JEE Main 2019 Paper 1
January 12, Shift 1
Section: Physics

1. Energy, $E = \frac{3}{2}PV = \frac{3}{2}nRT$

Where, R = Universal gas constant

T = Temperature of gas

Given, $P = 3 \times 10^6$ Pa; $V = 2$ m^3

$$E = \frac{f}{2}nRT = \frac{f}{2}PV$$

Let gas is monoatomic $f = 3$

Therefore, $E = \frac{3}{2} \times 3 \times 10^6 \times 2 = 9 \times 10^6$ J

Answer: (1)

2. We know that

$$y = a \sin(\omega t + kx)$$

Given, $y = 10^{-3}\sin(50t + 2x)$

On comparing both the equations, we get

$$a = 10^{-3},\ \omega = 50,\ k = 2$$

Thus, wave is propagating along negative x-axis with speed

$$v = \frac{\omega}{k} = \frac{50}{2} = 25 \text{ m/s}$$

Answer: (1)

3. Let i be the current flowing in the wire

$$i = \frac{V}{R} = \frac{4}{R+5}$$

If resistance of 10 m length of wire is r, then

$$r = 0.5\,\Omega = 5 \times \frac{0.1}{1}\Omega$$

Potential difference, $\Delta V = ir$

$$5 \times 10^{-3} = \left(\frac{4}{R+5}\right)0.5$$

$$\Rightarrow 5 \times 10^{-3} = \left(\frac{4}{R+5}\right) \times \frac{5}{100}$$

$$\Rightarrow \frac{4}{R+5} = 10^{-2}$$

$$\Rightarrow R + 5 = 400$$

$$\Rightarrow R = 400 - 5 = 395\ \Omega$$

Answer: (3)

4. Balanced condition of Meter bride is

$$\frac{P}{Q} = \frac{R}{S}$$

$$\Rightarrow \frac{(100 - l)}{l} = \frac{R}{S}$$

Given $\frac{dR}{dl} \propto \frac{1}{\sqrt{l}}$

$$\Rightarrow dR = k\frac{dl}{\sqrt{l}} \qquad (k = \text{constant})$$

For zero deflection,

$$\frac{R_1}{R_2} = 1$$

$$\Rightarrow R_1 = R_2 \qquad (1)$$

$$\Rightarrow R_1 = k\int_0^l \frac{dl}{\sqrt{l}} = k \times 2\sqrt{l}$$

And

$$R_2 = k\int_l^{100-l} \frac{dl}{\sqrt{l}} = k(2 - 2\sqrt{l})$$

Substitute the value of R_1 and R_2 in Eq. (1), we have

$$2k\sqrt{l} = k(2 - 2\sqrt{l})$$

$$\Rightarrow 2k\sqrt{l} = k_2 - 2k\sqrt{l}$$

$$\Rightarrow \sqrt{l} = 1 - \sqrt{l}$$

$$\Rightarrow \sqrt{l} + \sqrt{l} = 1$$

$$\Rightarrow 2\sqrt{l} = 1$$

$$\Rightarrow \sqrt{l} = \frac{1}{2}$$

$$\Rightarrow (\sqrt{l})^2 = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow l = \frac{1}{4} = 0.25 \text{ m}$$

Answer: (3)

5. Let passenger train and freight train be A and B and v_A is the velocity of passenger train and v_B is the velocity of freight train B.

Relative velocity of A with respect to B is (same direction)

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$= 80 - 30 = 50 \text{ km/h}$$

Time taken by the train A to cross the train B

$$t_1 = \frac{600 + 120}{50}$$

$$\Rightarrow t_1 = \frac{180}{50} \qquad (1)$$

Train moving in opposite direction

$$v_{AB} = v_A - (-v_B)$$

$$= 80 - (-30)$$

$$= 80 + 30 = 110$$

Time taken $t_2 = \frac{60 + 120}{110} \Rightarrow t_2 = \frac{180}{110}$

Therefore, required ratio is

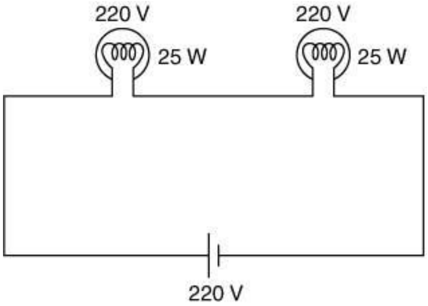
$$\frac{t_1}{t_2} = \frac{\frac{180}{50}}{\frac{180}{110}} = \frac{110}{50}$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{11}{5}$$

Answer: (1)

6. Suppose R_1 and R_2 are Resistance of bulb

$$R_1 = \frac{V_1'^2}{P_1'} = \frac{(220)^2}{25}\ ;\ R_2 = \frac{V_2'^2}{P_2'} = \frac{(220)^2}{100}$$



Current

$$i = \frac{220}{R_1 + R_2}$$

$$\Rightarrow i = \frac{220}{\left[\frac{(220)^2}{25} + \frac{(220)^2}{100}\right]}$$

$$= \frac{220}{\frac{(220)^2}{25}\left[1 + \frac{1}{4}\right]}$$

$$= \frac{220}{\frac{(220)^2}{25} + \frac{(220)^2}{100}}$$

$$= \frac{220}{\frac{(220)^2}{25}\left[1 + \frac{1}{4}\right]}$$

$$= \frac{220 \times 25 \times 4}{(220)^2 \times 5} = \frac{1}{11}$$

Now, Power

$$P_1 = i^2 R_1$$

$$= \frac{1}{11} \times \frac{1}{11} \times \frac{(220)^2}{25}$$

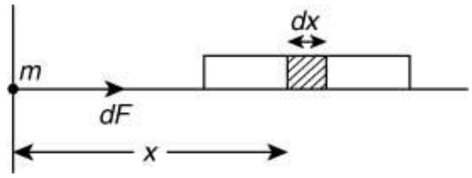
$$= \frac{400}{25} = 16\text{ W}$$

$$P_2 = i_2 R_2$$

$$= \frac{1}{11} \times \frac{1}{11} \times \frac{(220)^2}{100} \Rightarrow P_2 = \frac{400}{100} \Rightarrow P_2 = 4\,\text{W}$$

Answer: (1)

7.



The mass per unit length of the rod,

$$dm = (A + Bx^2)dx$$

Gravitational Force is

$$dF = \frac{GM\ dm}{r^2}$$

$$\Rightarrow dF = \frac{GM(A + B\lambda^2)}{\lambda^2}dx$$

Integrating both the sides, we get

$$\int dF = \int_a^{a+L} \frac{GM}{x^2}(A + Bx^2)dx$$

$$F = GM\left[-\frac{A}{x} + Bx\right]_a^{a+L}$$

$$= GM\left[-A\left(\frac{1}{a+L} - \frac{1}{a}\right) + B(a + L - a)\right]$$

$$= GM\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$$

Answer: (4)

8. As we know that lens formula is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

For first lens: $u = -20$; $f = 5$

$$\frac{1}{5} = \frac{1}{v} - \frac{1}{(-2.0)}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{5} - \frac{1}{20} = \frac{4-1}{20}$$

$$\Rightarrow \frac{1}{v} = \frac{3}{20} = \frac{20}{3}$$

For second lens refraction,

$$u = \frac{20}{3} - 2$$

$$u = \frac{14}{3}$$

And, $f = -5$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{-5} = \frac{1}{v} - \frac{1}{\frac{14}{3}}$$

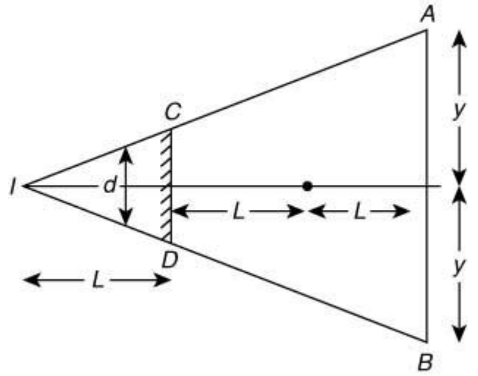
$$\Rightarrow \frac{1}{v} = \frac{1}{70}$$

$$\Rightarrow v = 70\,\text{cm}$$

Therefore, 70 cm from B at right, real.

