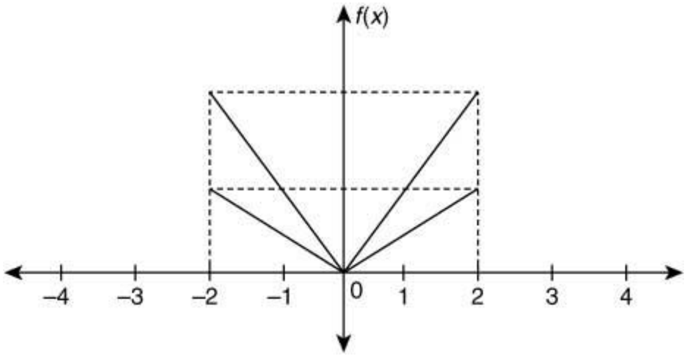
$2(3\lambda - 4) - 3(3k - 2) + 4 = 0$

Therefore, locus is $6x - 9y + 2 = 0$ whose slope is $\frac{2}{3}$.

Answer: (3).

79. Given,

$$f(x) = \begin{cases} \max\{|x|, x^2\} & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$$



Hence, from the above graph, we have

$S = (-2, -1, 0, 1, 2)$

Answer: (2).

80. Equation of the circle is

$x^2 + y^2 + 4x - 6y = 12 = 0$

Equation of the tangent at (1, −1) is

$x - y + 2(x + 1) - 3(y - 1) - 12 = 0$

$\Rightarrow 3x - 4y - 7 = 0$

Now, the equation of the circle is

$(x - 1)^2 + (y + 1)^2 + \lambda(3x - 4y - 7) = 0$

The circle passes through the point (4, 0), so

$9 + 1 + \lambda(12 - 7) = 0$

$\Rightarrow \lambda = -2$

Therefore, the required circle is $x^2 + y^2 - 8x + 10y + 16 = 0$.

$\text{Radius} = \sqrt{16+25-16} = 5$

Answer: (3).

81. Given function is

$f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$

$\Rightarrow f'(x) = 3x^2 + 2xf'(1) + f''(2)$

$\Rightarrow f''(x) = 6x + 2f'(1)$

$\Rightarrow f'''(x) = 6$

$f'(x) = f'(1) = 3 + 2f'(1) + f''(2)$ (1)

$f''(x) = f'(2) = 12 + 2f'(1)$ (2)

$f'''(x) = f'''(3) = 6$ (3)

From Eq. (1), (2) and (3), we get

$f'''(3) = 6$

$f''(2) = 2$

$f'(1) = -5$

Hence, $f(2) = 8 - 20 + 4 + 6$

$\Rightarrow f(2) = -2$

Answer: (3).

82. Given

$\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$

$\vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$

$\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$

So,

$4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k} = 4\hat{i} + 2\lambda_1\hat{j} + 6\hat{k}$

$\Rightarrow 3 - \lambda_2 = 2\lambda_1 \quad \Rightarrow 2\lambda_1 + \lambda_2 = 3$ (1)

Since, $\vec{a}\,.\,\vec{c} = 0$

Thus, $6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0 \quad \Rightarrow 2\lambda_1 + \lambda_3 = -1$ (2)

Hence, $(\lambda_1, \lambda_2, \lambda_3) = \left(\frac{-1}{2}, 4, 0\right)$

Answer: (2).

83. Given,

$\vec{r} = (1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k}$

Suppose point A is $(1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k}$ and the point B is (3, 2, 6).

So, $\overrightarrow{AB} = (2+3\mu)\hat{i} + (3-\mu)\hat{j} + (4-5\mu)\hat{k}$

Since, $\overrightarrow{AB}$ is parallel to the plane $x - 4y + 3z = 1$

Hence, $2 + 3\mu - 12 + 4\mu + 12 - 15\mu = 0$

$\Rightarrow \mu = \frac{1}{4}$

Answer: (1).