Answer: (4)

9. Consider ΔICD and ΔIAB.
Since, both are similar triangle so, ratio of identical side of triangle are equal.



$$\frac{2y}{d} = \frac{3L}{L}$$

$$\Rightarrow 2y = 3d$$

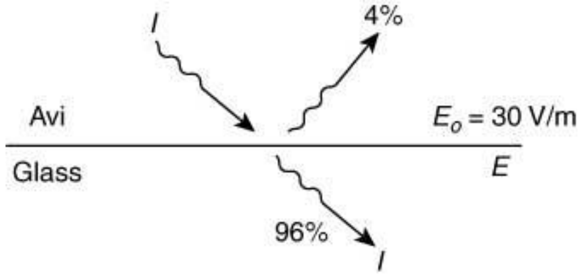
Answer: (3)

10. Let I be the incident energy and I' be the reflected incident energy.

$$I' = 0.96I \qquad (1)$$

Incident energy in air

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$



Incident energy in glass

$$I' = \frac{1}{2}\varepsilon E^2 v$$

Put these values in Eq. (1), we get

$$\frac{1}{2}\varepsilon E^2 v = 0.96\frac{1}{2}\varepsilon_0 E_0^2 c$$

$$\Rightarrow E^2 = 0.96\frac{\varepsilon_0}{\varepsilon}E_0^2\frac{c}{v}$$

$$\Rightarrow E^2 = 0.96\frac{\varepsilon_0}{\varepsilon_0\varepsilon_r}E_0^2\frac{c}{v} \qquad [\varepsilon = \varepsilon_0\varepsilon_r]$$

$$\Rightarrow E^2 = 0.96\frac{1}{\varepsilon_r}E_0^2\frac{c}{v}$$

$$\Rightarrow E^2 = 0.96\frac{1}{n^2}E_0^2 n$$

$$= 0.96\frac{E_0^2}{n}$$

$$\Rightarrow E^2 = 0.96\times\frac{(30)^2}{1.5}$$

$$\Rightarrow E^2 = 576$$

$$\Rightarrow E = \sqrt{576} = 24 \text{ V/m}$$

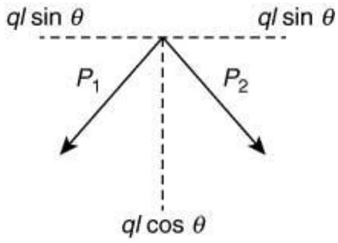
Answer: (3)

11. Work done by a gas is area under cyclic diagram.

Therefore, work done by gas = $\Delta W = \frac{1}{2} \times 4 \times 5 = 10$ J

Answer: (2)

12.



Dipole moment is

$$P_1 = ql \cos\theta$$
$$= ql \cos 30° \quad (1)$$
$$P_2 = ql \cos\theta$$
$$= ql \cos 30° \quad (2)$$

Now, net dipole moment is

$$P = 2 \times ql \cos 30°$$
$$= 2 \times ql \times \frac{\sqrt{3}}{2} = \sqrt{3}\, ql$$

Since, direction of dipole moment is negative y axis.

Therefore, $P = -\sqrt{3} ql\ \hat{j}$

Answer: (4)

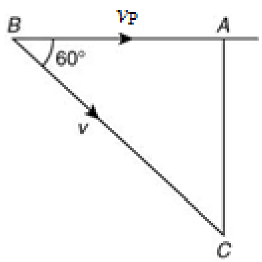
13. We have,

$$x_{cm} = \frac{2mL + 2mL + \frac{5mL}{2}}{4m}$$
$$= \frac{13mL}{8m} = \frac{13L}{8}$$
$$y_{cm} = \frac{2mL + m\frac{L}{2} + m \times 0}{4m}$$
$$= \frac{5mL}{4m} = \frac{5}{4}L$$

Therefore, $\vec{r} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

Answer: (1)

14.



$$AB = v_P \times t$$
$$BC = v \times t$$

In ΔABC,

$$\cos\theta = \frac{AB}{BC}$$

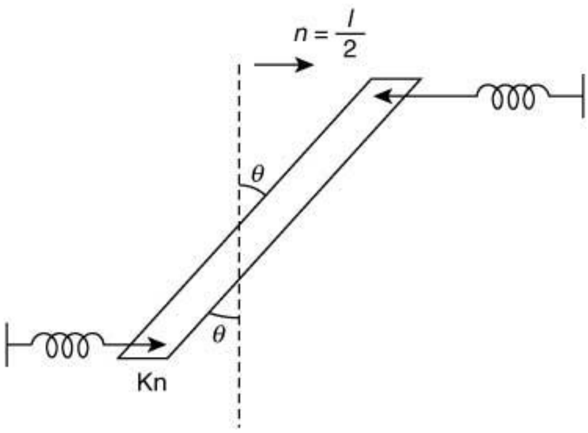
$$\cos 60^\circ = \frac{v_P \times t}{v \times t}$$

$$\Rightarrow \frac{1}{2} = \frac{v_P}{v}$$

$$\Rightarrow v_P = \frac{v}{2}$$

Answer: (4)

15.



According the relation between Torque and Moment of Inertia

$$\tau_H = I\alpha$$

$$\Rightarrow 2\left(k\frac{l\theta}{2}\frac{l}{2}\right) = \frac{ml^2}{12}\alpha$$

$$\Rightarrow 2\cdot\frac{kl^2\theta}{4} = \frac{ml^2}{12}\alpha$$

$$\Rightarrow \alpha = \left(\frac{6k}{m}\right)\theta \qquad (1)$$

We know that, $\alpha = \omega^2\theta$ (2)

On comparing Eq. (1) and Eq. (2), we get

$$\omega^2 = \frac{6k}{m}$$

$$\Rightarrow \omega = \sqrt{6k/m}$$

Therefore, frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

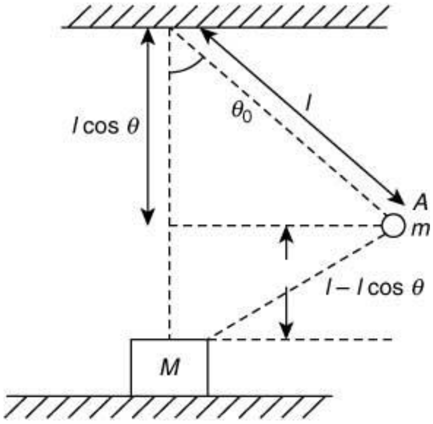
Answer: (3)

16. Let velocity of bob at B is *u*, which is released by conservation of energy at Point A and B

$$0 + mg(l - l\cos\theta_0) = \frac{1}{2}mv^2 + 0$$

$$\Rightarrow u^2 = 2g(l - l\cos\theta_0)$$

$$\Rightarrow u = \sqrt{2gl(1-\cos\theta_0)} \quad (1)$$



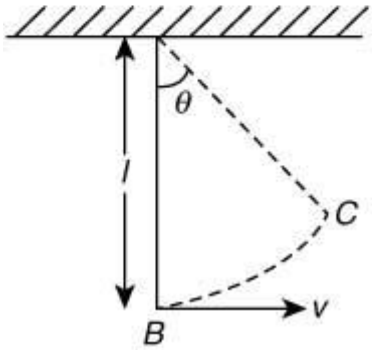
Let velocity of bob after the collision is *v*.

$$v = \left(\frac{m-M}{m+M}\right)u \quad (2)$$

By conservation of energy at Point B and C

$$0 = 0 + mg(l - l\cos\theta_0)$$

$$v = \sqrt{2gl(1-\cos\theta_0)} \quad (3)$$



Substituting the value of *v* and *u* in Eq. (2), we get

$$\sqrt{2gl(1-\cos\theta_1)} = \left(\frac{m-M}{m+M}\right)\sqrt{2gl(1-\cos\theta_0)}$$

$$\Rightarrow \frac{m-M}{m+1} = \sqrt{\frac{1-\cos\theta_1}{1-\cos\theta_0}}$$

$$\Rightarrow \frac{m-M}{m+M} = \frac{\sin\theta_1/2}{\sin\theta_0/2}$$

If $\sin\theta \simeq \theta$

then, $$\frac{m-M}{m+M} = \frac{\theta_1}{\theta_0}$$

$$\Rightarrow m\theta_0 - m\theta_0 = m\theta_1 + m\theta_2$$

$$\Rightarrow m(\theta_0 + \theta_1) = M(\theta_0 - \theta_1)$$

$$\Rightarrow M = \left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right) m$$

Answer: (3)

17. Modulation index is given by

$$\mu = \frac{E_m}{E_c} \quad (1)$$

Now, maximum amplitude

$$E_c + E_m = 160 \text{ V} \quad (2)$$

And minimum amplitude

$$E_c - E_m = 40 \text{ V} \quad (3)$$

Solving Eq. (2) and Eq. (3), we get

$$E_c = 100 \text{ V}; E_m = 60 \text{ V}$$

Substitute these values in Eq. (1), we get

$$\mu = \frac{60}{100} = 0.6$$

Answer: (3)

18. Thermal Resistance,

$$R = \frac{l}{KA} \qquad (K = \text{thermal conductivity})$$

Thermal Resistance of inner cylinder of Radius R

$$R_1 = \frac{l}{K_1(\pi R^2)} \quad (1)$$

Thermal Resistance of outer cylinder of radius $2R$

$$R_2 = \frac{l}{K_2(4\pi R^2 - \pi R^2)}$$

$$\Rightarrow R_2 = \frac{l}{K_2(3\pi R^2)} \quad (2)$$

Now, equivalent thermal Resistance

$$R_{eq} = \frac{l}{K_{eq}(4\pi R^2)} \quad (3)$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} - \frac{1}{R_2}$$

$$\Rightarrow \frac{K_{eq}(4\pi R^2)}{l} = \frac{K_1(\pi R^2)}{l} + \frac{K_2(3\pi R^2)}{l}$$

$$\Rightarrow 4\,K_{eq} = K_1 + 3K_2$$

$$\Rightarrow K_{eq} = \frac{K_1 + 3K_2}{4}$$

Answer: (4)

19. We have,

Least count = 5 μm = 5×10^{-6} m

Pitch = 1 mm = 10^{-3} m

Thus, least count $= \dfrac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

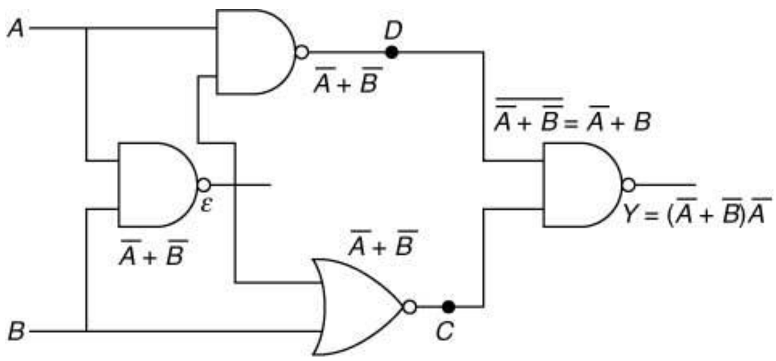
$$\Rightarrow 5\times 10^{-6} = \frac{10^{-3}}{\text{No. of division on circular scale}}$$

Therefore, number of divisions on circular scale $= \dfrac{10^{-3}}{5\times 10^{-6}}$

$$= \frac{1000}{5} = 200$$

Answer: (2)

20.



We have,

$$C = B + A\overline{B}$$

$$D = \overline{A.(\overline{A}B)}$$

$$E = \overline{AB}$$

Thus, $$Y = \overline{(\overline{A}+\overline{B})\overline{A}}$$

$$= \overline{\overline{A}+\overline{A}B}$$

$$= A(\overline{\overline{A}B}) = A(A+\overline{B})$$

$$= A + A\overline{B} = A\overline{B}$$

Answer: (3)

21. Magnetic field at O will be done by PS and QN
Given OP = OQ = 4 × 10^{-2}
Magnetic field at point O is $B_{net} = 10^{-4}$ T
Magnetic field at Point O due to wire LP and MQ is zero.
Let current in each wire = i

Thus, $$B_{net} = B_{PS} + B_{QN}$$ (both inward)

$$= \frac{\mu_0 i}{4\pi d} + \frac{\mu_0 i}{4\pi d}$$

$$\Rightarrow B_{net} = \frac{\mu_0 i}{2\pi d}$$

$$\Rightarrow 10^{-4} = \frac{2\times 10^{-7}\times i}{4\times 10^{-2}} = 20\text{ A}$$

Therefore, current is 20 A and it is perpendicular into the page.

Answer: (3)

22. Given $U(r) = \dfrac{1}{2}kr^2$

$$\left|\vec{F}\right| = \frac{-dU}{dr}$$

$$= \left| -\left(\frac{2kr}{2} \right) \right| = kr$$

In circular motion

$$F_c = kr$$

$$\frac{mv^2}{r} = kr$$

$$\Rightarrow mv^2 = kr^2 \qquad (1)$$

Bohr quantization is

$$mvr = \frac{nh}{2\pi}$$

$$\Rightarrow mv = \frac{nh}{2\pi r} \qquad (2)$$

From Eq. (1), we get

$$\frac{m^2v^2}{m} = kr^2$$

$$\Rightarrow \frac{1}{m}\left(\frac{nh}{2\pi r}\right)^2 = kr^2$$

$$\Rightarrow \frac{n^2h^2}{4\pi^2mr^2} = kr^2$$

$$\Rightarrow r^4 = \frac{n^2h^2}{4\pi^2mk}$$

$$\Rightarrow r^4 = \left(\frac{h^2}{4\pi^2mk}\right)^{1/4} n^{1/2}$$

$$\Rightarrow r \propto \sqrt{n}$$

From Eq. (1), we get

$$U \propto \sqrt{n}$$

Thus, kinetic energy $= \frac{1}{2}mv^2$

And potential energy $= \frac{1}{2}kr^2$

Total energy E = KE + PE

$$= \frac{1}{2}mv^2 + \frac{1}{2}kr^2$$

$$= kr^2 \propto n$$

Answer: (1)

23. Kinetic energy = $q\Delta V$

$$\Rightarrow r = \frac{mv}{qB} = \frac{p}{qB}$$

$$\Rightarrow r = \frac{\sqrt{2mqV}}{qB}$$

$$\Rightarrow r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\Rightarrow r \propto \sqrt{\frac{m}{q}}$$

Given $m_p : m_\alpha = 1 : 4$ and $q_p : q_\alpha = 1 : 2$

Therefore,

$$\frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p}} = \sqrt{\frac{1}{4} \times \frac{2}{1}} = \frac{1}{\sqrt{2}}$$

Answer: (1)

24. Moment of Inertia of Hollow sphere is

$$I = \frac{M(a^2 + b^2)}{2} \quad [a = \text{radius of inner cylinder}, b = \text{radius of outer cylinder}]$$

$$= \frac{M[(200)^2 + (10)^2]}{2}$$

$$= \frac{M[400 + 100]}{2} = 250M$$

Now, radius of gyration is

$$I = MK^2$$

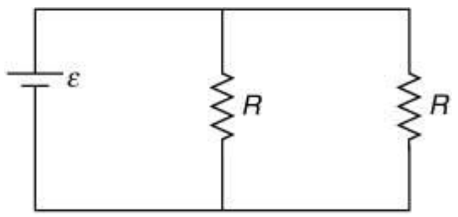
$$\Rightarrow 250M = MK^2$$

$$\Rightarrow K = \sqrt{250} = 15.81 \text{ cm}$$

$$\Rightarrow K = 16 \text{ cm}$$

Answer: (2)

25.



Let I be the current in the circuit. Ideal inductor will behave like zero, resistance long time after switch is closed

$$I = \frac{2E}{R}$$

$$\Rightarrow I = \frac{2 \times 15}{5}$$

$$\Rightarrow I = 6\text{A}$$

Answer: (4)

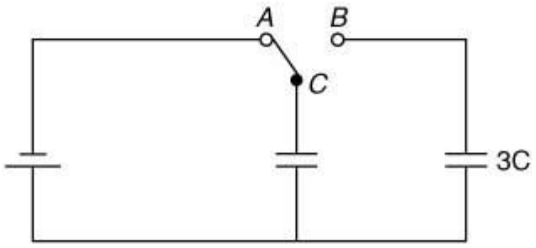
26. Change on capacitor C before turning the switch S

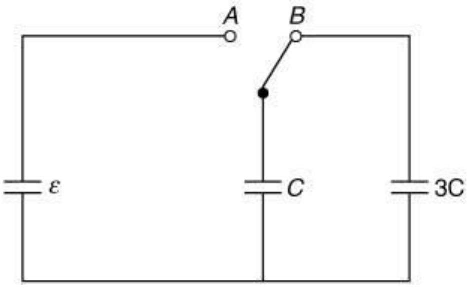
$$Q = CE \quad (1)$$

Since, when switch S is turned from A to B charge will be transferred from one capacitor to another capacitor until potential will not same.