84. We have

$$I = \int \frac{(\sin^n \theta - \sin\theta)^{\frac{1}{n}} . \cos\theta}{\sin^{n+1}\theta} d\theta = \int \frac{\sin\theta\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{1/n}}{\sin^{n+1}\theta} d\theta$$

Let, $1 - \frac{1}{\sin^{n-1}\theta} = t$

$\Rightarrow \frac{n-1}{\sin^n \theta} \cos\theta \, d\theta = dt$

Therefore,

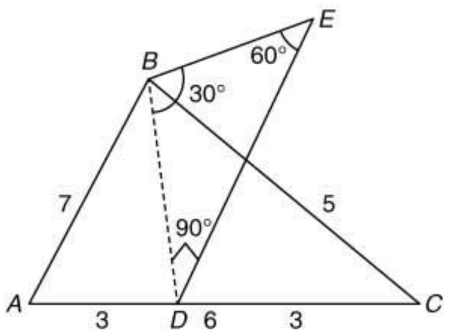
$$I = \frac{1}{n-1}\int (t)^{yn} dt = \frac{1}{n-1} \times \frac{t^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C$$

$$= \frac{1}{n-1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{1}{n}+1} + C$$

$$\Rightarrow I = \frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$$

Answer: (1).

85.



$BD = h\cot 30° = h\sqrt{3}$

Thus, $7^2 + 5^2 = 2(h\sqrt{3})^2 + 3^2$

$\Rightarrow 3h^2 = 28$

$\Rightarrow h = \frac{2}{3}\sqrt{21}$

Answer: (2).

86. Equation of hyperbola is

$$\frac{x^2}{5} - \frac{y^2}{4} = 1$$

Slope of tangent = 4

Equation of tangent is

$y = mx \pm \sqrt{5m^2 - 4}$

$\Rightarrow\ y = x \pm \sqrt{5-4}$

$\Rightarrow y = x \pm 1$

Therefore, $y = x + 1$ or $y = x - 1$

Answer: (1).

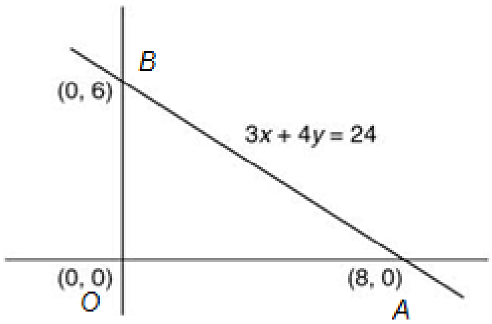
87. Equation of the given line is

$3x + 4y - 24 = 0$

x and y intercepts can be obtained by putting $y = 0$ and $x = 0$ respectively in the above equations. Hence,

x – intercept = 8

y – intercept = 6



We know the incentre of the triangle is given by

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right) = \left(\frac{8(6)+6(0)+10(0)}{24}, \frac{10(0)+6(8)+8(0)}{24}\right) = (2,2)$$

Hence, the incentre of the triangle is (2,2).

Answer: (2).

88. We have

$$\sum_{r=2}^{13}(7r+2) = 7\times\frac{2+13}{2}\times 6 + 2\times 12$$

$= 7 \times 90 + 24 = 654$

$$\Rightarrow \sum_{r=1}^{13}(7r+5) = 7\left(\frac{1+13}{2}\right)\times 13 + 5\times 13$$

$= 702$

Total = 654 + 702 = 1356

Answer: (4).

89. Area bounded by $y^2 = 4ax$ & $x^2 = 4by$, $a, b \neq 0$ is $\left|\frac{16ab}{3}\right|$

Since, $4a = \frac{1}{k} = 4b, k > 0$

$$\text{Area} = \left|\frac{16.\frac{1}{4k}.\frac{1}{4k}}{3}\right| = 1$$

$\Rightarrow k^2 = \frac{1}{3}$

$$\Rightarrow k = \frac{1}{\sqrt{3}}$$

Answer: (2).

90. $P(n) = x^2 - n + 41$ is prime

$P(5) = 5^2 - 5 + 41 = 25 - 5 + 41 = 61$ which is prime

$P(3) = 3^2 - 3 + 41 = 9 - 3 + 41 = 47$ which is also prime

Answer: (1).