Let common potential is ε'

$$\varepsilon' = \frac{C\varepsilon + 0}{C + 3C} = \frac{C\varepsilon}{4C} = \frac{\varepsilon}{4}$$





Initial energy, $E_{\text{i}} = \frac{1}{2}CE^2$

Final energy, $E_{\text{f}} = \frac{1}{2}C\left(\frac{\varepsilon}{4}\right)^2$

$$= \frac{1}{2}(3C)\left(\frac{\varepsilon}{4}\right)^2 = \frac{1}{8}CE^2$$

Energy dissipated, $E = E_{\text{i}} - E_{\text{f}}$

$$= \frac{1}{2}C\varepsilon^2\left[1 - \frac{1}{4}\right] = \frac{1}{2}C\varepsilon^2\left[\frac{3}{4}\right]$$

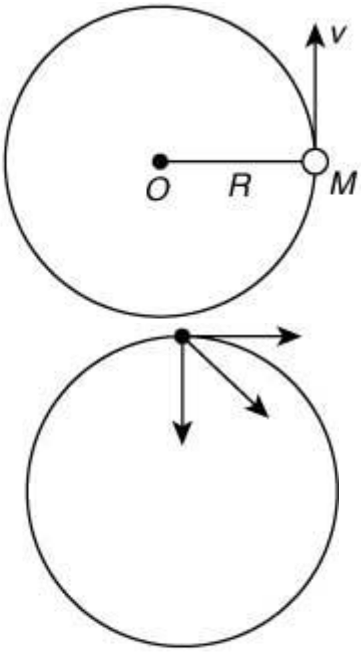
$$= \frac{3}{8}C\varepsilon^2 = \frac{3}{8}\frac{Q^2}{C}$$

Answer: (2)

27. Along x axis by conservation of momentum

$$0 + (-Mv) = -2Mv_x$$

$$\Rightarrow v_x = \frac{v}{2} \quad (1)$$



Along y axis by conservation of momentum

$$0 + Mv = 2Mv_y$$

$$\Rightarrow v_y = \frac{v}{2} \quad (2)$$

Total velocity $\vec{v}' = \vec{v}_x + \vec{v}_y = v_x\hat{i} - v_y\hat{j}$

$$\Rightarrow v' = \sqrt{v_x^2 + v_y^2} = \sqrt{\frac{v^2}{4} + \frac{v^2}{4}}$$

$$\Rightarrow v' = \frac{v}{\sqrt{2}}$$

So, it is an elliptical orbit.

Answer: (3)

28. Let R_g is resistance of galvanometer

Case I: $i_g = \frac{V}{R + R_g}$

$$\Rightarrow i_g = \frac{V}{220 + R_g} = C\theta_0 \qquad (1)$$

Case II:

$$i_g = \left[\frac{V}{220 + \frac{5\,R_g}{5 + R_g}}\right] \times \frac{5}{R+5} = \frac{C\theta_0}{5} \qquad (2)$$

$$\Rightarrow \frac{5V}{225\,R_g + 1100} = \frac{C\theta}{5}$$

$$\Rightarrow \frac{V}{220 + R_g} = \frac{C\theta}{5}$$

$$\Rightarrow \frac{225\,R_g + 1100}{1100 + 5R_g} = 5$$

$\Rightarrow$ $225R_g + 1100 = 5500 + 25R_g$

$\Rightarrow$ $225R_g - 25R_g - 5500 - 1100$

$\Rightarrow$ $200R_g = 4400$

$$\Rightarrow R_g = \frac{4400}{200}$$

$\Rightarrow$ $R_g = 22\ \Omega$

Answer: (2)

29. Wave length is given by

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}} \qquad [K = qV]$$

Thus, $\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{2m_B q_B V_B}{2m_A q_A V_A}}$

$$= \sqrt{\frac{4mq \times 2500}{mq \times 50}}$$

$$= 2\sqrt{50} = 2 \times 7.07$$

Therefore, $\frac{\lambda_A}{\lambda_B} = 14.14$

Answer: (3)

30. At any instant time t total energy of charge distribution is constant.

$$\frac{1}{2}mv^2 + \frac{kQ^2}{2R} = 0 + \frac{kQ^2}{2R_0}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{kQ^2}{2R_0} - \frac{kQ^2}{2R}$$

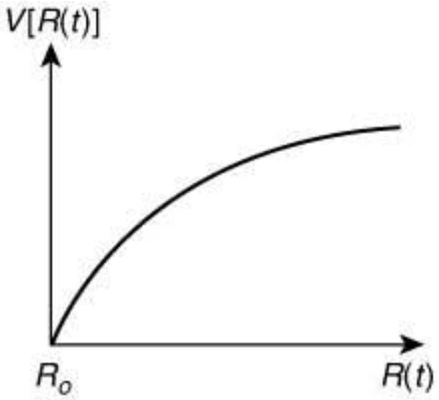
$$\Rightarrow v = \sqrt{\frac{2}{m}\left(\frac{kQ^2}{2R_0} - \frac{kQ^2}{2R}\right)}$$

$$\Rightarrow v = \sqrt{\frac{2}{m}.\frac{kQ^2}{2}\left(\frac{1}{R_0} - \frac{1}{R}\right)}$$

$$\Rightarrow v = \sqrt{\frac{kQ^2}{m}\left(\frac{1}{R_0} - \frac{1}{R}\right)}$$

$$\Rightarrow v = C\sqrt{\frac{1}{R_0} - \frac{1}{R}}$$

Also the slope of *V*-*R* curve will go on decreasing.



Answer: (3)

Section: Chemistry

31. In the Hall-Heroult process, the cathode is made of carbon. This process is widely used in extraction of aluminium.
Answer: (2)

32. The correct order of acidic strength is $CH \equiv CH > CH_3 - C \equiv CH > CH_2 = CH_2$
Being the most electronegative, the *sp*-hybridized carbon atom of ethyne polarizes its C—H bonds to the greatest extent, causing its hydrogens to be most positive. Therefore, ethyne donates a proton to a base more readily.
Answer: (4)

33.

$$\begin{array}{lcccc} & A & + \; 2B & \rightleftharpoons \; 2C & + \; D \\ t=0 & a_0 & 1.5a_0 & 0 & 0 \\ t=t_{eq} & a_0 - x & 1.5a_0 - x & 2x & x \end{array}$$

At equilibrium [A] = [B]

$$a_0 - x = 1.5a_0 - 2x \Rightarrow x = 0.5a_0$$

Therefore,

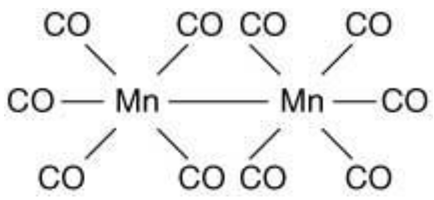
$$\begin{array}{lcccc} & A & + \; 2B & \rightleftharpoons \; 2C & + \; D \\ t = t_{eq} & 0.5a_0 & 0.5a_0 & a_0 & 0.5a_0 \end{array}$$

$$K_C = \frac{[C]^2[D]}{[A][B]^2} = \frac{(a_0)^2(0.5a)}{(0.5a_0)(0.5a_0)^2} = 4$$

Answer: (1)

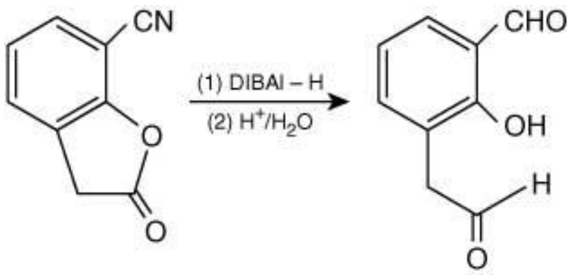
34. Smaller the value of critical temperature of gas, lesser is the extent of adsorption. So, least adsorbed gas is H_2.
Answer: (4)

35. Compounds having at least one bond between carbon and metal are known as organometallic compounds.



The presence of Mn–C bond makes $Mn_2(CO)_{10}$ an organometallic compound.
Answer: (1)

36. Both esters and nitriles can be reduced to aldehydes by DIBAL-H.



Answer: (3)

37.

$$\underset{x}{Rb} + O_2(\text{excess}) \rightarrow \underset{x}{RbO_2}$$

$$\underset{x}{RbO_2} + 2H_2O \rightarrow 2RbOH + H_2O_2 + O_2$$

Answer: (2)

38. N_2 does not play any role in the formation of photochemical smog while NO_2, O_3 and hydrocarbon are responsible for building up of photochemical smog.
Answer: (1)

39. $[V(H_2O)_6]Cl_2$: V is in +2 oxidation state. The electronic configuration of V^{2+} is $3d^3$. The number of unpaired electrons is 3. Hence, spin-only magnetic moment is given by

$$\mu = \sqrt{n(n+2)} = \sqrt{3(3+2)} = \sqrt{15} = 3.87\,\text{BM}$$

$[Co(H_2O)_6]Cl_2$: Co is in +2 oxidation state. The electronic configuration of Co^{2+} is $3d^7$. The number of unpaired electrons is 3. Hence, spin-only magnetic moment is given by

$$\mu = \sqrt{n(n+2)} = \sqrt{3(3+2)} = \sqrt{15} = 3.87\,\text{BM}$$

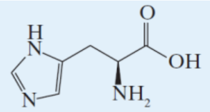
Answer: (1)

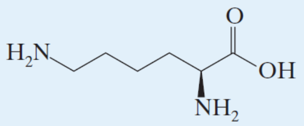
40. We know $C_p = \frac{7}{2}R$ (independent of p). Therefore, Graph (1) does not represent the correct variation of C_p vs p.

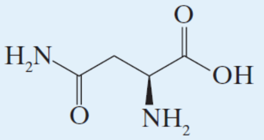
Also, $C_V = \frac{5}{2}R$ (independent of V)

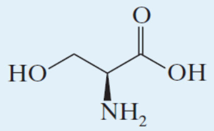
Answer: (1)

41. Asparagine and serine are neutral amino acids whereas both histidine and lysine are basic amino acid. Histidine is more than lysine due to the presence of imidazole ring.


Histidine


Lysine


Asparagine


Serine

ANSWER: (4)

42. Reaction of amines with alkyl halides is a nucleophilic substitution reaction. So, it depends on the availability of lone pair of electrons on N to attach the alkyl halide. From the given compounds, (B) will be least reactive as it is an imine group and lacks lone pair of electrons for attack. This is followed by A, where the lone pair on N is involved in resonance with adjoining C=O group, so it is not readily available for the reaction. In case of compound C, -CN is an electron withdrawing group, so it withdraws electrons from the pi system, thereby making the compound less nucleophilic. Aniline is known to be very reactive towards nucleophilic substitution reactions, due to lone pair on N.
Therefore, (B) < (A) < (C) < (D)
Answer: (1)

43. Among the given compounds, naphthalene has the lowest melting point of approx. 80°C.
Answer: (4)

44. The reaction involved is

$$H_2C_2O_4 + 2NaOH \rightarrow Na_2C_2O_4 + 2H_2O$$

$$\text{milliequiv. of } H_2C_2O_4 = \text{milliequiv. of NaOH}$$

$$50 \times 0.5 \times 2 = 25 \times M_{NaOH} \times 1$$

$$M_{NaOH} = 2\,M$$

Now, 1000 mL solution contains = 2 × 40 g = 80 g NaOH

Therefore, 50 mL solution contains $= \frac{80}{1000} \times 50 = 4\,g$ NaOH

Answer: (*).

45. Given: $V_A = 2V_B$; $Z_A = 3Z_B$; $n_A = n_B = n$; $T_A = T_B = T$

$$Z_A = \frac{p_A V_A}{nRT} \quad (1)$$

$$Z_B = \frac{p_B V_B}{nRT} \quad (2)$$

On dividing Eq. (1) from Eq. (2), we get

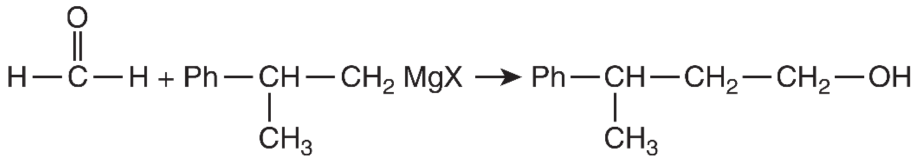
$$\frac{Z_A}{Z_B} = \frac{p_A \times V_B \times 2}{p_B \times V_B}$$

$$\frac{p_A}{p_B} = \frac{3}{2} \Rightarrow 2p_A = 3p_B$$

Answer: (2)

46. Hardness due to $CaCO_3 = (10^{-3} \times 10^3) \times 100 = 100$ ppm
Answer: (4)

47.



Answer: (4)

48. For the same freezing point, molality of both solutions should be same.

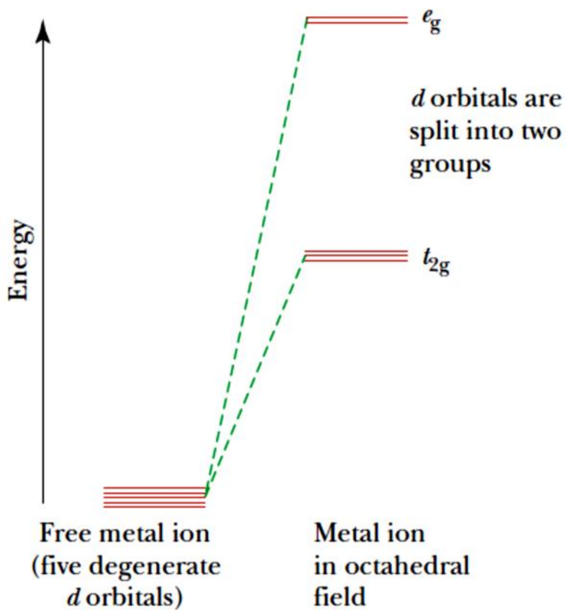
$$m_X = m_Y$$

$$\frac{4 \times 1000}{96 \times m_X} = \frac{12 \times 1000}{88 \times m_Y}$$

$$m_Y = \frac{96 \times 12A}{4 \times 88} = 3.27A \approx 3A$$

Answer: (1)

49. In an octahedral complex under the influence of an octahedral ligand field the *d*-orbitals split into two groups of different energies. The lobes of the e_g orbitals ($d_{x^2-y^2}$ and d_{z^2}) point along the axes *x*, *y* and *z*. The lobes of the t_{2g} orbitals (d_{xy}, d_{xz} and d_{yz}) point in between the axes.



Answer: (2)

50. Rate constant (*k*) = 0.05 μg/year. The unit of rate constant indicates the zero-order reaction.

$$t_{1/2} = \frac{a_0}{2k}$$

$$t_{1/2} = \frac{5\ \mu g}{2 \times 0.05\ \mu g/year} = 50\ year$$

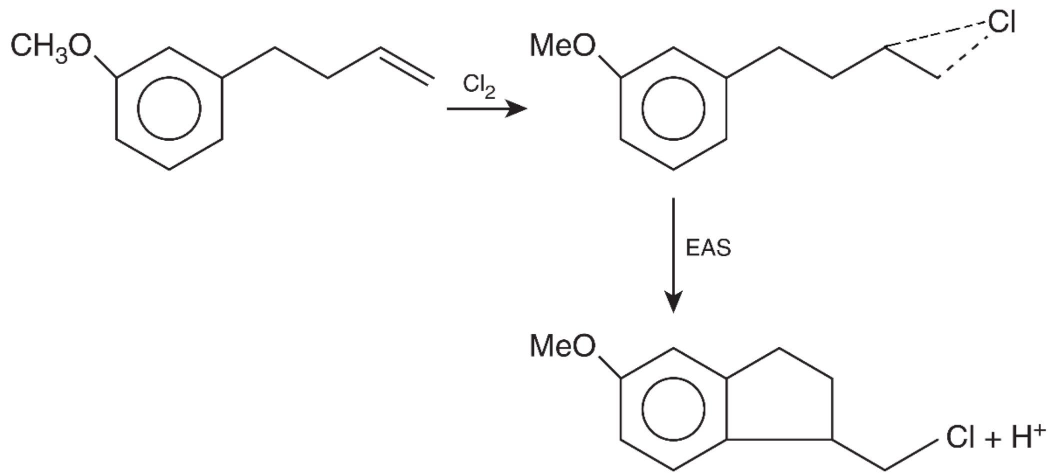
Answer: (1)

51. We have

$$\Delta H = nFT\left(\frac{\Delta E}{\Delta T}\right) - nF$$
$$= 2 \times 96000 \times 300(-5 \times 10^{-4}) - 2 \times 96000 \times 2$$
$$= -412.8\ kJ/mol$$

Answer: (1)

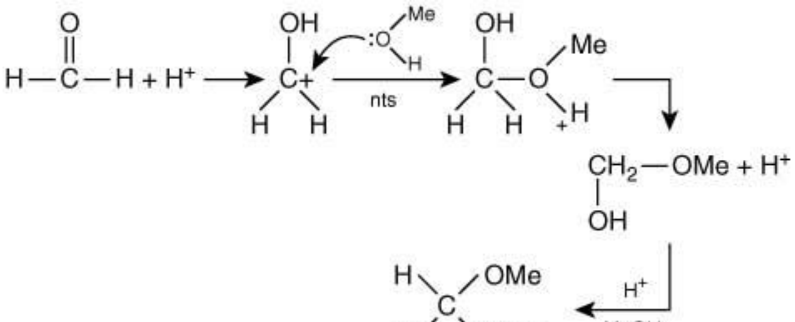
52.



Answer: (4)

53. In the periodic table, the element with Z = 120 will be a s-block element and alkaline earth metal.
Answer: (2)

54.



$$\text{rate} \propto \frac{1}{\text{steric chowding of aldehyde}}$$

Answer: (2)

55.

$$\underset{}{A(s)} \rightleftharpoons \underset{p_1}{B(g)} + \underset{p_1+p_2}{C(g)}$$

$$D(s) \rightleftharpoons \underset{p_1+p_2}{C(g)} + \underset{p_2}{E(g)}$$

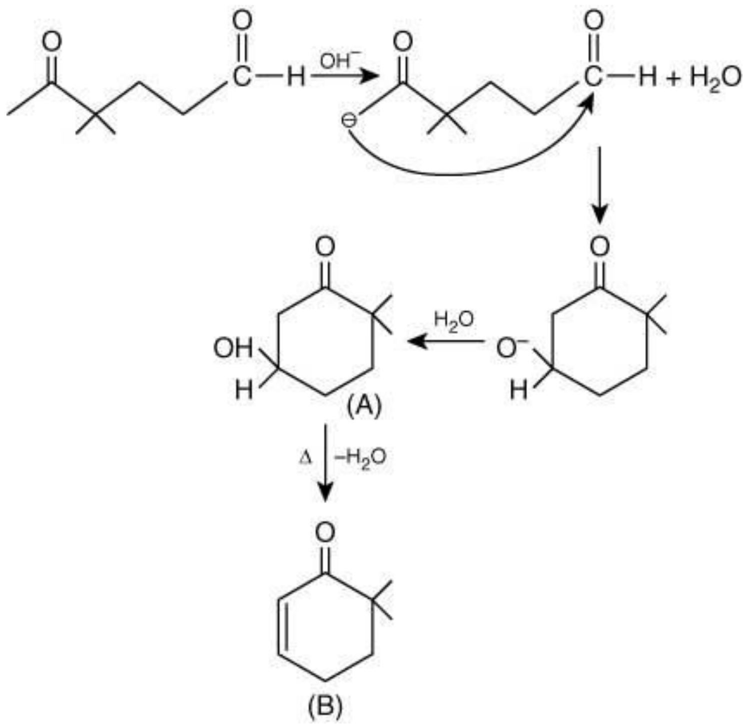
$$p_1(p_1 + p_2) = x \quad (1)$$
$$p_2(p_1 + p_2) = y \quad (2)$$

$$x + y = (p_1 + p_2)^2$$
$$p_1 + p_2 = \sqrt{x+y}$$

Therefore, total pressure = $\sqrt{x+y}$ atm

Answer: (2)

56.



Answer: (1)

57. We know $$h\nu = \phi + \frac{1}{2}mv^2$$
$$\phi = \frac{6.626\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}} - \frac{1}{2}\times9\times10^{-31}\times(6\times10^{5})^2$$
$$\phi = 3.35\times10^{-19}\ \text{J}$$
$$= \frac{3.35\times10^{-19}}{1.6021\times10^{-19}} \Rightarrow \phi \approx 2.1\,\text{eV}$$

Answer: (3)

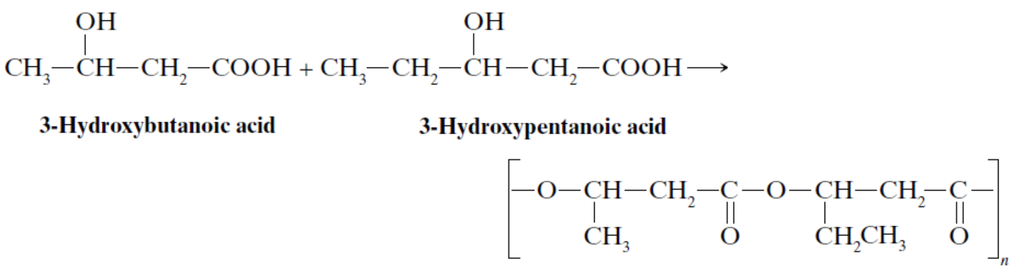
58. The reaction involved is

$$I_2 + 10HNO_3 \rightarrow 2H\overset{+5}{\underset{Y}{I}}O_3 + 10NO_2 + 4H_2O$$

In HIO_3, oxidation state of iodine is +5.

Answer: (1)

59. PHBV is obtained by copolymerization of 3-hydroxybutanoic acid and 3-hydroxypentanoic acid.



PHBV

Answer: (4)

60. Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

Answer: (3)

Section: Mathematics

61. We have,

$$(1+\alpha)x + \beta y + z = 0$$
$$\alpha x + (1+\beta)y + z = 0$$
$$\alpha x + \beta y + 2z = 0$$

For unique solution

$$\Delta \neq 0 \Rightarrow \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0 \Rightarrow \alpha + \beta \neq -2$$

$$\Rightarrow \alpha + \beta = -2$$

Hence, required ordered pair is (–4, 2).

Answer: (3)

62. Let three terms of G.P. are $\frac{a}{r}, a$ and ar.

Thus, $a^3 = 512 \Rightarrow a = 8$

So, three terms of GP are $\frac{8}{r}, 8$ and $8r$.

According to the question, $\frac{8}{r} + 4, 12$ and $8r$ are in A.P.

We know that the relationship between three numbers a, b and c which are in AP is

$$2b = a + c$$
$$\Rightarrow 24 = \frac{8}{r} + \frac{4}{1} + 8r$$
$$\Rightarrow 24r = 8 + 4r + 8r^2$$
$$\Rightarrow 8r^2 - 20r + 8 = 0$$
$$\Rightarrow 8r^2 - 16r - 4r + 8 = 0$$
$$\Rightarrow 8r(r-2) - 4(r-2) = 0$$
$$\Rightarrow r = \frac{4}{8}, r = 2$$

If $r = 2$ then the terms of G.P. are 4, 8, 16

If $r = \frac{1}{2}$ then G.P. are 16, 8, 4

Therefore, required sum = 28

Answer: (4)

63. We have,

$$((p \wedge q) \wedge p)) \vee ((p \wedge q) \vee \sim q) \wedge (p \vee q)$$
$$\Rightarrow (p \vee ((p \wedge \sim q)) \vee \sim (p \vee q)$$
$$\Rightarrow (p \vee (p \vee \sim q)) \wedge \sim (p \vee q)$$
$$\Rightarrow (p \vee \sim q) \wedge \sim (p \wedge q)$$
$$\Rightarrow \sim (p \vee q) \Rightarrow \sim p \wedge \sim q$$

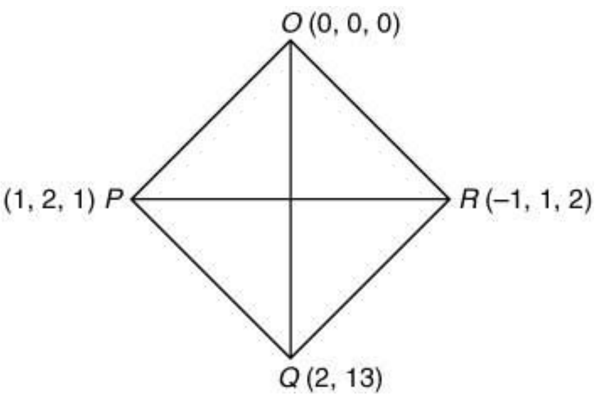
Answer: (3)

64. Number of ways of selecting 3 numbers

$$= {}^{10}C_3 = \frac{10!}{7!3!} = \frac{10 \times 9 \times 8 \times 7!}{7! \times 3 \times 2} = 120$$

Answer: (1)

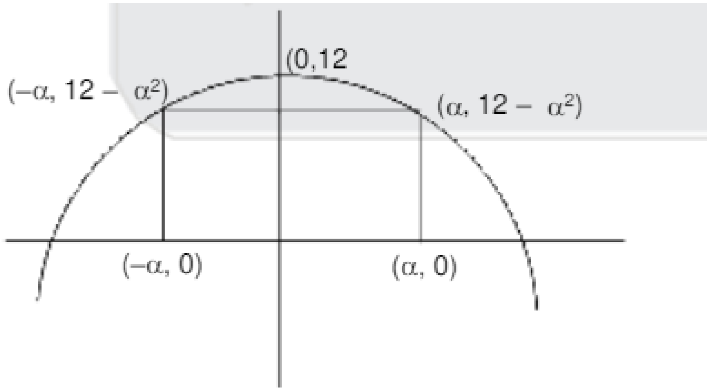
65.



$$\overrightarrow{OP} \times \overrightarrow{OQ} = (\hat{i} + 2\hat{j} + \hat{k}) \times (2\hat{i} + \hat{j} + 3\hat{k})$$
$$= 5\hat{i} - \hat{j} - 3\hat{k}$$
$$\overrightarrow{PQ} \times \overrightarrow{PR} = (\hat{i} - \hat{j} + 2\hat{k}) \times (-2\hat{i} - \hat{j} + \hat{k})$$
$$= \hat{i} - 5\hat{j} - 3\hat{k}$$
$$\Rightarrow \cos\theta = \frac{5 + 5 + 9}{(\sqrt{25 + 9 + 1})^2} = \frac{19}{35}$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

Answer: (2)

66.



Area of rectangle, $A = 2\alpha(12 - \alpha^2)$

Differentiating A with respect to α we get

$$\frac{dA}{d\alpha} = 2(12 - 3\alpha^2)$$

$$\frac{dA}{d\alpha} = 0 \Rightarrow \alpha = \pm 2$$

So, area of rectangle is maximum at $a = 2$.

Therefore, maximum area = 4(12 – 4) = 32 sq. units

Answer: (3)

67. Line perpendicular to $2x - 3y + 5 = 0$ is $3x + 2y + c = 0$ which satisfies (7, 17).

$$3(7) + 2(17) + c = 0$$
$$\Rightarrow 21 + 34 + c = 0$$
$$\Rightarrow c = -55$$

Equation of line is $3x + 2y - 55 = 0$

$$\Rightarrow 3(15) + 2(\beta) - 55 = 0$$
$$\Rightarrow 45 + 2\beta - 55 = 0$$
$$\Rightarrow 2\beta - 10 = 0$$
$$\Rightarrow 2\beta = 10$$
$$\Rightarrow \beta = 5$$

Answer: (4)

68. Consider

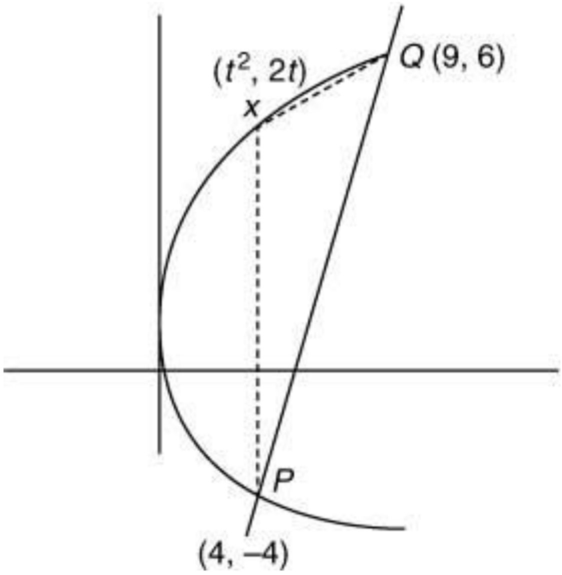
$$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$$\Rightarrow \mu(\mu^2 - 1) - 1(\mu - 1) + 1(1 - \mu) = 0$$
$$\Rightarrow \mu^3 - \mu - \mu + 1 + 1 - \mu = 0$$
$$\Rightarrow \mu^3 - 3\mu + 2 = 0$$
$$\Rightarrow \mu = 1, \mu^2 + \mu - 2 = 0$$
$$\Rightarrow \mu = 1, \mu = -2$$

Hence, sum of distinct real values of $\mu = 1 + (-2) = -1$

Answer: (1)

69.



Equation of parabola is

$$y^2 = 4x$$

Differentiating both the sides, we get

$$\Rightarrow 2yy' = 4$$

$$\Rightarrow y' = \frac{1}{t} = 2,\ t = \frac{1}{2}$$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} \frac{1}{4} & 1 & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix} = \frac{125}{4}$$

Answer: (4)

70. We have

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 9+9+9 & 3+3 & 1 \end{bmatrix}$$

$$\Rightarrow P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 6\times 9 & 3+3+3 & 1 \end{bmatrix}$$

From above we can make formula

$$P^n = \begin{bmatrix} 1 & 0 & 0 \\ 3n & 1 & 0 \\ \frac{n(n+1)}{2}.3^2 & 3n & 1 \end{bmatrix}$$

Thus, $P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5\times 3 & 1 & 0 \\ 15\times 9 & 5\times 3 & 1 \end{bmatrix}$

So, $Q = P^5 + I_3$

$$\Rightarrow Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

Hence,

$$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$$

Answer: (1)

71. Given differential equation is,

$$x\frac{dy}{dx} + y = x\log_e x,\ (x > 1)$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \ln .x$$

$$\text{I.F.} = e^{\int \frac{1}{x}dx} = x$$

Thus, $xy = \int x \ln x + C$

$$\Rightarrow \ln .x.\frac{x^2}{2} = \int \frac{1}{x}.\frac{x^2}{2}$$

$$\Rightarrow xy = \frac{x}{2}\ln x - \frac{x^2}{4} + C,$$

Since, $2y(2) = 2 \ln 2 - 1$

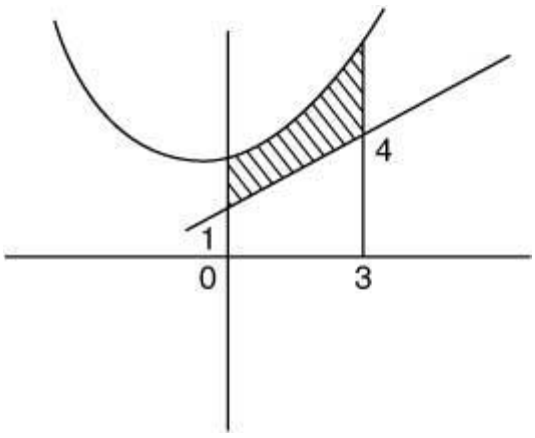
$$\Rightarrow C = 0$$

Therefore, $y = \frac{x}{2}\ln x - \frac{x}{4}$

$$\Rightarrow y(e) = \frac{e}{2}$$

Answer: (3)

72.



Equation of parabola is $y = x^2 + 2$ and the equations of lines are $y = x + 1$, $x = 0$ and $x = 3$.

$$\text{Required area } = \int_0^3 (x^2 + 2)dx - \frac{1}{2} \times 5 \times 3$$

$$= 9 + 6 - \frac{15}{2}$$

$$= 15 - \frac{15}{2} = \frac{15}{2}$$

Answer: (4)

73. Required probability = $\frac{1}{6^2}\left(\frac{5^3}{6^3}+\frac{{}^2C_1.5^2}{6^3}\right)$

$$=\frac{1}{6^2}\left(\frac{5^3+2\times 25}{6^3}\right)=\frac{125+50}{6^5}=\frac{175}{6^5}$$

Answer: (4)

74. Common chord is $S_1 - S_2 = 0$

$$(x^2+y^2-2x-2y-2)-(x^2+y^2-6x-6y+14)=0$$

$\Rightarrow -2x + 6x - 2y + 6y - 2 - 14 = 0$

$\Rightarrow 4x + 4y = +16$

$\Rightarrow 4(x + y) = 16$

$\Rightarrow x + y = 4$

Solving equation of circle and equation of chord, we have

$x^2 + (4 - x)^2 - 2x - 2(4 - x) - 2 = 0$

$\Rightarrow x^2+16+x^2-8x-2x-8+2x-2=0$

$\Rightarrow 2x^2 - 8x + 6 = 0$

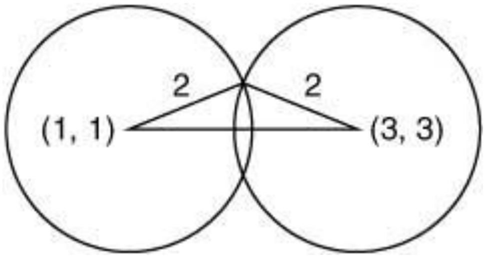
$\Rightarrow 2(x^2 - 4x + 3) = 0$

$\Rightarrow x^2 - 3x - x + 3 = 0$

$\Rightarrow x(x - 3) - 1(x - 3) = 0$

$\Rightarrow (x - 1)(x - 3) = 0$

$\Rightarrow x = 1, 3$



Therefore, required area $=2\times\frac{1}{\not 2}=4=4$ sq units

Answer: (4)

75. We have,

$3\cos\theta+5\sin\left(\theta-\frac{\pi}{6}\right)$

So, for real value of θ

$y=3\cos\theta+5\left(\sin\theta\times\frac{\sqrt{3}}{2}-\cos\theta\times\frac{1}{2}\right)$

$(\sin(A-b)-\sin A.\cos B-\cos A\sin B)$

Hence, for maximum value,

$\sin\theta\times\frac{5\sqrt{3}}{2}+\cos\theta\times\frac{1}{2}$

$y_{\max}=\sqrt{\frac{75}{4}+\frac{1}{4}}=\sqrt{\frac{76}{4}}=\sqrt{19}$

Answer: (1)

76. We have,

$$\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2}=1$$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow x=-1, \frac{1}{6}$$

As, $x \geq 0 \Rightarrow x=\frac{1}{6}$

Hence, it is a singleton set.

Answer: (3)

77. Let roots are α and β.

Now, $3m^2x^2 + m(m-4)x + 2 = 0$

$$\lambda+\frac{1}{\lambda}=1,\ \frac{\alpha}{\beta}+\frac{\beta}{\alpha}=1,\ \alpha^2+\beta^2=\alpha\beta$$

$$\Rightarrow (\alpha+\beta)^2 = 3\alpha\beta$$

$$\Rightarrow \left(-\frac{m(m-4)}{3m^2}\right)^2=\frac{3(2)}{3m^2}$$

$$\Rightarrow \frac{m^2(m-4)^2}{9m^4}=\frac{6}{3m^2}$$

$$\Rightarrow 3m^4(m-4)^2=54m^4$$

$$\Rightarrow (m-4)^2 = 18$$

$$m^2 - 8m + 16 = 18$$

$$m^2 - 8m - 2 = 0$$

$$m=\pm\sqrt{18}=4\pm 3\sqrt{2}$$

Therefore, the least value of m is $4-3\sqrt{2}$.

Answer: (2)

78. Centre of circles are opposite sides of line.

Thus, $(3 + 4 - \lambda)(27 + 4 - \lambda) < 0$

$$\Rightarrow (\lambda - 7)(\lambda - 31) < 0$$

$$\lambda \in (7,31)$$

Distance from S_1 is

$$\left|\frac{3+4-\lambda}{5}\right|\geq 1 \quad \Rightarrow \lambda\in(-\infty,2)\cup(12,\infty)$$

Distance from S_2 is

$$\left|\frac{27+4-\lambda}{5}\right|\geq 2 \Rightarrow \lambda\in(-\infty,21)\cup(41,\infty)$$

Therefore, $\lambda\in(12,21)$

Answer: (3)

79. Given

$$(2x)^{2y} = 4e^{2x-2y}$$

$$\Rightarrow 2y \log_e(2x) = \log_e(4) + 2x - 2y$$

$$\Rightarrow y=\frac{x+\log_e(2)}{1+\log_e(2x)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1+\log_e(2x)-[x+\log_e(2)]\times\frac{1}{x}}{(1+\log_e 2x)^2}$$

$$\Rightarrow \frac{dy}{dx}(1+\log_e .2x)^2=\left[\frac{x\log_e(2x)+\log_e(2)}{x}\right]$$

Answer: (1)

80. We have,

$$I=\int\cos(\log_e x)dx$$

$$I=\cos(\log_e x).x+\int\sin(\log_e x).dx$$

$$I=\cos(\log_e x).x+[\sin(\log_e x).x-\int\cos(\log_e x)dy$$

$$\frac{dA}{d\alpha}=0\Rightarrow\alpha=\pm 2$$

$$\Rightarrow I=\frac{x}{2}[\sin(\log_e x)+\cos(\log_e x)]+C$$

Answer: (3)

81.

$$\frac{T_5}{T_5^1}=\frac{{}^{10}C_4(2^{1/3})^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2(3^{1/3})}\right)^{10-4}(2^{1/3})^4}$$

$$=\frac{{}^{10}C_4\left(\frac{1}{2(3^{1/3})}\right)^4 2^{6/3}}{{}^{10}C_4(2)^{4/3}\left(\frac{1}{2(3^{1/3})}\right)^6}$$

$$=\frac{2^2}{2^4}\times\frac{2^{-2}3^{-4/3}}{2^{4/3-6}3^{-2}}=3^{2/3}.2^{8/3}=4(36)^{1/3}$$

So, required ratio $=4(36)^{1/3}:1$

Answer: (3)

82. We have,

$$S_k=\frac{k+1}{2}$$

$$\sum S_k^2=\frac{5}{12}A$$

$$\sum_{k=1}^{10}\left(\frac{k+1}{2}\right)^2=\frac{2^2+3^2+4^2\cdots\cdots+11^2}{4}=\frac{5}{12}A$$

$$=\frac{11\times 12\times 23}{6}-1=\frac{5}{3}A$$

$$\Rightarrow 505=\frac{5}{3}A$$

$$\Rightarrow A=\frac{505\times 3}{5}$$

$$\Rightarrow A=303$$

Answer: (3)

83. Consider,

$$\begin{vmatrix} i & j & k \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$$\Rightarrow \hat{i}(35-28) - \hat{j}(21-7) + \hat{k}(12-5)$$

$$\Rightarrow 7\hat{i} - 14\hat{j} + 7\hat{k}$$

$$\Rightarrow \hat{i} - 2\hat{j} + \hat{k}$$

Hence, from the first line, we have

$$1 \times (x+2) - 2 \times (y-2) + 1 \times (z+5) = 0$$

$$\Rightarrow x + 2 - 2y + 4 + z + 5 = 0$$

$$\Rightarrow x - 2y + z + 11 = 0$$

Therefore, required distance= $\frac{11}{\sqrt{4+1+1}} = \frac{11}{\sqrt{6}}$

Answer: (2)

84. Sum of deviations = 50

Number of observations = 50

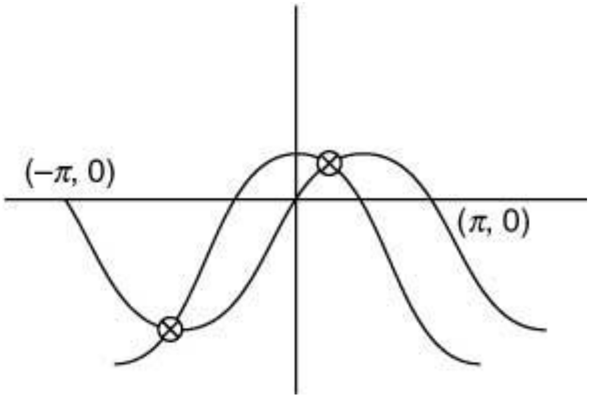
Therefore, mean of observations

$$= 30 + \frac{\text{Sum of deviations}}{\text{Number of deviations}}$$

$$= 30 + \frac{50}{50} = 30 + 1 = 31$$

Answer: (4)

85.



Thus, number of points where $f(x)$ is non-differentiable are 2, that is. $\frac{-3\pi}{4}$ and $\frac{\pi}{4}$.

Hence, S is a subset of $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$.

Answer: (2)

86.

$$I = \int_0^a f(x)g(x)dx = \int_0^a f(a-x)g(a-x)dx$$

We know that,

$$\int_a^b f(x)g(x)dx = \int_a^b f(a+b-x)g(a+b-x)dx$$

$$I = \int_0^a f(x)[4 - g(x)]dx$$

$$I = 4\int_0^a f(x)dx - \int_0^a f(x)g(x)dx$$

$$I = 4\int_0^a f(x)dx - I$$

$$\Rightarrow I = 2\int_0^a f(x)dx$$

Answer: (3)

87. Given expression is

$$\lim_{x \to \frac{\pi}{4}} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(1 - \tan^4 x)}{\cos(x + \pi/4)}$$

$$= 2 \lim_{x \to \frac{\pi}{4}} \frac{(1 - \tan^2 x)}{\frac{\cos - \sin x}{\sqrt{2}}} \frac{1}{\cos^2 x}$$

$$= 4\sqrt{2} \lim_{x \to \pi/4} (\cos x + \sin x) = 8$$

Answer: (4)

88. We have,

$$\frac{z - \alpha}{z + \alpha} + \frac{\bar{z} - \alpha}{\bar{z} + \alpha} = 0$$

$$\Rightarrow z\bar{z} + z\alpha - \alpha\bar{z} - \alpha^2 + z\bar{z} - z\alpha + \bar{z}\alpha - \alpha^2 = 0$$

$\Rightarrow |z^2| = \alpha^2$

$\Rightarrow \alpha = \pm 2$

Answer: (1)

89. S = {1, 2, 3, ….., 100}

Required number of non-empty subsets

= Total non-empty subsets – Subsets whose product of elements is odd

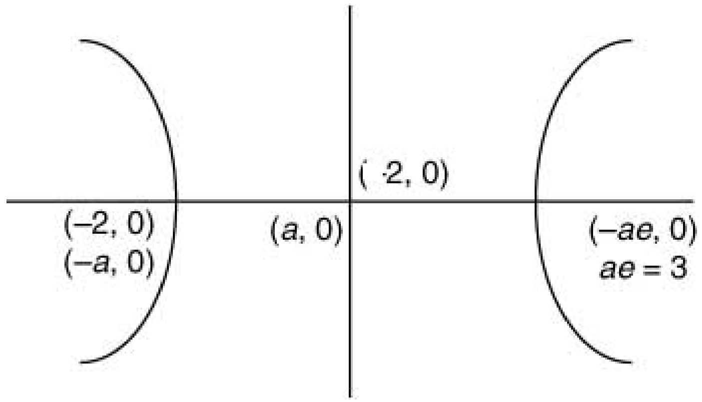
= $2^{100} - 1 - 1\ [(2^{50} - 1)]$

= $2^{100} - 2^{50}$

= $2^{50}\ (2^{50} - 1)$

Answer: (2)

90.



Here, $ae = 3$

$$\Rightarrow e = \frac{3}{2}$$

$$\Rightarrow b^2 = 4\left(\frac{9}{4} - 1\right), b^2 = 5$$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = 1$$

Hence $(6, 5\sqrt{2})$ does not lie on $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

Answer: (4)